The different faces of branes in DFT

based on 1903.05601 with

Eric Bergshoeff (Groningen U.), Axel Kleinschmidt (AEI MPG), Fabio Riccioni (Rome U.)

Edvard Musaev

Moscow Inst. of Physics and Technology

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Branes and strings

Spectrum of string theory is populated by various branes

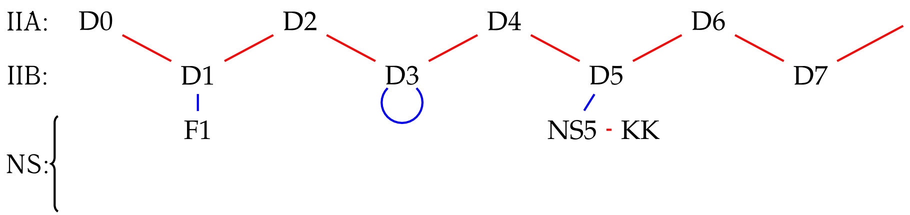
- The fundamental string F1, $T \sim g_s^0$: perturbative dynamics, spectrum, amplitudes, T-duality, source of $B_{(2)}$;
- D-branes, $T \sim g_s^{-1}$: position of open string ends, sources of RR gauge fields $C_{(p)}$;
- NS5-brane, $T \sim g_s^{-2}$: magnetic dual of F1, source of $B_{(6)}$;

- KK-monopole, $T \sim g_s^{-2}$: magnetic dual of the graviton, T-dual of the NS5;
- other exotic branes, $T \sim g_s^{-a}, a \geq 2$: T- and S-duals of geometric branes, sources of exotic potentials;

Dynamics

- F1: gauge invariant kinetic and Wess-Zumino actions;
- NS5 & KK5: gauge invariant kinetic and WZ, T-covariant kinetic action;
- Dp: DBI actions for various p's, gauge invariant WZ actions

Web of (some) branes



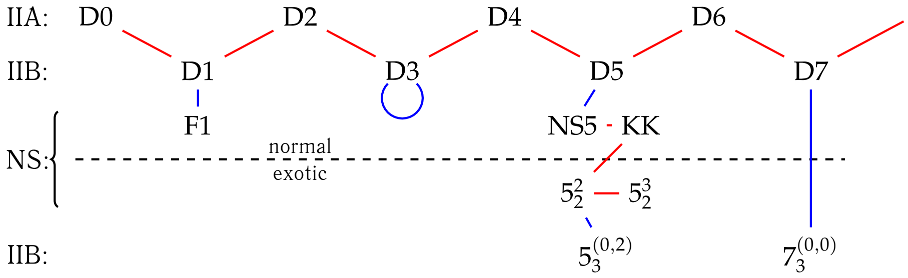
T-duality orbit of D-branes

$$
\begin{array}{rcccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
D0: & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
D1: & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
D2: & \times & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
D3: & \times & \times & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
D4: & \times & \times & \times & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet \\
D5: & \times & \times & \times & \times & \times & \times & \bullet & \bullet & \bullet & \bullet \\
D6: & \times & \times & \times & \times & \times & \times & \times & \bullet & \bullet & \bullet \\
D7: & \times & \times & \times & \times & \times & \times & \times & \times & \bullet & \bullet \\
D8: & \times & \times & \times & \times & \times & \times & \times & \times & \times & \bullet \\
D9: & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times
\end{array}
\tag{1}
$$

Convenient notation:

$$
Dp \equiv p_1, \quad T_p \sim g_s^{-1} \tag{2}
$$

Web of (some) branes



T-duality orbit of NS branes

	0	1	2	3	4	5	6	7	8	9
$\mathrm{NS5} \equiv 5_2^0$:	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
	world-volume						transverse			
$\mathrm{KK5} \equiv 5_2^1$:	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\odot$
	world-volume						transverse			special
5_2^2 :	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\odot$	$\odot$
5_2^3 :	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\odot$	$\odot$	$\odot$
5_2^4 :	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\odot$	$\odot$	$\odot$	$\odot$

T_9 (NS5 ↔ KK5), T_8 (KK5 ↔ 5_2^2) (3)

[deBoer, Shigemori]

