One Particle States in Curved Spacetime

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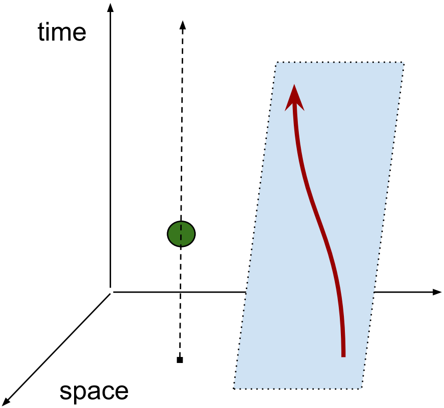
Overview

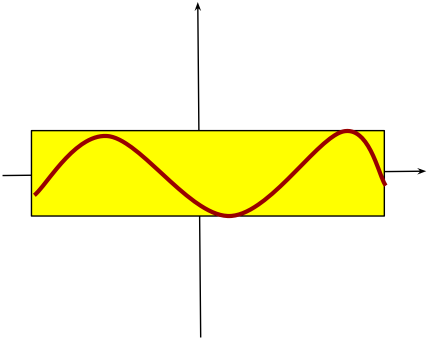
Particles and the cosmological constant

1. Particles localized on hypersurfaces of the bulk geometry.
2. Particles in non-stationary curved spacetime
3. Cosmological constants

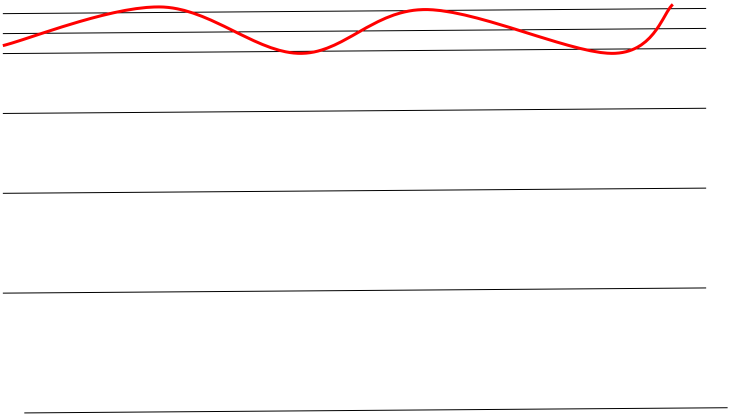
Localization



Potential Well



Warping the spacetime



Scalar creepers in D-dimensional Minkowski spacetime are scalar fields whose propagating modes are localized on $d \leq D$ dimensional subspaces.

Action

$$\mathcal{S} = -\int d^D x \left(\sum_{a,b=0}^{d-1} \frac{1}{2} \eta^{ab} \partial_a \phi \partial_b \phi + V(\phi) \right),$$

where $\partial_a := \frac{\partial}{\partial x^a}$,

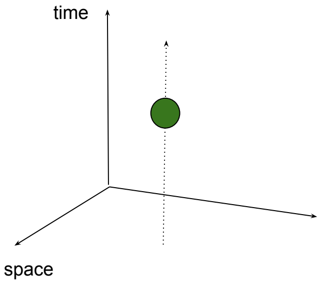
$$V(\phi) = \frac{1}{2} m^2 \phi^2 + V_{\text{int}}(\phi),$$

m^2 is a constant, and $V_{\text{int}}(\phi)$ gives self-interaction.

Non interacting $d = 1$ creeper

Action

$$\mathcal{S} := \frac{1}{2}\int d^D x (\partial_0 \phi)^2,$$



Correlation function

$$D_F(x-x') := \mathcal{Z}^{-1} \int \mathcal{D}\phi \, e^{i\mathcal{S}} \phi(x)\phi(x'),$$

where $\mathcal{Z} := \int \mathcal{D}\phi \, e^{i\mathcal{S}}$ is the partition function.

Classical field equation

$$\delta\mathcal{S}/\delta\phi = 0,$$

in which, $\frac{\delta\mathcal{S}}{\delta\phi} = \left(\Box^{(d)} - m^2\right)\phi$ and $\Box^{(d)} := \sum_{a,b=0}^{d-1}\eta^{ab}\partial_a\partial_b$.

$$\left(\Box^{(d)} - m^2\right) D_F(x - x') = i\delta^D(x - x').$$

Solution

$$D_F(x - x') = D_F^{(d)}(x_{\shortparallel} - x'_{\shortparallel})\delta^{D-d}(x_\perp - x'_\perp),$$

where $x^a_{\shortparallel} := x^a$ for $a = 0, \cdots, d-1$, $x^a_\perp := x^a$ for $a = d, \cdots, D-1$, and $D_F^{(d)}(x_{\shortparallel} - x'_{\shortparallel})$ denotes the celebrated Feynman propagator in d-dimensional Minkowski spacetime.

