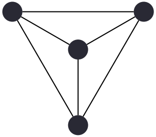
M. Kontsevich's graph complexes and universal structures on graded symplectic manifolds



Supersymmetries and Quantum Symmetries – SQS'19

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Motivations

Formality

In his seminal 97' preprint, ***"Deformation quantization of Poisson manifolds I"***, M. Kontsevich constructs an explicit Lie_∞quasi-isomorphism

$$\mathcal{U} : \mathcal{T}_{\mathrm{poly}}(\mathbb{R}^m) \xrightarrow{\sim} \mathcal{D}_{\mathrm{poly}}(\mathbb{R}^m) \qquad \text{Formality map}$$

between

- $\mathcal{T}_{\mathrm{poly}}$ the Schouten graded Lie algebra of polyvector fields on the affine space $\mathbb{R}^m$
- $\mathcal{D}_{\mathrm{poly}}$ the Hochschild differential graded Lie algebra of multidifferential operators on $\mathbb{R}^m$.

Quantization

An important corollary of the formality theorem is that it provides an explicit bijective map:

$$\hat{\mathcal{U}} : \mathsf{FPoiss} \xrightarrow{\sim} \mathsf{Star} \qquad \text{Quantization map}$$

between the sets

- FPoiss of (equivalence classes of) formal Poisson structures on $\mathbb{R}^m$
- Star of (equivalence classes of) star products on $\mathbb{R}^m$.

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- It provides a complete solution to the quantization problem formulated in Berezin 75', Bayen, Flato, Fronsdal, Lichnerowicz, Sternheimer 78'.
- The Taylor coefficients of the formality morphism $\mathcal{U}$ are:
  - **universal** *i.e.* are written in terms of graphs and independently of the dimension.
  - **transcendental** *i.e.* involve (hard) integrals over configuration spaces of points (or more generally, Drinfel'd associators).
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Universal model for deformation theory

- On general grounds, any dg Lie algebra $\mathfrak{g}$ is quasi-isomorphic (as a Lie_∞-algebra) to its cohomology $H(\mathfrak{g})$ endowed with a certain Lie_∞-structure obtained from the dg Lie algebra structure on $\mathfrak{g}$ via the **homotopy transfer theorem**.
- This allows in particular to address formality questions by studying the space of Lie_∞-structures deforming the graded Lie structure on $H(\mathfrak{g})$. The relevant deformation theory is therefore controlled by the **Chevalley–Eilenberg** dg Lie algebra $\mathsf{CE}\big((H(\mathfrak{g})\big)$.
- M. Kontsevich introduced a **universal version** of the Chevalley–Eilenberg complex associated to the Schouten algebra of polyvector fields $\mathsf{CE}(\mathcal{T}_{\mathrm{poly}})$ in the guise of a dg Lie algebra of **graphs**, denoted fGC_2, together with an injective morphism:

$$\mathsf{fGC}_2 \hookrightarrow \mathsf{CE}(\mathcal{T}_{\mathrm{poly}})$$

given by local formulas.
- **Existence**: Obstructions to universal formality live in $H^1(\mathsf{fGC}_2)$.
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- The morphism $\mathsf{fGC}_2 \hookrightarrow \mathsf{CE}(\mathcal{T}_{\mathrm{poly}})$ is given by explicit local formulas making implicit use of the supergeometric interpretation of the Schouten algebra as the algebra of functions on a graded symplectic manifold (or NP-manifold) of degree 1, *i.e.* $\mathcal{T}_{\mathrm{poly}} \simeq \mathscr{C}^\infty(T^*[1]\mathscr{M})$.
- In this context, Poisson manifolds are interpreted as differential graded symplectic manifolds (or NPQ-manifolds) of degree 1.
- More generally, NPQ-manifolds of positive degree n naturally form the target space of AKSZ-type σ-models over a source of dimension $d = n + 1$:
  - $d = 2$: Poisson σ-model Ikeda 93', Schaller-Strobl 94'
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  - $\vdots$
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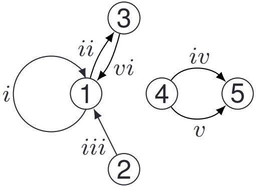

- Flipping of edges:

- Relabelling of edges:

The symmetric group $\mathbb{S}_N$ acts naturally on $\mathsf{Gra}_d(N)$ by permuting the label of vertices.

