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TIMELIKE AND SPACELIKE CHARACTERISTICS OF INCLUSIVE PROCESSES

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We apply the nonperturbative a-expansion method to analyze spacelike and timelike characteristics of some inclusive processes. In particular, we show that the "experimental" D-functions corresponding to the e^+e^- annihilation into hadrons and the inclusive τ decay data are both in good agreement with results obtained in the framework of the method.

1 Introduction

Perturbation theory combined with the renormalization procedure is now a basic method for computations in quantum field theory. Perturbative series for many interesting models including realistic models are not convergent. In spite of that at small values of the coupling constant these series may be considered as asymptotic series and could provide a useful information. However, a specific feature of quantum field theory is that a sufficiently complete study of the structure of a quantum field model within the framework of perturbative approach is not enough, even in theories with a small coupling constant.

We use the method of constructing the so-called floating or variational series in quantum chromodynamics (see reviews [1, 2]). This approach is based on the idea of variational perturbation theory (VPT) [3], which in the case of QCD leads to a new small expansion parameter [4, 5]. Within this method, a quantity under consideration can be represented in the form of a series, which is different from the conventional perturbative expansion and can be used to go beyond the weak-coupling regime [6]. This allows one to deal with considerably lower energies than in the case of perturbation theory [7–9].

In the VPT a certain variational procedure is combined with the possibility of calculating corrections to the principal contribution which allows the possibility of probing the validity of the leading contribution and the region of applicability of the results obtained. At present, this idea finds many applications in the development of various approaches, which should enable us to go beyond perturbation theory. The a-expansion method, which we will use here, is the nonperturbative expansion technique suggested in [4, 5].

2 Variational perturbation theory in QCD

In the usual version of perturbation theory, the total action corresponding to a physical system is split into a free part and a part describing the interaction. The latter is treated as a perturbation, and the coupling constant entering into it is viewed as the small expansion parameter. As a rule, this treatment leads to asymptotic series which, albeit not "well behaved," nevertheless widely used in physics and allow useful information about the system in question to be extracted for weak coupling. As the interaction constant grows, the perturbation theory becomes worse and worse. The reason for this is understood: now the treatment of the interaction term as a perturbation of the free system is no longer adequate, since the physical system in question has properties far from those of a free system. In order to have a method of performing calculations in this case, it is necessary to split the total action in different way, such that the new "interaction term" can be treated as a perturbation not only when the coupling constant is small, but for a wider range of its value. Of course, here one must worry about whether this procedure, which is similar to ordinary perturbation theory, allows the possibility of calculating a main contribution and corrections.

How is it possible to seek a functional which can be used as a perturbation with more justification than the usual interaction term? One possibility, realized in variational perturbation theory (VPT) [3, 10–18]. is to probe the system by using a variational type functional to study a system's response to a change of the trial parameters. In the VPT, a given quantity can be approximated by constructing series, different from those of ordinary perturbation theory, which allow quantum systems to be studied not only in the weak-coupling region, but also far beyond it. The VPT method allows *ab initio* the determination of an algorithm for calculating corrections, therefore the effect of corrections on the main contribution can be studied. Moreover, the VPT series is not a strict construction specified once and for all. Special parameters characterizing the variational probe allow the convergent properties of the VPT expansion to be controlled. Series of this type whose convergence properties can be influenced by varying special parameters are referred to as variational or floating series.

In the case of QCD the VPT ideology leads to a new expansion parameter a, which according to [4,5] is connected with the initial coupling constant g in the Lagrangian by the relation

$$\lambda = \frac{g^2}{(4\pi)^2} = \frac{\alpha_s}{4\pi} = \frac{1}{C}\frac{a^2}{(1-a)^3}, \tag{2.1}$$

where a positive constant C plays a role of a variational parameter. As follows from (2.1), for any value of the coupling constant g, the expansion parameter a obeys the inequality

$$0 \leq a < 1. \tag{2.2}$$

While remaining within the range of applicability of the a-expansion, one can deal with low-energy processes where g is no longer small.

