String Scattering Amplitudes and Tachyon Condensations

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String Scattering Amplitudes and Tachyon Condensations

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2. Tachyon Condensation in Open String Theory (TL, hep-th 1905.11879)
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   2. Three-Open-String Scattering Amplitudes
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   1. Four-Tachyon Scattering Amplitude: Virasoro-Shapiro Amplitude
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   3. Virasoro-Shapiro Amplitude in S-channel and Feynman Diagrams
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4. Hatefi (2013), Garousi and Hatefi (2008)
   Tachyon potential based on string scattering amplitudes.

Tachyons in String Theories: Review

1. Dual Models and Sponteneous Breaking (Bardakci, 1974)
2. Time-dependent (tachyon condensation in open string theory)
    1. Unstable D-Branes
    2. Rolling Tachyons
    3. Unstable D-$\bar{D}$ Systems
3. Local (inhomogeneous) and time-dependent
    1. Closed string tachyon (Adams, Polchinski and Silverstein)
4. Homogeneous and static tachyon condensation and tachyon potential
5. GSO Projection in Super-symmetric string theories (Discusssions)

QUESTION: Should we get rid of Tachyons ?

Four-Tachyon Scattering Amplitudes in Open String Theory

1. Veneziano Amplitude (Veneziano, G. (1968))

$$\mathcal{F}^{\mathrm{T}}_{[4]}(s,t) = g^2 \frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(-\frac{s}{2}-\frac{t}{2}-2\right)}.$$

2. Expansions in terms of s-channel poles,

$$\begin{aligned}\mathcal{F}^{\mathrm{T}}_{[4]}(s,t) &= -g^2\sum_{n=0}^{\infty}\frac{(\alpha(t)+1)(\alpha(t)+2)\cdots(\alpha(t)+n)}{n!}\frac{1}{\alpha(s)-n} \\ &= -\frac{\boxed{2g^2}}{s+2}-\frac{\boxed{g^2(t+4)}}{s}-\frac{\boxed{g^2(t+4)(t+6)/4}}{s-2}+\cdots\end{aligned}$$

where $\alpha(s)=1+s/2$ and $\alpha(t)=1+t/2$.

Three-Open-String Scattering Amplitudes

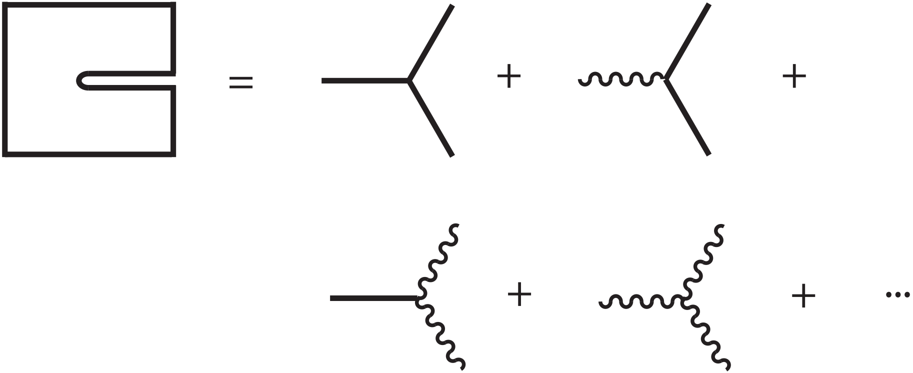
The three-string scattering amplitude in the proper-time gauge (TL, Phys. Lett. B 768 (2017) 248)

