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S-MATRIX DESCRIPTION OF FINITE-TEMPERATURE NONEQUILIBRIUM MEDIA

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The article describes the «inclusive» approach to the nonequilibrium dissipative system on the early (kinetic) stage of evolution, when the temperature is distributed nonuniformly. The perturbation theory is formulated in terms of the space-time local temperature Green functions.

В статье описан «инклюзивный» подход к неравновесным диссипативным системам, находящимся в ранней (кинетической) стадии эволюции, когда температура распределена неравномерно. Теория возмущения формулируется в терминах функций Грина, зависящих от локальной в пространстве-времени температуры.

INTRODUCTION

The aim of this article is to construct the relativistic perturbation theory formalism for the nonequilibrium media description. As an example of interesting system one can have in mind the hadrons very high-multiplicity (VHM) production process. It will be assumed to distinguish this processes as the hadron number

$$n \gg \bar{n}(s), \tag{1}$$

where $\bar{n}(s)$ is the mean multiplicity at the energy $\sqrt{s}$[***]. The phenomenology of high-multiplicity processes was given in [1]. One can offer also the following VHM condition:

$$\sqrt{s} \gg \bar{\varepsilon}_f, \tag{2}$$

where $\bar{\varepsilon}_{fin}$ is the mean energy of the final state.

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[***]The CM frame will be used everywhere and the energy is measured in π-on mass m_π units.

The process (1) is a rear one since the primary kinetic energy is completely dissipated into the mass of secondaries. From thermodynamical point of view it looks like the processes of total dissipation of the high-temperature space-time local fluctuation in a low temperature media. Noting that the multiplicity n is a measure of incident energy dissipation, it is natural to assume that the system becomes equilibrium if n is sufficiently large.

We understand the equilibrium state as the state with uniform energy distribution over all degrees of freedom and with small energy fluctuations near its uniform value, $\bar{\varepsilon}_f$. In other terms, the equilibrium-states energy distribution should be described by the exponent, $\exp\{-\beta\varepsilon\}$, of the Gibbs–Boltzmann law, where $1/\beta$ is the produced particle mean energy, $\beta = 1/\bar{\varepsilon}_f$. Therefore, to describe at least the energy property of such completely thermalized system, it is enough to know (measure) only the particle mean energy $1/\beta$.

So, the tendency to equilibrium with the rising multiplicity is the important proposition, see also Appendix A.

The temperature evolves in the inelastic processes and, generally speaking, is not distributed uniformly at least on the early stages. In the present paper we will consider the general theory of such processes in the frame of various boundary conditions. This will allow one to use the approach not only in the particle physics.

Symmetry Constraints. One should take into account that the hadron inelastic processes have additional special characteristics. This becomes evident noting that the hadron mean multiplicity rises with energy, $\bar{n}(s) \sim \ln^2 s \gg 1$, but is sufficiently smaller than its boundary value $n_{\max}$, $\bar{n}(s) \ll n_{\max} = \sqrt{s}$.

This effect may be explained as the consequence of high symmetry constraints «hidden» in the underlying field theory. So, the symmetry consequence is the integrals of motion in involution and one constraint can reduce a number of degrees of freedom by two units. It is the so-called Liouville–Arnold theorem [2]. Therefore, if the number of constraints exceed the number of the phase space degrees of freedom, then there would not be any thermalization process. This phenomenon was noted firstly by Fermi, Pasta and Ulam [3] in the fifties and was explained theoretically by Zakharov [4] in the seventies. The fact that $\bar{n}(s) \sim \ln^2 s$ means therefore that the number of constraints in the hadron field theory is insufficient to suppress the thermalization effect completely. But these constraints exist and they are able to prevent complete thermalization for which $\bar{n}(s) \simeq n_{\max} = \sqrt{s}$ is natural. Notice here that the first consistent model of multiple production of Fermi–Landau [5] was based on the complete thermalization assumption and the symmetry constraints were ignored in it.

Therefore, it is impossible to hope to observe the equilibrium state in the experimentally observed processes with $n \ll n_{\max}$. It was shown [1] that to have

the thermalized final state the multiple energy correlators should be small:

$$R_l(n,s) = \frac{|K_l(n,s)|^{2/l}}{|K_2(n,s)|} \ll 1,\ l = 3,4,\ldots, \tag{3}$$

where

$$K_l = \sum_{\{\varepsilon\}} \langle \prod_{i=1}^{l} (\varepsilon_i - \langle \varepsilon \rangle) \rangle$$

and the bracket $\langle \ldots \rangle$ means the averaging over existing inelastic states for given multiplicity and incident energy. The model analysis and the experiment show that $R_l(n,s) \gtrsim 1$, $l \geqslant 3$ if $n \sim \bar{n}(s)$ [6]. This result is natural since, as was explained above, at multiplicity $n \sim \bar{n}(s)$ the symmetry induces the preventing thermalization constraints and in the result the correlations should be high, i.e., $R_l(n,s)$, $l > 2$ should be large.

We may offer the following scenario of the multiple production event. At the low multiplicity processes, $n \sim \bar{n}(s)$, the thermalization is depressed by the symmetry constraints. The probability of such processes is the largest and the multiplicity is comparatively small. One may assume that with rising n the mechanism of particle production changes. The possibility that the production dynamics is multicomponent was considered in [7].