• The corresponding one-particle states are localized on the d dimensional subspace.

Comments

1. Creepers on spacelike hypersurfaces can be introduced similarly.
2. D-dimensional creepers in D-dimensional Minkowski spacetime are the ordinary scalar fields.
3. For $d < D$, the classical field equation $\left(\Box^{(d)} - m^2\right)\phi = 0$ is not deterministic, if not meaningless altogether, because it is silent about the behavior of the classical field in directions $x^a_\perp$ perpendicular to the hypersurface. But classical fields do not participate in particle physics. The particle interpretation of physical states comes from quantum fields whose correlation function is well-defined and can be interpreted in terms of the Feynman propagator of one-particle states **confined to the hypersurface**.

Ordinary scalars in curved spacetime

Action

$$\mathcal{S} := -\frac{1}{2}\int d^D y\, \mathfrak{e}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi,$$

in which $\mathfrak{e} := \sqrt{|\det g|}$.

Local frames

Consider the tetrad $e^\mu{}_a$ satisfying $\eta^{ab}e^\mu{}_a e^\nu{}_b = g^{\mu\nu}$ and the vector fields $\partial_{e_a} := e^\mu{}_a\partial_\mu$,

$$\mathcal{S} := -\frac{1}{2}\int d^D y\, \mathfrak{e}\, \eta^{ab}\partial_{e_a}\phi\partial_{e_b}\phi.$$

Difficulty

Equation of Motion

$$\frac{\delta \mathcal{S}}{\delta \phi} = \mathfrak{e}\eta^{ab} \left(\partial_{e_a}\partial_{e_b} + (\nabla_\mu {e_a}^\mu)\partial_{e_b}\right)\phi,$$

where ∇_μ denotes the Levi-Civita connection, and we have used the identity $\nabla_\mu v^\mu = \mathfrak{e}^{-1}\partial_\mu(\mathfrak{e}\, v^\mu)$.

The roots of the difficulty

The vector fields ∂_{e_a} are not necessarily divergence free and they do not commute with each other in general.

Creepers in curved spacetime

Action

$$\mathcal{S} = \int d^D y \, \mathfrak{e} \mathcal{L}(\phi; \mathfrak{g}_{(d)}).$$

The Lagrangian density $\mathcal{L}(\phi; \mathfrak{g}_{(d)})$ is diffeomorphism invariant though it is independent of the spacetime metric g.

Lagrangian

$$\mathcal{L}(\phi;\mathfrak{g}_{(d)}) := -\frac{1}{2}\mathfrak{g}_{(d)}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi),$$

where $\partial_\mu := \frac{\partial}{\partial y^\mu}$,and

$$\mathfrak{g}_{(d)}^{\mu\nu} := \sum_{a,b=0}^{d-1} \eta^{ab} {v_a}^\mu {v_b}^\nu,$$

v_a's are divergence-free vector fields commuting with each other, with v_0 being timelike asymptotically.

A straightforward approach to obtain such vector fields is to work with coordinate systems x^μ used in **unimodular gravity** in which $\mathfrak{e} = 1$. In these coordinates ${v_a}^\mu = {\delta_a}^\mu$, i.e.,

$$\partial_{v_a} := {v_a}^\mu \partial_\mu = \frac{\partial}{\partial x^a}.$$

Action

$$\mathcal{S} := -\int d^D x \left(\frac{1}{2} \sum_{a,b=0}^{d} \eta^{ab} \partial_a \phi \partial_b \phi + V(\phi) \right),$$

where $\partial_a := \frac{\partial}{\partial x^a}$, and $\eta = \mathrm{diag}(-1, 1, \cdots, 1)$.

Equation of motion

$$\frac{\delta \mathcal{S}}{\delta \phi} = \left(\Box^{(d)} - m^2 \right) \phi,$$

where

$$\Box^{(d)} := \sum_{a,b=0}^{d-1} \eta^{ab} \partial_a \partial_b,$$

Stress tensor

$$T_{\mu\nu} := -2\mathfrak{e}^{-1}\frac{\delta\mathcal{S}}{\delta g^{\mu\nu}} = \mathcal{L}\, g_{\mu\nu},$$

which resembles a bare cosmological constant term λ_B in the Einstein field equation suggesting that

Bare cosmological constant

$$\lambda_B = -8\pi G\, \mathcal{L}\big|_{\text{on}-\text{shell}}\,.$$

Feature

1. Similarly to ordinary scalars, they are natural extensions of scalars in Minkowski spacetime to curved spacetime. Their actions are diffeomorphism invariant.
2. They havea well-defined notion of one-particle states in nonstationary curved spacetimes, localized to $d \leq D$ dimensional hypersurfaces without using warp factors or potential wells, hence the moniker.
3. Their stress tensor resembles a bare cosmological constant, i.e., they all act like perfect fluid with equation of state $w = -1$. So they do not describe ordinary matter.

