The linear dilaton: from the clockwork mechanism to its supergravity embedding

Chrysoula Markou

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Max-Planck-Institut für Physik

Overview

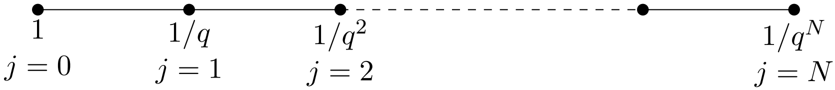
1. **The clockwork mechanism**
   - The clockwork scalar
   - Motivation and basic references
   - The clockwork graviton
   - Continuum
2. **Little String Theory**
   - Definition
   - Geometry
   - Phenomenology
3. **Supergravity embedding**
   - Structure of $\mathcal{N} = 2$, $D = 5$ Supergravity
   - Effective supergravity

Setup

1. massless scalar $\pi(x)$, symmetry $U(1)$
2. N+1 copies: $\pi_j(x)$, $j = 0, 1, \ldots, N$, symmetry $U(1)^{N+1}$ (at least)
3. explicit breaking $U(1)^{N+1} \to U(1)$ via a mass–mixing that involves **near–neighbours** only

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N}\partial_\mu\pi_j\partial^\mu\pi_j - \frac{1}{2}m^2\sum_{j=0}^{N-1}(\pi_j - q\pi_{j+1})^2 + \ldots$$

q treats the site j and the site $j+1$ asymmetrically



Eigenmodes $\pi = Oa$

1. massless Goldstone a_0, $O_{j0} = \frac{\mathcal{N}_0}{q^j}$
2. massive Clockwork gears a_k, $O_{jk} = \mathcal{N}_k \left[q \sin \frac{jk\pi}{N+1} - \sin \frac{(j+1)k\pi}{N+1} \right]$

now couple eg. topological term to the N–**th site only**

$$\frac{1}{f} \pi_N G_{\mu\nu} \widetilde{G}^{\mu\nu} = \left\{ \frac{1}{f_0} a_0 - \sum_{k=1}^{N} (-)^k \frac{1}{f_k} a_k \right\} G_{\mu\nu} \widetilde{G}^{\mu\nu}$$

1. $f_0/f \sim q^N$: *exponential* enhancement
2. $f_k/f \sim N^{3/2}/k$: mild N dependance

- the clockwork as a *renormalizable* scalar field theory:
  Choi and Im '15
  Kaplan and Rattazzi '15
  (see also natural inflation and relaxion models)

- the clockwork as an *effective* low–energy field theory:
  Giudice and McCullough '16
  - Recipe: $N+1$ of particle P whose masslessness is protected by symmetry S & explicit breaking via near–neighbour interactions
  - Known examples for the following spins: 0, 1/2, 1, 2

- growing pheno literature (LHC signatures, inflation, ...)

- group theory of the clockwork:
  Craig, Garcia Garcia, Sutherland '17
  Giudice and McCullough '17

1. @ the linear level: use Fierz–Pauli 1939 mass term

$$\mathcal{L}_{int} = -\frac{1}{2}m^2 \sum_{j=0}^{N-1} \left[(h_j^{\mu\nu} - q h_{j+1}^{\mu\nu})^2 - \left[\eta_{\mu\nu}(h_j^{\mu\nu} - q h_{j+1}^{\mu\nu})\right]^2 \right]$$

Giudice and McCullough '16

2. @ the nonlinear level: use Hinterbichler and Rosen '12 interaction

$$\mathcal{S}_V = -\sum_{i,j,k,l} \int T_{ijkl}\, \varepsilon_{abcd}\, (E_i)^a \wedge (E_j)^b \wedge (E_k)^c \wedge (E_l)^d$$

Niedermann, Padilla, Saffin '18

in both cases: *asymmetric* gear shifts $\xi_i^\mu = \frac{1}{q^i}\xi^\mu$

Setup

discrete and finite lattice dimension $\rightarrow$ continuous y on $S^1/\mathbb{Z}_2$