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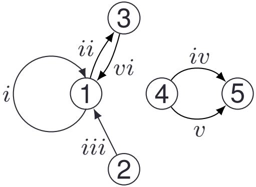

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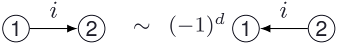

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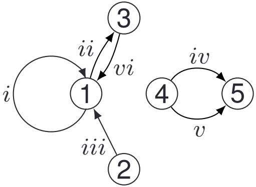
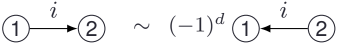
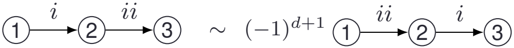


- Flipping of edges: 

- Relabelling of edges: 

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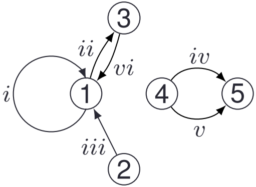
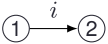
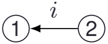


- Flipping of edges:  $\sim \quad (-1)^d$ 
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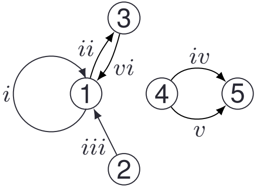

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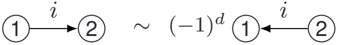

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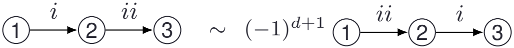

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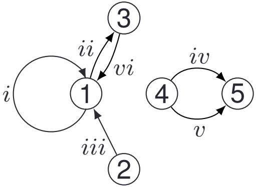
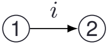
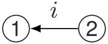


- Flipping of edges:  $\sim \; (-1)^d$ 
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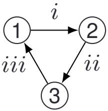
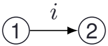
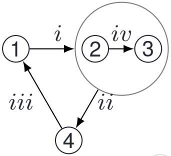
 $\circ_2$  $=$ 

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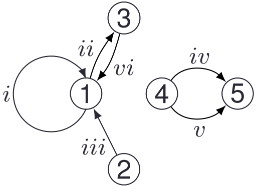

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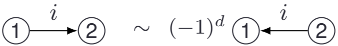

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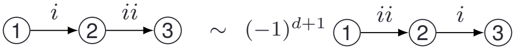

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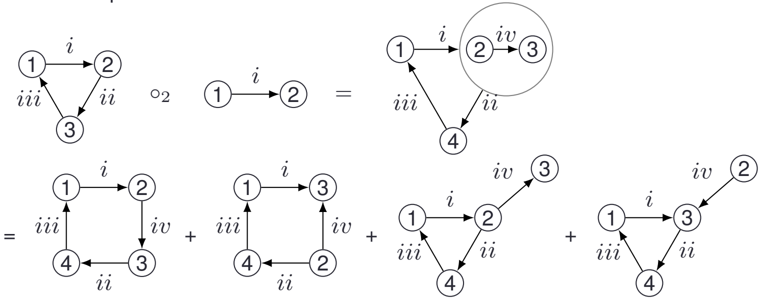

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The partial composition operations $\circ_i$ endow the total space

$$\mathsf{fGC}_d := \prod_{N \geq 1} \left(\mathsf{Gra}_d(N) \otimes \mathrm{sgn}_N^{\otimes d}[d(1-N)] \right)^{\mathbb{S}_N}$$

with a structure of graded Lie algebra where:

- A graph γ with N vertices and k edges has degree $|\gamma| = d(N-1) + k(1-d)$.
- The sign conventions depend on the parity of d as:

	Edge direction	Edge label	Vertex label
d even	$+$	$-$	$+$
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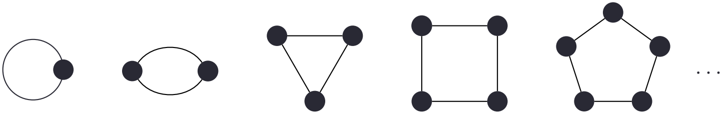

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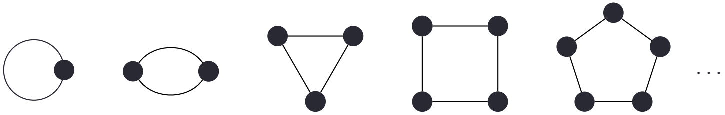

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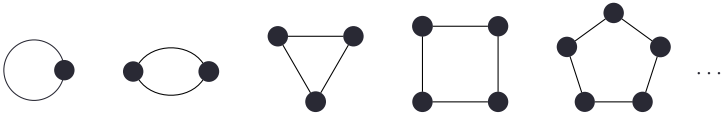

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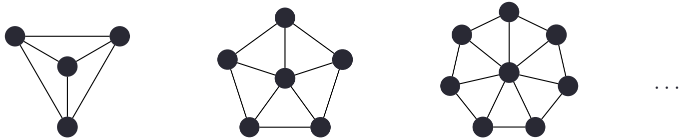

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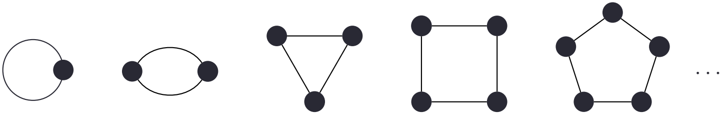