The parameter C is an auxiliary parameter of a variational type, which is associated with the use of a floating series. The original quantity, which is approximated by this expansion, does not depend on the parameter C. However, any finite approximation depends on it due to an inevitable truncation of the series. Here we will fix this parameter using some further information, coming from the potential approach to meson spectroscopy. An important feature of the a-expansion method is the fact that its use can ensure the reflecting the general principles of local quantum field theory correct analytic properties of the running expansion parameter. An analytic approach in QCD which combines the Q^2-analyticity and renormalization group resummation has been proposed in [19,20]. In [21,22] it has been argued that a requirement of the Källén–Lehmann type analyticity allows one to define a variational parameter the value of which is agreed well with nonperturbative data coming from the meson spectroscopy.

Consider the following approximations to the renormalization group β-function, the functions $\beta^{(3)}$ and $\beta^{(5)}$, which are obtained if one takes into consideration the terms $O(a^3)$ and $O(a^5)$ in the corresponding renormalization constant Z_λ. As has been shown in [5], C is determined by requiring that $-\beta^{(k)}(\lambda)/\lambda$ tends to 1 for sufficiently large λ, which gives $C_3 = 4.1$ and $C_5 = 21.5$. The increase of C_k with the order of the expansion is explained by the necessity to compensate for the higher order contributions. A similar phenomenon takes place also in zero- and one-dimensional models. The behavior of the functions $-\beta^{(k)}(\lambda)/\lambda$ gives evidence for the convergence of the results, in accordance with the phenomenon of induced convergence.[1] The behavior of the β-function at large value of the coupling constant, $-\beta^{(k)}(\lambda)/\lambda \simeq 1$, corresponds to the infrared singularity of the running coupling: $\alpha_s(Q^2) \sim Q^{-2}$ at small Q^2. In the potential quark model this Q^2 behavior is associated with the linear growth of the quark-antiquark potential.

The renormalization group β-function of the expansion parameter a is

$$\beta_a(a) = \mu^2 \frac{\partial a}{\partial \mu^2} = \frac{2\beta_0}{C} \frac{1}{F'(a)}, \tag{2.3}$$

where $\beta_0 = 11 - 2f/3$ is the one-loop coefficient of the β-function in the usual perturbative expansion, and f is the number of active quarks, has a zero at $a = 1$ that demonstrates the existence of the infrared fixed point of the expansion parameter and its freezing-like behavior in the infrared region. By finding the renormalization constants in the massless renormalization scheme with an accuracy $O(a^3)$, we find for the function $F(a)$

$$F^{(3)}(a) = \frac{2}{a^2} - \frac{6}{a} - 48 \ln a - \frac{18}{11}\frac{1}{1-a} + \frac{624}{121} \ln(1-a) + \frac{5184}{121} \ln\left(1 + \frac{9}{2}a\right). \tag{2.4}$$

By solving the renormalization group equation (2.3) one finds the momentum dependence of the running expansion parameter $a(Q^2)$ as a solution of the following transcendental equation

$$Q^2 = Q_0^2 \exp\left\{\frac{C}{2\beta_0}\left[F(a) - F(a_0)\right]\right\}. \tag{2.5}$$

For any values of Q^2, this equation has a unique solution $a = a(Q^2)$ in the interval between 0 and 1.

By working at $O(a^5)$ one obtains a more complicated result

$$F^{(5)}(a) = \frac{1}{5(5+3B)} \sum_{i=1}^{3} x_i \, J(a, b_i) \tag{2.6}$$

[1] It has been observed empirically [23, 24] that the results seem to converge if the variational parameter is chosen, in each order, according to some variational principle. This induced-convergence phenomenon is also discussed in [25].

with $B = \beta_1/(2C\beta_0)$, where the two-loop coefficient $\beta_1 = 102 - 3f/3$, and

$$J(a,b) = -\frac{2}{a^2b} - \frac{4}{ab^2} - \frac{12}{ab} - \frac{9}{(1-a)(1-b)} + \frac{4+12b+21b^2}{b^3}\ln a + \frac{30-21b}{(1-b)^2}\ln(1-a) - \frac{(2+b)^2}{b^3(1-b)^2}\ln(a-b) \qquad (2.7)$$

with

$$x_i = \frac{1}{(b_i - b_j)(b_i - b_k)}. \qquad (2.8)$$

Here indices $\{ijk\}$ are $\{123\}$ and cyclic permutations. The values of b_i are the roots of the equation $\psi(b_i) = 0$, where the function $\psi(a)$ is related to the β-function and is

$$\psi(a) = 1 + \frac{9}{2}a + 2(6+a)a^2 + 5(5+3B)a^3. \qquad (2.9)$$

It should be stressed, in contrast to many nonperturbative approaches, in the VPT the quantity under consideration from the very beginning is written in the form of a series which makes it possible to calculate the needed corrections. The VPT method thereby allows for the possibility of determining the degree to which the principal contribution found variationally using some variational principle adequately reflects the problem in question and determining the region of applicability of the results obtained.

