Higher derivatives couplings from maximal conformal supergravity

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Based on:

F.C. and B. Sahoo, *JHEP* 01 (2015) 59

D. Butter, F.C., B. de Wit and B. Sahoo, *PRL* 118 (2017) 081602

D. Butter, F.C., B. de Wit and B. Sahoo, *in progress*

Supersymmetries and Quantum symmetries, Yerevan

26th of August 2019

Supergravities in 4 dimensions

- Supergravity theories are supersymmetric extensions of Einstein's gravity.

Freedman, van Nieuwenhuizen, Ferrara, 1976

→ **Local symmetries:** supersymmetry + $\underbrace{\text{Poincaré symmetries}}_{\text{translations + Lorentz}}$

- **Extended** supergravity: N independent supersymmetry parameters ϵ^i. $i = 1, \cdots, N$

van Nieuwenhuizen, Ferrara, 1976

→ There is always a so-called gravity multiplet: $(\underbrace{e_\mu{}^a}_{\text{vierbein}}, \underbrace{\psi_\mu{}^i}_{\text{gravitini}}, \cdots)$

Physical states of the **gravity multiplets** (massless multiplets)

helicity	N=1	N=2	N=3	N=4	N=5	N=6	N=7	N=8	N=9
+5/2									1
+2	1	1	1	1	1	1	1	1	10
+3/2	1	2	3	4	5	6	8	8	45
+1		1	3	6	10	16	28	28	120
+1/2			1	4	11	26	56	56	210
+0				2	10	30	70	70	252
-1/2			1	4	11	26	56	56	210
-1		1	3	6	10	16	28	28	120
-3/2	1	2	3	4	5	6	8	8	45
-2	1	1	1	1	1	1	1	1	10
-5/2									1

Higher-spin field

Supergravity limit

<u>*Lagrangians:*</u>

$$\mathcal{L} = \det(e)R + \ldots$$

SUSY completion

Supergravities in 4 dimensions

- As Einstein's gravity, supergravities typically **suffer from** unrenormalizable **ultraviolet divergences** at the quantum level.
- N=8 and N=4 are *special*:
  - N=8 is the maximal theory. Latest result: finite up to five loops!

    Bern, Carrasco, Chen, Edison, Johansson, Parra-Martinez, Roiban, Zeng, 2018

  - N=4 has its first divergence at four loops. It is believed to be tied to a rigid $\mathrm{U}(1) \subset \mathrm{SU}(1,1)$ duality symmetry anomaly in the theory.

    Grisaru, 1978
    Marcus, 1985
    Bern, Davis, Dennen, A.Smirnov, V.Smirnov, 2013

helicity	N=1	N=2	N=3	N=4	N=5	N=6	N=7	N=8	N=9
+5/2									1
+2	1	1	1	1	1	1	1	1	10
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-2	1	1	1	1	1	1	1	1	10
-5/2									1

Conformal supergravity

$i = 1, \cdots, N$

- Conformal supergravities are supersymmetric extensions of conformal gravity.

Kaku, Townsend, van Nieuwenhuizen, 1978

→ **Local symmetries:**

N supersymmetries + conformal symmetries + internal R-symmetries

Q- and S- supersymmetries

Poincaré
+ dilatations
+ conformal boosts

Only for N>0

Conformal supergravity

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Kaku, Townsend, van Nieuwenhuizen, 1978

→ **Local symmetries:**

conformal symmetries

Poincaré
+ dilatations
+ conformal boosts

- Conformal gravity (N=0):

The fields of theory are the gauge fields associated to the local conformal symmetries:

$$\big(\underbrace{e_\mu{}^a\,,\,\boxed{\omega_\mu{}^{ab}}}_{\text{Poincaré}}\,,\,\underbrace{b_\mu}_{\text{dilatations}}\,,\,\underbrace{\boxed{f_\mu{}^a}}_{\text{conformal boosts}}\big)$$

As in gravity, some fields are not independent as a result of algebraic constraints.

The **unique invariant Lagrangian** is:

$$\mathcal{L} = \det(e)\, C^{\mu\nu}{}_{ab}\, \boxed{C_{\mu\nu}{}^{ab}}$$

← **Higher-derivative Lagrangian**

Weyl tensor (traceless Riemann tensor)

Conformal supergravity

$i = 1, \cdots, N$

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→ **Local symmetries:**

$$\underbrace{\text{N supersymmetries}}_{\text{Q- and S-supersymmetries}} + \underbrace{\text{conformal symmetries}}_{\text{Poincaré + dilatations + conformal boosts}} + \underbrace{\text{internal R-symmetries}}_{\text{Only for N>0}}$$

- Conformal supergravity ($N \neq 0$) :

The fields of theory include the gauge fields associated to the local superconformal symmetries. They organise into a so-called

Weyl multiplet: $(e_\mu{}^a, \omega_\mu{}^{ab}, b_\mu, f_\mu{}^a, \psi_\mu{}^i, \underbrace{\phi_\mu{}^i}_{\text{S-susy gauge field}}, \cdots)$

The invariant Lagrangian is the superconformal completion of the Weyl tensor squared:

$$\mathcal{L} = \det(e)\, C^{\mu\nu}{}_{ab}\, \boxed{C_{\mu\nu}{}^{ab}} + \cdots$$

← **Higher-derivative Lagrangian**

Weyl tensor (traceless Riemann tensor)

D=4 conformal supergravities

Weyl multiplets (off-shell):

	N=1	N=2	N=3	N=4	N=5
spin-5/2					1
spin-2	1	1	1	1	10
spin-3/2	2	4	6	8	44
spin-1	1	6	15	27	110
spin-1/2		4	20	48	165
spin-0		1	14	42	132

Higher-spin field

Conformal supergravity limit

- The N=1,2 conformal supergravities have been *studied extensively*.

[Butter, de Wit, Ferrara, Gates, Kaku, Kuzenko, Novak, Siegel, Stelle Tartaglino-Mazzucchelli, Townsend, Van Holten, Van Proeyen, West …]

Powerful: **off-shell formalism** to construct higher-derivatives invariants.

After gauge fixing the conformal symmetries, they provide insights into the higher-derivatives structure of the Poincaré theories.

→ String theory effective action, subleading corrections to black hole entropy,…

- N=4 conformal supergravity is the maximal theory.

Until now' still no Lagrangian…

N=4 conformal supergravity

$i = 1, \cdots, 4$

The full field content was derived 35 years ago:

Bergshoeff, de Roo, de Wit, 1981

→ **N=4 Weyl multiplet**: gauge fields + 'matter' fields.

