Towards light scalar meson structure in terms of quark and gluon degrees of freedom

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The origin of the lightest scalar mesons is studied in the framework of instanton liquid model (ILM) of the QCD vacuum. The impact of vacuum excitations on the σ-meson features is analyzed in detail. In particular, it is noticed that the changes produced in the scalar sector may unexpectedly become quite considerable in spite of insignificant values of corrections to the dynamical quark masses and then the medley of σ-meson and those excitations may reveal itself as broad resonance states of vitally different masses.

Nowadays well-known theoretical results of existing scalar mesons, the non-zero vacuum expectation value of which is strongly argued by chiral symmetry breaking, find earnest experimental support. It comes from studying the low energy S-wave of π–π-scattering where the presence of solitary low mass scalar resonance looks inevitable [1] and the radiative ϕ-meson and heavy quarkonia decays as seen from the view point of chiral shielding idea [2]. Thus, the present situation in the subject entirely justifies the theoretical expectation that the physics of scalar mesons is driven by the Goldstone dynamics [3].

However, it is still difficult to understand the properties of scalar mesons (resonances) in terms of the QCD basic fields, in particular, the quark and gluon origin of light σ-meson ($f_0(400-1000$ MeV)) and broad resonance $f_0(1000-1600$ MeV) [4]. In a sense, the current status of the lightest scalars could be summarized by the strange assertion that people know where the scalars are but do not know what they are. Meanwhile, the solution of this still pending problem can be found by studying the QCD vacuum structure [5] and searching putative quark-gluon plasma state and in recent time the critical point in ultra-relativistic heavy ion collisions [6]. An idea to exploit σ-meson (precisely, its coupling to photons) as a key guide to exploring the existence and features of mixed phase of strongly interacting matter created in nuclei-nuclei collisions at lower energies [7] requires such an insight as well. Attempts to find an explanation of splitting two lowest scalar mesons in the framework of rather sophisticated models mixing the quark-antiquark states with the glueballs (see, for example [8, 9]) are not fully successful and the results are ambiguous.

In the present paper we consider the origin of lowest scalar mesons in the instanton liquid model (ILM) of the QCD vacuum [10] mixing its phonon-like excitations [11] with the scalar mesons treated in a standard way as bound quark-antiquark states. The effective Lagrangian of phonon-like excitations is similar to the dilaton one [8] and describes the state with the quantum numbers $J^{PC} = 0^{++}$ (J = total angular momentum, P = parity, C = charge-conjugation eigenvalue) of glueball. One specific feature of this Lagrangian is the form of its kinetic term and rather strong interactions with quarks (resulting in the strong mixing with the σ-meson field [12]).

Avoiding the technical details of constructing this effective Lagrangian we mention briefly here the major steps necessary for the developed approach. In particular, it is grounded on the hypothesis the vacuum field configurations are stabilized at the certain characteristic scale and their action develops a well-defined minimum at the point of average configuration size. The generating functional in ILM

$$Z = \int D[\mathcal{A}]\; e^{-S(\mathcal{A})} ,$$

where $S(\mathcal{A})$ is the Yang-Mills action, is supposed to be saturated by instanton superposition

$$\mathcal{A}^a_\mu(x) = \sum_{i=1}^{N} A^a_\mu(x;\gamma_i) . \tag{1}$$

Here $A^a_\mu(x;\gamma_i)$ is the (anti-)instanton field in the singular gauge

$$A^a_\mu(x) = \frac{2}{g}\; \omega^{ab}\bar\eta_{b\mu\nu}\; \frac{\rho^2}{y^2+\rho^2}\, \frac{y_\nu}{y^2} , \quad y = x - z , \tag{2}$$

where $\gamma_i = (\rho_i, z_i, \omega_i)$ are the parameters characterizing the i-th (anti-)instanton of size ρ with a matrix of colour orientation ω and with coordinates of its center position at z and g is the strong coupling constant. For anti-instanton the 't Hooft tensor should be substituted according to $\bar\eta \to \eta$. We do not discuss here the mechanism of instanton ensemble stabilization at the scale of $\bar\rho$ (see, for example [13]) but focus on extracting some phenomenological results in the context of our interest.