$$
\begin{aligned}
\mathcal{I}_{[3]} &= \int \prod_{r=1}^{3} dp^{(r)} \delta\left(\sum_{r=1}^{3} p^{(r)}\right) \frac{2g}{3} \langle \Psi_1, \Psi_2, \Psi_3 | E[1,2,3] | 0 \rangle, \\
E[1,2,3]|0\rangle &= \exp\left\{ \frac{1}{2} \sum_{r,s=1}^{3} \sum_{n,m\geq 1} \bar{N}^{rs}_{nm}\, \alpha^{(r)\dagger}_{n\mu} \alpha^{(s)\dagger}_{m\nu} \eta^{\mu\nu} + \sum_{r=1}^{3} \sum_{n\geq 1} \bar{N}^{r}_{n} \alpha^{(r)\dagger}_{n\mu} \boldsymbol{P}^{\mu} \right. \\
& \left. \quad + \tau_0 \sum_{r=1}^{3} \frac{1}{\alpha_r} \left(\frac{(p^{(r)}_{\mu} p^{(r)\mu}}{2} - 1 \right) \right\} |0\rangle,
\end{aligned}
$$

where $\boldsymbol{P} = p^{(2)} - p^{(1)}$. Here $\bar{N}^{rs}_{nm}$ and $\bar{N}^{r}_{n}$ are the Neumann functions.

Three-Open-String Scattering Amplitudes

Expansion of string scattering amplitude in string theory in terms of field theoretical scattering amplitudes



Local Three-Particle Interactions

Choosing the external string state,

$$\langle \Psi^{(1)}, \Psi^{(2)}, \Psi^{(3)}| = \langle 0| \prod_r^3 \left(\phi(r) + \boldsymbol{A}(r)\right)$$

where $\phi(r) = \phi(p^{(r)}),\ \boldsymbol{A}(r) = A_\mu(p^{(r)}) a_1^{(r)\mu},\ r = 1, 2, 3.$

$$\mathcal{I}_{[3]} \ = \ \mathcal{I}_{\phi\phi\phi} + \mathcal{I}_{\phi\phi A} + \mathcal{I}_{\phi AA} + \mathcal{I}_{AAA}.$$

Three-tachyon interaction

$$\mathcal{I}_{\phi\phi\phi} = \frac{2g}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta\left(\sum_{r=1}^{3} p^{(r)}\right) \mathrm{tr}\, \phi(1)\phi(2)\phi(3).$$

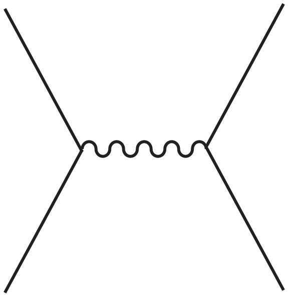
Local Three-Particle Interactions

$$\begin{aligned}
\mathcal{I}_{\phi\phi A} &= \frac{2g}{3}\int \prod_{r=1}^{3} dp^{(r)}\delta\left(\sum_{r=1}^{3} p^{(r)}\right) \mathrm{tr}\,\langle 0|\big(\phi(1)\phi(2)\boldsymbol{A}(3) + \\
&\quad \phi(1)\boldsymbol{A}(2)\phi(3) + \boldsymbol{A}(1)\phi(2)\phi(3)\big)\, e^{\tau_0 \sum_{r=1}^{3}\frac{1}{\alpha_r}\left(\frac{(p^{(r)})^2}{2}-1\right)} \\
&\quad \left(\sum_{r=1}^{3} \bar{N}^r_1 a^{(r)\dagger}_1 \cdot \boldsymbol{P}\right)|0\rangle \\
&= g\int \prod_{r=1}^{3} dp^{(r)}\delta\left(\sum_{r=1}^{3} p^{(r)}\right) \mathrm{tr}\left\{\left(p^{(1)}_\mu - p^{(2)}_\mu\right)\phi(1)\phi(2)A(3)^\mu\right\}.
\end{aligned}$$

If ϕ is real, we may write this cubic interaction as follows

$$\mathcal{I}_{\phi\phi A} = -gi\int d^d x\, \mathrm{tr}\, \partial_\mu\phi\, [A^\mu, \phi]\,.$$

Local Three-Particle Interactions



$$-\frac{\boxed{g^2(t+4)}}{s}$$

Complex Scalar Field Theory for Tachyons

To be consistent with the Veneziano amplitude,

the tachyon fields must be complex or matrix valued !

On D-$\bar{D}$ system, complex tachyon
On multiple D-branes, $U(N)$ matrix valued tachyon.