Special directions

Manifestations of special directions

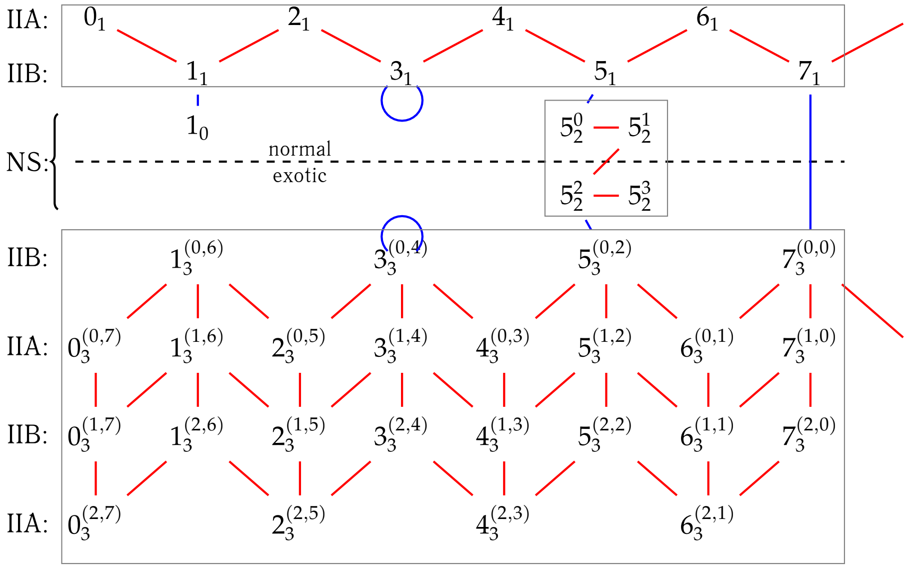
- Number of isometric non-worldvolume directions with non-trivial monodromy;
- Dependence of background on dual coordinates;
- Mass formula for states of 3D supergravity: fully wrapped branes

$$M[b_n^c] = \frac{R_1 \dots R_b R_{b+1}^2 \dots R_{b+c}^2}{g_s^n l_s^{b+2c+1}} = \text{tension of the brane} \tag{4}$$

- Interaction with mixed symmetry potentials

$$b_n{}^c \iff A_{\mu_1 \dots \mu_{b+c+1}, \nu_1 \dots \nu_c} \equiv A_{(b+c+1,c)} \tag{5}$$

Web of (some) branes



The results

T-duality orbits for $\alpha = -1, -2, -3, -4$

- T-covariant Wess-Zumino actions for these T-duality orbits has been constructed. Embedding into DFT.
- For the D-branes orbit depending on orientation these project down to the actions for normal branes.
- Coupling of some exotic branes to massive IIA backgrounds
- The results suggest non-geometric effects for D-branes

T-duality and DFT

- String on $\mathbb{T}^d$ — T-duality group $O(d,d)$ (drop the torus in DFT);
- Doubled coordinates $X^M = (x^m, \tilde{x}_m)$; T-duality: $T_x : x \longleftrightarrow \tilde{x}$;
- section constraint for consistency of the theory, kills half of X^M;
- Potentials combine into irreps of $O(d,d)$

α	Potential	Object
1	C_α (spinor)	Dp-branes
2	$D^{MNPQ} = D^{[MNPQ]}$	NS 5-branes
3	$E_{MN\alpha}$ (gamma-traceless tensor-spinor)	ex. branes (S-dual of D7)
4	$F^+_{M_1\dots M_{10}} = F^+_{[M_1\dots M_{10}]}$ (self-dual)	ex. branes (S-dual of D9)
4	$F^{M_1\dots M_4, N_1 N_2}$ ((4,2)-tensor)	exotic branes
4	$F^{M_1\dots M_7, N_1}$ ((7,1)-tensor)	exotic branes

Covariant potentials

- D-brane potentials $C_{(p+1)}$ can be combined:

$$|\chi\rangle = \sum_{p=0}^{10} C_{m_1\dots m_p}\,\Gamma^{m_1\cdots m_p}|0\rangle \tag{6}$$

- $O(10,10)$ algebra that includes $GL(10)$ as $T^M = (T^m, T_m)$:

$$\begin{aligned} \{\Gamma_M, \Gamma_N\} &= 2\eta_{MN} \quad \Longrightarrow \quad \{\Gamma_m, \Gamma^n\} = \delta_m{}^n, \\ \text{Clifford vacuum:} & \qquad\qquad \Gamma_m|0\rangle = 0 \end{aligned} \tag{7}$$

- For each brane one defines **a charge** $\langle Q|$

$$\begin{aligned} &\text{if } \langle Q| = \langle 0| &&\Longrightarrow \quad \langle Q|\Gamma_{m_1\dots m_{10}}|\chi\rangle = C_{m_1\dots m_{10}} \\ &\text{if } \langle Q| = \langle 0|\Gamma_{\hat m} &&\Longrightarrow \quad \langle Q|\Gamma_{m_1\dots m_9}{}^{\hat m}|\chi\rangle = C_{m_1\dots m_9}. \end{aligned} \tag{8}$$