Gauge fields

	Field	Symmetry (Generator)	Name/Restrictions	SU(4)	w	c
Bosons	$e_\mu{}^a$	Translations (P)	vierbein	$\mathbf{1}$	-1	0
	$\omega_\mu{}^{ab}$	Lorentz (M)	spin connection	$\mathbf{1}$	0	0
	b_μ	Dilatation (D)	dilatational gauge field	$\mathbf{1}$	0	0
	$V_\mu{}^i{}_j$	SU(4) (V)	SU(4) gauge field	$\mathbf{15}$	0	0
			$V_{\mu i}{}^j \equiv (V_\mu{}^i{}_j)^* = -V_\mu{}^j{}_i$			
			$V_\mu{}^i{}_i = 0$			
	$f_\mu{}^a$	Conformal boosts (K)	K-gauge field	$\mathbf{1}$	1	0
	a_μ	U(1)	U(1) gauge field	$\mathbf{1}$	0	0
Fermions	ϕ_{μ_i}	S-supersymmetry (S)	S-gauge field	$\bar{\mathbf{4}}$	$\frac{1}{2}$	$\frac{1}{2}$
			$\gamma_5\, \phi_{\mu_i} = \phi_{\mu_i}$			
	$\psi_\mu{}^i$	Q-supersymmetry (Q)	gravitino; $\gamma_5\, \psi_\mu^i = \psi_\mu^i$	$\mathbf{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$

Gauge symmetries
=
conformal symmetries
+
internal symmetries:
$\mathrm{SU}(4) \times \mathrm{U}(1)$
+
Q- and S-Supersymmetries

N=4 conformal supergravity

$i = 1, \cdots, 4$
$\alpha = 1, 2$

In addition to gauge fields, the N=4 Weyl multiplet contains "auxiliary fields":

'Matter' fields

	Field	Properties	SU(4)	w	c
Bosons	ϕ_α	$\phi^\alpha = \eta^{\alpha\beta}(\phi_\alpha)^\star$, $\phi_\alpha\,\phi^\alpha = 1$	$\mathbf{1}$	0	-1
	E_{ij}	$E_{ij} = E_{ji}$	$\overline{\mathbf{10}}$	1	-1
	$T_{ab}{}^{ij}$	$\frac{1}{2}\varepsilon_{ab}{}^{cd}T_{cd}{}^{ij} = -T_{ab}{}^{ij}$	$\mathbf{6}$	1	-1
		$T_{ab}{}^{ij} = -T_{ab}{}^{ji}$			
	$D^{ij}{}_{kl}$	$D^{ij}{}_{kl} = \frac{1}{4}\varepsilon^{ijmn}\varepsilon_{klpq}D^{pq}{}_{mn}$	$\mathbf{20'}$	2	0
		$D_{kl}{}^{ij} \equiv (D^{kl}{}_{ij})^* = D^{ij}{}_{kl}$			
		$D^{ij}{}_{kj} = 0$			
Fermions	Λ_i	$\gamma_5\Lambda_i = \Lambda_i$	$\overline{\mathbf{4}}$	$\frac{1}{2}$	$-\frac{3}{2}$
	$\chi^{ij}{}_k$	$\gamma_5\chi^{ij}{}_k = \chi^{ij}{}_k$; $\chi^{ij}{}_k = -\chi^{ji}{}_k$	$\mathbf{20}$	$\frac{3}{2}$	$-\frac{1}{2}$
		$\chi^{ij}{}_j = 0$			

The complex scalars ϕ_α, ϕ^α transform under rigid $\mathrm{SU}(1,1)$ transformations and satisfy:

$$\phi_1\phi^1 + \phi_2\phi^2 = 1$$

and parametrize $\mathrm{SU}(1,1)$ matrices:

$$g = \begin{pmatrix} \phi_1 & -\phi^2 \\ \phi_2 & \phi^1 \end{pmatrix}$$

→ The scalars ϕ_α are also subject to the local U(1) symmetry and therefore describe two physical degrees of freedom associated with an $\mathrm{SU}(1,1)/\mathrm{U}(1)$ coset space.

→ They carry no Weyl weight and will play a central role.

Lagrangian(s?)

The full ***off-shell* superconformal transformation rules** have been derived:

Bergshoeff, de Roo, de Wit, 1981

$$\delta_Q\, e_\mu{}^a = \bar{\epsilon}^i \gamma^a \psi_{\mu i} + \mathrm{h.c.}\,,$$

$$\delta_Q\, \psi_\mu{}^i = 2\, \mathcal{D}_\mu \epsilon^i - \tfrac{1}{2}\, \gamma^{ab} T_{ab}{}^{ij} \gamma_\mu \epsilon_j + \varepsilon^{ijkl}\, \boxed{\bar{\psi}_{\mu j} \epsilon_k\, \Lambda_l}\,,$$

$$\delta_Q\, E_{ij} = 2\, \bar{\epsilon}_{(i} \gamma^\mu D_\mu \Lambda_{j)} - 2\, \bar{\epsilon}^k \chi^{mn}{}_{(i}\, \varepsilon_{j)kmn} - \boxed{\bar{\Lambda}_i \Lambda_j\, \bar{\epsilon}_k \Lambda^k} + \boxed{2\, \bar{\Lambda}_k \Lambda_{(i}\, \bar{\epsilon}_{j)} \Lambda^k}\,,$$

$$\ldots$$

Their non-linearity makes it *very tedious* to construct the **invariant Lagrangian** by directly supersymmetrising the Weyl tensor squared:

$$\mathcal{L}_{N=4} = \det(e)\, R(M)^{\mu\nu}{}_{ab}\, \underbrace{R(M)_{\mu\nu}{}^{ab}}_{\text{supercovariant}} + \overset{?}{\ldots}$$

"Minimal" *N=4 conformal SUGRA*

$= C_{\mu\nu}{}^{ab} + \mathrm{fermions}$

→ **Partial construction:** with this iterative ("Noether") method, the Lagrangian was derived up to quadratic order in fermions.

FC, Sahoo, 2015

→ Unlike for N<4, there is *no multiplet calculus available…*

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$$\delta_Q\, E_{ij} = 2\,\bar{\epsilon}_{(i} \gamma^\mu D_\mu \Lambda_{j)} - 2\,\bar{\epsilon}^k \chi^{mn}{}_{(i}\, \varepsilon_{j)kmn} - \boxed{\bar{\Lambda}_i \Lambda_j\, \bar{\epsilon}_k \Lambda^k} + \boxed{2\,\bar{\Lambda}_k \Lambda_{(i}\, \bar{\epsilon}_{j)} \Lambda^k}\,,$$

$$\ldots$$

Their non-linearity makes it *very tedious* to construct the **invariant Lagrangian** by directly supersymmetrising the Weyl tensor squared:

$$\mathcal{L}_{N=4} = \det(e)\, \mathcal{H}(\phi_\alpha, \phi^\alpha)\, R(M)^{\mu\nu}{}_{ab}\, \underbrace{R(M)_{\mu\nu}{}^{ab}}_{\text{supercovariant}} + \overset{?}{\ldots}$$

$$= C_{\mu\nu}{}^{ab} + \text{fermions}$$

"Non-minimal" *N=4 conformal SUGRA*

In fact, it was suggested already a long time ago that there could/should exist a large class of actions, that depend on a function of the coset scalars $\mathcal{H}(\phi_\alpha, \phi^\alpha)$.