We would like to remind now that the symmetry constraints have a «long range» character. Let us consider then the situation when the one of the multiple production channels is «short range», where the constraints are not effective. Such a process should be a «fast» one, i.e., the particle-production «speed» should be larger than the particle-dissipation velocity. In such conditions only we would have the VHM final state. It should be stressed once more that the VHM final states are produced in the result of special *initial* conditions, when the incoming particles interact hardly. That is why the VHM final state is so rear.

The S-Matrix Interpretation of Thermodynamics. The only way to introduce the temperature in the S-matrix formalism is to follow the microcanonical approach. The temperature T, in the frame of this approach, is introduced as the Lagrange multiplier of the energy conservation law. Then the physical (measurable) value of T will be defined by the equation of states. It should be assumed also that the fluctuations near this physical values of T are sufficiently small.

So, we will attach the physical sense to temperature T assuming that the fluctuations near T are Gaussian. Just this condition leads to the inequality (3). Moreover, it can be shown that if this condition (3) is satisfied, then the *probability* of the initial states transition into the final one looks like* the *equilibrium partition*

*Compare Sec. 1 and Sec. 3.

function of Matsubara [1, 9] if there is no special correlations on the remote hypersurface [9].

Therefore, saying that given parameter is measurable, i.e., is a slowly fluctuating quantity, we get to the equilibrium over this parameter. Such an approach seems for us to be interesting since it does not force one to introduce the canonical definition of the equilibrium notion through the condition of equilibrium with the hypothetical external heat bath (thermostat)*. In this case the system under consideration should have the same value of thermodynamical parameters as the thermostat.

The inclusive formalism has one more important property. So, to formulate the theory of irreversible processes in the linear response approximation one may introduce the *mechanical* disturbances [10]. They are considered as an addition of the energy of given controlled environment to the Hamiltonian of the system under consideration. But there are also the *thermal* disturbances and they cannot be defined unambiguously as the part of Hamiltonian since the *thermal* property assumes averaging over appropriate degrees of freedom, i.e., is the *collective* property.

Such an approach to the irreversible-processes description creates the problem which becomes apparent for quantum case. The most problem consists in the necessity to describe the influence of *mechanical* and *thermal* flows on a subsystem of macroscopic system [11]. Then, it is not hard to understand that in quantum case this two flows are noncommutativity operators, i.e., the result depends on the order of their action. The attempt to resolve this problem was given in [12]. But it was shown that distinguishing the external *thermal* flow leads to «*thermal*» renormalization of propagators even in the case when the interaction constant is equal to zero. Detailed analysis of this problem is given in Appendix B.

Let us consider now the inclusive approach. The temperature is introduced in this case as the mean energy of *produced* particles, i.e., when all interactions were finished, if the final state temperature is considered, or not yet started, if the initial state is described. Therefore, there is no necessity to distinguish the *mechanical* and *thermal* flows.

The above described programme was realized in [13] for uniform temperature distribution. This formalism may be naturally extended also over the case of nonuniform temperature distributions. Introducing cells of *measuring device* and introducing the energy-momentum shells of each cell separately, we can introduce the individual temperatures in each cell. This can be done since the measurement performed by free (mass-shell) particles in the S-matrix theory, i.e., the measurement of energy (and momentum), can be performed in each cell separately.

*Which is by itself «equilibrium»!

The quantum uncertainty principle leads to impossibility of taking the 4-dimension of measurement cell, δR, arbitrary small. This is the main difficulty of the considered approach. We will offer the solution introducing, based on the inclusive approach formalism, the Lorentz-covariant interpretation of the Wigner-functions theory [14] given by Carruthers and Zachariasen [15]. This gives the theoretical possibility to fix the canonically conjugate variables simultaneously, i.e., to take the theoretical limit $\delta R = 0$.

Boundary Conditions. The usual Kubo–Martin–Schwinger (KMS) periodic boundary conditions [16] cannot be applied if the nonequilibrium case is considered [12] (see also [1]). We will introduce the boundary conditions «by hands», modelling the environment of the system. Supposing that the system is in a vacuum we will have a usual field-theoretical vacuum boundary condition (Sec. 1). We will consider also the system in the background field of black-body radiation and the latter restores in the equilibrium limit the theory with KMS boundary condition.

1. VACUUM BOUNDARY CONDITION

The probability $r(P)$ of in- into out-states transition with the fixed total 4-momentum P can be calculated using the n- into m-particle transition amplitude $a_{n,m}$. It looks as follows [13]:

$$a_{n,m}((q)_n,(p)_m) = \prod_{k=1}^{n} \hat{\phi}(q_k) \prod_{k=1}^{m} \hat{\phi}^*(p_k) Z(\phi), \tag{4}$$

where $q_k(p_k)$ are the momentum of in(out)-going particles, and the annihilation operator

$$\hat{\phi}(q) = \int d^4x \, \mathrm{e}^{-iqx} \hat{\phi}(x), \; \hat{\phi} = \frac{\delta}{\delta\phi(x)}, \tag{5}$$

was introduced. Correspondingly, $\hat{\phi}^*(p)$ is the creation operator. One can put the auxiliary field $\phi(x)$ equal to zero at the end of calculation. The vacuum-into-vacuum transition amplitude in the presence of external field ϕ

$$Z(\phi) = \int D\Phi \exp^{(iS_{C_+}(\Phi) - iV_{C_+}(\Phi+\phi))} \tag{6}$$

is defined on the Mills complex time contour C_f [14], i.e., $C_f : t \to t + i\hat{e}, \hat{e} > 0$. In Eq. (6), S_{C_f} is the free part of the action and V_{C_f} describes the interactions.