The local action of $\mathcal{I}_{\phi\phi A}$ may be written with the complex tachyon field as

$$\begin{aligned}
\mathcal{I}_{\phi\phi A} &= g \int \prod_{r=1}^{3} dp^{(r)} \delta\left(\sum_{r=1}^{3} p^{(r)}\right) \mathrm{tr}\left\{\left(p_\mu^{(1)} - p_\mu^{(2)}\right) \phi^\dagger(1)\phi(2)A(3)^\mu\right\} \\
&= ig \int d^d x \,\mathrm{tr}\left(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi\right) A^\mu .
\end{aligned}$$

Complex Scalar Field Theory for Tachyons

Accordingly, we should also rewrite the action for tachyon up to a cubic term as

$$\begin{aligned} S_{\text{tach, cubic}} &= \frac{1}{2}\int d^d x\, \text{tr}\left(\partial_\mu \phi^\dagger \partial^\mu \phi + \phi^\dagger \phi\right) \\ &+ \frac{g}{3}\int d^d x\, \text{tr}\left\{\left(\phi + \phi^\dagger\right)\left(\phi^\dagger \phi\right)\right\}. \end{aligned}$$

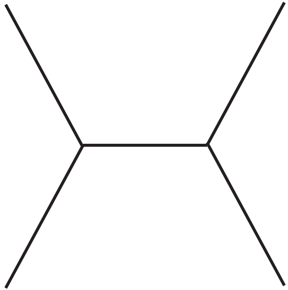
Cubic interactions $\mathcal{I}_{\phi AA}$ and $\mathcal{I}_{AAA}$

$$
\begin{aligned}
\mathcal{I}_{\phi AA} &= \frac{g}{2}\int d^d x \, \mathrm{tr}\left\{\left(\phi+\phi^\dagger\right) A_\mu A^\mu + \left(\phi+\phi^\dagger\right)\partial_\mu A^\nu \partial_\nu A^\mu\right\}, \\
\mathcal{I}_{AAA} &= gi\int d^d x \;\; \mathrm{tr}\Big\{(\partial_\mu A_\nu - \partial_\nu A_\mu)\,[A^\mu, A^\nu] \\
&\quad + \frac{1}{3}(\partial_\mu A^\nu - \partial^\nu A_\mu)\left(\partial_\nu A^\lambda - \partial^\lambda A_\nu\right)(\partial_\lambda A^\mu - \partial^\mu A_\lambda)\Big\}.
\end{aligned}
$$

Veneziano amplitude and Feynman diagrams

$$\mathcal{F}^{\mathrm{T}}_{[4]}(s,t) \;=\; -\frac{\boxed{2g^2}}{s+2} - \frac{\boxed{g^2\,(t+4)}}{s} - \frac{\boxed{g^2(t+4)(t+6)/4}}{s-2} + \cdots$$

Four-tachyon scattering amplitude with a tachyon pole

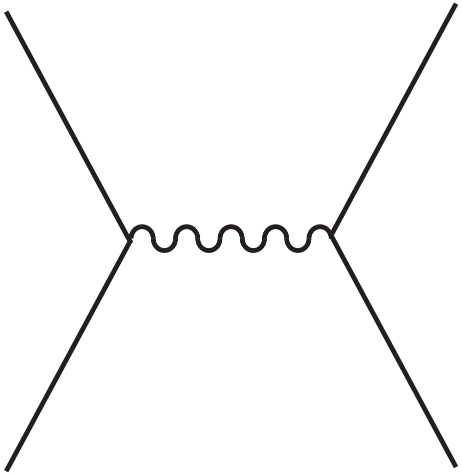


Veneziano amplitude and Feynman diagrams

Four-tachyon scattering amplitude with a tachyon pole

$$
\begin{aligned}
\mathcal{S}^{\mathrm{T}}_{[4]}\Big|_{\mathrm{tachyon}} &= 2g^2 \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \\
&\quad \mathrm{tr}\left(\phi(p^{(1)})^{\dagger}\phi(p^{(2)})\frac{1}{\left(p^{(1)}+p^{(2)}\right)^2-2}\phi(p^{(3)})\phi^{\dagger}(p^{(4)})\right) \\
&= \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right)\left(-\frac{\boxed{2g^2}}{s+2}\right) \\
&\quad \mathrm{tr}\left(\phi(p^{(1)})^{\dagger}\phi(p^{(2)})\phi(p^{(3)})\phi^{\dagger}(p^{(4)})\right)
\end{aligned}
$$