Giudice and McCullough '16

1. Consider real scalar ϕ in 5D with metric Ansatz:
$ds^2 = X(|y|)dx^2 + Y(|y|)dy^2$

2. Discretize: $y \rightarrow y_j = ja \quad , \quad \phi, X, Y \rightarrow \phi_j, X_j, Y_j$

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N} \partial_\mu \phi_j \partial^\mu \phi_j - \frac{1}{2}\sum_{j=0}^{N-1} \frac{X_j}{a^2 Y_j}\left(\phi_j - \frac{X_j^{1/2} Y_j^{1/4}}{X_{j+1}^{1/2} Y_{j+1}^{1/4}}\phi_{j+1}\right)^2$$

3. Identify: $X_j \sim Y_j \sim e^{-\frac{4}{3}\frac{k\pi R}{N} j}$ with $q = e^{ka}$

Eigenmodes

1. zero–mode $\psi_0 \sim e^{-k|y|}$
2. Kaluza–Klein excitations $\psi_n \sim \left(\frac{kR}{n}\sin\frac{n|y|}{R} + \cos\frac{n|y|}{R}\right)$

now couple eg. topological term to $y = 0$ **only**
(flipped k sign for pheno reasons)

$$\frac{1}{f}\int dy\,\delta(y)\,\phi\,G_{\mu\nu}\widetilde{G}^{\mu\nu} = \left\{\frac{1}{f_0}\hat{\phi}_0 + \sum_{n=1}^{N}\frac{1}{f_n}\hat{\phi}_n\right\} G_{\mu\nu}\widetilde{G}^{\mu\nu}$$

1. $f_0/f \sim e^{k\pi R}/\sqrt{k\pi R}$: *exponential* enhancement
2. $f_n \sim f$: roughly equal

Origin and comparison

- Continuum clockwork
  - 5D metric: $ds^2 = e^{\frac{4}{3}k|y|}(dx^2 + dy^2)$
  - KK spectrum: $m_n^2 = k^2 + \left(\frac{n}{R}\right)^2$

$\Rightarrow$ precisely those of the 5D toy model for the holographic dual of 6D Little String Theory as we will see now!

- Discretize 5th dimension of other models:
  - large extra dimensions: no warping and *no* "clockworking"
  - Randall–Sundrum: warping and "clockworking"
  - but $\nexists$ simple one–parameter deformation from the clockwork to Randall–Sundrum
- to explain eg. hierarchy, need only $\mathcal{O}(10)$ sites and $q \sim 4$

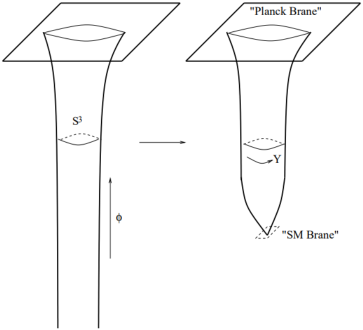
- there exist two inequivalent limits of string theory:
  1. $E << m_S$: low–energy limit
  2. $g_S \to 0$: "little string theories"

  Berkooz, Rozali, Seiberg '97, Seiberg '97, ...

- simple example of LST: take $g_S \to 0$ for a stack of N $NS5$–branes in type IIB
  1. m_S and N are the only parameters
  2. bulk dynamics decouple

- Properties of LST
  1. non–gravitational, since $M_{Pl} \to \infty$ as $g_S \to 0$
  2. non–local, eg. Hagedorn density of states at high energies, T–duality
  3. interacting, with $\frac{1}{g_N^2} = m_S^2$

- “holographic” dual of 6D LST: string theory on $\mathbb{R}^{5,1} \times \mathbb{R}_y \times S^3_N$
  Aharony, Berkooz, Kutasov, Seiberg ’98
  - infinite throat connected to asymptotically flat spacetime
  - linear dilaton $\Phi = -\frac{m_S}{\sqrt{N}} y$
- compactify y on interval (therby treating strong coupling problem and obtaining a large but finite M_{Pl} in the weak coupling regime): cigar–like geometry, Antoniadis, Dimopoulos, Giveon ’01



- 5D graviton–dilaton toy model for cigar: Antoniadis, Arvanitaki, Dimopoulos, Giveon '11

$$e^{-1}\mathcal{L}_{LST} = e^{-\frac{\Phi}{M_5^{3/2}}}\left(M_5^3\mathcal{R} + (\nabla\Phi)^2 - \Lambda\right) + \text{branes}$$

- impose linear dilaton background $\Phi = \alpha|y|$:
  - background metric: $ds^2 = e^{-\frac{2}{3}\alpha|y|}(dx^2 + dy^2)$
  - KK spectrum: $m_n^2 = \left(\frac{\alpha}{2}\right)^2 + \left(\frac{n\pi}{r_c}\right)^2$

$\Rightarrow$ same as continuum clockwork!