In this section we will propose the vacuum boundary condition:

$$\int_{\sigma_\infty} d\sigma_\mu \Phi \partial^\mu \Phi = 0, \tag{7}$$

where σ_∞ is the infinitely far hypersurface.

Let us consider now the (unnormalized) *probability*

$$r(P) = \sum_{n,m} r(P;n,m) = \sum_{n,m} \frac{1}{n!m!} \int d\omega_n(q) d\omega_m(p) \times \\ \times \delta^{(4)} \left(Q - \sum_{k=1}^{n} q_k \right) \delta^{(4)} \left(P - \sum_{k=1}^{m} p_k \right) |a_{n,m}|^2 , \quad (8)$$

where

$$d\omega_n(q) = \prod_{k=1}^{n} d\omega(q_k) = \prod_{k=1}^{n} \frac{d^3 q_k}{(2\pi)^3 2\varepsilon(q_k)}, \ \varepsilon(q) = (q^2 + m^2)^{1/2}. \quad (9)$$

Equation (8) is the basic formula of our calculations. The microcanonical description was introduced in [12] considering the Fourier transformation of δ functions of (8).

We start the consideration with the assumption that the temperature fluctuations are large scale. In a cell the dimension of which is much smaller than the fluctuation scale of temperature, we can assume that the temperature is a «good» parameter. (The «good» parameter means that the corresponding fluctuations are Gaussian.)

Let us surround the interaction region, i.e., the system under consideration, by N cells with the known space-time position and let us suppose that we can measure the energy and momentum of groups of in- and out-going particles in each cell. The 4-dimension of cells cannot be arbitrary small because of the quantum uncertainty principle.

We decompose δ functions in (8) on the product of $(N+1)$ δ functions:

$$\delta \left(P - \sum_{k=1}^{n} q_k \right) = \int \prod_{\nu=1}^{N} \left\{ dQ_\nu \delta \left(Q_\nu - \sum_{k=1}^{n_\nu} q_{k,\nu} \right) \right\} \delta \left(P - \sum_{\nu=1}^{N} Q_\nu \right), \quad (10)$$

$$\sum_{\nu=1}^{N} n_\nu = n,$$

where $q_{k,\nu}$ are the momentum of kth in-going particle in the νth cell and Q_ν is the total 4-momenta of n_ν in-going particles in this cell. The same decomposition will be used for the second δ function in (8). Inserting this decomposition into (8), we must take into account the multinomial character of particle decomposition on N groups. This will give the coefficient

$$\frac{n!}{n_1! \cdots n_N!} \ \delta_K \left(n - \sum_{\nu=1}^{N} n_\nu \right) \frac{m!}{m_1! \cdots m_N!} \ \delta_K \left(m - \sum_{\nu=1}^{N} m_\nu \right), \quad (11)$$

where δ_K is Kronecker's δ function.

As a result, the quantity

$$r\left((Q)_N,(P)_N\right)=\sum_{(n.m)}\int|a_{(n,m)}|^2\times$$

$$\times\prod_{\nu=1}^{N}\left\{\prod_{k=1}^{n_\nu}\frac{d\omega(q_{k,\nu})}{n_\nu!}\delta\left(Q_\nu-\sum_{k=1}^{n_\nu}q_{k,\nu}\right)\prod_{k=1}^{m_\nu}\frac{d\omega(p_{k,\nu})}{m_\nu!}\delta\left(P_\nu-\sum_{k=1}^{m_\nu}p_{k,\nu}\right)\right\} \tag{12}$$

describes the probability that in the νth cell we observe the flows of in-going particles with the total 4-momentum Q_ν and of out-going particles with the total 4-momentum P_ν. The sequence of these two measurements is not fixed.

The Fourier transformation of δ functions in (12) gives the formula:

$$r((Q)_N,(P)_N)=$$
$$=\int\prod_{\nu=1}^{N}\frac{d^4\alpha_{i,\nu}}{(2\pi)^4}\frac{d^4\alpha_{f,\nu}}{(2\pi)^4}\exp\left(i\sum_{\nu=1}^{N}(Q_\nu\alpha_{i,\nu}+P_\nu\alpha_{f,\nu})\right)R((\alpha_i)_N,(\alpha_f)_N), \tag{13}$$

where $R((\alpha_i)_N,(\alpha_f)_N)\equiv R(\alpha_{i,1},\alpha_{i,2},\ldots,\alpha_{i,N};\alpha_{f,1},\alpha_{f,2},\ldots,\alpha_{f,N})$ has the form:

$$R((\alpha_i)_N,(\alpha_f)_N)=$$
$$=\int\prod_{\nu=1}^{N}\left\{\prod_{k=1}^{n_\nu}\frac{d\omega(q_{k,\nu})}{n_\nu!}\,\mathrm{e}^{-i\alpha_{i,\nu}q_{k,\nu}}\prod_{k=1}^{m_\nu}\frac{d\omega(p_{k,\nu})}{m_\nu!}\,\mathrm{e}^{-i\alpha_{f,\nu}p_{k.\nu}}\right\}|a_{(n,m)}|^2. \tag{14}$$