The possibility of performing calculations is based on the fact that the VPT, like standard perturbation theory, uses only Gaussian functional quadratures. Here, of course, the VPT series possesses a different structure and, in addition, some of the Feynman rules are modified at the level of the propagators and vertices. The form of diagrams themselves does not change, which is very important technically. The diagrams contributing to the Nth order of the VPT expansion are of the same form as those contributing to the Nth order of ordinary perturbation theory. The variational parameters arising in the VPT method allow the convergence properties of the VPT series to be controlled.

3 Threshold singularities

In the threshold region one cannot truncate the perturbative series. Threshold singularities of the Feynman diagrams of the form $(\alpha/v)^n$ have to be summarized. This resummation, performed on the basis of the nonrelativistic Schrödinger equation with the Coulomb potential $V(r) = -\alpha/r$, leads to the Sommerfeld-Sakharov factor [26, 27] which is related to the wave function of the continuous spectrum at the origin. An expansion of this factor in a power series in the coupling constant α reproduces the threshold singularities of the Feynman diagrams in the form $(\alpha/v)^n$.

A description of quark-antiquark systems near threshold also requires such type of resummation. The S-factor appears in the parametrization of the imaginary part of the quark current correlator, the Drell ratio $R(s)$, which can be approximated in terms of the Bethe-Salpeter (BS) amplitude of two charged particles $\chi_{\rm BS}(x)$ at $x = 0$ [28]. The nonrelativistic replacement of this amplitude by the wave function which obeys the Schrödinger equation with the Coulomb potential, leads to the S-factor with $\alpha \to 4\alpha_s/3$, for QCD.

In the relativistic theory the nonrelativistic approximation needs to be modified. To use the S-factor within such a relativistic regime one usually uses the simple substitution $v_{\rm nr} \to v$ with $v = \sqrt{1 - 4m^2/s}$. However, the corresponding relativistic generalization of the S-factor is obviously not unique, for there are numerous ways of expressing the nonrelativistic velocity in terms of the relativistic energy $\sqrt{s}$. For a systematic relativistic analysis of quark-antiquark systems, it is essential from the very beginning to have a relativistic generalization of the S-factor. Here we will use a new form for this relativistic factor proposed in [29].

The starting point of the consideration performed in [29] is the quasipotential (QP) approach proposed by Logunov and Tavkhelidze [30], in the form suggested by Kadyshevsky [31]. To find an explicit form for the relativistic S-factor one uses a transformation of the QP equation from momentum space into relativistic configuration space [32]. The local Coulomb potential defined in this representation, as it has been demonstrated by Savrin and Skachkov [33], has a QCD-like behavior in momentum space. Solutions of the QP equation with the Coulomb potential have been investigated in [34].

The possibility of using the QP approach to define the relativistic S-factor is based on the fact that the BS amplitude, which parameterizes the physical quantity $R(s)$, is taken at $x = 0$, therefore, in particular, at relative time $\tau = 0$. The QP wave function is defined as the BS amplitude at $\tau = 0$, and the R-ratio can be expressed through the QP wave function

$\psi_{\rm QP}(\mathbf{p})$ by using the relation

$$\chi_{\rm BS}(x=0) = \int d\Omega_p \, \psi_{\rm QP}(\mathbf{p}), \tag{3.10}$$

where $d\Omega_p = (d\mathbf{p})/[(2\pi)^3 E_p]$ is the relativistic three-dimensional volume element in the Lobachevsky space realized on the hyperboloid $E_p^2 - \mathbf{p}^2 = m^2$.

The relativistic S-factor obtained in [29] has the form

$$S(\chi) = \frac{X(\chi)}{1 - \exp[-X(\chi)]}, \qquad X(\chi) = \frac{\pi\,\alpha}{\sinh\chi}, \tag{3.11}$$

where χ is the rapidity which related to s by $2m\cosh\chi = \sqrt{s}$.