Four-tachyon scattering amplitude with a massless pole



Four-tachyon scattering amplitude with a massless pole

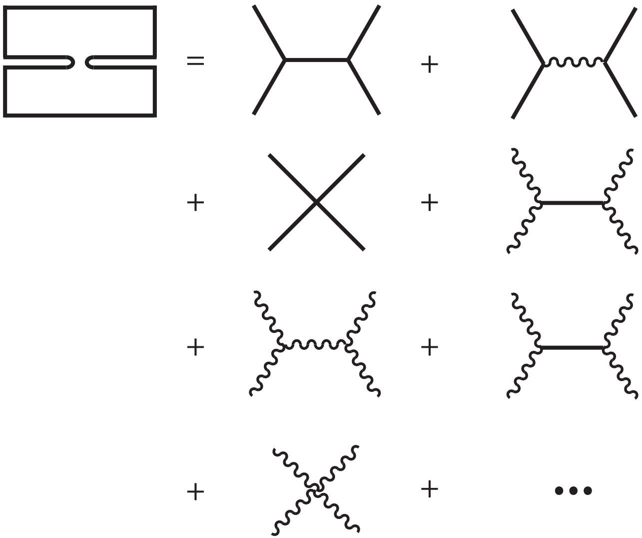
$$
\begin{aligned}
\mathcal{S}^{T}_{[4]}|_{\text{gauge}} &= -g^2 \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \\
&\quad \left(\phi(p^{(1)})^{\dagger}\phi(p^{(2)})\phi(p^{(3)})\phi^{\dagger}(p^{(4)})\right)(p^{(1)} - p^{(2)})_{\mu} \\
&\quad G^{\mu\nu}\left(p^{(1)} + p^{(2)}\right)(p^{(3)} - p^{(4)})_{\nu} \\
&= -g^2 \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \\
&\quad \left(\phi(p^{(1)})^{\dagger}\phi(p^{(2)})\phi(p^{(3)})\phi^{\dagger}(p^{(4)})\right)\left\{\frac{2(t+4)+s}{s}\right\} \\
G^{\mu\nu}(p) &= \frac{1}{p^2}\left(\eta^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{p^2}\right).
\end{aligned}
$$

Local quartic tachyon potential

$$
\begin{aligned}
S_{\phi^4} &= \mathcal{S}^{Veneziano}_{[4]}|_{\text{gauge}} - \mathcal{S}^{T}_{[4]}|_{\text{gauge}} \\
&= -g^2 \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \frac{(t+4)}{s} \\
&\quad \left(\phi(p^{(1)})^\dagger \phi(p^{(2)}) \phi^\dagger(p^{(3)}) \phi(p^{(4)})\right) \\
&\quad + \frac{g^2}{2!} \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \left(\frac{2(t+4)+s}{s}\right) \\
&\quad \left(\phi(p^{(1)})^\dagger \phi(p^{(2)}) \phi^\dagger(p^{(3)}) \phi(p^{(4)})\right) \\
&= \frac{g^2}{2} \int d^d x \left(\phi^\dagger \phi\right)^2 .
\end{aligned}
$$

We need a local quartic tachyon interaction term to match the Veneziano amplitude.