Inserting (4) into (14) we find:

$$\ln R((\alpha_i)_N,(\alpha_f)_N)=-i\sum_{\nu=1}^{N}\int dxdx'\left[\hat\phi_f(x)D_{fi}(x-x';\alpha_{f,\nu})\hat\phi_i(x')-\right.$$
$$\left.-\hat\phi_i(x)D_{if}(x-x';\alpha_{i,\nu})\hat\phi_f(x')\right]Z(\phi_f)Z^*(\phi_i), \tag{15}$$

where ϕ_i is defined on the complex conjugate contour $C_i: t\to t-i\hat e$ and

$$D_{fi}(x-x';\alpha)=-i\int d\omega(q)\,\mathrm{e}^{iq(x-x')}\,\mathrm{e}^{-i\alpha q}, \tag{16}$$

$$D_{if}(x-x';\alpha)=i\int d\omega(q)\,\mathrm{e}^{-iq(x-x')}\,\mathrm{e}^{-i\alpha q} \tag{17}$$

are the positive and negative frequency correlation functions.

We must integrate over sets $(Q)_N$ and $(P)_N$ if the distribution of flows momentum is unknown. As a result,

$$r(P) = \int D^4\alpha_i(P) D^4\alpha_f(P) R((\alpha_i)_N, (\alpha_f)_N), \tag{18}$$

where the differential measure

$$D^4\alpha(P) = \prod_{\nu=1}^{N} \frac{d^4\alpha_\nu}{(2\pi)^4} K(P, (\alpha)_N) \tag{19}$$

takes into account the energy-momentum conservation laws:

$$K(P, (\alpha)_N) = \int \prod_{\nu=1}^{N} d^4Q_\nu \exp\left(i \sum_{\nu=1}^{N} \alpha_\nu Q_\nu\right) \delta\left(P - \sum_{\nu=1}^{N} Q_\nu\right). \tag{20}$$

The explicit integration gives

$$K(P, (\alpha)_N) \sim \prod_{\nu=1}^{N} \delta^{(3)}(\alpha - \alpha_\nu), \tag{21}$$

where the 3-coordinate α is conjugate to the total 3-momentum $\mathbf{P}$.

To simplify the consideration let us put $\alpha = (-i\beta, \mathbf{0})$. As a result,

$$K(E, (\beta)_N) = \int_0^\infty \prod_{\nu=1}^{N} dE_\nu \, \exp\left(\sum_{\nu=1}^{N} \beta_\nu E_\nu\right) \delta\left(E - \sum_{\nu=1}^{N} E_\nu\right). \tag{22}$$

At the same time,

$$r(E) = \int D\beta_f(E) D\beta_i(E) R((\beta_f)_N, (\beta_i)_N), \tag{23}$$

where

$$D\beta(E) = \prod_{\nu=1}^{N} \frac{d\beta_\nu}{2\pi i} K(E, (\beta)_N) \tag{24}$$

and $R((\beta)_N)$ was defined in (15) with $\alpha_{k,\nu} = (-i\beta_{k,\nu}, \mathbf{0})$, $\operatorname{Re}\beta_{k,\nu} > 0$, $k = f, -$.

We will calculate integrals over β_k using the stationary phase method. The equations for mostly probable values of β_k:

$$-\frac{1}{K(E, (\beta_k)_N)} \frac{\partial}{\partial\beta_{k,\nu}} K(E, (\beta_k)_N) = \frac{1}{R((\beta_1)_N)} \frac{\partial}{\partial\beta_{k,\nu}} R((\beta)_N), \quad k = f, i, \tag{25}$$

always have the unique positive solutions $\tilde{\beta}_{k,\nu}(E)$. We propose that the fluctuations of β_k near $\tilde{\beta}_k$ are small, i.e., are Gaussian. This is the basis of the local-equilibrium hypothesis [13]. In this case $1/\tilde{\beta}_{i,\nu}$ is the temperature in the initial state in the measurement cell ν, and $1/\tilde{\beta}_{f,\nu}$ is the temperature of the final state in the νth measurement cell.

The last formulation (18) implies that the 4-momenta $(Q)_N$ and $(P)_N$ cannot be measured. It is possible to consider another formulation also. For instance, we can suppose that the initial set $(Q)_N$ is fixed (measured) but $(P)_N$ is not. In this case we will have mixed experiment: $\tilde{\beta}_{i,\nu}$ is defined by the equation

$$E_\nu = -\frac{1}{R}\frac{\partial}{\partial\beta_{i,\nu}}R \tag{26}$$

and $\tilde{\beta}_{f,\nu}$ is defined by the second equation in (25).

Considering limit $N \to \infty$, the dimension of cells tends to zero. In this case we are forced by quantum uncertainty principle to propose that the 4-momenta sets (Q) and (P) are not fixed. This formulation becomes pure thermodynamical: we must assume that (β_i) and (β_f) are measurable quantities. For instance, we can fix (β_i) and try to find (β_f) as the function of E and (β_i). In this case Eqs. (25) become the functional equations.