4 Smeared quantities and D-functions

It is important to determine in QCD "simplest" objects which allow one to check direct consequences of the theory without using model assumptions in an essential manner. Comparison of theoretical results for these objects with experimental data allows us to justify transparently the validity of basic statements of the theory, and make some conclusions about completeness and efficiency of the theoretical methods used. Some single-argument functions which have a straightforward connection with experimentally measured quantities can play the role of these objects. A theoretical description of inclusive processes can be expressed in terms of functions of this sort. Let us mention among them moments $\mathrm{M}_n(Q^2)$ of the structure functions in inelastic lepton-hadron scattering and the hadronic correlator $\Pi(s)$ (or the corresponding Adler D-function), which appear in the processes of e^+e^- annihilation into hadrons or the inclusive decay of the τ lepton.

The cross-section for e^+e^- annihilation into hadrons or its ratio to the leptonic cross-section, $R(s)$, have a resonance structure that is difficult to describe, at the present stage of a theory, without model considerations. Moreover, the basic method of calculations in quantum field theory, perturbation theory, becomes ill-defined due to the so-called threshold singularities. These problems can, in principle, be avoided if one considers a "smeared" quantity [35]

$$R_\Delta(s) = \frac{\Delta}{\pi}\int_0^\infty ds' \frac{R(s')}{(s-s')^2 + \Delta^2}. \tag{4.12}$$

However, a straightforward usage of conventional perturbation theory to calculate R_Δ is not possible. Indeed, if the QCD contribution to the function $R(s)$ in Eq. (4.12) is, as usual, parametrized by the perturbative running coupling that has unphysical singularities, it is difficult to define the integral on the right-hand side. Moreover, the standard method of the renormalization group gives a Q^2-evolution law of the running coupling in the Euclidean region, and there is the question of how to parametrize a quantity, for example, $R(s)$, defined for timelike momentum transfers [36, 37]. To perform this procedure self-consistently, it is important to maintain correct analytic properties of the hadronic correlator which are violated in perturbation theory. Within the nonperturbative a-expansion it is possible to maintain such analytic properties and to self-consistently determine the effective coupling in the Minkowskian region [9]. Note, that the analytic approach to QCD [19, 20, 38] also leads to a well-defined procedure of analytic continuation [39, 40].

Another function, which characterizes the process of e^+e^- annihilation into hadrons and can be extracted from experimental data, is the Adler function

$$D(Q^2) = -Q^2 \frac{d\Pi}{dQ^2} = Q^2 \int_0^\infty ds \frac{R(s)}{(s+Q^2)^2} \,. \tag{4.13}$$

The D-function defined in the Euclidean region for a positive momentum Q^2 is a smooth function, and thus it is not necessary to apply any "smearing" procedure in order to be able to compare theoretical results with experimental data. An "experimental" curve for this function which is related to the process of e^+e^- annihilation into hadrons has been obtained in [41]. We will also consider the "light" D-function corresponding to the τ decay data.

For massless quarks, one can write down the timelike (Minkowskian) quantity $R(s)$ in the form

$$R(s) = 3 \sum_f q_f^2 \left[1 + r_0 \lambda_s^{\text{eff}}(s)\right] \,, \tag{4.14}$$

where the sum runs over quark flavors, q_f are quark charges and r_0 is the first perturbative coefficients that is renormalization-scheme independent. This expression includes the effective coupling defined in the Minkowskian region or, as we will say, in the s-channel, which is reflected in the subscript s. It should be stressed that, as it has been argued from general principles, the behavior of the effective couplings in the spacelike and the timelike domains cannot be symmetric [42].

Within the a-expansion method the s-channel running coupling can be written as

$$\lambda_s^{(i)}(s) = \frac{1}{2\pi \mathrm{i}} \frac{1}{2\beta_0} \left[\phi^{(i)}(a_+) - \phi^{(i)}(a_-) \right], \tag{4.15}$$

where $a_\pm$ obey the equation

$$F(a_\pm) = F(a_0) + \frac{2\beta_0}{C} \left(\ln \frac{s}{Q_0^2} \pm \mathrm{i}\,\pi \right). \tag{4.16}$$

At the level $O(a^3)$, the function $\phi(a)$ has the form

$$\phi^{(3)}(a) = -4\ln a - \frac{72}{11}\frac{1}{1-a} + \frac{318}{121}\ln(1-a) + \frac{256}{363}\ln\left(1+\frac{9}{2}a\right). \tag{4.17}$$

Similarly, a more complicated expression for the $O(a^5)$ level, we will use, can be derived.