The form of action with peculiar minimum for the saturating configuration equilibrated makes an existence of "oscillations" (one should keep in mind we are working in the Euclidean space) of this configuration around the very natural size $\bar\rho$. In principle, such a description should be done by the corresponding Green function. However, calculating it is not simple and eventually impracticable because the Green function is found to be singular [14]. It was noticed in Ref.[15] that if one is interested in the major terms of a generating functional exponent then less information on interrelation between the saturating configuration $\mathcal{A}$ and field of "oscillations" $\dfrac{\partial\rho}{\partial x}$ at the characteristic scale $\bar\rho$ is necessary to calculate the respective kinetic term of the effective Lagrangian. The instanton ensemble excitations generated by a certain impact might naturally be described by more general saturating configurations of the form of Eq.(2) if the instanton size and its colour space orientation are varied ($\rho \to R(x,z)$, $\omega^{ab} \to \Omega^{ab}(x,z)$) and the deformation fields could be defined by dealing with minimal action requirement $\delta S = 0$. As a result it leads to a more accurate (than plain superposition Eq.(1)) solution and allows to determine the interrelation looked for. In fact, it can be carried out directly since the deformations are defined by the multipole expansion done in the center point of instanton

$$\begin{aligned} R_{in}(x,z) &= \rho + c_\mu\, y_\mu + c_{\mu\nu}\, y_\mu\, y_\nu + \dots , \qquad |y| \le L \\ R_{out}(x,z) &= \rho + d_\mu\, \frac{y_\mu}{y^2} + d_{\mu\nu}\, \frac{y_\mu}{y^2}\, \frac{y_\nu}{y^2} + \dots , \qquad |y| > L , \end{aligned} \tag{3}$$

(similar expansion should be done for instanton orientation in the "isotopic" space $\Omega(x,z)$), here L is a parameter which fixes a sphere radius where the increasing multipole expansion with distance increase changes its behaviour for the decreasing one because of the requirement of deformation regularity. The coefficient c_μ in Eq.(3) just corresponds to the function $\dfrac{\partial\rho}{\partial z_\mu}$ which we are interested in (by the way, for the sample of deformation field Eq.(3) it is valid $\dfrac{\partial R}{\partial x_\mu} \simeq -\dfrac{\partial\rho}{\partial z_\mu}$ and the interrelation between two fields is defined by the solution of Eq.(2) with $R(x,z)$ included).

Now calculating an action of this crumpled (as was called in Ref.[11]) configuration we are able to determine the kinetic term of effective Lagrangian at the characteristic scale $\bar\rho$ (certainly, we can not profess a higher precision in this quasi-classical approximation as it was mentioned before) in the form

$$S_{kin} = \int dx\; \frac{1}{4}\; G^a_{\mu\nu}(A)G^a_{\mu\nu}(A) - \beta = \frac{\kappa}{2}\; (\delta_\mu\rho)^2 , \quad \kappa = \frac{9}{10}\,\beta , \tag{4}$$

here $\beta = 8\pi^2/g^2$ is the single (anti-)instanton action at the $\bar\rho$ scale which in its general form can be presented as

$$s(\rho) = \beta(\rho) + 5\ln(\Lambda\rho) - \ln \widetilde{\beta}^{2N_c} + \beta\xi^2\ n\bar\rho^2\rho^2\ , \tag{5}$$

with the function $\beta(\rho) = -\ln C_{N_c} - b\ln(\Lambda\rho)$, $\Lambda = \Lambda_{\overline{MS}} = 0.92\Lambda_{P.V.}$, and the constant C_{N_c} depending on the renormalization scheme $C_{N_c} \approx \frac{4.66\ \exp(-1.68N_c)}{\pi^2(N_c-1)!(N_c-2)!}$, $\nu = \frac{b-4}{2}$, $b = \frac{11\ N_c - 2\ N_f}{3}$, N_f is the number of flavours, N_c is the number of colours and $\beta = \beta(\bar\rho)$, $\widetilde{\beta} = \beta + \ln C_{N_c}$ are the magnitudes of $\beta(\rho)$ function at the fixed value of $\bar\rho$, ξ is a constant characterizing the repulsive power of pseudoparticles $\xi^2 = \frac{27}{4}\frac{N_c}{N_c^2-1}\pi^2$ and n is the instanton liquid density. Holding the terms of second order in the small deviations from the point of action minimum $\left.\frac{ds(\rho)}{d\rho}\right|_{\rho_c} = 0$ we receive the approximate result

$$s(\rho) \simeq s(\bar\rho) + \frac{s^{(2)}(\bar\rho)}{2}\ \widetilde{\varphi}^2, \tag{6}$$