Fradkin, Tseytlin, 1982

Harvey, Moore, 1998

[Dijkgraaf, Verlinde^2; Cardoso, de Wit, Kappeli, Mohaupt; Jatkar, Sen…]

Bossard, Howe, Stelle, 2013

How to proceed…?

How we construct the invariant Lagrangian(s)?

We directly propose a **density formula** (or a ‘skeleton’) **for the Lagrangian** using the ‘ectoplasm method’.

Gates, Grisaru, Knutt-Wehlau, Siegel, 1998

→ Entirely written in terms of 4-forms. Schematically:

$$e^4 = e^a \wedge e^b \wedge e^c \wedge e^d$$

$$\mathcal{L}_{N=4} = \underline{e^4}\,\underline{\boxed{}} + \underline{e^3\psi}\,\underline{\boxed{}} + e^2\psi^2\,\boxed{} + e\psi^3\,\boxed{} + \underline{\psi^4}\,\underline{\boxed{}},$$

Weyl weight w: -4 +4 -7/2 +7/2 ... -2 +2

The $\boxed{}$ denote supercovariant coefficient functions (or **‘composites’**) that will ultimately depend on the Weyl multiplet fields.

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⟶ Entirely written in terms of 4-forms. Schematically:

$$e^4 = e^a \wedge e^b \wedge e^c \wedge e^d$$

$$\mathcal{L}_{N=4} = e^4 \square + e^3\psi \square + e^2\psi^2 \square + e\psi^3 \square + \boxed{\psi^4 \square,}$$

Weyl weight w**:** **-4** **+4** **-7/2** **+7/2** **...** **-2** **+2**

The $\square$ denote supercovariant coefficient functions (or **‘composites’**) that will ultimately depend on the Weyl multiplet fields.

Two assumptions:

- The 4-forms that appear only involve the gravitini and the vierbein.
- The bottom composites transform in the $20'$ of $\mathrm{SU}(4)$:

$$\boxed{\begin{aligned}\mathcal{L}_{N=4} = \cdots &- i\,\varepsilon^{\mu\nu\rho\sigma}\,\bar\psi_{\mu i}\,\psi_{\nu j}\,\bar\psi_\rho{}^k\,\psi_\sigma{}^l\,A^{ij}{}_{kl} \\ &- \tfrac{i}{4}\,\varepsilon^{\mu\nu\rho\sigma}\,\bar\psi_{\mu i}\,\psi_{\nu j}\,\bar\psi_{\rho k}\,\psi_{\sigma l}\,\varepsilon^{klrs}\,C^{ij}{}_{rs} + \mathrm{h.c.}\end{aligned}} \quad 20'$$

Require $\delta_Q\big($***density formula***$\big) = 0$ ***at each order in*** e ***and*** ψ***:*** ⟶

Cancellation of $\epsilon\psi^4$ variations

We parametrise the Q-supersymmetry transformations of the bosonic composites:

$$\delta C^{ij}{}_{kl} = \bar{\epsilon}^m \, \Xi^{ij}{}_{kl,m} + \bar{\epsilon}_m \, \Xi^{ij,m}{}_{kl} \,,$$
$$\delta A^{ij}{}_{kl} = \bar{\epsilon}^m \, \Omega^{ij}{}_{kl,m} + \bar{\epsilon}_m \, \Omega^{ij,m}{}_{kl} \,,$$

in terms of fermionic composites.

Gathering all the Q-supersymmetry variations proportional to $\epsilon\psi^4$ and requiring them to vanish, leads to the following **constraints on the variations of the composites** $C^{ij}{}_{kl}$ **and** $A^{ij}{}_{kl}$:

→ $$[\Xi^{ij}{}_{kl,m}]_{\overline{\mathbf{60}}} = [2\Lambda_m A^{ij}{}_{kl}]_{\overline{\mathbf{60}}} \;, \qquad [\Xi^{ij,m}{}_{kl}]_{\mathbf{60}} = 0 \;,$$
$$[\Omega^{ij}{}_{kl,m}]_{\overline{\mathbf{60}}} = [\Lambda_m \bar{C}^{ij}{}_{kl}]_{\overline{\mathbf{60}}}$$

$\Lambda_m \in \text{Weyl multiplet}$ (not a composite)

→ $$[\Xi^{ij}{}_{kl\,,m} + 2\,\Lambda_m\, A^{ij}{}_{kl}]_{\overline{\mathbf{20}}} \propto \boxed{}$$
$$[\Omega^{ij}{}_{kl\,,m} + 3\,\Lambda_m\, \bar{C}^{ij}{}_{kl}]_{\overline{\mathbf{20}}} \propto \boxed{}$$

where $\boxed{}$ and $\boxed{}$ are the fermionic composites coming from:

$$\mathcal{L}_{N=4} = \cdots + e\psi^3 \big(\boxed{} \,, \boxed{} \big) + \cdots$$

Fixes the Q-supersymmetry transformations of $C^{ij}{}_{kl}$ and $A^{ij}{}_{kl}$: determines the expressions of some higher composites.

Repeat for all other variations: $\epsilon\, e\, \psi^3,\ \epsilon\, e^2\, \psi^2,\ \ldots$

Completing the density formula

By requiring all variations to vanish, we obtain further constraints which determine the supersymmetry transformation rules of all the composites.

⚠ Final (and necessary) check:

We then verify that the transformations of the composites satisfy the same algebra as those of the Weyl multiplet fields.

→ We make use the computer algebra package ***Cadabra***

Peeters, 1998

Summary at this point:

We have **determined an invariant Lagrangian** (as a density formula) which is expressed **in terms of supercovariant composites** that do not yet depend explicitly on the Weyl multiplets fields.

Constructing the composites

Last step: Express the composites in terms of the Weyl multiplet fields.

$$\mathcal{L}_{N=4} = e^4 \, ? + e^3\psi \, ? + e^2\psi^2 \, ? + e\psi^3 \, ? + \psi^4 \, \boxed{?} \, ,$$

δ_Q δ_Q δ_Q δ_Q

Start with $C^{ij}{}_{kl}$ *and* $A^{ij}{}_{kl}$

→ Determine the other composites via the SUSY transformations rules.