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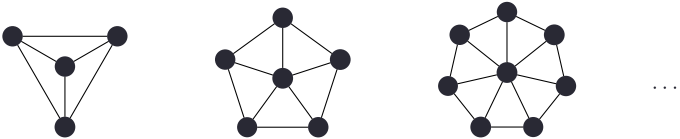

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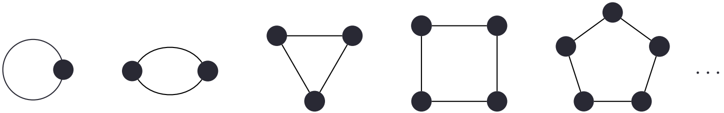

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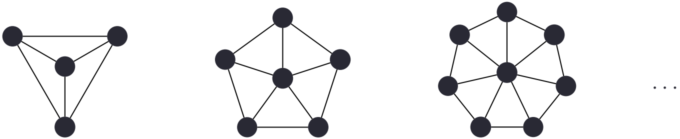

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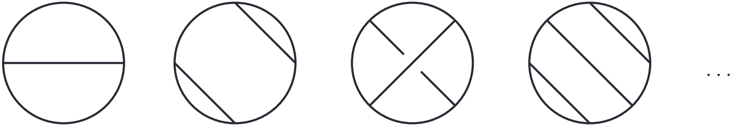

Main result

Let $(\mathcal{V}, \omega)$ be a NP-manifold of degree n and denote $d = n + 1$.

Proposition

The graded algebra of functions on $\mathcal{V}$ is endowed with a structure of a Gra_d*-algebra.*

Explicitly, we construct a tower of injective morphisms $\mathsf{Gra}_d \hookrightarrow \mathsf{End}_{\mathscr{C}^\infty(\mathcal{V})}$ mapping graphs to multidifferential operators on $\mathscr{C}^\infty(\mathcal{V})$.

This generalises the result of Kontsevich 94', Willwacher 10' from $d = 2$ to any d.

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The graded algebra of functions on $\mathcal{V}$ is endowed with a structure of a Gra_d-algebra.

Explicitly, we construct a tower of injective morphisms $\mathsf{Gra}_d \hookrightarrow \mathsf{End}_{\mathscr{C}^\infty(\mathcal{V})}$ mapping graphs to multidifferential operators on $\mathscr{C}^\infty(\mathcal{V})$.

This generalises the result of Kontsevich 94', Willwacher 10' from $d = 2$ to any d.

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- For all d :
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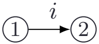
    - The graph 

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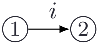
  - The graph 

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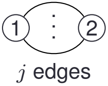
  - The sum of graphs $\sum_{j \geq 0} \frac{1}{j!}$ 

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$$\mathsf{fGC}_d \hookrightarrow \mathsf{CE}(\mathcal{T}_{\mathrm{poly}}^{(n)})$$

where $\mathcal{T}_{\mathrm{poly}}^{(n)} = \mathscr{C}^\infty(\mathcal{V})[n]$ *stands for the* n*-suspension of the algebra of functions on* $\mathcal{V}$*.*

This defines a universal model for the deformation theory of the graded Lie algebra $\mathcal{T}_{\mathrm{poly}}^{(n)}$.

As a by-product, we obtain a classification of universal structures on NP-manifolds:

- $H^1(\mathsf{fGC}_d)$: Universal deformations of $\mathcal{T}_{\mathrm{poly}}^{(n)}$ as a Lie_∞-algebra (obstructions to formality).
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Universal deformations of Poisson manifolds *cf.* A. V. Kiselev's talk

Let $(\mathscr{M}, \pi)$ be a Poisson manifold defining the Hamiltonian function $\mathscr{H} = \frac{1}{2}\pi^{\mu\nu}\, p_\mu\, p_\nu$ on the associated NP-manifold of degree 1.

The zeroth cohomology of the graph complex fGC_2 is infinite dimensional and isomorphic to the Grothendieck–Teichmüller algebra $\mathfrak{grt}_1$ (conjecturally) generated by wheel cocycles.

Examples

- The simplest wheel cocycle is given by the tetrahedron graph $\gamma_3 =$.

  The associated tetrahedral Hamiltonian flow maps the Hamiltonian function $\mathscr{H}$ to the associated universal Lichnerowicz cocycle defined as $\overset{\circ}{\mathscr{H}} = \frac{1}{2}\overset{\circ}{\pi}{}^{\mu\nu}\, p_\mu\, p_\nu$ where:

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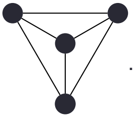
- The simplest wheel cocycle is given by the tetrahedron graph $\gamma_3 =$  .