In the considered microcanonical description the finiteness of temperature does not touch the quantization mechanism. Really, one can see from (15) that all thermodynamical information is confined in the operator exponent

$$\prod_\nu \prod_{i\neq j} \mathrm{e}^{i\int \hat{\phi}_i D_{ij} \hat{\phi}_j} = \mathrm{e}^{N(\hat{\phi}_i \hat{\phi}_j)}, \tag{27}$$

the expansion of which describes the environment, and the «mechanical» perturbations are described by the amplitude $Z(\phi)$. This factorization was achieved by introduction of auxiliary field ϕ and is independent of the choice of boundary conditions, i.e., of the choice of the considered systems environment.

2. THE DISTRIBUTION FUNCTIONS

In the previous section the generating functional $R((\beta)_N)$ was calculated by means of dividing the «measuring device» (calorimeter) on the N cells. It was assumed that the dimension of the device cells tends to zero ($N \to \infty$). Now we will specify the cells coordinates using Wigner's description [1, 2].

Let us introduce the distribution function F_n which defines the probability to find n particles with definite momentum and with arbitrary coordinates. These

probabilities (cross section) are usually measured in particle physics. The corresponding Fourier-transformed generating functional can be deduced from (15):

$$F(z,(\beta_f)_N,(\beta_i)_N) = \\ = \prod_{\nu=1}^{N}\prod_{k\neq j}\exp\left\{\int d\omega(q)\hat{\phi}_k^*(q)\,\mathrm{e}^{-\beta_{j,\nu}\varepsilon(q)}\hat{\phi}_j(q)z_{kj}^{\nu}(q)\right\}Z(\phi_f)Z^*(\phi_i). \quad (28)$$

The variation of F over $z_{kj}^{\nu}(q)$ generates corresponding distribution functions. One can interpret $z_{ij}^{\nu}(q)$ as the local activity: the logarithm of $z_{ij}^{\nu}(q)$ is conjugate to the particles number in the cell ν with momentum q for the initial ($kj=+-$) or final ($kj=-+$) states. Note that $z_{kj}^{\nu}(q)\hat{\phi}_k^*(q)\hat{\phi}_j(q)$ can be considered as the operator of activity.

The Boltzmann factor $\mathrm{e}^{-\beta_{j,\nu}\varepsilon(q)}$ can be interpreted as the probability to find a particle with the energy $\varepsilon(q)$ in the final state ($j=f$) and in the initial state ($j=i$). The total probability, i.e., the process of creation and further absorption of n particles, is defined by multiplication of these factors.

The generating functional (28) is normalized as follows:

$$F(z=1,(\beta)) = R(\beta), \quad (29)$$

$$F(z=0,(\beta)) = |Z(0)|^2 = R_0(\phi)|_{\phi=0}, \quad (30)$$

where

$$R_0(\phi) = Z(\phi_f)Z^*(\phi_i) \quad (31)$$

is the «probability» of the vacuum into vacuum transition in the presence of auxiliary fields $\phi_{i(f)}$. The one-particle distribution function

$$F_1\left((\beta_f)_N,(\beta_i)_N;q\right) = \left.\frac{\delta}{\delta z_{kj}^{\nu}(q)}F\right|_{z=0} = \\ = \left\{\hat{\phi}_k^*(q)\,\mathrm{e}^{-\beta_k^{\nu}\varepsilon(q)/2}\right\}\left\{\hat{\phi}_j(q)\,\mathrm{e}^{-\beta_k^{\nu}\varepsilon(q)/2}\right\}R_0(\phi_{\pm}) \quad (32)$$

describes the probability to find one free particle.

Using definition (5), we have

$$F_1((\beta_f)_N,(\beta_i)_N;q) = \int dxdx'\,\mathrm{e}^{iq(x-x')}\,\mathrm{e}^{-\beta_{k,\nu}\varepsilon(q)}\hat{\phi}_k(x)\hat{\phi}_j(x')R_0(\phi_{\pm}) = \\ = \int dY\left\{dy\,\mathrm{e}^{iqy}\,\mathrm{e}^{-\beta_{i,\nu}\varepsilon(q)}\right\}\hat{\phi}_i(Y+y/2)\hat{\phi}_j(Y-y/2)R_0(\phi_{\pm}). \quad (33)$$

We introduce using this definition the «one-particle Wigner function» W_1 [2]:

$$F_1((\beta_f)_N,(\beta_i)_N;q) = \int dY W_1((\beta_f)_N,(\beta_i)_N;Y,q), \quad (34)$$

so,

$$W_1((\beta_f)_N,(\beta_i)_N;Y,q) = \\ = \int dy\, \mathrm{e}^{iqy}\, \mathrm{e}^{-\beta_{k,\nu}\varepsilon(q)} \hat{\phi}_k(Y+y/2)\hat{\phi}_j(Y-y/2)R_0(\phi_\pm). \quad (35)$$

This distribution function describes the conditional probability to find the free particle with momentum q at the point Y in the cell ν.

It is natural to adjust the cell coordinate to the coordinate of measurement Y since the choice of the device coordinates is in our hands:

$$W_1((\beta_f)_N,(\beta_i)_N;Y,q) = \\ = \int dy\, \mathrm{e}^{iqy}\, \mathrm{e}^{-\beta_k(Y)\varepsilon(q)} \hat{\phi}_k(Y+y/2)\hat{\phi}_j(Y-y/2)R_0(\phi_\pm). \quad (36)$$

We will find in concluding Section the Liouville equation for W_1 to make more exact the physical sense of the phase space coordinate (Y,q).