- supergravity embedding
  Kehagias, Riotto '17, Antoniadis, Delgado, CM, Pokorski '17

$\mathcal{N}=2,\ D=5$ Supergravity

- pure supergravity multiplet: (e_M^m, ψ_M^i, A_M^0)
- vector multiplet: (ϕ, λ^i, A_M^1)
- Maxwell–Einstein supergravity: $(e_M^m, \psi_M^i, A_M^I, \lambda_i^a, \phi^x)$
    1. $\{\phi^x\}$: coordinates of $\mathcal{M}$
    2. $\{\lambda_i^a\}$: tangent space group $SO(n)$
- Lagrangian:

$$
\begin{aligned}
e^{-1}\mathcal{L}_{ME} \;=\;& \frac{1}{2\kappa^2}\left[R(\omega)-\overline{\psi}^i_M\Gamma^{MNP}D_N\psi_{Pi}\right]-\frac{1}{4}G_{IJ}F^I_{MN}F^{MNJ}\\
&-\frac{1}{2}\overline{\lambda}^{ia}\left(\not{D}\delta^{ab}+\Omega_x{}^{ab}\not{\partial}\phi^x\right)\lambda^b_i-\frac{1}{2}g_{xy}(\partial_M\phi^x)(\partial^M\phi^y)\\
&-\frac{i}{2}\overline{\lambda}^{ia}\Gamma^M\Gamma^N\psi_{Mi}\,f^a_x\partial_N\phi^x+\frac{1}{4}h^a_I\,\overline{\lambda}^{ia}\Gamma^M\Gamma^{\Lambda P}\psi_{Mi}\,F^I_{\Lambda P}\\
&+\frac{i\kappa}{4}\Phi_{Iab}\,\overline{\lambda}^{ia}\Gamma^{MN}\lambda^b_i\,F^I_{MN}+\frac{\kappa e^{-1}}{6\sqrt{6}}C_{IJK}\,\epsilon^{MNP\Sigma\Lambda}F^I_{MN}F^J_{P\Sigma}A^K_\Lambda\\
&-\frac{3i}{8\kappa\sqrt{6}}h_I\left(\overline{\psi}^i_M\Gamma^{MNP\Sigma}\psi_{Ni}F_{P\Sigma}+2\overline{\psi}^{Mi}\psi^N_iF_{MN}\right)+\ldots
\end{aligned}
$$

Günaydin–Sierra–Townsend '84

Constraints

1. Algebraic:

$$h^I_x h_I = h_{Ix} h^I = 0 \quad , \quad h^I_x h^J_y G_{IJ} = g_{xy}$$

$$G_{IJ} = h_I h_J + h^x_I h^y_J g_{yx} = h_I h_J + h^x_I h_{Jx}$$

$$h^I h_I = 1 \quad , \quad h^I_x h^y_I = \delta^y_x$$

$$\Rightarrow \quad h^I G_{IJ} = h_J \quad , \quad h^I_x G_{IJ} = h_{Jx}$$

2. Differential:

$$h_{I,x} = \sqrt{\tfrac{2}{3}} h_{Ix} \quad , \quad h^I_{,x} = -\sqrt{\tfrac{2}{3}} h^I_x$$

$$h_{Ix;y} = \sqrt{\tfrac{2}{3}} \left(g_{xy} h_I + T_{xyz} h^z_I \right) \quad , \quad h^I_{x;y} = -\sqrt{\tfrac{2}{3}} \left(g_{xy} h^I + T_{xyz} h^{Iz} \right)$$

Embedding

- $\mathcal{E}$ with coordinates $X^I = X^I(\phi^x, \mathcal{N})$
- hypersurfaces $\mathcal{M}_k$ defined by $\ln \mathcal{N} = k$ with normal vectors $n_I = \partial_I \ln \mathcal{N}$
- **orthonormality** relations:

$$X^I_{,x} n_I = 0 \quad , \quad \frac{3}{2}\beta^2 \, X^I_{,x} X_I^{;y} = \delta_x^y$$

