Partially-massless spin-2 fields :
An interacting theory in 4D

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Plan of the talk

1. Introduction

2. UIRs of $so(1,d+1)$: Dictionary for physicists

[Th. Basile, X. Bekaert, N.B. 2017]
1612.08166

3. A theory for multiple PM spin-2 fields

[N.B., C. Deffayet, S. Garcia-Saenz, L. Traina, 1906.03868]

1 Introduction

- De Sitter and anti-de Sitter spacetimes allow for partially massless (PM) gauge fields, with no counterpart in Minkowski spacetime.

↳ Possess a mass m (⇝ eigenvalue of (A)dS covariant d'Alembertien)

$$(\Box - m^2)\varphi = 0$$

intermediate between those of massive and massless fields

$$(\Box - m^2)\,\varphi = 0$$

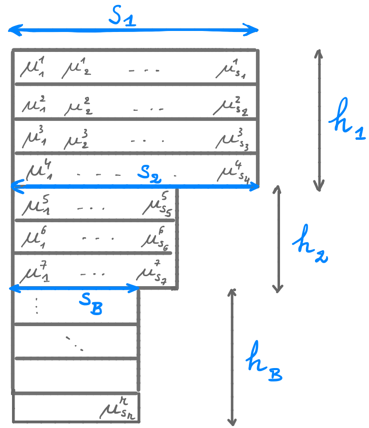
↳ Non-zero mass m proportional to λ, the (square root of) the cosmological constant $\Lambda = -\sigma \frac{d(d-1)}{2} \lambda^2$

where $AdS_{d+1} \rightsquigarrow \sigma = +1$

$dS_{d+1} \rightsquigarrow \sigma = -1$

and Einstein-Hilbert $S^{EH} = \frac{1}{\kappa^2} \int_{M_D} \sqrt{-g}\,(R - 2\Lambda)\, d^D x$

• A field in $(A)dS_{d+1}$: $\varphi_{\mathbb{Y}}$ characterised by a Young tableau

• $\mathbb{Y} =$ 

of $Gl(d+1)$

s.t. $c_1 + c_2 \leqslant d$.

• $p_I := \sum_{J=1}^{I} h_J$ • $p := p_B = \sum_{I=1}^{B} h_I$ • $r := \mathrm{rank}(so(d)) = [\frac{d}{2}]$.

the height of $\mathbb{Y}$

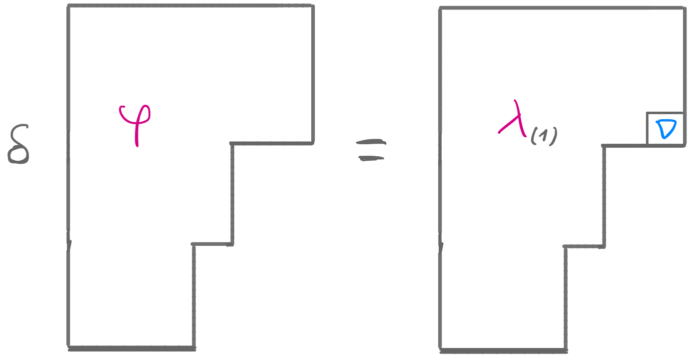
AdS_{d+1} $\quad so(2,d)$	dS_{d+1} $\quad so(1,d+1)$
• $\mathcal{D}(e_0, \mathbb{Y}) \rightsquigarrow so(2) \oplus so(d)$	• $\mathcal{D}(\Delta_c, \mathbb{Y}) \rightsquigarrow so(1,1) \oplus so(d)$
• $C_2 = e_0(e_0 - d) + C_2[so(d)]$	• $C_2 = \Delta_c(\Delta_c - d) + C_2[so(d)]$