This choice of the device coordinates leads to the following generating functional:

$$\ln F(z,\beta) = i\int dydY \left[\hat{\phi}_f(Y+y/2)D_{fi}(y;\beta_f(Y),z)\hat{\phi}_i(Y-y/2) - \right. \\ \left. - \hat{\phi}_i(Y+y/2)D_{if}(y;\beta_i(Y),z)\hat{\phi}_f(Y-y/2)\right] R_0(\phi_\pm), \quad (37)$$

where

$$D_{fi}(y;\beta_f(Y),z) = -i\int d\omega(q) z_{fi}(Y,q)\, \mathrm{e}^{iqy}\, \mathrm{e}^{-\beta_f(Y)\varepsilon(q)}, \quad (38)$$

$$D_{if}(y;\beta_f(Y),z) = i\int d\omega(q) z_{if}(Y,q)\, \mathrm{e}^{-iqy}\, \mathrm{e}^{-\beta_i(Y)\varepsilon(q)} \quad (39)$$

are the modified positive and negative correlation functions (16), (17).

3. THE CLOSED-PATH BOUNDARY CONDITION

The developed in Sec. 1 formalism allows one to introduce the more general boundary conditions instead of (6). Considering the probability R which has the double-path integral representation we will introduce integration over closed path. This allows one to introduce the equality:

$$\int_{\sigma_\infty} d\sigma_\mu(\Phi_f\partial^\mu\Phi_f - \Phi_i\partial^\mu\Phi_i) = 0, \quad (40)$$

as the boundary condition, where σ_∞ is the infinitely far hypersurface. The general solution of this equation is:

$$\Phi_\pm(\sigma_\infty) = \Phi(\sigma_\infty) \tag{41}$$

where $\Phi(\sigma_\infty)$ is the «turning-point» field. The result of this changing of boundary condition was analyzed in [12] for the case of uniform temperature distribution.

In terms of S matrix the field $\Phi(\sigma_\infty)$ represents the background flow of mass-shell particles. We will propose that the probability to find a particle of the background flow is determined by the energy-momentum conservation law only. In another words, we will propose that the system under consideration is surrounded by the black-body radiation.

Presence of additional flow will reorganize the differential operator $\exp\{N(\hat\phi_i\hat\phi_j)\}$ only, and new generating functional $R_{\rm cp}$ has the form:

$$R_{\rm cp}(\alpha_f,\alpha_i) = \mathrm{e}^{N(\hat\phi_i\hat\phi_j)}R_0(\phi_\pm). \tag{42}$$

The calculation of operator $N(\hat\phi_i\hat\phi_j)$ is strictly the same as in [12]. Introducing the cells in the Y space we will find that

$$N(\hat\phi_i\hat\phi_j) = \int dY dy \hat\phi_i(Y+y/2)\tilde n_{ij}(Y,y)\hat\phi_j(Y-y/2), \tag{43}$$

where the occupation number $\tilde n_{ij}$ carries the cells index Y:

$$\tilde n_{ij}(Y,y) = \int d\omega(q)\,\mathrm{e}^{iqy}n_{ij}(Y,q) \tag{44}$$

and ($q_0 = \varepsilon(q)$)

$$n_{++}(Y,q_0) = n_{--}(Y,q_0) = \tilde n(Y,(\beta_f+\beta_i)|q_0|/2) = \\ = \frac{1}{\exp\left((\beta_f+\beta_i)(Y)|q_0|/2\right)-1}, \tag{45}$$

$$n_{+-}(Y,q_0) = \Theta(q_0)(1+\tilde n(Y,\beta_f q_0)) + \Theta(-q_0)\tilde n(Y,-\beta_i q_0), \tag{46}$$

$$n_{-+}(Y,q_0) = n_{+-}(Y,-q_0). \tag{47}$$

For simplicity the CM system was used.

Calculating R_0 perturbatively we will find that

$$R_{\rm cp}(\beta) = \exp\left\{-iV(-i\hat j_f) + iV(-i\hat j_i)\right\}\times \\ \times\exp\left\{i\int dY dy[\hat j_i(Y+y/2)G_{ij}(y,(\beta(Y))\hat j_j(Y-y/2)\right\}, \tag{48}$$

where, using the matrix notations,

$$iG(q,(\beta(Y))) = \begin{pmatrix} \dfrac{i}{q^2-m^2+i\hat{e}} & 0 \\ 0 & -\dfrac{i}{q^2-m^2-i\hat{e}} \end{pmatrix} +$$

$$+ 2\pi\delta(q^2-m^2) \begin{pmatrix} n\left(\dfrac{(\beta_f+\beta_i)(Y)}{2}|q_0|\right) & n(\beta_f(Y)|q_0|)a_f(\beta_f) \\ n(\beta_i(Y)|q_0|)a_i(\beta_i) & n\left(\dfrac{(\beta_f+\beta_i)(Y)}{2}|q_0|\right) \end{pmatrix}, \quad (49)$$

and

$$a_\pm(\beta) = -\mathrm{e}^{\beta(|q_0|\pm q_0)/2}. \quad (50)$$

Formally these Green functions obey the standard equations in the y space:

$$\begin{aligned} (\partial^2-m^2)_y G_{ii} &= \delta(y), \\ (\partial^2-m^2)_y G_{ij} &= 0, \quad i \neq j \end{aligned} \quad (51)$$

since $\Phi(\sigma_\infty) \neq 0$ reflects the mass-shell particles. But the boundary conditions for these equations are not evident.