Task: $C^{ij}{}_{kl}$ and $A^{ij}{}_{kl}$ have to be in the $\mathbf{20}'$ of $\mathrm{SU}(4)$ and S-supersymmetric.
They ***must satisfy****:*

$$[\Xi^{ij}{}_{kl,m}]_{\overline{\mathbf{60}}} = [2\Lambda_m A^{ij}{}_{kl}]_{\overline{\mathbf{60}}} \, , \quad [\Xi^{ij,m}{}_{kl}]_{\mathbf{60}} = 0 \, ,$$
$$[\Omega^{ij}{}_{kl,m}]_{\overline{\mathbf{60}}} = [\Lambda_m \bar{C}^{ij}{}_{kl}]_{\overline{\mathbf{60}}}$$

with

$$\delta C^{ij}{}_{kl} = \bar{\epsilon}^m \, \Xi^{ij}{}_{kl,m} + \bar{\epsilon}_m \, \Xi^{ij,m}{}_{kl} \, ,$$
$$\delta A^{ij}{}_{kl} = \bar{\epsilon}^m \, \Omega^{ij}{}_{kl,m} + \bar{\epsilon}_m \, \Omega^{ij,m}{}_{kl} \, ,$$

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Last step: Express the composites in terms of the Weyl multiplet fields.

$$\mathcal{L}_{N=4} = e^4 \, \boxed{?} + e^3\psi \, \boxed{?} + e^2\psi^2 \, \boxed{?} + e\psi^3 \, \boxed{?} + \psi^4 \, \boxed{\boxed{?}} \, ;$$

δ_Q δ_Q δ_Q δ_Q

Start with $C^{ij}{}_{kl}$ *and* $A^{ij}{}_{kl}$

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They ***must satisfy****:*

$$[\Xi^{ij}{}_{kl,m}]_{\overline{\mathbf{60}}} = [2\Lambda_m A^{ij}{}_{kl}]_{\overline{\mathbf{60}}} \,, \quad [\Xi^{ij,m}{}_{kl}]_{\mathbf{60}} = 0 \,,$$
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with

$$\delta C^{ij}{}_{kl} = \bar{\epsilon}^m \, \Xi^{ij}{}_{kl,m} + \bar{\epsilon}_m \, \Xi^{ij,m}{}_{kl} \,,$$
$$\delta A^{ij}{}_{kl} = \bar{\epsilon}^m \, \Omega^{ij}{}_{kl,m} + \bar{\epsilon}_m \, \Omega^{ij,m}{}_{kl} \,,$$

We find only one solution:

- Depends on a holomorphic function $\mathcal{H}(\phi_\alpha)$ which is homogeneous of zero-th degree in the coset scalars:

Predicted by Bossard, Howe, Stelle, 2012, 2013

→ $\mathcal{H}(\phi_\alpha)$ only depends on powers of ϕ_1/ϕ_2 .

→ The expressions of all the other composites depend on $\mathcal{H}(\phi_\alpha)$.

N=4 conformal supergravity Lagrangians

$$\mathcal{D} = -\phi^{\alpha}\varepsilon_{\alpha\beta}\frac{\partial}{\partial\phi_{\beta}}$$

The bosonic terms of this class of Lagrangians are, up to a total derivative:

$$\begin{aligned}
\det(e)^{-1}\mathcal{L} = \mathcal{H}\Big[\, & \tfrac{1}{2}\, R(M)^{abcd}\, R(M)^{-}_{abcd} + R(V)^{ab\,i}{}_{j}\, R(V)^{-\,j}_{ab}{}_{i} + \tfrac{1}{8}\, D^{ij}{}_{kl}\, D^{kl}{}_{ij} + \tfrac{1}{4}\, E_{ij}\, D^{2} E^{ij} - 4\, T_{ab}{}^{ij} D^{a} D_{c}\, T^{cb}{}_{ij} \\
& - \bar{P}^{a} D_{a} D_{b} P^{b} + P^{2} \bar{P}^{2} + \tfrac{1}{3}(P^{a}\bar{P}_{a})^{2} - \tfrac{1}{6}\, P^{a}\bar{P}_{a}\, E_{ij}\, E^{ij} - 8\, P_{a}\, \bar{P}^{c}\, T^{ab}{}_{ij}\, T_{bc}{}^{ij} - \tfrac{1}{16}\, E_{ij}\, E^{jk}\, E_{kl}\, E^{li} \\
& + \tfrac{1}{48}\, [E_{ij}\, E^{ij}]^{2} + T^{ab}{}_{ij}\, T_{ab\,kl}\, T^{cd\,ij}\, T_{cd}{}^{kl} - T^{ab}{}_{ij}\, T_{cd}{}^{jk}\, T_{ab\,kl}\, T^{cd\,li} - \tfrac{1}{2}\, E^{ij}\, T^{ab\,kl}\, R(V)_{ab}{}^{m}{}_{i}\, \varepsilon_{jklm} \\
& + \tfrac{1}{2}\, E_{ij}\, T^{ab}{}_{kl}\, R(V)_{ab}{}^{i}{}_{m}\, \varepsilon^{jklm} - \tfrac{1}{16}\, E_{ij} E_{kl}\, T^{ab}{}_{mn}\, T_{ab\,pq}\, \varepsilon^{ikmn}\, \varepsilon^{jlpq} - \tfrac{1}{16}\, E^{ij} E^{kl}\, T^{ab\,mn}\, T_{ab}{}^{pq}\, \varepsilon_{ikmn}\varepsilon_{jlpq} \\
& - 2\, T^{ab\,ij}\left(P_{[a} D_{c]} T_{b}{}^{c\,kl} + \tfrac{1}{6}\, P^{c} D_{c} T_{ab}{}^{kl} + \tfrac{1}{3}\, T_{ab}{}^{kl} D_{c} P^{c}\right)\varepsilon_{ijkl} - 2\, T^{ab}{}_{ij}\left(\bar{P}_{[a} D_{c]} T_{b}{}^{c}{}_{kl} - \tfrac{1}{2}\, \bar{P}^{c} D_{c} T_{\boldsymbol{ab}\,kl}\right)\varepsilon^{ijkl}\Big] \\
+ \mathcal{D}\mathcal{H}\Big[\, & \tfrac{1}{4}\, T_{ab}{}^{ij}\, T_{cd}{}^{kl} R(M)^{abcd}\, \varepsilon_{ijkl} + E_{ij}\, T^{ab\,ik}\, R(V)_{ab}{}^{j}{}_{k} - \tfrac{1}{8}\, D^{ij}{}_{kl}\left(T^{ab\,mn}\, T_{ab}{}^{kl}\, \varepsilon_{ijmn} - \tfrac{1}{2}\, E_{im}\, E_{jn}\, \varepsilon^{klmn}\right) \\
& + T^{ab\,ij}\, T_{a}{}^{c\,kl}\, R(V)_{bc}{}^{m}{}_{k}\, \varepsilon_{ijlm} - \tfrac{1}{24}\, E_{ij}\, E^{ij}\, T^{ab\,kl}\, T_{ab}{}^{mn}\, \varepsilon_{klmn} - \tfrac{1}{6}\, E^{ij}\, T_{ab}{}^{kl}\, T^{ac\,mn}\, T^{b}{}_{c}{}^{pq}\, \varepsilon_{iklm}\, \varepsilon_{jpqn}\Big] \\
+ \mathcal{D}^{2}\mathcal{H}\Big[\, & \tfrac{1}{6}\, E_{ij}\, T_{ab}{}^{ik}\, T^{ac\,jl}\, T^{b}{}_{c}{}^{mn}\, \varepsilon_{klmn} - \tfrac{1}{8}\, E_{ij}\, E_{kl}\, T_{ab}{}^{ik}\, T^{ab\,jl} + \tfrac{1}{384}\, E_{ij}\, E_{kl}\, E_{mn}\, E_{pq}\, \varepsilon^{ikmp}\, \varepsilon^{jlnq} \\
& + \tfrac{1}{32}\, T^{ab\,ij}\, T^{cd\,pq}\, T_{ab}{}^{mn}\, T_{cd}{}^{kl}\, \varepsilon_{ijkl}\, \varepsilon_{mnpq} - \tfrac{1}{64}\, T^{ab\,ij}\, T^{cd\,pq}\, T_{ab}{}^{kl}\, T_{cd}{}^{mn}\, \varepsilon_{ijkl}\, \varepsilon_{mnpq}\Big] \\
+ 2\, \mathcal{H}\, & e_{a}{}^{\mu} f_{\mu}{}^{c}\, \eta_{cb}\left[P^{a}\, \bar{P}^{b} - P^{d}\, \bar{P}_{d}\, \eta^{ab}\right] + \text{h.c}
\end{aligned}$$

Weyl tensor

+

...