$$\boxed{(\nabla^2 - \lambda^2 m^2_{\mathbb{Y}})\varphi_{\mathbb{Y}} = 0}$$

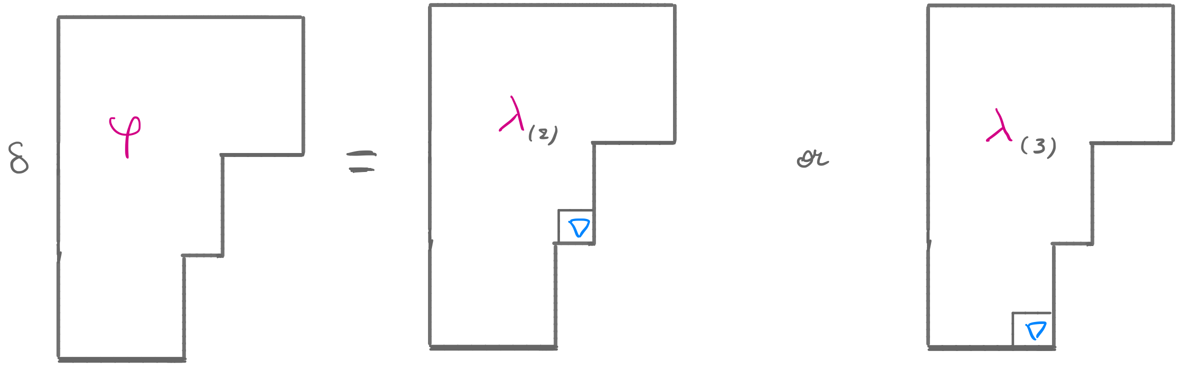
+ extra conditions
tracelessness - divergenceless

• $m^2_{\mathbb{Y}} = e_0(e_0 - d) - \sum_{k=1}^{r} s_k$	• $m^2_{\mathbb{Y}} = -\Delta_c(\Delta_c - d) + \sum_{k=1}^{r} s_k$
• (Partially) massless for	• Various cases of "masslessness":
$e_0 = e^I_t := s_I - p_I + d - t$	Exceptional & Discrete UIR series
$1 \leq t \leq s_I - s_{I+1}$	

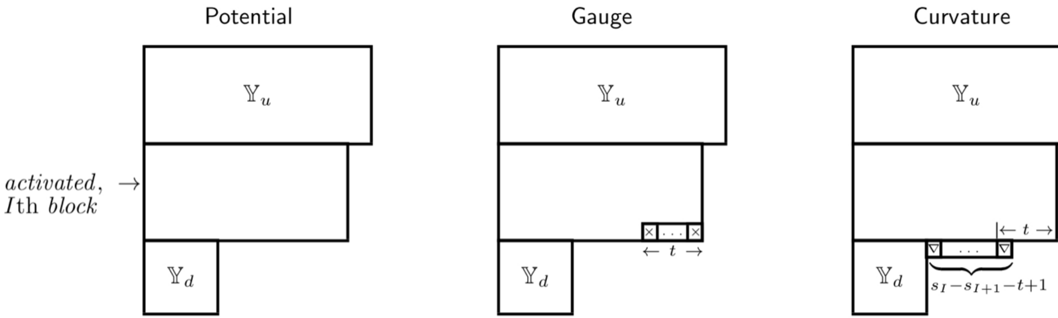
In field-theoretical terms, "massless" in $(A)dS_{d+1}$ means ($t = 1$)



or



For a partially massless field in (A)dS [N.B., C. Iazeolla, P. Sundell 2008]



• Unitary in AdS_{d+1}: 1^{st} block activated

② UIRs of $so(1,d+1)$: Dictionary for physicists

- **Principal series** : $\Delta_c = \frac{d}{2} + i\rho$, Y & $\overset{\mathbb{R}}{\underset{\in}{\rho}}$ arbitrary

[scalar : $\nabla^2 \varphi_o = (-\lambda^2)\Delta_c(\Delta_c - d)\varphi_o$ where $-\Delta_c(\Delta_c - d) = \rho^2 + \frac{d^2}{4}$ => $\nabla^2 \geq 0$ in dS_{d+1}]

- **Complementary series** : $p < \Delta_c < d-p$, $p \in \{0, 1, \dots, [\frac{d-1}{2}]\}$

$s_i = 0$ for $p+1 \leq i \leq r$

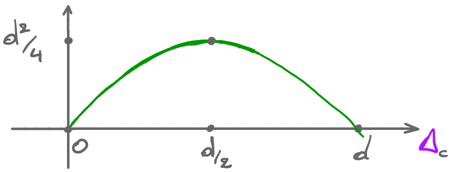
Rem : s_r may be $\neq 0$ for $d = 2r+1$, but then $s_r \in \mathbb{N}$