where $s^{(2)}(\bar\rho) \simeq \left.\frac{d^2 s(\rho)}{d\rho^2}\right|_{\rho_c} = \frac{4\nu}{\bar\rho^2}$, and the scalar field $\widetilde{\varphi} = \delta\rho = \rho - \rho_c \simeq \rho - \bar\rho$ just realizes the field of deviations from the equillibrium value $\rho_c = \bar\rho\ \left(1 - \frac{1}{2\nu}\right)^{1/2} \simeq \bar\rho$. Finally the deformation field could be described by the following effective Lagrangian density [11],[15]

$$\mathcal{L}_\varphi = \frac{n\kappa}{2}\left\{\ \left(\frac{\partial\widetilde{\varphi}}{\partial z}\right)^2 + M_\varphi^2\widetilde{\varphi}^2\right\}\ , \tag{7}$$

with the mass gap of phonon-like excitations as $M_\varphi^2 = \frac{s^{(2)}(\bar\rho)}{\kappa} = \frac{4\nu}{\kappa\bar\rho^2}$ (the coefficient $\kappa = 0.9\beta$ generates the scale of order 1 GeV with $\Lambda \simeq 280$ MeV).

Then the quark determinant $\mathcal{Z}_\psi$ for the stochastic ensemble of pseudo-particles looks like

$$\mathcal{Z}_\psi\ \simeq\ \int D\psi^\dagger D\psi\ \langle\langle\ e^{S(\psi,\psi^\dagger,A)}\ \rangle\rangle_A\ ,$$

where $S(\psi,\psi^\dagger,A)$ is the action of QCD with massless quarks which should describe the spontaneous breaking of chiral symmetry and generation of dynamical quark mass of order 300 MeV [16]. Eventually the quark generating functional (after absorbing the variations of (anti-)instanton average size) takes the following form (for $N_f = 2$) of the integral over saddle point parameters [12]

$$\begin{aligned}
\mathcal{Z}_\psi &= \int d\lambda\ DM\ D\psi^\dagger D\psi\ e^{-\mathcal{L}_{\psi,\widetilde{\varphi}}}\ ,\\
\mathcal{L}_{\psi,\widetilde{\varphi}} &= -N\ln\frac{N}{\lambda V} + 2\int dx\ (N_f-1)\ \lambda^{-\frac{1}{N_f-1}}\ (Det M)^{\frac{1}{N_f-1}} - \\
&- \int\frac{dkdl}{(2\pi)^8}\ \psi_f^\dagger(k)\left[(2\pi)^4\delta(k-l)\left(-\hat{k} + i\ m_{fg}\ v(k,k)\right) + i\ m_{fg}\ u(k,l)\ \widetilde{\varphi}(k-l)\right]\psi_g(l)\ ,
\end{aligned} \tag{8}$$

where N is the total number of pseudo-particles in the volume V $(n = N/V)$, M is the $N_f \times N_f$-matrix of scalar boson fields in the flavour space and f,g are the flavour indecies. The vertex functions $v(k,k)$ and $u(k,l)$ are defined by the zero-modes (those are the solutions of the Dirac equation in the field of (anti-)instanton of zero energy) and have the following forms

$$v(k,k) = G^2(k), \qquad u(k,l) = \frac{d\ v(k,l)}{d\rho} = G(k)G'(l) + G'(k)G(l)\ ,$$

moreover $G(k,\rho) = 2\pi\rho F(k\rho/2)$, $G'(k,\rho) = \frac{dG(k,\rho)}{d\rho}$, where

$$F(x) = 2x\ [I_0(x)K_1(x) - I_1(x)K_0(x)] - 2\ I_1(x)K_1(x)\ ,$$

and $I_i,\ K_i\ (i=0,1)$ are the modified Bessel functions.

After executing the bosonisation procedure the Lagrangian density takes the following form

$$\mathcal{L} = \mathcal{L}_{\widetilde{\varphi}} + \mathcal{L}_{\psi,\widetilde{\varphi}}\ , \tag{9}$$

which describes the quarks and phonon-like excitations of instanton liquid. The Lagrangian parameters are fixed by saddle point available as

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0\ \ ,\qquad \frac{\partial \mathcal{L}}{\partial M} = 0\ .$$

In the particular case of negligible impact of field $\widetilde{\varphi}$, saddle point values of parameters are fixed by the requirements

$$\lambda^{-\frac{1}{N_f-1}} \prod_{i=1}^{N_f} m_i^{\frac{1}{N_f-1}} = \frac{N}{2V}\ ,$$

$$4N_c \int \frac{dk}{(2\pi)^4}\ \frac{m_f^2\ v^2(k)}{k^2+m_f^2\ v^2(k)} = \frac{N}{V}\ \ , \tag{10}$$

here $v(k)=v(k,k)$.