The cosmological constant problem

Local Lorentz symmetry implies that

$$\langle T_{\mu\nu} \rangle = - \langle \rho \rangle \, g_{\mu\nu}$$

where $\rho \sim \Lambda^4$ is the vacuum energy density and Λ is the high energy cutoff of the ordinary QFT. For $\Lambda \sim 1\text{TeV}$

$$8\pi G \langle \rho \rangle \sim M_{\text{Pl}}^{-2} \Lambda^4 \sim 10^{-56} M_{\text{Pl}}^2.$$

The incredible fine-tuning of λ_B

$\lambda_{\text{eff}} = \lambda_B + 8\pi G \langle \rho \rangle \sim 10^{-122} M_{\text{Pl}}^2.$

Recently Wang and Unruh have shown that the cosmological constant problem can be resolved if fluctuations of ρ are taken into account and λ_B has taken a large negative value $-\lambda_B \gg \Lambda^2$.

- Q. Wang, "Fine-tuning of the cosmological constant is not needed," arXiv:1904.09566 [gr-qc].
- Q. Wang and W. G. Unruh, "Vacuum fluctuation, micro-cyclic "universes" and the cosmological constant problem," arXiv:1904.08599 [gr-qc].

Results

1. $d = 1$ creepers correspond to $\lambda_B < 0$. $|\lambda_B| \sim 1 M_{\rm Pl}^2$.
2. For $d = 2$ the symmetry of the parameter space of classical solutions corresponding to $\lambda_B \neq 0$ is $O(1,1)$ which enhances to $\mathbb{Z}_2 \times \mathrm{Diff}(\mathbb{R}^1)$ at $\lambda_B = 0$.
3. For $d > 2$ we obtain $O(d-1,1)$, $O(d-1) \times \mathrm{Diff}(\mathbb{R}^1)$ and $O(d-1,1) \times O(d-2) \times \mathrm{Diff}(\mathbb{R}^1)$ corresponding to, respectively, $\lambda_B < 0$, $\lambda_B = 0$ and, $\lambda_B > 0$.

Spin $\frac{1}{2}$ Weinberg field

$$\mathcal{S}_{\mathrm{W}}^{(\frac{1}{2})} = \int d^4y\, \mathfrak{e}\overline{\psi}(i\gamma^a\partial_{v_a} - m)\psi.$$

Equation of motion

$$(i\gamma^a\partial_{v_a} - m)\psi = 0,$$
$$(i\gamma^a\partial_{v_a} + m)\overline{\psi} = 0.$$

Massless spin 1 Weinberg field

$$\mathcal{S}^{(1)}_{\mathrm{W}} = -\frac{1}{4}\int d^4y\, \mathfrak{e} \mathcal{F}_{ab}\mathcal{F}^{ab},$$

where $\mathcal{F}_{ab} := \partial_{v_a}\mathcal{A}_b - \partial_{v_b}\mathcal{A}_a$, $\mathcal{A}_a := v_a{}^{\mu}A_{\mu}$, and $\mathcal{F}^{ab} := \eta^{ac}\eta^{bd}\mathcal{F}_{cd}$.

$U(1)$ gauge symmetry

$$\mathcal{A}_a \to \mathcal{A}_a + \partial_{v_a}\varphi,$$

The Lorentz gauge

$$\partial_{v_a} A^a = 0,$$

where $\mathcal{A}^a := \eta^{ab}\mathcal{A}_b$.

Field equation

$$\eta^{ab}\partial_{v_a}\partial_{v_b}\mathcal{A}^c = 0.$$

Discussion

Weinberg's interpretation of particles and interactions in 1960's, gives a particle interpretation of states of quantum field theory in general nonstationary curved spacetimes only if

1. We understand the x-coordinates in his work, as a coordinate system in which $|\det g| = 1$.
2. We interpret the time-ordering as ordering with respect to x^0 though ∂_0 is not timelike everywhere,
3. Suppose that quantum fields located at x_1 and x_2 (anti)commute for $\eta_{ab}(x_1 - x_2)^a(x_1 - x_2)^b < 0$, though Minkowski metric is not necessarily the metric of spacetime in the x-coordinates.

Conclusion

Quanta of the dark energy

The scalar creepers and the Weinberg particles add to the cosmological constant and can be considered as a dynamical source for the bare cosmological constant.