Butter, FC, de Wit, Sahoo, 2017

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\det(e)^{-1}\mathcal{L} = \mathcal{H}\Big[&\tfrac{1}{2}\,\underline{R(M)^{abcd}}\,R(M)^{-}_{abcd} + R(V)^{ab\,i}{}_{j}\,R(V)^{-}_{ab}{}^{j}{}_{i} + \tfrac{1}{8}\,D^{ij}{}_{kl}\,D^{kl}{}_{ij} + \tfrac{1}{4}\,E_{ij}\,D^2E^{ij} - 4\,T_{ab}{}^{ij}D^aD_c\,T^{cb}{}_{ij} \\
&- \bar{P}^a D_aD_bP^b + P^2\bar{P}^2 + \tfrac{1}{3}(P^a\bar{P}_a)^2 - \tfrac{1}{6}\,P^a\bar{P}_a\,E_{ij}\,E^{ij} - 8\,P_a\,\bar{P}^c\,T^{ab}{}_{ij}\,T_{bc}{}^{ij} - \tfrac{1}{16}\,E_{ij}\,E^{jk}\,E_{kl}\,E^{li} \\
&+ \tfrac{1}{48}\,[E_{ij}\,E^{ij}]^2 + T^{ab}{}_{ij}\,T_{ab\,kl}\,T^{cd\,ij}\,T_{cd}{}^{kl} - T^{ab}{}_{ij}\,T_{cd}{}^{jk}\,T_{ab\,kl}\,T^{cd\,li} - \tfrac{1}{2}\,E^{ij}\,T^{ab\,kl}\,R(V)_{ab}{}^{m}{}_{i}\,\varepsilon_{jklm} \\
&+ \tfrac{1}{2}\,E_{ij}\,T^{ab}{}_{kl}\,R(V)_{ab}{}^{i}{}_{m}\,\varepsilon^{jklm} - \tfrac{1}{16}\,E_{ij}E_{kl}\,T^{ab}{}_{mn}\,T_{ab\,pq}\,\varepsilon^{ikmn}\,\varepsilon^{jlpq} - \tfrac{1}{16}\,E^{ij}E^{kl}\,T^{ab\,mn}\,T_{ab}{}^{pq}\,\varepsilon_{ikmn}\varepsilon_{jlpq} \\
&- 2\,T^{ab\,ij}\left(P_{[a}D_{c]}T_b{}^{c\,kl} + \tfrac{1}{6}\,P^cD_cT_{ab}{}^{kl} + \tfrac{1}{3}\,T_{ab}{}^{kl}D_cP^c\right)\varepsilon_{ijkl} - 2\,T^{ab}{}_{ij}\left(\bar{P}_{[a}D_{c]}T_b{}^{c}{}_{kl} - \tfrac{1}{2}\,\bar{P}^cD_cT_{ab\,kl}\right)\varepsilon^{ijkl}\Big] \\
+ \mathcal{D}\mathcal{H}\Big[&\tfrac{1}{4}\,T_{ab}{}^{ij}\,T_{cd}{}^{kl}R(M)^{abcd}\,\varepsilon_{ijkl} + E_{ij}\,T^{ab\,ik}\,R(V)_{ab}{}^{j}{}_{k} - \tfrac{1}{8}\,D^{ij}{}_{kl}\left(T^{ab\,mn}\,T_{ab}{}^{kl}\,\varepsilon_{ijmn} - \tfrac{1}{2}\,E_{im}\,E_{jn}\,\varepsilon^{klmn}\right) \\
&+ T^{ab\,ij}\,T_a{}^{c\,kl}\,R(V)_{bc}{}^{m}{}_{k}\,\varepsilon_{ijlm} - \tfrac{1}{24}\,E_{ij}\,E^{ij}\,T^{ab\,kl}\,T_{ab}{}^{mn}\,\varepsilon_{klmn} - \tfrac{1}{6}\,E^{ij}\,T_{ab}{}^{kl}\,T^{ac\,mn}\,T^{b}{}_{c}{}^{pq}\,\varepsilon_{iklm}\,\varepsilon_{jpqn}\Big] \\
+ \mathcal{D}^2\mathcal{H}\Big[&\tfrac{1}{6}\,E_{ij}\,T_{ab}{}^{ik}\,T^{ac\,jl}\,T^{b}{}_{c}{}^{mn}\,\varepsilon_{klmn} - \tfrac{1}{8}\,E_{ij}\,E_{kl}\,T_{ab}{}^{ik}\,T^{ab\,jl} + \tfrac{1}{384}\,E_{ij}\,E_{kl}\,E_{mn}\,E_{pq}\,\varepsilon^{ikmp}\,\varepsilon^{jlnq} \\
&+ \tfrac{1}{32}\,T^{ab\,ij}\,T^{cd\,pq}\,T_{ab}{}^{mn}\,T_{cd}{}^{kl}\,\varepsilon_{ijkl}\,\varepsilon_{mnpq} - \tfrac{1}{64}\,T^{ab\,ij}\,T^{cd\,pq}\,T_{ab}{}^{kl}\,T_{cd}{}^{mn}\,\varepsilon_{ijkl}\,\varepsilon_{mnpq}\Big] \\
+ 2\,\mathcal{H}\,e_a{}^{\mu}f_{\mu}{}^{c}\,\eta_{cb}&\left[P^a\,\bar{P}^b - P^d\,\bar{P}_d\,\eta^{ab}\right] + \text{h.c}
\end{aligned}$$

Weyl tensor + ...