As it was mentioned above the Lagrangian density (9) is quite similar to the dilaton one. However, there are, at least, two substantial distinctions. First, it is the presence of factor $n\kappa$ in the kinetic term. This factor is analogous to the factor F_π^2 of chiral π-meson Lagrangian density $\mathcal{L}_\pi = \frac{F_\pi^2}{2}\,\partial_\mu\widetilde{\pi}\partial_\mu\widetilde{\pi}$, and sets up the scale of phonon-like excitations. Average instanton size $\bar{\rho}$ ($\widetilde{\varphi}=\bar{\rho}\varphi$) is quite relevant dimensional unit for phonon-like field and, hence, the "correct" dimension for the factor in kinetic term could be

$$F_\varphi^2 = n\kappa\bar{\rho}^2\ .$$

The second distinction concerns the interrelation of phonon-like field and quarks which does not disappear in the chiral limit and is defined by the vertex function $u(k,l)$.

Then a nontrivial solution of saddle point equation Eq.(10) fixes the dynamical quark mass $M(k)=m_f v(k)$ together with the quark condensate

$$-i\langle\psi^\dagger\psi\rangle = -4N_c\ \int \frac{dk}{(2\pi)^4}\frac{M(k)}{k^2+M^2(k)}\ .$$

The excitations of quark condensate with the respective quantum numbers are seen as σ- and π-mesons (in this paper we are dealing with $N_f=2$ only) and

$$M = m\ (1+\widetilde{\sigma})\ e^{i\widetilde{\pi}^a\tau^a}\ ,$$

where τ^a are the Pauli matrices. The corresponding correlation functions R_σ and R_π are given, as known, in the form [16]

$$R_\pi(p) = 2N_fN_c\int \frac{dk_1}{(2\pi)^4}\ \frac{M_1^2}{k_1^2+M_1^2} - 2N_fN_c\int \frac{dk_1}{(2\pi)^4}\ \frac{((k_1k_2)+M_1M_2)\ M_1M_2}{(k_1^2+M_1^2)(k_2^2+M_2^2)}\ \ , \tag{11}$$

$$R_\sigma(p) = n\bar{\rho}^4 - 2N_fN_c\int \frac{dk_1}{(2\pi)^4}\ \frac{((k_1k_2)-M_1M_2)\ M_1M_2}{(k_1^2+M_1^2)(k_2^2+M_2^2)}\ \ , \tag{12}$$

with $k_2=k_1+p$, $M_1=M(k_1)$, $M_2=M(k_2)$. Expanding the integrals around small values of momentum p as

$$R_\pi(p) = \beta_\pi\ p^2\ ,\quad R_\sigma(p) = \alpha_\sigma + \beta_\sigma\ p^2\ ,$$

we are able to calculate the following decay constants

$$\begin{aligned}
\beta_\pi &= \frac{F_\pi^2}{2} = N_c N_f \int \frac{dk}{(2\pi)^4}\; \frac{M^2(k) - \frac{k}{2}\, M'(k)M(k) + \frac{k^2}{4}\, (M'(k))^2}{(k^2+M^2(k))^2}\;, \\
\beta_\sigma &= \frac{F_\sigma^2}{2} = \beta_\pi + 4N_c N_f \int \frac{dk}{(2\pi)^4}\; \left[\frac{2M^2(k)}{(k^2+M^2(k))^2}\, \Delta_1 - \frac{M^4(k)}{(k^2+M^2(k))^4}\, \Delta_2\right]\;, \\
\Delta_1 &= \frac{1}{16}\, \left(\frac{3}{k}\, M(k)M'(k) - M'(k)M'(k) + M(k)M''(k)\right)\;, \\
\Delta_2 &= \frac{k^2+M^2(k)}{2}\, \left(1+\frac{M(k)M'(k)}{k}\right) - \frac{k^2}{4}\, \left(1+\frac{M(k)M'(k)}{k}\right)^2 + \\
&+ \frac{k^2+M^2(k)}{8}\, \left(M'(k)M'(k) + M(k)M''(k) - \frac{M(k)M'(k)}{k}\right)\;,
\end{aligned}$$

(here the prime denotes the derivative calculation) and the σ-meson mass as well

$$\begin{aligned}
M_\sigma^2 &= \frac{\alpha_\sigma}{\beta_\sigma}\;, \\
\alpha_\sigma &= n\bar{\rho}^4 - 2N_f N_c \int \frac{dk}{(2\pi)^4}\; \frac{(k^2-M^2(k))\; M^2(k)}{(k^2+M^2(k))^2}\;.
\end{aligned}$$