Rem : A scalar field can sit in these two UIRs

A scalar in the complementary series : $p = 0$ and $\Delta_c \in \,]0, d[$

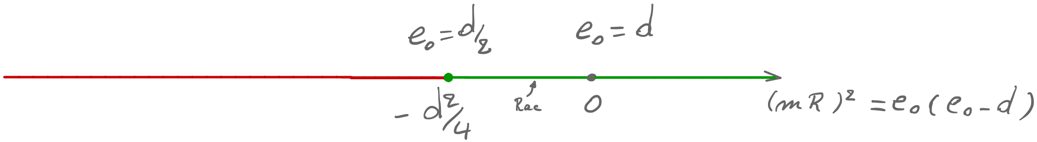
Example : scalar field

$$-\Delta_c(\Delta_c - d) =: R^2 m^2$$



$\boxed{dS_{d+1}}$

$\boxed{AdS_{d+1}}$



$e_0 \geq s - p + d - 1$ for $s > 0$

$e_0 \geqslant \frac{d-2}{2}$ for $s = 0$

$e_0 \geqslant \frac{d-1}{2}$ for $s = 1/2$

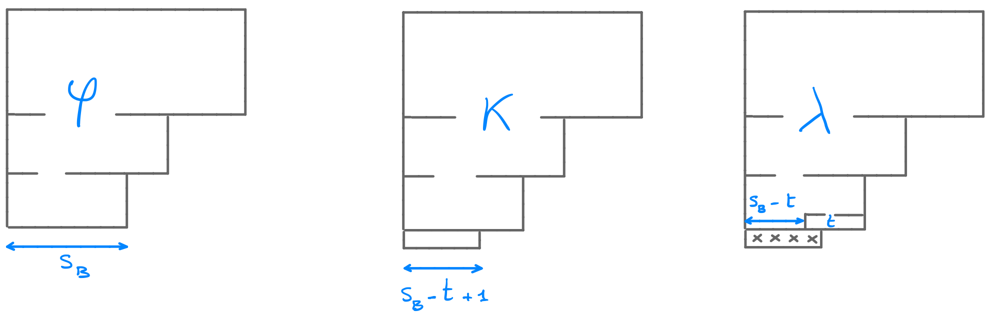
• $m^2_{Rac} = -\frac{1}{4}(d^2 - 4)$

• Exceptional series : (partially) massless fields with less-than-maximal height

↳ Unitarity: only the last block must be activated contrary to the first one in AdS.

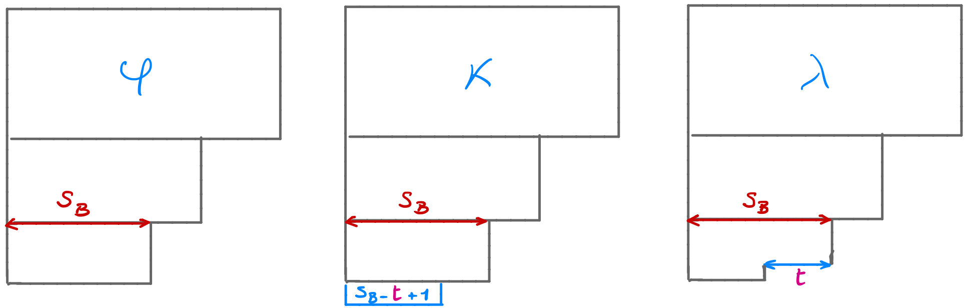
Rem: The weights $(\Delta_c, \mathbb{Y})$ labelling the UIR ⇝ curvature and not potential

$$\Delta_c = s_B - p + d - t \qquad p \equiv p_B$$



- Discrete series : Only for $d = 2r+1$, i.e. dS_{d+1} in even $D = d+1$

↳ massless field φ with maximal height , $s_r \neq 0$



potential curvature gauge para.