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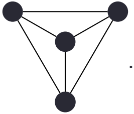
- The simplest wheel cocycle is given by the tetrahedron graph $\gamma_3 =$  .

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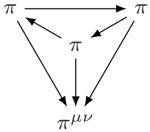
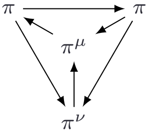
$\overset{\circ}{\pi}{}^{\mu\nu} =$ 

$- 6$ 

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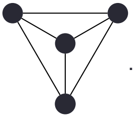
- The simplest wheel cocycle is given by the tetrahedron graph $\gamma_3 =$  .

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Kontsevich 94', Bouisaghouane & Kiselev 16', Bouisaghouane, Buring & Kiselev 16'

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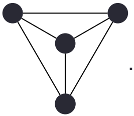
- The simplest wheel cocycle is given by the tetrahedron graph $\gamma_3 =$  .

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Universal deformations of Courant algebroids

Let C be a Courant algebroid defining the Hamiltonian function $\mathscr{H} = \rho_a{}^\mu\, \xi^a p_\mu + \frac{1}{6}\, T_{abc}\, \xi^a \xi^b \xi^c$ on the associated NP-manifold of degree 2.

The zeroth cohomology of the graph complex fGC_3 is one dimensional and spanned by the triangle loop class $H^0(\mathsf{fGC}_3) = \mathbb{K}\,\langle L_3 \rangle$ where $L_3 =$.

The associated triangle Hamiltonian flow maps the Hamiltonian function $\mathscr{H}$ to the associated universal cocycle defined as $\mathring{\mathscr{H}} = \mathring{\rho}_a{}^\mu\, \xi^a p_\mu + \frac{1}{6}\, \mathring{T}_{abc}\, \xi^a \xi^b \xi^c$ where:

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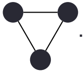
The zeroth cohomology of the graph complex fGC_3 is one dimensional and spanned by the triangle loop class $H^0(\mathsf{fGC}_3) = \mathbb{K} \langle L_3 \rangle$ where $L_3 =$ .

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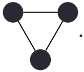
The zeroth cohomology of the graph complex fGC_3 is one dimensional and spanned by the triangle loop class $H^0(\mathsf{fGC}_3) = \mathbb{K}\,\langle L_3 \rangle$ where $L_3 =$  .

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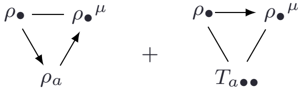
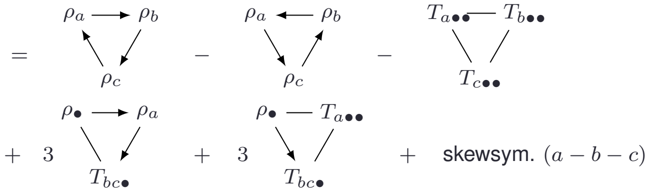
- $\overset{\circ}{\rho}_a{}^{\mu} =$ 

- $\overset{\circ}{T}_{abc} =$ 

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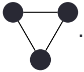
The zeroth cohomology of the graph complex fGC_3 is one dimensional and spanned by the triangle loop class $H^0(\mathsf{fGC}_3) = \mathbb{K}\, \langle L_3 \rangle$ where $L_3 =$  .

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It can be checked that $\left\{\mathscr{H}, \mathring{\mathscr{H}}\right\}_\omega = 0$ as a consequence of $\left\{\mathscr{H}, \mathscr{H}\right\}_\omega = 0$.

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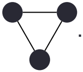
The zeroth cohomology of the graph complex fGC_3 is one dimensional and spanned by the triangle loop class $H^0(\mathsf{fGC}_3) = \mathbb{K}\,\langle L_3 \rangle$ where $L_3 =$  .

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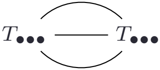
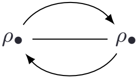
-  $\mapsto \Omega =$ 

$+\,6$ 

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Deformation

- The Kontsevich universal model for the deformation theory of Poisson manifolds (for $d = 2$) is generalised to all NPQ-manifolds (for any positive d).
  - Classification of universal structures on NP(Q)-manifolds.
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- This construction provide new insights regarding the deformation quantization problem for NPQ-manifolds of higher d.
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- Globalisation *à la* Fedosov See *e.g.* Jost 12' for $d = 2$
- Considering (multi)-oriented graph complexes will allow to see some incarnation of $\mathfrak{grt}_1$ (or equivalently Drinfel'd associators) within the quantization of NPQ-manifolds of higher d.

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Perspectives

- Globalisation *à la* Fedosov See *e.g.* Jost 12' for $d = 2$
- Considering (multi)-oriented graph complexes will allow to see some incarnation of $\mathfrak{grt}_1$ (or equivalently Drinfel'd associators) within the quantization of NPQ-manifolds of higher d.