Butter, FC, de Wit, Sahoo, 2017

Corresponds to the bosonic part of

$$\mathcal{L}_{N=4} = \boxed{e^4\,\square} + e^3\psi\,\square + e^2\psi^2\,\square + e\psi^3\,\square + \psi^4\,\square,$$

N=4 conformal supergravity Lagrangians

$$\mathcal{D} = -\phi^{\alpha}\varepsilon_{\alpha\beta}\frac{\partial}{\partial\phi_{\beta}}$$

The bosonic terms of this class of Lagrangians are, up to a total derivative:

$$
\begin{aligned}
\det(e)^{-1}\mathcal{L} = \mathcal{H}\Big[& \tfrac{1}{2}\,\underline{R(M)^{abcd}}\,R(M)^{-}_{abcd} + R(V)^{ab\,i}{}_{j}\,R(V)^{-}_{ab}{}^{j}{}_{i} + \tfrac{1}{8}\,D^{ij}{}_{kl}\,D^{kl}{}_{ij} + \tfrac{1}{4}\,E_{ij}\,D^{2}E^{ij} - 4\,T_{ab}{}^{ij}\,D^{a}D_{c}\,T^{cb}{}_{ij} \\
& - \bar{P}^{a}D_{a}D_{b}P^{b} + P^{2}\bar{P}^{2} + \tfrac{1}{3}(P^{a}\bar{P}_{a})^{2} - \tfrac{1}{6}\,P^{a}\bar{P}_{a}\,E_{ij}\,E^{ij} - 8\,P_{a}\,\bar{P}^{c}\,T^{ab}{}_{ij}\,T_{bc}{}^{ij} - \tfrac{1}{16}\,E_{ij}\,E^{jk}\,E_{kl}\,E^{li} \\
& + \tfrac{1}{48}\,[E_{ij}\,E^{ij}]^{2} + T^{ab}{}_{ij}\,T_{ab\,kl}\,T^{cd\,ij}\,T_{cd}{}^{kl} - T^{ab}{}_{ij}\,T_{cd}{}^{jk}\,T_{ab\,kl}\,T^{cd\,li} - \tfrac{1}{2}\,E^{ij}\,T^{ab\,kl}\,R(V)_{ab}{}^{m}{}_{i}\,\varepsilon_{jklm} \\
& + \tfrac{1}{2}\,E_{ij}\,T^{ab}{}_{kl}\,R(V)_{ab}{}^{i}{}_{m}\,\varepsilon^{jklm} - \tfrac{1}{16}\,E_{ij}E_{kl}\,T^{ab}{}_{mn}\,T_{ab\,pq}\,\varepsilon^{ikmn}\,\varepsilon^{jlpq} - \tfrac{1}{16}\,E^{ij}E^{kl}\,T^{ab\,mn}\,T_{ab}{}^{pq}\,\varepsilon_{ikmn}\varepsilon_{jlpq} \\
& - 2\,T^{ab\,ij}\left(P_{[a}D_{c]}T_{b}{}^{c\,kl} + \tfrac{1}{6}\,P^{c}D_{c}T_{ab}{}^{kl} + \tfrac{1}{3}\,T_{ab}{}^{kl}D_{c}P^{c}\right)\varepsilon_{ijkl} - 2\,T^{ab}{}_{ij}\left(\bar{P}_{[a}D_{c]}T_{b}{}^{c}{}_{kl} - \tfrac{1}{2}\,\bar{P}^{c}D_{c}T_{ab\,kl}\right)\varepsilon^{ijkl}\Big] \\
+ \mathcal{D}\mathcal{H}\Big[& \tfrac{1}{4}\,T_{ab}{}^{ij}\,T_{cd}{}^{kl}R(M)^{abcd}\,\varepsilon_{ijkl} + E_{ij}\,T^{ab\,ik}\,R(V)_{ab}{}^{j}{}_{k} - \tfrac{1}{8}\,D^{ij}{}_{kl}\left(T^{ab\,mn}\,T_{ab}{}^{kl}\,\varepsilon_{ijmn} - \tfrac{1}{2}\,E_{im}\,E_{jn}\,\varepsilon^{klmn}\right) \\
& + T^{ab\,ij}\,T_{a}{}^{c\,kl}\,R(V)_{bc}{}^{m}{}_{k}\,\varepsilon_{ijlm} - \tfrac{1}{24}\,E_{ij}\,E^{ij}\,T^{ab\,kl}\,T_{ab}{}^{mn}\,\varepsilon_{klmn} - \tfrac{1}{6}\,E^{ij}\,T_{ab}{}^{kl}\,T^{ac\,mn}\,T^{b}{}_{c}{}^{pq}\,\varepsilon_{iklm}\,\varepsilon_{jpqn}\Big] \\
+ \mathcal{D}^{2}\mathcal{H}\Big[& \tfrac{1}{6}\,E_{ij}\,T_{ab}{}^{ik}\,T^{ac\,jl}\,T^{b}{}_{c}{}^{mn}\,\varepsilon_{klmn} - \tfrac{1}{8}\,E_{ij}\,E_{kl}\,T_{ab}{}^{ik}\,T^{ab\,jl} + \tfrac{1}{384}\,E_{ij}\,E_{kl}\,E_{mn}\,E_{pq}\,\varepsilon^{ikmp}\,\varepsilon^{jlnq} \\
& + \tfrac{1}{32}\,T^{ab\,ij}\,T^{cd\,pq}\,T_{ab}{}^{mn}\,T_{cd}{}^{kl}\,\varepsilon_{ijkl}\,\varepsilon_{mnpq} - \tfrac{1}{64}\,T^{ab\,ij}\,T^{cd\,pq}\,T_{ab}{}^{kl}\,T_{cd}{}^{mn}\,\varepsilon_{ijkl}\,\varepsilon_{mnpq}\Big] \\
+ 2\,\mathcal{H}\,e_{a}{}^{\mu}f_{\mu}{}^{c}\,\eta_{cb} & \left[P^{a}\,\bar{P}^{b} - P^{d}\,\bar{P}_{d}\,\eta^{ab}\right] + \text{h.c}
\end{aligned}
$$

Weyl tensor
+
...

Butter, FC, de Wit, Sahoo, 2017

Corresponds to the bosonic part of

$$\mathcal{L}_{N=4} = \boxed{e^{4}} + e^{3}\psi + e^{2}\psi^{2} + e\psi^{3} + \psi^{4},$$

~ 900 terms

Fermions still being massaged…
Butter, FC, de Wit, Sahoo

N=4 conformal supergravity Lagrangians

$$\mathcal{D} = -\phi^{\alpha}\varepsilon_{\alpha\beta}\frac{\partial}{\partial\phi_{\beta}}$$

The bosonic terms of this class of Lagrangians are, up to a total derivative:

$$\begin{aligned}
\det(e)^{-1}\mathcal{L} = \mathcal{H}\Big[& \tfrac{1}{2}\,\underline{R(M)^{abcd}}\,R(M)^{-}_{abcd} + R(V)^{ab\,i}{}_{j}\,R(V)^{-}_{ab}{}^{j}{}_{i} + \tfrac{1}{8}\,D^{ij}{}_{kl}\,D^{kl}{}_{ij} + \tfrac{1}{4}\,E_{ij}\,D^2E^{ij} - 4\,T_{ab}{}^{ij}D^aD_cT^{cb}{}_{ij} \\
& - \bar{P}^aD_aD_bP^b + P^2\bar{P}^2 + \tfrac{1}{3}(P^a\bar{P}_a)^2 - \tfrac{1}{6}\,P^a\bar{P}_a\,E_{ij}\,E^{ij} - 8\,P_a\,\bar{P}^c\,T^{ab}{}_{ij}\,T_{bc}{}^{ij} - \tfrac{1}{16}\,E_{ij}\,E^{jk}\,E_{kl}\,E^{li} \\
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& - 2\,T^{ab\,ij}\left(P_{[a}D_{c]}T_b{}^{c\,kl} + \tfrac{1}{6}\,P^cD_cT_{ab}{}^{kl} + \tfrac{1}{3}\,T_{ab}{}^{kl}D_cP^c\right)\varepsilon_{ijkl} - 2\,T^{ab}{}_{ij}\left(\bar{P}_{[a}D_{c]}T_b{}^{c}{}_{kl} - \tfrac{1}{2}\,\bar{P}^cD_cT_{\boldsymbol{ab}\,kl}\right)\varepsilon^{ijkl}\Big] \\
& + \cancel{\mathcal{D}\mathcal{H}}\Big[\cancel{\tfrac{1}{4}\,T_{ab}{}^{ij}\,T_{cd}{}^{kl}\,R(M)^{abcd}\,\varepsilon_{ijkl} + E_{ij}\,T^{ab\,ik}\,R(V)_{ab}{}^{j}{}_{k} - \tfrac{1}{8}\,D^{ij}{}_{kl}\left(T^{ab\,mn}\,T_{ab}{}^{kl}\,\varepsilon_{ijmn} - \tfrac{1}{2}\,E_{im}\,E_{jn}\,\varepsilon^{klmn}\right)} \\
& \qquad \cancel{+ T^{ab\,ij}\,T_a{}^{c\,kl}\,R(V)_{bc}{}^{m}{}_{k}\,\varepsilon_{ijlm} - \tfrac{1}{24}\,E_{ij}\,E^{ij}\,T^{ab\,kl}\,T_{ab}{}^{mn}\,\varepsilon_{klmn} - \tfrac{1}{6}\,E^{ij}\,T_{ab}{}^{kl}\,T^{ac\,mn}\,T^b{}_c{}^{pq}\,\varepsilon_{iklm}\,\varepsilon_{jpqn}}\Big] \\
& + \cancel{\mathcal{D}^2\mathcal{H}}\Big[\cancel{\tfrac{1}{6}\,E_{ij}\,T_{ab}{}^{ik}\,T^{ac\,jl}\,T^b{}_c{}^{mn}\,\varepsilon_{klmn} - \tfrac{1}{8}\,E_{ij}\,E_{kl}\,T_{ab}{}^{ik}\,T^{ab\,jl} + \tfrac{1}{384}\,E_{ij}\,E_{kl}\,E_{mn}\,E_{pq}\,\varepsilon^{ikmp}\,\varepsilon^{jlnq}} \\
& \qquad \cancel{+ \tfrac{1}{32}\,T^{ab\,ij}\,T^{cd\,pq}\,T_{ab}{}^{mn}\,T_{cd}{}^{kl}\,\varepsilon_{ijkl}\,\varepsilon_{mnpq} - \tfrac{1}{64}\,T^{ab\,ij}\,T^{cd\,pq}\,T_{ab}{}^{kl}\,T_{cd}{}^{mn}\,\varepsilon_{ijkl}\,\varepsilon_{mnpq}}\Big] \\
& + 2\,\mathcal{H}\,e_a{}^{\mu}f_{\mu}{}^{c}\,\eta_{cb}\left[P^a\,\bar{P}^b - P^d\,\bar{P}_d\,\eta^{ab}\right] + \text{h.c}
\end{aligned}$$

Weyl tensor

+

…

When $\mathcal{H}(\phi_\alpha) = \text{ constant}$, the Lagrangian reduces to the result of: ***FC, Sahoo, 2015***

N=4 conformal supergravity Lagrangians

$$\mathcal{D} = -\phi^\alpha \varepsilon_{\alpha\beta} \frac{\partial}{\partial \phi_\beta}$$

The bosonic terms of this class of Lagrangians are, up to a total derivative:

$$\begin{aligned}
\det(e)^{-1}\mathcal{L} = \mathcal{H}\Big[& \tfrac{1}{2}\, R(M)^{abcd}\, R(M)^-_{abcd} + R(V)^{ab\,i}{}_j\, R(V)^{-}_{ab}{}^j{}_i + \tfrac{1}{8}\, D^{ij}{}_{kl}\, D^{kl}{}_{ij} + \tfrac{1}{4}\, E_{ij}\, D^2 E^{ij} - 4\, T_{ab}{}^{ij} D^a D_c\, T^{cb}{}_{ij} \\
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& + \tfrac{1}{2}\, E_{ij}\, T^{ab}{}_{kl}\, R(V)_{ab}{}^i{}_m\, \varepsilon^{jklm} - \tfrac{1}{16}\, E_{ij} E_{kl}\, T^{ab}{}_{mn}\, T_{ab\,pq}\, \varepsilon^{ikmn}\, \varepsilon^{jlpq} - \tfrac{1}{16}\, E^{ij} E^{kl}\, T^{ab\,mn}\, T_{ab}{}^{pq}\, \varepsilon_{ikmn}\varepsilon_{jlpq} \\
& - 2\, T^{ab\,ij}\,\big(P_{[a} D_{c]} T_b{}^{c\,kl} + \tfrac{1}{6}\, P^c D_c T_{ab}{}^{kl} + \tfrac{1}{3}\, T_{ab}{}^{kl} D_c P^c\big)\, \varepsilon_{ijkl} - 2\, T^{ab}{}_{ij}\,\big(\bar P_{[a} D_{c]} T_b{}^c{}_{kl} - \tfrac{1}{2}\, \bar P^c D_c T_{ab\,kl}\big)\varepsilon^{ijkl}\Big] \\
+ \mathcal{D}\mathcal{H}\Big[& \tfrac{1}{4}\, T_{ab}{}^{ij}\, T_{cd}{}^{kl} R(M)^{abcd}\, \varepsilon_{ijkl} + E_{ij}\, T^{ab\,ik}\, R(V)_{ab}{}^j{}_k - \tfrac{1}{8}\, D^{ij}{}_{kl}\,\big(T^{ab\,mn}\, T_{ab}{}^{kl}\, \varepsilon_{ijmn} - \tfrac{1}{2}\, E_{im}\, E_{jn}\, \varepsilon^{klmn}\big) \\
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+ 2\,\mathcal{H}\, & e_a{}^\mu f_\mu{}^c \eta_{cb}\Big[P^a\, \bar P^b - P^d\, \bar P_d\, \eta^{ab}\Big] + \text{h.c}
\end{aligned}$$

Weyl tensor
+
...

When $\mathcal{H}(\phi_\alpha) \neq \text{ constant}$, the function breaks the rigid $\mathrm{SU}(1,1)$ invariance.

Applications…?

Applications

Provide a *class of higher-derivatives invariants for N=4 Poincaré supergravity*:

How to transition to Poincaré ? $\longrightarrow$ use ‘compensating’ matter.