4. CONCLUDING REMARKS

One may note that the natural generalization of inclusive approach consisted in offering to consider the particles beam in the cell as a one particle. This idea is based on the possibility to have the particle subsystems in the «pre-equilibrium» state.

In other words, the «pre-equilibrium» media consist of macroscopic-size equilibrium domains $\Omega(Y)$. We offer to consider a subset of particles $\Omega_q(Y) \in \Omega(Y)$ as a particle with coordinate Y. It can have some additional property q. We would like to take into account here the fact that all particles, or arbitrary set of particles, have the same thermal properties.

This generalization has evident benefits from experimental point of view. It is important since the VHM processes are rear and it is hard to believe in possibility to have a large statistics. The above idea means that the «thermodynamical» rough measurement may give complete information about the system under consideration.

Another interesting question regards the possibility of using the phase space description of the state produced in the high-energy particles collision. The (relativistic) phase space distribution function can be introduced through the Wigner function W_1, see (34). This interpretation follows from the fact that W_1 should

obey the Liouville equation, i.e., it conserves the phase space volume. This statement may be proven using just the functional integral representation (36) (see Appendix C).

Appendix A
VHM ASYMPTOTICS

Returning to (8),

$$r(P) = \sum_{n,m} r(P;n,m) =$$

$$= \sum_{n,m} \frac{1}{n!m!} \int d\omega_n(q)\omega_m(p)\delta\left(P - \sum_{k=1}^{n} q_k\right)\delta\left(P - \sum_{k=1}^{m} p_k\right)|a_{n,m}|^2, \quad \text{(A.1)}$$

let us assume that a_{nm} is a constant over particle momenta. If the hadron (actually of the π meson) mass is not equal to zero, then the zero momentum limit should exist. So, we will assume that $m_\pi \neq 0$. Leaving in the sum over m one term $m = 2$, we will consider the asymptotics over $n \to n_{\max}$ of the expression:

$$r(E,n) = \frac{\bar{a}_n(E)}{n!} \int d\omega_n(q)\delta\left(E - \sum_{k=1}^{n} \varepsilon_k\right). \quad \text{(A.2)}$$

Notice that the momentum conservation function was omitted. Introducing the Fourier transform of the remaining δ function, we can write:

$$r(E,n) = \frac{\bar{a}_n(E)}{n!} \int \frac{d\beta}{2\pi} \mathrm{e}^{\beta E} \mathrm{e}^{-F(\beta,n)}, \quad \text{(A.3)}$$

where

$$F(\beta,n) = -n \ln\left\{\int d\omega_1(p)\, \mathrm{e}^{-\beta\varepsilon(p)}\right\},\ \varepsilon(p) = m_\pi + \frac{p^2}{2m_\pi}. \quad \text{(A.4)}$$

The integral over β will be calculated using the saddle-point method. To find mostly probable value of β one should solve the equation (of state):

$$E = \frac{\partial}{\partial\beta}F(\beta,n) = n\frac{\int d\omega_1(p)(m_\pi + p^2/2m_\pi)\,\mathrm{e}^{-\beta\varepsilon(p)}}{\int d\omega_1(p)\,\mathrm{e}^{-\beta\varepsilon(p)}} \simeq n\left(m_\pi + \frac{a}{\beta}\right), \quad \text{(A.5)}$$

where a is the inessential positive constant. Therefore, the «physical» value

$$\beta_c \simeq \frac{na}{E - m_\pi n} = \frac{a}{m_\pi}\frac{n}{n_{\max} - n} \to \infty \text{ at } n \to n_{\max}. \quad \text{(A.6)}$$

Expanding $F(\beta, n)$ near β_c:

$$F(\beta, n) = F(\beta_c, n) + \frac{1}{2!}F^{(2)}(\beta_c, n)(\beta - \beta_c)^2 + \frac{1}{3!}F^{(3)}(\beta_c, n)(\beta_c - \beta)^3 + \dots$$

we must expand over $F^l(\beta_c, n)$. Resulting perturbation series has the zero radii of convergence. Assuming that we are dealing with asymptotic series, one *must* assume that at least

$$\frac{|F^{(3)}(\beta_c, n)|^{2/3}}{|F^{(2)}(\beta_c, n)|} \ll 1. \tag{A.7}$$

It is not hard to see that this inequality leads to (3). In the considered asymptotics (A.6), we would have that

$$R_3(n, s) \sim \frac{1}{n} \ll 1. \tag{A.8}$$

Therefore, we can offer the proposition: *if the theory is finite in the soft particles limit, then the final state described by such theory appears to be equilibrium in the limit* $n \to n_{\max}$.

Appendix B
ZUBAREV'S NONSTATIONARY OPERATOR FORMALISM

One cannot expect the evident connection between the above considered and Zubarev's [11] approaches. The reason is as follows.