The convenient way to incorporate quark mass effects is to use an approximate expression [35]

$$\tilde{R}(s) = 3\sum_f q_f^2\, \Theta(s-4m_f^2)\, \mathcal{R}_f(s)\,, \mathcal{R}_f(s) = T(v_f)\ [1\ +\ g(v_f) r_f(s)] \tag{4.18}$$

where

$$T(v) = v\frac{3-v^2}{2}\,,\ \ g(v) = \frac{4\pi}{3}\left[\frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right],$$

$$v_f = \sqrt{1 - \frac{4m_f^2}{s}}\,. \tag{4.19}$$

The quantity $r_f(s)$ is defined by the s-channel effective coupling $\lambda_s^{\mathrm{eff}}(s)$. The smeared quantity (4.12) and the D-function (4.13) can be calculated by using (4.18) in the corresponding integrands. For MS-like renormalization schemes, one has to consider some matching procedure. To perform this matching procedure, we can require the s-channel running coupling and its derivative to be continuous functions in the vicinity of the threshold [9, 43].

To take into account the threshold resummation factor, we, following [44], modify the expression (4.18) by using the ansatz

$$\mathcal{R}(s) = T(v)\left[S(\chi) - \frac{1}{2}X(\chi) + g(v)\, r(s)\right]. \tag{4.20}$$

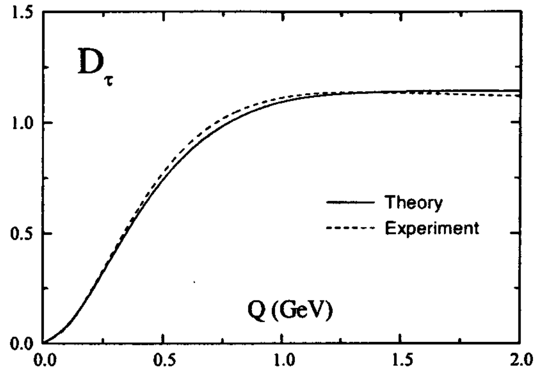

Fig. 1: *The plot of the "light" D-function. The experimental curve corresponding to ALEPH data is taken from [47].*

As the mass $m \to 0$, this expression leads to Eq. (4.14). We will use Eq. (4.20) in our analysis.

The non-strange vector contribution for the inclusive τ-lepton decay can be described in analogy with the e^+e^- annihilation into hadrons process. Using the theoretical expression for R_τ-ratio [45]

$$R_\tau^V = R^{(0)} \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \mathcal{R}(s)\,, \tag{4.21}$$

where $R^{(0)}$ corresponds to the parton level, and measured value $R_\tau^V = 1.775 \pm 0.017$ [46], as an input, we extracted the value of parameter a_0 in Eq. (4.16) at the τ mass scale, $Q_0 = M_\tau$.

The "light" D-function with three active quarks is shown in Fig. 1, where we draw the experimental curve, as dashed line, which was extracted in [47] from the ALEPH data, and our theoretical result (solid line) obtained by using the following effective masses of light quarks $m_u = m_d = 260\,\text{MeV}$ and $m_s = 400\,\text{MeV}$. Virtually, the same values were used in [38, 48–50] to describe the region of low lying mesons. These values are close to the constituent quark masses and incorporate some nonperturbative effects. The shape of the infrared tail of the D-function is sensitive to the value of these masses.