The width Γ_σ of σ-meson decay into $\pi\pi$ is defined by the set of graphs α—δ (see Fig.1) for the anharmonic term $\sim \tilde{\sigma}\tilde{\pi}^2$ of effective Lagrangian

$$\mathcal{L}_{int} = \int \frac{dmdl}{(2\pi)^8} \int \frac{dk}{(2\pi)^4}\; W_\sigma(k,m,l)\; \tilde{\sigma}(m)\; \tilde{\pi}^a(l)\; \tilde{\pi}^a(m-l)\;,$$

$$\begin{aligned}
\frac{W_\sigma(k,m,l)}{4N_c} &= -\frac{M^2(k)}{k^2+M^2(k)} - \\
&- \frac{k(k-m) - M(k)M(k-m)}{(k^2+M^2(k))((k-m)^2+M^2(k-m))}\; M(k)M(k-m) + \\
&+ 2\, \frac{k(k-l) + M(k)M(k-l)}{(k^2+M^2(k))((k-l)^2+M^2(k-l))}\; M(k)M(k-l) + \\
&+ 2\, \frac{k(k-m)M(k-l) - k(k-l)M(k-m) - ((k-m)(k-l)M(k) - M(k)M(k-m)M(k-l)}{(k^2+M^2(k))((k-m)^2+M^2(k-m))((k-l)^2+M^2(k-l))} \times \\
&\times M(k)M(k-m)M(k-l)\;.
\end{aligned}$$

(four terms of this expression are in explicit correspondence with the four graphs of Fig.1).

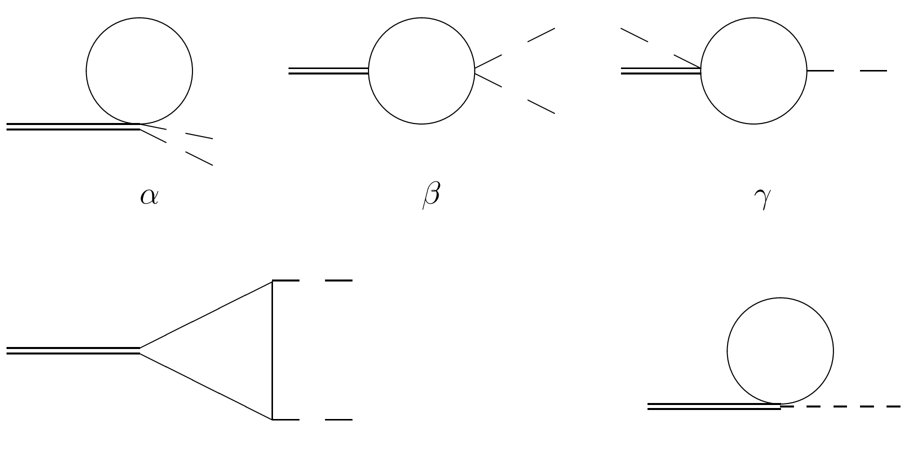

Figure 1: The single line corresponds to the quark contribution, the double one corresponds to σ-meson, the short dashed line shows the phonon-like field and the long dashed one shows π-meson.

Then for the decay width of σ-meson in its rest frame we find that after integrating the function $W_\sigma(k,m,l)/F_\sigma/F_\pi^2$ over the loop at fixed momenta of π-meson [17] we have

$$\Gamma_\sigma = \frac{3}{8\pi}\frac{W_\sigma^2}{M_\sigma}\left(1-\frac{4M_\pi^2}{M_\sigma^2}\right)^{1/2} ,$$

here W_σ denotes the integral over anharmonic contribution of type $W_\sigma\ \sigma\pi^2$. At zero outgoing momenta the anharmonic contribution disappears $W_\sigma(k,0,0) = 0$ because in the chiral limit π-meson fields may appear in the combination with the derivative term $(\partial_\mu\pi)^2$ only. As known, this fact, in some extent, results in the small σ-meson width.