String Scattering Amplitude and Feynman Diagrams



Tachyon potential

$$V_T(\phi) = -\phi^\dagger\phi - \frac{g}{3}\text{tr}\left(\phi^\dagger + \phi\right)\left(\phi^\dagger\phi\right) + \frac{g^2}{2}\text{tr}\left(\phi^\dagger\phi\right)^2.$$

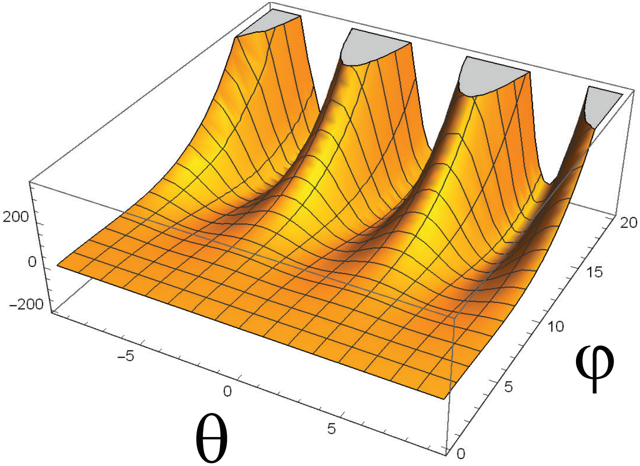
If for $N = 1$, we may write ϕ in terms of two real functions $\theta(x)$ and $\varphi(x)$, $\phi(x) = e^{i\theta(x)}\varphi(x)$,

$$V_T(\phi) = -\varphi^2 - \frac{2g}{3}\cos\theta\varphi^3 + \frac{g^2}{2}\varphi^4.$$

Local minina are located at

$$\theta = 2n\pi, \quad \text{and} \quad \varphi = \frac{(1 \pm \sqrt{5})}{2|g|}, \quad \text{where} \quad n = 0, \pm 1, \pm 2, \dots.$$

Tachyon Potential and Condensation



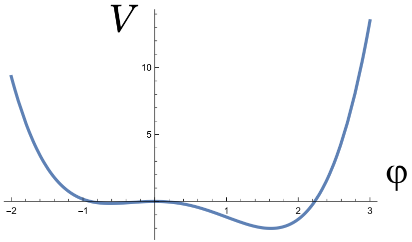
Stable minima at $\theta = 2n\pi$, and $\varphi = \varphi_0 = \frac{(1+\sqrt{5})}{2|g|}$.

Tachyon Condensation

Near the stable minima, we may expand the tachyon potential as follows

$$V_T(\theta, \varphi) = \frac{1+\sqrt{5}}{12}\theta^2 + \frac{(5+\sqrt{5})}{2}\varphi^2 - \frac{(29+13\sqrt{5})}{12g^2} + \dots$$

where $\theta \to \theta/\varphi_0$. Both sclalars θ and φ disappear from spectrum in low energy, becoming massive at Planck scale.
Tachyon potential at $\theta = 0$



Four-Tachyon Scattering Amplitude in Closed String Theory: Virasoro-Shapiro Amplitude

Virasoro-Shapiro Amplitude (1969)

$$\mathcal{A}_{\text{Tachyon}}[1,2,3,4] \;=\; 2\pi g^2 \frac{\Gamma\left(-1-\frac{s}{8}\right)\Gamma\left(-1-\frac{t}{8}\right)\Gamma\left(-1-\frac{u}{8}\right)}{\Gamma\left(2+\frac{s}{8}\right)\Gamma\left(2+\frac{t}{8}\right)\Gamma\left(2+\frac{u}{8}\right)}.$$

Expansion of the Shapiro-Virasoro amplitude in terms of s-channel poles:

$$\begin{aligned}\mathcal{A}_{\text{Tachyon}} \;=\; & -\frac{g^2}{(s/8+1)} - g^2\frac{(t/8+2)^2}{s/8} \\ & -\frac{g^2}{s/8-1}\frac{(t/8+3)^2(t/8+2)^2}{2^2} + \cdots .\end{aligned}$$

Closed-String-Scattering Amplitude

Polyakov string path integral over the pants diagram in the proper-time gauge

$$\begin{aligned} \mathcal{W}_{[3]} &= g^2 \int D[X]D[h] \exp\left(iS + i\int_{\partial M} \sum_{r=1}^{3} P^{(r)} \cdot X^{(r)} d\sigma \right), \\ S &= -\frac{1}{4\pi} \int_M d\tau d\sigma \sqrt{-h} h^{\alpha\beta} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta} \eta_{\mu\nu}, \end{aligned}$$