$\Rightarrow$ identify: $$h_I \overset{!}{=} \alpha n_I \quad , \quad h^I \overset{!}{=} \beta X^I$$

- can also show that:

$$\mathcal{N} = \beta^3 C_{IJK} X^I X^J X^K$$

- $\mathcal{M}$: special case $k=0 \quad \Rightarrow \quad \beta^3 C_{IJK} X^I X^J X^K = 1$

Gauging

- gauge $U(1)$ subgroup of $SU(2)$ w.r.t. $A_M = v_I A_M^I$
- new Lagrangian terms:

$$e^{-1}\mathcal{L}' = -\frac{g^2}{\kappa^4} P - \frac{i\sqrt{6}}{8}\frac{g}{\kappa^3}\overline{\psi}^i_M \Gamma^{MN}\psi^j_N \delta_{ij} P_0 - \frac{1}{\sqrt{2}}\frac{g}{\kappa^2}\overline{\lambda}^{ia}\Gamma^M \psi^j_M \delta_{ij} P_a + \frac{i}{2\sqrt{6}}\frac{g}{\kappa}\overline{\lambda}^{ia}\lambda^{jb}\delta_{ij} P_{ab}$$

- new terms in the fermion transformations:

$$\delta'\psi_{Mi} = \frac{i}{2\kappa\sqrt{6}} g P_0 \Gamma_M \epsilon_{ji}\delta^{jk}\epsilon_k$$

$$\delta'\lambda^a_i = \frac{1}{\kappa^2\sqrt{2}} g P^a \epsilon_{ji}\delta^{jk}\epsilon_k$$

- algebraic and differential constraints

Günaydin–Sierra–Townsend ’84, ’85

Embedding

- use a $U(1)$ gauging of $\mathcal{N}=2$, $D=5$ Maxwell–Einstein supergravity with *one* vector multiplet

- Ansatz:

$$\mathcal{N} = st^2 + at^3 \quad , \quad X^0 \equiv s\,,\, X^1 = t$$

  Klemm, Lerche, Mayr '95, Antoniadis, Ferrara, Taylor, '95

- calculate potential:

$$P = -3A_P\Big(\frac{A_P}{4}t^2 + B_P\frac{1}{t}\Big)$$

- redefine $\sqrt{3}\ln t = \Phi$, set $\frac{3}{4}g^2 A_P^2 = -\Lambda\,,\, B_P = 0$

$$\Rightarrow e^{-1}\mathcal{L}_{LST} = \frac{1}{2}\mathcal{R} - \frac{1}{2}(\partial_M\Phi)(\partial^M\Phi) - e^{\frac{2}{\sqrt{3}}\Phi}\Lambda$$

Partial breaking

- $\Phi = Cy$, fine–tuning condition: $C = \pm\frac{gA_P}{\sqrt{2}}$

- fermion transformations:

$$\widetilde{\delta}\psi_{\mu i} = \frac{i}{2\sqrt{3}}\Gamma_\mu\Big(iC\Gamma^5\epsilon_i + \frac{gA_P}{\sqrt{2}}\varepsilon_{ji}\delta^{jk}\epsilon_k\Big)$$

$$\widetilde{\delta}\lambda_i = -\frac{1}{2}e^{\frac{1}{\sqrt{3}}Cy}\Big(iC\Gamma^5\epsilon_i + \frac{gA_P}{\sqrt{2}}\varepsilon_{ji}\delta^{jk}\epsilon_k\Big)$$

so that

$$\widetilde{\delta}\big(\lambda_1 - i\Gamma^5\lambda_2\big) = 0$$

$$\widetilde{\delta}\big(\lambda_1 + i\Gamma^5\lambda_2\big) \sim \epsilon_2 - i\Gamma^5\epsilon_1$$

$\Rightarrow$ *partial supersymmetry breaking*

- three free parameters: $g, \upsilon_0, \upsilon_1$. Mass gap: $\sim g$

Summary

- Gauged $\mathcal{N} = 2$, $D = 5$ supergravity can accommodate the linear dilaton model (**minimal** case)

- Partial breaking *due to the linear dilaton bkg*

- 5D supergravity embedding of other models
  Im, Nilles, Olechowski '18

- Branes compatible with the unbroken supersymmetry
  Antoniadis, Delgado, CM, Pokorski (ongoing work)

- deeper link between LST and the clockwork?