In Fig. 2, we have presented the smeared function $R_\Delta(s)$ for $\Delta = 3\ \text{GeV}^2$. We use the same masses for the light quarks as before and

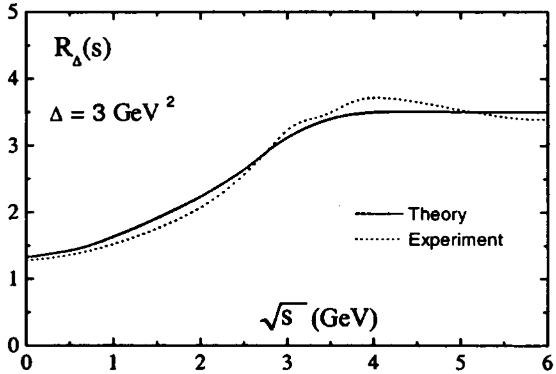

Fig. 2: *The smeared quantity $R_\Delta(s)$ for $\Delta = 3$ GeV^2. The solid curve is our result. The smeared experimental curve is taken from [51, 52].*

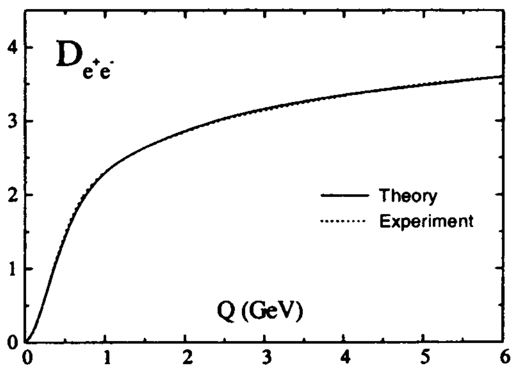

Fig. 3: *The D-function for the process of e^+e^- annihilation into hadrons. The solid curve is our result for five active quarks. The experimental curve is taken from [41].*

the following masses for heavy quarks $m_c = 1.3\,\text{GeV}$ and $m_b = 4.7\,\text{GeV}$. The smeared $R_\Delta(s)$ function for $\Delta \simeq 1$–$3\ \text{GeV}^2$ is less sensitive to the value of light quark masses as compared with the infrared tail of the D-function. The result for the D-function of the e^+e^--annihilation process which includes both the light and heavy quarks is plotted in Fig. 3.

5 Conclusions

The method we used here is the non-perturbative approach based on an idea of variational perturbation theory which combines an optimization procedure of variational type with a regular method of calculating corrections. In the case of QCD the non-perturbative expansion parameter, a, obeys an equation whose solutions are always smaller than unity for any value of the original coupling constant. An important feature of this approach is the fact that for sufficiently small value of the running coupling the a-expansion reproduces the standard perturbative expansion, and, therefore, the perturbative high-energy physics is preserved. In moving to low energies, where ordinary perturbation theory breaks down ($\bar{\alpha}_s \simeq 1$), the parameter a remains small and we still stay within the region of applicability of the a-expansion method.

To summarize the threshold singularities we have used the new relativistic form of the threshold resummation factor. This relativistic factor could have a significant impact in interpreting strong-interaction physics. In many physically interesting cases, $R(s)$ occurs as a factor in an integrand, as, for example, for the case of inclusive τ decay, for smearing quantities, and for the Adler D-function. Here the behavior of S at intermediate values of v becomes important. In the nonrelativistic limit, $v \ll 1$, the relativistic S-factor reproduces the nonrelativistic result. In the ultrarelativistic limit, the bound state spectrum vanishes as $m \to 0$ because the particle mass is the only dimensional parameter. This feature reflects an essential difference between potential models and quantum field theory, where an additional dimensional parameter appears. One can conclude that within a potential model, the S-factor which corresponds to the continuous spectrum should go to unity in the limit $m \to 0$. Thus, the relativistic resummation factor S obtained here reproduces both the expected nonrelativistic and ultrarelativistic limits and corresponds to a QCD-like Coulomb potential.

The Minkowskian and Euclidean physical quantities obtained from the

e^+e^- annihilation and τ decay experimental data that have been considered here are the smeared "timelike" function $R_\Delta(s)$ and the "spacelike" D-functions. The experimental D-function turned out to be a smooth function without traces of the resonance structure of $R(s)$. One can expect that this object more precisely reflects the quark-hadron duality and is convenient for comparing theoretical predictions with experimental data. Note here that any finite order of the operator product expansion fails to describe the infrared tail of the D-function. Within the framework of nonperturbative a-expansion technique with the relativistic threshold factor, we have obtained a good agreement between our results and the experimental data down to the lowest energy scale both for Minkowskian and Euclidean quantities.

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