D9-brane WZ term

The conventional Dp-brane Wess-Zumino term

$$S_{WZ}^{Dp} = q \int_{\Sigma_{p+1}} e^{\mathcal{F}_2} \wedge C, \quad \text{where} \begin{cases} C = C_0 + C_1 + C_2 + \dots, \\ \mathcal{F}_2 = db_1 + B_2 \end{cases} \tag{9}$$

Generalize this for D9 to a DFT expression

$$S_{WZ}^{D9} = \int d^{10}\xi \, \overline{Q}_{10} S_{\mathcal{F}}^{-1} C, \quad \text{where} \begin{cases} \overline{Q}_{10} = \frac{q}{2^{10}} \langle 0 | \Gamma_0 \cdots \Gamma_9, \\ S_{\mathcal{F}} = e^{-\frac{1}{2}\mathcal{F}_{mn}\Gamma^m \Gamma^n} \end{cases} \tag{10}$$

Invariant under gauge transformations

$$\begin{aligned} \delta B_2 = d\Sigma_1, \quad \delta b_1 = -\Sigma_1, \\ \delta C = \not{\partial}\lambda + S_{\mathcal{F}} \not{\partial} S_{\mathcal{F}}^{-1} \lambda \quad , \end{aligned} \tag{11}$$

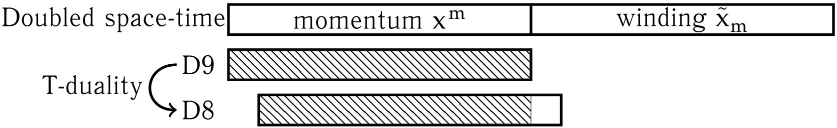
WZ term for $\alpha = 1$ branes

- D9-brane gauge fixing: $x^m = \xi^m \Longrightarrow$ avoid doubled world-volume
- T-duality acts on the brane charge Q_{10}

$$\overline{Q}_9 = \frac{q}{2^9}\langle 0|\Gamma_0 \cdots \Gamma_8 = \overline{Q}_{10}\Gamma^9\,, \qquad (12)$$

- The brane extends along the winding direction $\tilde{x}_9$
- Wess-Zumino term

$$S^{D8}_{WZ} = \int d^{10}\xi\, \overline{Q}_9 S_{\mathcal{F}}^{-1} C = \int d^9\xi\, \overline{Q}_9 S_{\mathcal{F}}^{-1} C \qquad (13)$$



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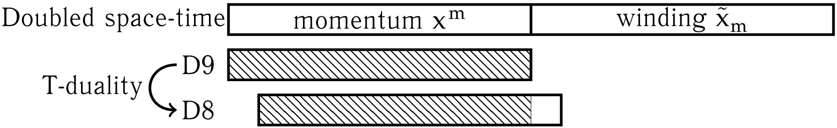
$$S_{WZ}^{Dp} = \int d^{10}\xi \, \overline{Q}_p S_{\mathcal{F}}^{-1} C \tag{13}$$



$\alpha = 2$ branes (NS5 and friends)

NS5 interacts with the field D_6 whose field strength is

$$\begin{aligned} \text{IIA}: \quad & H_7 = dD_6 \qquad - C_1 \wedge G_6 + C_3 \wedge G_4 - C_5 \wedge G_2; \\ \text{IIB}: \quad & H_7 = dD_6 + C_0 \wedge G_7 - C_2 \wedge G_5 + C_4 \wedge G_3 - C_6 \wedge G_1. \end{aligned} \tag{14}$$

Gauge invariant RR field strengths:

$$G_{p+1} = dC_p + H_3 \wedge C_{p-2} \tag{15}$$

Covariant potential, that contains $D_6 \Longrightarrow D_{m_1 \dots m_6} = \varepsilon_{m_1 \dots m_6 mnkl} D^{mnkl}$

$$D^{MNKL} = (D^{mnkl}, D^{mnk}{}_l, D^{mn}{}_{kl}, D^{m}{}_{nkl}, D_{mnkl}) \tag{16}$$

Also need D_{MN}, D

$\alpha = 2$ branes (NS5 and friends)

DFT language for field strengths

$$H^{MNP} = \partial_Q D^{QMNP} + \overline{G}\Gamma^{MNP}C \tag{17}$$

Remind:

$$C = \sum_{p=0}^{10} \frac{1}{p!} C_{m_1\dots m_p} \Gamma^{m_1\dots m_p} |0\rangle , \quad G = \sum_{p=0}^{10} \frac{1}{p!} G_{m_1\dots m_p} \Gamma^{m_1\dots m_p} |0\rangle , \tag{18}$$

! H^{MNP} knows about isometries of exotic 5_2^p-branes with $p > 0$.

