The isospin symmetry breaking effects in K_{e4} decays

S.R.Gevorkyan[*], **A.N.Sissakian**, **A.V.Tarasov**, **H.T.Torosyan**, **O.O.Voskresenskaya**[†]

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Joint Institute for Nuclear Research, 141980 Dubna, Russia

Abstract

The Fermi-Watson theorem is generalized to the case of two coupled channels with different masses. The proposed approach is applied to final state interaction in K_{e4} decay, but can be easily generalized for the arbitrary two channel task. The impact of considered effect on the pions phase shifts is essential in the vicinity of charged pions production threshold and can be crucial for scattering lengths extraction from experimental data on K_{e4} decay.

1 Introduction

The $\pi\pi$ scattering at low energies provides a testing ground for strong interaction study [1]. As the free pion targets cannot be created the experimental evaluation of $\pi\pi$ scattering characteristics is restricted to the study of a dipion system in a final state of more complicated reactions. One of the most suitable reactions for such study are the K_{e4} decays i.e.

$$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu \tag{1}$$

$$K^{\pm} \to \pi^{0}\pi^{0}e^{\pm}\nu \tag{2}$$

For many years [2, 3] the decay (1) was considered as the cleanest method to determine the isospin zero scattering length a_0. At present the value

[*]On leave of absence from Yerevan Physics Institute

[†]On leave of absence from Siberian Physical Technical Institute

of a_0 is predicted by Chiral Perturbation Theory with high precision [4] $a_0 = 0.22 \pm 0.005$, thus the extraction of this quantity from experimental data with highest possible accuracy becomes an actual task. From the other hand, the appearance of new precise experimental data [5, 6, 7] demands the more detailed approaches taking into account the effects which can be neglected up to now in comparison of experimental data with theoretical models.

The usual method used for extraction of the scattering length a_0 from decays (1) and (2) is based on the classical works [8, 9]. The decay rates of K_{e4} are determined by three form factors F,G,H. Making the partial-wave expansion of the hadronic current with respect to the angular momentum of the dipion system and restricted to s and p waves[1] these form factors can be cast in the following form:

$$\begin{aligned} F &= f_s e^{i\delta_0^0(s)} + f_p e^{i\delta_1^1(s)} \cos\theta_\pi \\ G &= g_p e^{i\delta_1^1(s)}; \quad H = h_p e^{i\delta_1^1(s)} \end{aligned} \tag{3}$$

Here $s = M^2_{\pi\pi}, \theta_\pi$ are invariant mass squared of the dipion and the polar angle of pion in the dipion rest frame measured with respect to the flight direction of dipion in the K meson rest frame. The coefficients f_s, f_p, g_p, h_p can be parameterized as a functions of pions momenta q in dipion rest system and invariant mass of lepton pair $s_{e\nu}$ in a known way [11]. The phases δ_l^I relevant to certain isospin I and orbital momenta l of dipion system due to Fermi—Watson theorem [15] coincide with the corresponding phase shifts in elastic $\pi\pi$ scattering.

Recently, in experiment NA48/2 at CERN [12] in the $\pi^0\pi^0$ mass distribution from the decays $K^\pm \to \pi^\pm\pi^0\pi^0$ the effect of cusp was observed which as was pointed by N. Cabibbo [13] is the result of isospin breaking in final state $\pi^0\pi^0$ interaction, provided by inelastic $\pi\pi$ reactions and difference in masses of neutral and charge pions[2].

Obviously, the same effects can take place in K_{e4} decays. Up to now the final state interaction of two pions in K_{e4} decay are considered using the Fermi-Watson theorem [15] which is valid only in the isospin symmetry limit i.e. at $m_c = m_0$. The main result of present work is the generalization of usually accepted approach to K_{e4} decays, taking into account the inelastic processes in the final state and different masses of neutral and charged pions.

[1]As was shown in [10], the contribution of higher waves are small and can be safely neglected.

[2]The possibility of cusp in $\pi^0\pi^0$ scattering due to different pion masses in charge exchange reaction $\pi^+\pi^- \to \pi^0\pi^0$ was firstly predicted in [14].

2 Final state interactions and isospin breaking

First of all, let us noted that the phase shift δ_0^0 which is connected with scattering length a_0^0,has impact only on hadronic form factor F whereas the form factors G and H depends only on p-wave phase shift δ_1^1. If as usual one confines to consideration only lowest s and p waves the inelastic process $\pi^+\pi^- \to \pi^0\pi^0$ and vice versa are forbidden for $\pi\pi$ system in l=1 states due to identity of neutral pions. Thus, the inelastic transitions can change only the first term in the form factor F which is relevant to production of s-wave pions in the state with isospin $I = 0$.