In the meantime, if the phonon-like excitations are not taken into account π and σ-mesons are defined by the following fundamental characteristics

$$F_\pi \simeq 121 \text{ MeV} ,\quad F_\sigma \simeq 96 \text{ MeV} ,$$
$$M_\sigma \simeq 550 \text{ MeV} ,\quad \Gamma_\sigma \simeq 2.8 \text{ MeV} ,$$

and π-meson mass can be extracted from Gell-Mann-Oakes-Renner (GMOR) relation.The saddle-point parameter m taken as dimensionless quantity is equal to $m = 4.817{\cdot}10^{-3}$. Then we have for the instanton liquid density $n/\Lambda^4 = 1.03$, for the average pseudo-particle size $\bar\rho\Lambda = 0.28$ and eventually for single instanton action at this characteristic scale $\beta(\bar\rho) = 19$. The dynamical quark mass and quark condensate read as

$$M \simeq 385 \text{ MeV} ,\quad -i\langle\psi^\dagger\psi\rangle \simeq -(381)^3 \text{ MeV}^3 ,$$

at $\Lambda = 280$ MeV. Surprisingly, the small width of σ-meson is a result of interference of all terms contributing. In particular, in dimensionless units the corresponding contributions are as follows

$$\alpha = -1.199{\cdot}10^{-2} ,\ \beta = -7.137{\cdot}10^{-3} ,\ \gamma = 2.066{\cdot}10^{-2} ,\ \delta = -1.695{\cdot}10^{-3} ,\ \alpha+\beta+\gamma+\delta = -1.648{\cdot}10^{-4} .$$

Actually, many quantitative estimates are based on the calculations of δ graph only and it results in the magnitude of width Γ_σ which is declared to be a few hundred MeV. Assuredly, there is an unpleasant drawback of this calculation. It comes from the opposite signs of contributions of α and γ diagrams because the obvious compensation produces rather small value which is poorly controlled by one-loop calculations. Unfortunately, for the time being two-loop calculations are hardly realized technically.

In order to include the changes coming from phonon-like fields entering the game, the perturbation scheme was proposed in Ref.[12] in the tadpole approximation. Actually, it becomes possible, because of the quark condensate presence in the following sum

$$\psi^\dagger\psi\ \varphi = \langle\psi^\dagger\psi\rangle\ \varphi + (\psi^\dagger\psi - \langle\psi^\dagger\psi\rangle)\ \varphi ,$$

to keep the first term only in the respective inserts into quark Green functions and vertices. Then the dynamical quark mass is defined not only by the vertex v but the tadpole contribution with vertex u as well (see Eq.(8)). Amazingly, the dependence of diagram contributions on the coefficient κ is cancelled in this approximation. Generally, there is hardly any impact on the instanton liquid (it is negligible), average instanton size and parameter $\beta(\bar\rho)$ do not change but instanton liquid density slightly increases $n/\Lambda^4 = 1.11$. The saddle point parameter is getting smaller $m = 3.481{\cdot}10^{-3}$ which results in the dynamical quark mass and quark condensate decreasing (see Fig. 2).

$$M \simeq 324 \text{ MeV} ,\quad -i\langle\psi^\dagger\psi\rangle \simeq -(343)^3 \text{ MeV}^3 .$$

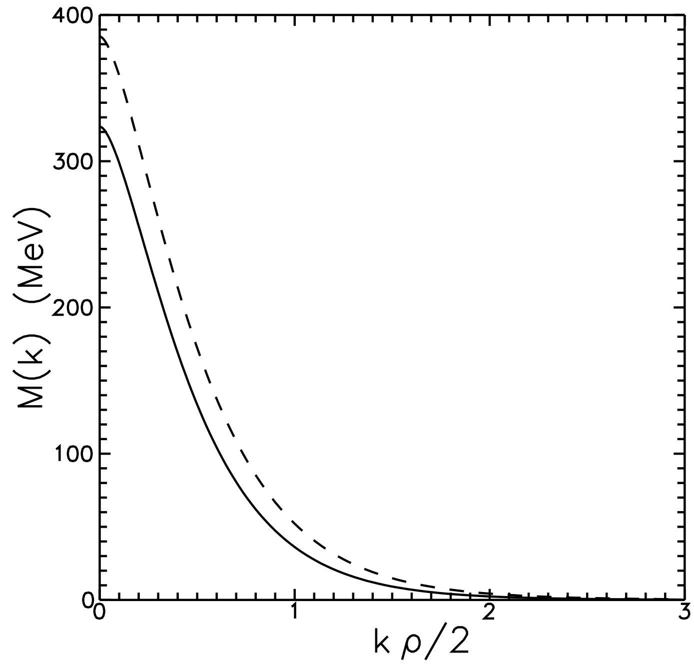

Figure 2: The dynamical quark mass as a function of momentum. The dashed line shows that behaviour when the phonon-like fields are not taken into account.