- $\Delta_c = s_r - r + d - t$
  - $P_B \equiv P = r$
    $s_B \equiv s_r$

Massless ⇝ $t = 1$; PM : $1 < t \leq s_B$

Rem : The weights $(\Delta_c, \mathbb{Y})$ labelling the UIR in the discrete series ⇝ potential ,

as is the case in AdS_{d+1}

• Special case : $dS_4 \longleftrightarrow so(1,4) \longleftrightarrow r = [\frac{3}{2}] = 1$

↳ (Partially) Massless fields with $\varphi \sim \boxed{s = s_r}$ are in the discrete series

$$\Delta_c = s_r - r + d - t = s + 2 - t \longleftrightarrow (\Box - \lambda^2 m^2)\varphi = 0$$

where $1 \leq t \leq s$ $\qquad m^2 = -\Delta_c(\Delta_c - d) + s$

e.g. $\underline{s=1}$: $\Delta_c = 3 - t \to t = 1 \rightsquigarrow$ massless only , $m^2 = -2(2-3)+1 = 3$

$\underline{s=2}$: $\Delta_c = 4 - t \to t = 1$: massless (graviton) $m^2 = -3(3-3)+2 = 2$

$\to t = 2$: PM spin-2 $\qquad m^2 = -2(2-3)+2 = 4$

$\underline{s=3}$: $\Delta_c = 5 - t$

$\to t = 1$ Fronsdal's $m^2 = -4(4-3)+3 = -1$

$\to t = 2$ PM $m^2 = -3(3-3)+3 = 3$

$\to t = 3$ PM $m^2 = -2(2-3)+3 = 5$

3. A theory for multiple PM spin-2 fields

$\hookrightarrow \delta \varphi_{\mu\nu} = \nabla_\mu \nabla_\nu \epsilon - \sigma \lambda^2 \bar{g}_{\mu\nu} \epsilon \rightsquigarrow \delta \underset{\varphi}{\widetilde{\square\square}} = \underset{\epsilon}{\widetilde{\boxed{\triangledown}\boxed{\triangledown}} \bullet}$, $K \sim \begin{array}{|c|c|}\hline & \\ \hline & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array}$

<u>Observations</u> : $m^2 \lambda^2$ is small, so is λ if $g_{\mu\nu} = \varphi_{\mu\nu}$ PM $(m^2 = 4)$

- Several endeavours to find a consistent theory of non-linear PM spin-2 fields [Y. Zinoviev 2006, C. de Rham - S. Renaux-Petel 2012, S.F. Hassan, A. Schmidt-May, M. von Strauss 2012, S. Deser, E. Joung and A. Waldron 2012, Deser - Sandora - Waldron 2013, E. Joung, K. Mkrtchyan and G. Poghosyan 2019]

$\Rightarrow \exists$ 2-derivative (cubic) vertex for a single PM field in 4D [Y. Zinoviev 2006]

however it does not admit any consistent higher-order completion.

- A systematic analysis about a possible non-abelian deformation of PM spin-2 theory was suggested by the comment

[26] In fact, since $s = 2$, $d = 4$ PM is gauge invariant, propagates on light cone [14], is conformally [15] and duality invariant [16], and couples consistently to charged matter, it might make more sense to search for non-abelian Yang–Mills-like interactions.

in [Deser - Joung - Waldron 2012] & [S. Deser - M. Sandora - A. Waldron 1301.5621]

- For a set of PM spin-2 [S. Garcia-Saenz, K. Hinterbichler, A. Joyce, E. Mitsou & R.A. Rosen 2015] show that there is NO non-abelian deformation $\boxed{\overset{(1)}{\delta_\epsilon} \varphi_{\mu\nu} = \vec{R}_{\mu\nu}(\epsilon)}$

with assumptions on # derivatives (max. 2) and taking $\vec{R}_{\mu\nu}$ linear in [h].

Felt the necessity to revisit this problem with more powerful methods

↪ BRST-BV from [G. Barnich & M. Henneaux 1993] : cohomological reformulation of [Berends-Burgers-van Dam 1985]

since the no-go result of [S. Garcia-Saenz, K. Hinterbichler, A. Joyce, E. Mitsou & R.A. Rosen 2015]

does not rule out non-abelian gauge algebras starting at higher orders in φ,

nor does it rule out transformations with more (than 2) derivatives.

→ We find that the abelian PM symetry admits no nonabelian deformation without any assumption on order of $\vec{R}_{\mu\nu}$ in $[\varphi]$ nor in the number of derivatives.