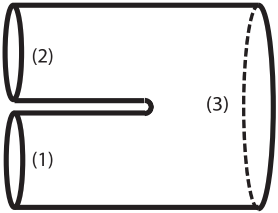
where $\sigma^1 = \tau$, $\sigma^2 = \sigma$ and $\mu, \nu = 0, \dots, d-1$.

$$\begin{aligned} X_L(\tau,\sigma) &= x_L + \sqrt{\frac{1}{2}} p_L(\tau+\sigma) + i\sqrt{\frac{1}{2}} \sum_{n\neq 0} \frac{1}{n} \alpha_n e^{-in(\tau+\sigma)}, \\ X_R(\tau,\sigma) &= x_R + \sqrt{\frac{1}{2}} p_R(\tau-\sigma) + i\sqrt{\frac{1}{2}} \sum_{n\neq 0} \frac{1}{n} \tilde{\alpha}_n e^{-in(\tau-\sigma)}. \end{aligned}$$

Three-Closed-String-Scattering Amplitude

$$\mathcal{W}_3 = \langle \mathbf{P} | \exp\left(\sum_r \xi_r L_0^{(r)}\right) | V[3] \rangle,$$

$$| V[3]_{\text{closed}} \rangle = \exp\left\{ E_{[3]\text{closed}}[1,2,3] \right\} |0; p\rangle.$$

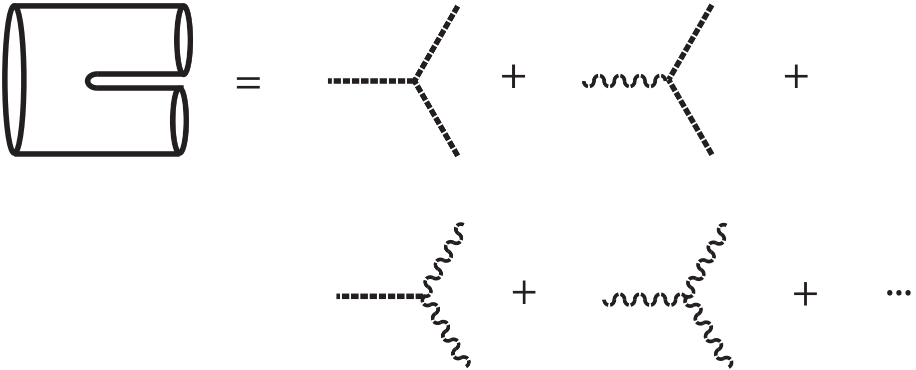


Three-Closed-String-Scattering Amplitude

Fock space representation of the three-string-vertex (TL, Web of Conferences **168**, 07004 (2018))

$$
\begin{aligned}
|V[3]_{\text{closed}}\rangle &= \exp\Bigg\{\sum_{r,s}\Bigg(\sum_{n,m\geq 1}\frac{1}{2}\bar{N}^{rs}_{nm}\,\alpha^{(r)\dagger}_n\cdot\alpha^{(r)\dagger}_m+\sum_{n\geq 1}\bar{N}^{rs}_{n0}\,\alpha^{(r)\dagger}_n\cdot p^{(s)}\Bigg)\Bigg\}\\
&\quad \exp\Bigg\{\tau_0\sum_r\frac{1}{\alpha_r}\left(\frac{1}{2}\left(p^{(r)}\right)^2-1\right)\Bigg\}\\
&\quad \exp\Bigg\{\sum_{r,s}\Bigg(\sum_{n,m\geq 1}\frac{1}{2}\bar{N}^{rs}_{nm}\,\tilde{\alpha}^{(r)\dagger}_n\cdot\tilde{\alpha}^{(r)\dagger}_m+\sum_{n\geq 1}\bar{N}^{rs}_{n0}\,\tilde{\alpha}^{(r)\dagger}_n\cdot p^{(s)}\Bigg)\Bigg\}\\
&\quad \exp\Bigg\{\tau_0\sum_r\frac{1}{\alpha_r}\left(\frac{1}{2}\left(p^{(r)}\right)^2-1\right)\Bigg\}|0;p\rangle.
\end{aligned}
$$