Conformal gravity (N=0) example:

2-derivatives Lagrangian: Free scalar ϕ in a conformal gravity background $\longleftarrow$ **gauge equivalence** $\longrightarrow$ Einstein's gravity

$$e^{-1}\mathcal{L} = -\phi D_\mu D^\mu \phi = \partial_\mu \phi\, \partial^\mu \phi - \underbrace{\frac{1}{6}}_{= f_\mu{}^\mu} R\, \phi^2$$

Invariant under local *conformal* trans.

conformal gauge fixing

$$b_\mu = 0 \qquad \phi = -\sqrt{3/\kappa} \longrightarrow e^{-1}\mathcal{L} = \frac{1}{2\kappa} R$$

Einstein's constant

Invariant under local *Poincaré* trans.

ϕ: compensator for the dilatation symmetry

For N=4: same principle, but ***additional subtleties***.

From N=4 conformal to Poincaré (1)

<u>For N=4</u>: use compensating vector multiplets: $(\; A^I_\mu \; , \; \lambda^I , \; \phi^I_{ij} \;)$

spin: 1 1/2 0

on-shell multiplet

$I, J = 1, \ldots, 6$

2-derivatives Lagrangian:

6 abelian vector multiplets in a N=4 conformal supergravity background ($\mathcal{L}_V$)

de Roo, 1985

← **gauge equivalence** →

N=4 Poincaré supergravity ($\mathcal{L}_\mathrm{P}$)

From N=4 conformal to Poincaré (1)

For N=4: use compensating vector multiplets: $\left(A^I_\mu \,, \boxed{\lambda^I}, \boxed{\phi^I_{ij}} \right)$

S-gauge D-gauge

on-shell multiplet

spin: 1 1/2 0

$I, J = 1, \ldots, 6$

2-derivatives Lagrangian: 6 abelian vector multiplets in a N=4 conformal supergravity background ($\mathcal{L}_V$) ⟵ **gauge equivalence** ⟶ N=4 Poincaré supergravity ($\mathcal{L}_{\mathrm{P}}$)

de Roo, 1985

$\dfrac{\mathcal{L}_V}{\text{Rigid } \mathrm{SU}(1,1) \text{ invariance}}$ —— **superconformal gauge fixing (D, K, S)** **+** **integrating out some of the 'auxiliary' fields of the N=4 Weyl multiplet** ⟶ $\dfrac{\mathcal{L}_{\mathrm{P}}}{\text{Rigid } \mathrm{SU}(1,1) \text{ invariance}}$

Electric-magnetic duality rotations

$F^I_{\mu\nu} \xrightarrow[\text{U(1)}]{} G^I_{\mu\nu}$

Straightforward: $\mathcal{L}_V$ is at most **quadratic** in these background fields, and their field equations are **algebraic**.

From N=4 conformal to Poincaré (2)

Add the 4-derivatives invariant $\mathcal{L}_{\mathrm{CSG}}$:

Butter, FC, de Wit, Sahoo, in progress

$$\mathcal{L}_V \;+\alpha\,\mathcal{L}_{\mathrm{CSG}} \xrightarrow{\text{superconformal gauge fixing + integrating out some of the 'auxiliary' fields of the N=4 Weyl multiplet}} \mathcal{L}_{\mathrm{P}} + \mathcal{L}'_{\mathrm{P}}$$

Depends on $\mathcal{H}(\phi_\alpha)$

Depends on $\mathcal{H}(\phi_\alpha)$

Integrating out the Weyl multiplet fields is now **non-trivial** since their field equations receive non-linear deformations from $\mathcal{L}_{\mathrm{CSG}}$.

→ Iterative procedure: solve the field equations order by order in α .

Toy example for a scalar A : $A = A_{(0)} + \alpha\, A^2$ $\xrightarrow{\text{solve}}$ $A = \sum_{n=0}^{\infty} A_{(n)}$, with

field equation

$$\begin{aligned} A_{(1)} &= \alpha\, A_{(0)}^2 \\ A_{(2)} &= 2\,\alpha\, A_{(1)} A_{(0)} \\ \ldots\; &= 2\,\alpha^2\, A_{(0)}^3 \end{aligned}$$

From N=4 conformal to Poincaré (2)

Add the 4-derivatives invariant $\mathcal{L}_{\mathrm{CSG}}$:

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$\mathcal{L}_V \quad + \alpha\, \mathcal{L}_{\mathrm{CSG}}$ —— **superconformal gauge fixing + integrating out some of the 'auxiliary' fields of the N=4 Weyl multiplet** ⟶ $\mathcal{L}_{\mathrm{P}} \quad + \mathcal{L}'_{\mathrm{P}}$

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$$
\begin{aligned}
A_{(1)} &= \alpha\, A_{(0)}^2 \\
A_{(2)} &= 2\,\alpha\, A_{(1)} A_{(0)} \\
&= 2\,\alpha^2\, A_{(0)}^3 \\
\ldots
\end{aligned}
$$

In practice: $T_{\mu\nu} \sim \underbrace{F_{\mu\nu}}_{\text{Field strength for spin 1}} + \alpha\big(\partial^2 T_{\mu\nu} + T\cdot T + \ldots\big)$

The field equations are not algebraic anymore.

- $\mathcal{L}'_P$ contains an infinity of terms: series in F and ∂F . → **Exclusive to SUGRA (Differs from Born-Infeld)**
- Involved in practice but the result is guaranteed to be supersymmetric.

Future directions

- Relevance for the potential duality anomaly of the N=4 Poincaré theory:

$$\underline{\mathcal{L}_{\rm P}} \quad + \mathcal{L}'_{\rm P}$$

Rigid $\mathrm{SU}(1,1)$ invariance

$U(1) \subset \mathrm{SU}(1,1)$ broken at one-loop: **anomaly?**

Depends on $\mathcal{H}(\phi_\alpha)$

$\mathcal{L}'_{\rm P}$ should be seen as a counterterm deforming the classical 2-derivatives Lagrangian $\mathcal{L}_{\rm P}$.

self-dual Riemann tensor

For $\mathcal{H}(\tau) = \, i\, \tau$,

with $\tau = i\dfrac{\phi_1 + \phi_2}{\phi_1 - \phi_2}$

$$\mathcal{L}'_{\rm P} = i\,\tau(R^-_{\mu\nu\rho\sigma})^2 - i\,\bar{\tau}(R^+_{\mu\nu\rho\sigma})^2 + \mathrm{SUSY}$$

Fully cancels the U(1) anomaly of N=4 Poincaré up to two loops: Based on loop computations of : *Carrasco, Kallosh, Roiban, Tseytlin, 2013*
Bern, Parra-Martinez, Roiban, 2017

Implications for finiteness of N=4 Poincare supergravity?!

Future directions

- Relevance for the potential duality anomaly of the N=4 Poincaré theory:

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Implications for finiteness of N=4 Poincare supergravity?!

- Study subleading corrections to N=4 black holes entropy.
  - ➜ Existing computations have been performed in a truncated N=2 setting.

- Higher-dimensional origin of the holomorphic function $\mathcal{H}(\phi_\alpha)$?
  - ➜ Connection with (2,0), D=6 conformal supergravity.

Butter, Kuzenko, Novak, Theisen, 2016

Butter, Novak, Tartaglino-Mazzucchelli, 2017

Thank you for your attention.

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