Keeping this in mind let us denote by M_{+-} the decay amplitude corresponding to two charged pions in the final state with quantum numbers $I = 0$, $l = 0$ whereas the amplitude for K meson decay to two neutral pions in the dipion state $I = 0$, $l = 0$ we denote as M_{00}. It is easy to show that in one loop approximation of nonperturbative effective field theory (see e.g. [16]) these amplitudes takes the form:

$$\begin{aligned} M_{00} &= \tilde{M}_{00}(1 + ik_1 a_{00}) + ik_2 a_x \tilde{M}_{+-} \\ M_{+-} &= \tilde{M}_{+-}(1 + ik_2 a_{+-}) + ik_1 a_x \tilde{M}_{00} \end{aligned} \tag{4}$$

Here, $\tilde{M}_{00}, \tilde{M}_{+-}$ are the so called "unperturbed" amplitudes of decays (1) and (2); $2k_1 = \sqrt{M^2 - 4m_0^2}$, $2k_2 = \sqrt{M^2 - 4m_c^2}$ are the momenta of $\pi^0\pi^0$ and $\pi^+\pi^-$ systems with the same invariant mass M. The a_{00}, a_{+-}, a_x are the s-waves amplitudes of the reactions $\pi^0\pi^0 \to \pi^0\pi^0$,$\pi^+\pi^- \to \pi^+\pi^-$ and $\pi^+\pi^- \to \pi^0\pi^0$ respectively. In the isospin symmetry limit they are connected with scattering lengths a_0, a_2 through the relations [3]:

$$a_{00} = \frac{a_0 + 2a_2}{3}; \quad a_{+-} = \frac{2a_0 + a_2}{3}; \quad a_x = \frac{\sqrt{2}}{3}(a_0 - a_2) \tag{5}$$

From the rule $\Delta I = 1/2$ for semileptonic decays it follows the simple relation between the "unperturbed" amplitudes $\tilde{M}_{+-} = \sqrt{2}\tilde{M}_{00}$. Using this fact one obtains from the expressions (4) in the isospin symmetry limit ($k_1 = k_2$) [4] :

$$\begin{aligned} M_{00} &= \tilde{M}_{00}(1 + ika_0) = \tilde{M}_{00}\sqrt{1 + k^2 a_0^2}\, e^{i\delta_0^0} \\ M_{+-} &= \tilde{M}_{+-}(1 + ika_0) = \tilde{M}_{+-}\sqrt{1 + k^2 a_0^2}\, e^{i\delta_0^0} \end{aligned} \tag{6}$$

[3]Our definition of amplitudes coincide with one adopted in [17], and differs from accepted in [16, 18]

[4]In the isospin symmetry limit the scattering lengths a_I corresponding to $\pi\pi$ states with certain isospin $I = 0, 2$ are connected with elements of K-matrix by the relation $e^{2i\delta_I} = (1 + ika_I)/(1 - ika_I)$.

These equations are nothing else as the Fermi—Watson theorem for the pion-pion interaction in final states.
We see that in the real world, where $m_c \neq m_0$ the Fermi—Watson theorem in its original form is not valid and the two channel problem in this case demands the special consideration.

The considered picture can be generalized to higher orders [19]. Summing all subsequent loops of $\pi\pi$ scattering we obtain:

$$\begin{aligned} M_{00} &= \frac{\tilde{M}_{00}(1 - ik_2 a_{+-}) + ik_2 a_x \tilde{M}_{+-}}{D} \\ M_{+-} &= \frac{\tilde{M}_{+-}(1 - ik_1 a_{00}) + ik_1 a_x \tilde{M}_{00}}{D} \\ D &= (1 - ik_1 a_{00})(1 - ik_2 a_{+-}) + k_1 k_2 a_x^2 \end{aligned} \tag{7}$$

It is convenient to rewrite these equations in the form

$$\begin{aligned} M_{00} &= \frac{\tilde{M}_{00}\sqrt{1 + k_2^2(a_{+-} - \sqrt{2}a_x)^2}}{|D|} e^{i\delta_{00}} \\ M_{+-} &= \frac{\tilde{M}_{+-}\sqrt{1 + k_1^2(a_{00} - \frac{1}{\sqrt{2}}a_x)^2}}{|D|} e^{i\delta_{+-}} \\ \delta_{00} &= \arctan \frac{k_1 a_{00} + k_2 a_{+-}}{1 + k_1 k_2 (a_x^2 - a_{00}a_{+-})} - \arctan k_2(a_{+-} - \sqrt{2}a_x) \\ \delta_{+-} &= \arctan \frac{k_1 a_{00} + k_2 a_{+-}}{1 + k_1 k_2 (a_x^2 - a_{00}a_{+-})} - \arctan k_1(a_{00} - \frac{1}{\sqrt{2}}a_x) \end{aligned} \tag{8}$$

In the case of exact isospin symmetry $(m_c = m_0)$ these equations become:

$$M_{00} = \frac{\tilde{M}_{00}}{\sqrt{1 + k^2 a_0^2}} e^{i\delta_0^0} \tag{9}$$

which is nothing else than Fermi—Watson theorem.Let us note that unlike the common wisdom (9) generalized decay amplitudes (8) depend not only on a_0, but also on a_2.