Cubic Couplings in Closed String Theory



Cubic Couplings in Closed String Theory

Choosing the three external closed string states

$$\langle \Psi^{(1)}, \Psi^{(2)}, \Psi^{(3)} | = \langle 0 | \prod_r \left(\phi(r) + \boldsymbol{h}(r) \right)$$

where $\phi(r) = \phi(p^{(r)})$, $\boldsymbol{h}(r) = h_{\mu\nu}(p^{(r)}) a_1^{(r)\mu} \tilde{a}_1^{(r)\nu}$, $\quad r = 1, 2, 3.$

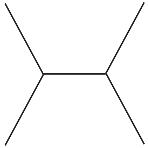
$$\mathcal{I}_{[3]} = \frac{2g}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left(\sum_{r=1}^{3} p^{(r)} \right) \langle \Psi^{(1)}, \Psi^{(2)}, \Psi^{(3)} | \exp \left\{ E_{[3]\text{closed}}[1, 2, 3] \right\} | 0; p \rangle .$$

Cubic Couplings in Closed String Theory

$$
\begin{aligned}
\mathcal{I}_{\phi\phi\phi} &= \frac{2g}{3}\int \prod_{r=1}^{3} dp^{(r)}\delta\left(\sum_{r=1}^{3} p^{(r)}\right)\ \phi(p^{(1)})\phi(p^{(2)})\phi(p^{(3)}) \\
\mathcal{I}_{\phi\phi h} &= g\int \prod_{r=1}^{3} dp^{(r)}\delta\left(\sum_{r=1}^{3} p^{(r)}\right) \\
&\quad h(1)_{(\mu\nu)}\left(p^{(2)\mu}p^{(2)\nu} + p^{(3)\mu}p^{(3)\nu} - 2p^{(2)\mu}p^{(3)\nu}\right)\phi(2)\phi(3).
\end{aligned}
$$

Four-Tachyon Scattering Amplitude with a Tachyon Pole

$$
\begin{aligned}
\mathcal{I}^{\text{pert}}_{\phi^4}\Big|_{\text{tachyon}} &= 4g^2 \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \\
&\quad \left(\phi(p^{(1)})\phi(p^{(2)}) \frac{1}{\left(p^{(1)}+p^{(2)}\right)^2/2-4} \phi(p^{(3)})\phi(p^{(4)})\right) \\
&= g^2 \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \\
&\quad \left\{-\frac{1}{(s/8+1)}\right\} \left(\phi(p^{(1)})\phi(p^{(2)})\phi(p^{(3)})\phi(p^{(4)})\right).
\end{aligned}
$$



Four-Tachyon Scattering Amplitude with Poles of Massless Particles

$$
\begin{aligned}
\mathcal{I}^{P}_{\phi^4}\Big|_{\rm massless} &= -g^2 \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \phi(p^{(1)})\phi(p^{(2)})\phi(p^{(3)})\phi(p^{(4)}) \\
&\quad \frac{1}{2}(p^{(1)} - p^{(2)})^{\mu_1}(p^{(1)} - p^{(2)})^{\nu_1} G_{\mu_1\nu_1\mu_2\nu_2}\left(p^{(1)} + p^{(2)}\right) \\
&\quad \frac{1}{2}(p^{(3)} - p^{(4)})^{\mu_2}(p^{(3)} - p^{(4)})^{\nu_2} \\
&= -\frac{g^2}{16} \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \phi(p^{(1)})\phi(p^{(2)})\phi(p^{(3)})\phi(p^{(4)}) \\
&\quad \frac{\left((t/8+2) + s/16\right)^2}{s/8}.
\end{aligned}
$$