• Revisiting these analyses in the BV BRST-cohomological formulation

Start from $S_0[h^a_{\mu\nu}] = -\frac{1}{4}\int d^nx\,\sqrt{-g}\;k_{ab}\left[K^{a\,\mu\nu|\rho}K^b{}_{\mu\nu|\rho} - 2K^{a\,\mu}K^b{}_\mu\right]$

$K^a{}_{\mu\nu|\rho} := 2\nabla_{[\mu}h^a{}_{\nu]\rho}$ curvature for PM

$\overset{(0)}{\delta}_\epsilon S_0 = 0$ under $\overset{(0)}{\delta}_\epsilon h^a_{\mu\nu} = \nabla_\mu\nabla_\nu\epsilon^a - \sigma\lambda^2 g_{\mu\nu}\epsilon^a$

1) We prove that the most general deformation of the gauge algebra:

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]\,h^a_{\mu\nu} = \overset{(0)}{\delta}_\chi h_{\mu\nu} \qquad (\text{off-shell})$$

where $\chi = (m^a{}_{bc}\,\epsilon_1^b\,\epsilon_2^c + n^a{}_{bc}\,\nabla^\mu\epsilon_1^b\,\nabla_\mu\epsilon_2^c) \;\rightarrow$ no field dependence

2) Consistency requires $m^a{}_{bc} = 0 = n^a{}_{bc} \Rightarrow$ Abelian

3) We prove that there are NO higher-order corrections!

4) Deformation of gauge symmetry (but abelian g), if 2 ∂'s :

Consistency gives only (out of 6 candidates)

$$\boxed{\overset{(1)}{\delta}_\epsilon h^a_{\mu\nu} = \alpha\, f^a{}_{b,c}\, F^b{}_{e(\mu|\nu)}\, \nabla^e \epsilon^c}$$, only in $D=4$.

5) Corresponding cubic vertex with 2 ∂'s : $S_1 = \int d^4x \sqrt{-g}\, h^a_{\mu\nu} J^{\mu\nu}_a$

where $J^{\mu\nu}_a = f_{bc,a}\left[F^{b\mu}{}_{e|\sigma} F^{c\,\nu e|\sigma} - \frac{1}{4} g^{\mu\nu} F^{b\,e\sigma|\lambda} F^c_{e\sigma|\lambda} + \text{improvements}\right]$

$\Rightarrow$ # independent deformation : $\frac{1}{2}N^2(N+1) \rightsquigarrow f_{ab,c} \sim \boxed{a|b} \otimes \boxed{c}$

$\longrightarrow$ Uniqueness result (since existence not new)

- Conservation : Obviously $\nabla_\mu \nabla_\nu J^{\mu\nu}_a - \frac{\sigma}{L^2} g_{\mu\nu} J^{\mu\nu}_a \approx 0$

but also, since $D=4$: $\boxed{\nabla_\mu J^{\mu\nu}_a \approx 0}$

$\Longrightarrow \mathcal{J}^\mu{}_{ab} := \sqrt{-g}\, J^{\mu\nu}_a \nabla_\nu \bar{\epsilon}_b$ Noether current $\partial_\mu \mathcal{J}^\mu{}_{ab} \approx 0$ in 4D

rigid symmetry $\delta h^a_{\mu\nu} = f^a{}_{b,c} F^b_{e(\mu|\nu)} \nabla^e \bar{\epsilon}^c$
Killing

6) Higher-order consistency :

Provided $f_{ae,b} f^e{}_{c,d} = 0$ (1) & $f_{ab,e} f^e{}_{c,d} = 0$ (2)

$S := S_0 + S_1$ fully consistent to all orders (!) .

But (1) & (2) non-trivial solution only if $k_{ab} \not\succ 0$

i.e. "wrong" relative signs.

$\Rightarrow$ First consistent interacting theory for PM spin-2.

- Analogous to (but not all obtainable from) conformal gravity and its multi-conformal graviton extensions [N.B., M. Henneaux 2001] & [N.B., M. Henneaux, P. van Nieuwenhuizen 2002]

- Possibility that coupling to gravity or more general Einstein background might cure unitarity issue.