However this quite noticeable decrease of quark dynamical mass (and quark condensate as well) should not be taken very seriously because the scale of Λ in the ILM is not strictly fixed and it is normalized by calculating the observables. Thus, if one considers, for example, π-meson there is only the possibility to fit parameters (for both with phonon-like excitations of instanton liquid and without them) in such a way to get the satisfactory value of constant F_π, the GMOR relation and etc. Then characteristic parameters of field scales in the tadpole approximation are the following

$$F_\pi \simeq 106 \text{ MeV} , \quad F_\sigma \simeq 86 \text{ MeV} , \quad F_\varphi \simeq 337 \text{ MeV} .$$

Peculiarly, the field φ develops remarkably larger scale comparing to π- and σ-mesons.

The width Γ_φ of phonon-like field φ decay into $\pi\pi$-mode is defined by the diagrams similar to α and γ but with changing σ-meson line for the line of phonon-like field. Then the corresponding anharmonic term of effective Lagrangian density $\widetilde{\varphi}\widetilde{\pi}^2$ takes the form as

$$\mathcal{L}_{int} = \int \frac{dmdl}{(2\pi)^8} \int \frac{dk}{(2\pi)^4} \; W_\varphi(k,m,l) \; \widetilde{\varphi}(m) \; \widetilde{\pi}^a(l) \; \widetilde{\pi}^a(m-l) \; ,$$

$$\frac{W_\varphi(k,m,l)}{4N_c} = -\frac{M(k)\;\Delta M(k)}{k^2+M^2(k)} + \\ +2\frac{k(k-l)+M(k)\;M(k-l)}{[k^2+M^2(k)][(k-l)^2+M^2(k-l)]}\; M(k,k-l)\;\Delta M(k-l,k) \; ,$$

two terms of the sum here correspond to the diagrams mentioned and $\Delta M(k) = m_f \; u(k,k)$. Finally, we have

$$\Gamma_\varphi = \frac{3}{8\pi}\frac{W_\varphi^2}{M_\varphi} \left(1-\frac{4M_\pi^2}{M_\varphi^2}\right)^{1/2} \; ,$$

where W_φ looks similar to W_σ but with changing the factor in denominator providing the dimensionless result for $F_\varphi F_\pi^2$). The mass of phonon-like field and its width are

$$M_\varphi \simeq 827 \text{ MeV} , \quad \Gamma_\varphi \simeq 340 \text{ MeV} ,$$

but these characteristics for σ-meson become as

$$M_\sigma \simeq 1.004 \text{ MeV} \ , \quad \Gamma_\sigma \simeq 270 \text{ MeV} \ ,$$

(for quark loop without the tadpole contribution the width of phonon-like excitation is estimated to be $\Gamma_\varphi \simeq 780$ MeV). The partial contributions of diagrams to the σ-meson width read as

$$\alpha = -7.508{\cdot}10^{-3}\,,\ \beta = -2.205{\cdot}10^{-3}\,,\ \gamma = 8.567{\cdot}10^{-3}\,,\ \ \delta = -2.999{\cdot}10^{-4}\,,\ \ \alpha{+}\beta{+}\gamma{+}\delta = -1.447{\cdot}10^{-3}\,.$$

It is instructive to compare the magnitudes of α_σ and β_σ integrals contributing dominantly to the σ-meson mass (upper line corresponds to the contribution without phonon-like field)

$$\alpha_\sigma = 1.333{\cdot}10^{-3}\,,\ \beta_\sigma = 1.812{\cdot}10^{-2}\,,\ \beta_\pi = 2.872{\cdot}10^{-2} \ ,$$

$$\alpha_\sigma = 3.508{\cdot}10^{-3}\,,\ \beta_\sigma = 1.427{\cdot}10^{-2}\,,\ \beta_\pi = 2.202{\cdot}10^{-2} \ ,$$

(for all that the packing fraction parameters are $n\bar\rho^4 = 5.997{\cdot}10^{-3}$ for the upper line and $n\bar\rho^4 = 6.494{\cdot}10^{-3}$ for the lower one). As seen, the coefficient α_σ gets the strongest impact.