Four-Tachyon Scattering Amplitude with Poles of Massless Particles

Propagators of massless particles in de Donder gauge

$$
\begin{aligned}
G_{\mu_1\nu_1\mu_2\nu_2}(k) &= G^{\mathrm{G}}_{\mu_1\nu_1\mu_2\nu_2}(k) + G^{\mathrm{B}}_{\mu_1\nu_1\mu_2\nu_2}(k) + G^{\mathrm{D}}_{\mu_1\nu_1\mu_2\nu_2}(k) \\
&= \eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\frac{i}{\frac{1}{2}k^2 + i\epsilon}, \\
G^{\mathrm{G}}_{\mu_1\nu_1\mu_2\nu_2}(k) &= \frac{1}{2}\left(\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{d-2}\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\right)\frac{i}{\frac{1}{2}k^2 + i\epsilon} \\
G^{\mathrm{A}}_{\mu_1\nu_1\mu_2\nu_2}(k) &= \frac{1}{2}\left(\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} - \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2}\right)\frac{i}{\frac{1}{2}k^2 + i\epsilon} \\
G^{\mathrm{D}}_{\mu_1\nu_1\mu_2\nu_2}(k) &= -\frac{1}{d-2}\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\frac{i}{\frac{1}{2}k^2 + i\epsilon}.
\end{aligned}
$$

Quartic Tachyon Potential

$$
\begin{aligned}
\mathcal{I}^{\text{local}}_{\phi^4} &= \mathcal{I}^{VS}_{\phi^4}\Big|_{\text{gauge}} - \mathcal{I}^{\text{pert}}_{\phi^4}\Big|_{\text{gauge}} \\
&= g^2 \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \left(\phi(p^{(1)})\phi(p^{(2)})\phi(p^{(3)})\phi(p^{(4)})\right) \\
&\quad \left\{ \frac{((t/8+2)+s/16)^2}{s/8} - \frac{(t/8+2)^2}{s/8} \right\} \\
&= 2g^2 \int \prod_{r=1}^{4} dp^{(r)}\, \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \left(\phi(p^{(1)})\phi(p^{(2)})\phi(p^{(3)})\phi(p^{(4)})\right) + \cdots
\end{aligned}
$$

where we treat s and t as small, $|s|, |t| \ll 1$.

Closed String Tachyon Potential and Condensation

The potential for closed string tachyon would be

$$V(\phi) \;=\; -4\phi^2 - \frac{2g}{3}\phi^3 + 2g^2\phi^4.$$

Local extrema at

$$\phi = 0,\; \frac{(1+\sqrt{5})}{2g},\; \frac{(1-\sqrt{5})}{2g}.$$

Near the stable minimum at $\phi = \phi_0 = \frac{(1+\sqrt{5})}{2g}$,

$$V = V(\phi_0) + \left(13 + 5\sqrt{5}\right)\phi^2 + \cdots\,.$$

The scalar tachyon becomes massive at Planck scale due to condensation, $m_\phi^2 = \left(13 + 5\sqrt{5}\right)/4$.

Conclusions and Discussions

Homogeneous and Static Tachyon Condensations

1. Quartic tachyon potentials are obtained from four-tachyon-scattering amplitudes
2. Tachyons become stable particles with masses at Planck scale
3. Thanks to the tachyon condensation and cubic couplings, massless particles also become massive at Planck scale.
4. Extensions to non-Abelian theories ($N \geq 2$, Multiple Dp-branes).
5. Rich structures of vacua thanks to non-trivial tachyon condensation.
6. Tachyons in super-string theories.
7. Phenomenological applications in particle physics.
8. Tail effects in low energy region
9. GSO projects $\Rightarrow$ Tachyon condensations.

Conclusions and Discussions

Homogeneous and Static Tachyon Condensations

1. GSO projects $\Rightarrow$ Tachyon condensations.
2. Sector projected out by GSO $\Rightarrow$ No massless particles, all particles become massive at Planck scale, Symmetries are broken.
3. Sector projected in by GSO $\Rightarrow$ Massless particles in spectrum, Symmetries are mostly kept intact, Weakly coupled to the Secter GSO proected out where tachyon condensations occur.