- NS5: for D^{mnpq} completely gauge invariant;
- KK5: for $D^{mnp}{}_q$ must set x^q isometric;
- 5_2^2: for $D^{mn}{}_{pq}$ need two isometric directions;

$\alpha = 2$ branes (NS5 and friends)

"Gauge invariant" and $O(4,4)$ invariant Wess-Zumino term:

$$S_{WZ}^{NS5} = \int d^6\xi \; Q_{\hat{M}\hat{N}\hat{P}\hat{Q}}[D^{\hat{M}\hat{N}\hat{P}\hat{Q}} + \overline{\mathcal{G}}\Gamma^{\hat{M}\hat{N}\hat{P}\hat{Q}}C] \quad . \tag{19}$$

- Brane embedding breaks $O(10,10) \hookleftarrow O(4,4) \times O(6,6)$

$$\Gamma_M = (\Gamma_A, \Gamma_{\hat{M}}\Gamma^*)\,, \tag{20}$$

- Charge $Q_{\hat{M}\hat{N}\hat{P}\hat{Q}}$ encodes the chosen brane
- T-duality flips chirality of the Clifford vacuum $|0\rangle$, i.e. IIA $\leftrightarrow$ IIB
- T-duality in transverse directions changes charge:

$$Q^{\hat{m}\hat{n}\hat{p}\hat{q}} \quad \longleftrightarrow \quad Q^{\hat{m}\hat{n}\hat{p}}{}_{\hat{q}} \tag{21}$$

$\alpha = 2$ branes (NS5 and friends)

- T-duality in transverse directions changes charge:

$$Q^{\hat{m}\hat{n}\hat{p}\hat{q}} \quad \longleftrightarrow \quad Q^{\hat{m}\hat{n}\hat{p}}{}_{\hat{q}} \tag{22}$$

- Gauge invariance requires $x^{\hat{q}}$ be an isometric direction (expected for the KK5 monopole)

	x^0	x^1	x^2	x^3	x^4	x^5	y^1	y^2	y^3	y^4	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$
5_2^0	×	×	×	×	×	×	•	•	•	•	k	k	k	k
5_2^1	×	×	×	×	×	×	•	•	•	k	k	k	k	•
5_2^2	×	×	×	×	×	×	•	•	k	k	k	k	•	•
5_2^3	×	×	×	×	×	×	•	k	k	k	k	•	•	•
5_2^4	×	×	×	×	×	×	k	k	k	k	•	•	•	•

(23)

• — localization direction, k — isometry direction

[1409.6314, 1607.05450, 1712.01739] and older papers by Jensen, Harvey

Coupling to massive IIA

In the DFT language Romans mass is introduced by simple generalized Scherk-Schwarz deformation [Hohm, Kwak]

$$C \quad \longrightarrow \quad C + \frac{m}{2} S_B \tilde{x}_1 \Gamma^1 |0\rangle, \tag{24}$$

WZ term for $\alpha = 1$

$$S_{\mathcal{F}}^{-1} C \quad \longrightarrow \quad S_{\mathcal{F}}^{-1} C + m \frac{1}{2^{\frac{p-1}{2}}} \frac{1}{(\frac{p+1}{2})!} b_{a_1} f_{a_2 a_3} \cdots f_{a_{p-1} a_p} \Gamma^{a_1} \cdots \Gamma^{a_p} |0\rangle \,, \tag{25}$$

! p has to be chosen manually at each step

WZ term for $\alpha = 2$

$$S_{WZ}^{NS5m} = \int d^6 \xi \; Q_{\hat{M}\hat{N}\hat{P}\hat{Q}} [D^{\hat{M}\hat{N}\hat{P}\hat{Q}} + \overline{\mathcal{G}} \Gamma^{\hat{M}\hat{N}\hat{P}\hat{Q}} C - m\, \bar{c} \Gamma^{\hat{M}\hat{N}\hat{P}\hat{Q}} (S_B + S_{\mathcal{F}}) |0\rangle] \tag{26}$$

Discussion

The results

- Gauge invariant and DFT-covariant Wess-Zumino for $\alpha = 1, 2, 3, 4$ have been constructed;
- Explicit expressions for gauge invariant field strengths and couplings to the massive Type IIA theory;

Questions and speculations

- Static gauge has been chosen, no doubled worldvolume;
- Manifest DFT-covariant kinetic term is still needed;
- For D-branes all this suggests localization in dual space, need microscopic picture;
- Consistency between localized backgrounds and the constructed WZ terms;
- Interpretation of brane creation process massive IIA in terms of E_{11} constructions;

The last slide

Thank you!