The relations (8) are valid in the region above the charged pions production threshold $M = 2m_c$. To go under threshold one has to do in the expression for M_{00} the simple substitution $k_2 \to i\kappa$. As a result under threshold it reads:

$$\begin{aligned} M_{00} &= \frac{\tilde{M}_{00}[1 + \kappa(a_{+-} - \sqrt{2}a_x)]}{D} e^{i\delta_{00}} \\ \delta_{00} &= \arctan k_1 \left(a_{00} - \frac{\kappa a_x^2}{1 + \kappa a_{+-}} \right) \end{aligned} \tag{10}$$

Thus, as in the case of $K \to 3\pi$ [12] in the decay (2), the cusp phenomenon has to appear.

The expressions (8,10) completely solve the problem of generalization of Fermi—Watson theorem to the case of two coupled channels with different masses in the final state. To estimate the numerical difference of proposed approach from the usually accepted one let us consider the ratios

$$R_{00} = \frac{\tan\delta_{00}}{k_1 a_0}; \quad R_{+-} = \frac{\tan\delta_{+-}}{k_2 a_0}.$$

The dependence of R on dipion invariant mass is depicted on the fig.1. As one can see from the figure the isospin breaking effects drastically change phase shifts particularly in the vicinity of charged pions production threshold $M_{\pi\pi} = 2m_c$.

The obtained results are crucial for scattering length a_0 extraction from K_{e4} decays. Moreover,the developed approach allows one to extract from these decays also the scattering length a_2, the challenge which is absent in common approach. At present the high quality data on K_{e4} from NA48/2 experiment at CERN are in progress and their fitting by the expressions of present work can be very useful and can shed light not only on the true values of scattering lengths a_0, a_2, but also help to understand the limits and validity of proposed approach.

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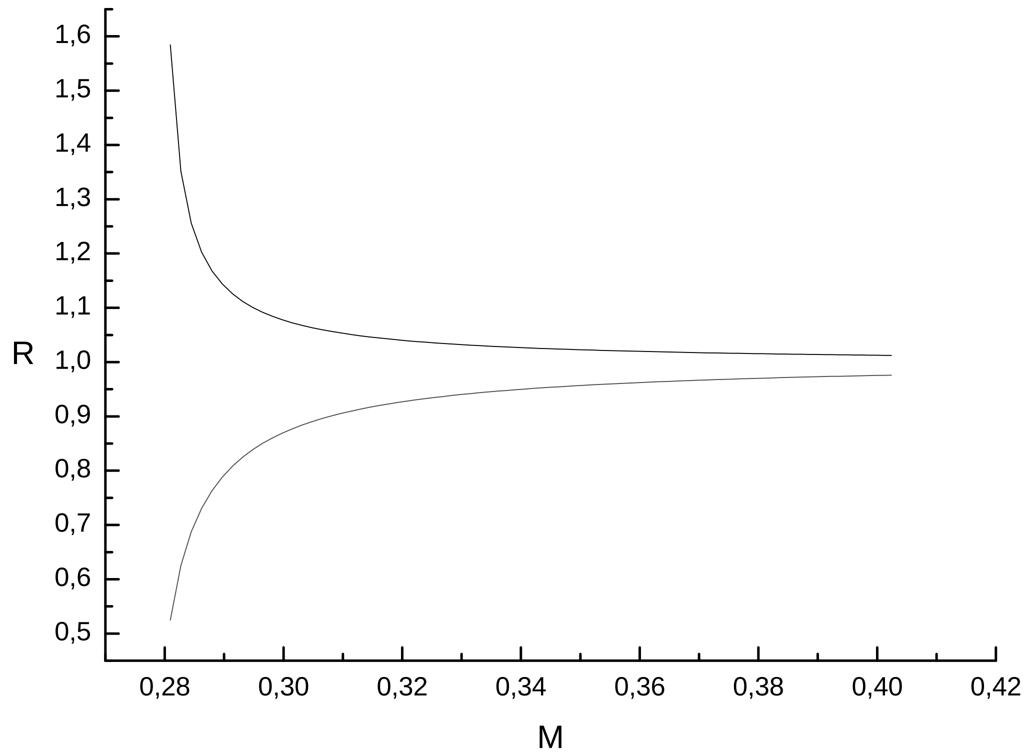

Figure 1: The dependence of phase shifts ratios for $\pi^+\pi^-$ (upper curve) and $\pi^0\pi^0$ (lower curve) on invariant mass of pion pair.