Now let us turn to the problem of mixing two scalar fields if their Lagrangian density tolerating the decay into two π-mesons mode has the following form

$$\begin{aligned} -\widetilde{\mathcal{L}} = &-\frac{F_\varphi^2}{2}\,(k^2+M_\varphi^2)\,\widetilde\varphi^2 - \frac{F_\sigma^2}{2}\,(k^2+M_\sigma^2)\,\widetilde\sigma^2 - \\ &-\frac{F_\pi^2}{2}\,(k^2+M_\pi^2)\,\widetilde\pi^2 + \widetilde\Delta\ \widetilde\varphi\widetilde\sigma + W_\varphi\ \widetilde\varphi\widetilde\pi^2 + W_\sigma\ \widetilde\sigma\widetilde\pi^2\ . \end{aligned} \tag{13}$$

Here the parameter regulating the component mixing $\widetilde\Delta$ is given by the tadpole graph of Fig.1 (and the sign of mixing term coincides with the sign of quark condensate) and could be presented as

$$\widetilde\Delta = 4N_cN_f \int \frac{dk}{(2\pi)^4}\ \frac{m_f\ u(k)\ M(k)}{k^2+M^2}\ .$$

As it was mentioned above, unlike the dilaton Lagrangian [8] in this scheme the field mixing survives in the chiral limit and for the instanton liquid parameters used above we have $\Delta = \dfrac{\widetilde\Delta}{F_\sigma F_\varphi} = (483 \text{ MeV})^2$. The masses of diagonal fields are given by

$$M_\pm^2 = \frac{M_\varphi^2+M_\sigma^2}{2} \pm \frac{((M_\varphi^2+M_\sigma^2)^2+4\Delta^2)^{1/2}}{2}\ . \tag{14}$$

Then the minimal mass of light scalar component is limited by constraint $M_- = 2M_\pi$, otherwise it could be stable in strongly interacting mode, and the upper limit of the Δ magnitude responsible for mixing looks like

$$\Delta^2_{max} \le (M_\varphi^2 - 4M_\pi^2)(M_\sigma^2-4M_\pi^2)\ ,$$

what for existing parameters gives $\Delta_{max} = (868 \text{ MeV})^2$. Comparing to the standard Δ value one may conclude the mixing effect is quite significant.

Passing through the standard procedure with θ as a mixing angle

$$\begin{aligned} \varphi &= \cos\theta\ \varphi' + \sin\theta\ \sigma'\ , \\ \sigma &= -\sin\theta\ \varphi' + \cos\theta\ \sigma'\ , \end{aligned}$$

we receive for corresponding widths of φ' and σ' the following results

$$W_{\varphi'} = W_\varphi\cos\theta - W_\sigma\sin\theta\ ,\quad W_{\sigma'} = W_\varphi\sin\theta + W_\sigma\cos\theta\ ,$$

where the value of mixing angle could be obtained from

$$tg(2\theta) = \frac{2\ \Delta}{M_\sigma^2 - M_\varphi^2}\ .$$

It becomes clear from Eq.(13) the mass of lighter components is getting smaller whereas the heavier one gains more mass. Remembering the phonon-like excitation was lighter before mixing we find out for φ' the following characteristics

$$M_{\varphi'} \simeq 749 \text{ MeV}\ ,\quad \Gamma_{\varphi'} \simeq 265 \text{ MeV}\ .$$

The heavier component σ' becomes as

$$M_{\sigma'} \simeq 1.063 \text{ MeV}\ ,\quad \Gamma_{\sigma'} \simeq 175 \text{ MeV}\ .$$

The mixing angle is $\theta = 27.5$ degrees. Unfortunately, the precision of calculating the particle decay widths is not high and allows to rely on the order of magnitude obtained only. The reason is tightly linked to the contributions of the α and γ graphs which are considerably larger than others. Calculating the α and γ contributions within 10 % precision gains a much poorer precision for the total result. In general it is rooted in that fact the estimates are obtained at the limit (sometimes beyond it in other approaches) of applicability of expansions used.

Summarizing, we would like to underline the approach proposed here sheds more light on the possible origin of scalar mesons. Assuredly, many other models have expressed the similar intends and succeeded to different extents. However, the calculations carried out in this paper demonstrate a quite robust effect of phonon-like excitations of instanton liquid in the scalar sector and capacity of such a model to successfully absorb new results and hints of running experiments. It concerns, first of all, the meson state splitting and its strong coupling scale. For example, both quark-antiquark and glueball 0^{++} operators mixed to create the scalar state in lattice QCD calculations give the mass of the lightest meson essentially suppressed with respect to the mass of corresponding (0^{++}) glueball and demonstrate strong lattice spacing dependence [18]. Besides, the model developed in this paper can be easily adapted for studying the influence of hot and dense medium which appears in ultrarelativistic heavy ion collisions and what could be fairly crucial step in accomplishing the objective.

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