In Zubarev's theory the «local-equilibrium» hypothesis was adopted as the boundary condition. It is assumed that in the suitably defined cells of a system at a given temperature distribution $T(\mathbf{x}, t) = 1/\beta(\mathbf{x}, t)$, where $(\mathbf{x}, t)$ is the index of the cell, the entropy is maximum. The corresponding nonequilibrium statistical operator

$$R_z \sim \mathrm{e}^{-\int d^3x \beta T_{00}} \tag{B.1}$$

describes evolution of a system. Here $T_{\mu\nu}$ is the energy-momentum tensor. It is assumed that the system «follows» to $\beta(\mathbf{x}, t)$ evolution and the local temperature $T(\mathbf{x}, t)$ is defined as the external parameter which is the regulator of the systems dynamics. For this purpose the special $i\varepsilon$ prescription was introduced [13].

But the KMS periodic boundary condition cannot be applied and by this reason the decomposition

$$\beta(\mathbf{x}, t) = \beta_0 + \beta_1(\mathbf{x}, t) \tag{B.2}$$

was offered [11]. Here β_0 is the constant and the inequality

$$\beta_0 \gg |\beta_1(\mathbf{x}, t)| \tag{B.3}$$

is assumed. Then,

$$R_z \sim \mathrm{e}^{-\beta_0(H_0+V+B)} \tag{B.4}$$

where H_0 is the free part of the Hamiltonian; V describes the interactions; and the linear over β_1/β_0 term B is connected with the deviation of temperature from the «equilibrium» value $1/\beta_0$. Considering V and B as the perturbations one can calculate the observables averaging over equilibrium states, i.e., adopting the KMS boundary condition. Using standard terminology [9] one can consider V as the «mechanical» and B as the «thermal» perturbations.

The quantization problem of operator (B.4) is connected with definition of the space-time sequence of mechanical (V) and thermal (B) excitations. It is necessary since the mechanical excitations affect the thermal ones and vice versa. It was assumed in [11] that V and B are commuting operators, i.e., the sequence of V and B perturbations is not sufficient. This solution leads to the particle propagator renormalization by the interactions with the external field $\beta(\mathbf{x},t)$ even without interactions among fundamental fields. (Note the absence of these renormalizations in our formalism.)

In [11], the operators V and B are noncommutative ones and B perturbations were switched on after V perturbations. In this formulation, the «nondynamical» renormalizations are also present but it is unlikely that they are cancelled at the very end of calculations.

This formulation with $\beta(\mathbf{x},t)$ as the external field remained the old, firstly quantized, field theory in which matter is quantized but fields are not. It is known that consistent quantum field theory requires the second quantization. Following this analogy, if we want to take into account consistently the reciprocal influence of V and B perturbations, the field $\beta(\mathbf{x},t)$ must be fundamental, i.e., must be quantized. But this is the evidently wrong idea in the canonical Gibbs formalism. So, as in the firstly quantized theory, the theory with operator (B.1) must have the restricted range of validity [10].

Therefore, we must reduce our formalism just to the hydrodynamic accuracy to find the quantitative connection with Zubarev's approach.

Appendix C
LIOUVILLE EQUATION FOR WIGNER FUNCTION

Let us consider (36):

$$W_1((\beta_f)_N,(\beta_i)_N;Y,q) = \\ = \int dy\,\mathrm{e}^{iqy}\,\mathrm{e}^{-\beta_k(Y)\varepsilon(q)}\hat{\phi}_k(Y+y/2)\hat{\phi}_j(Y-y/2)R_0(\phi_\pm). \tag{C.1}$$

We would like to investigate under what conditions W_1 obey the Liouville equation.

The functional integral representation for W_1 has a form:

$$W_1(\beta;Y,q)=\int dy\,\mathrm{e}^{iqy}\,\mathrm{e}^{-\beta_k(Y)\varepsilon(q)}\hat{\phi}_k(Y+y/2)\hat{\phi}_j(Y-y/2)\int D\Phi_+D\Phi_-\times$$
$$\times\,\mathrm{e}^{\left(iS^0_{C_+(t_{\mathrm{in}})}(\Phi_+)-iV_{C_+(t_{\mathrm{in}})}(\Phi_++\phi_f)-iS^0_{C_-(t_{\mathrm{in}})}(\Phi_-)+iV_{C_-(t_{\mathrm{in}})}(\Phi_--\phi_i)\right)}=$$
$$=\int dy\,\mathrm{e}^{iqy}\,\mathrm{e}^{-\beta_k(Y)\varepsilon(q)}\int D\Phi_+D\Phi_-V'_{C_+(t_{\mathrm{in}})}(\Phi_+;Y+y/2)V'_{C_-(t_{\mathrm{in}})}\times$$
$$\times\,(\Phi_-;Y-y/2)\mathrm{e}^{\left(iS_{C_+(t_{\mathrm{in}})}(\Phi_+)-iS_{C_-(t_{\mathrm{in}})}(\Phi_-)\right)},\quad\text{(C.2)}$$

where $S^0_{C_\pm(t_{\mathrm{in}})}$ is the free part of the total action, $S_{C_\pm(t_{\mathrm{in}})}(\Phi_\pm)=S^0_{C_\pm(t_{\mathrm{in}})}(\Phi_\pm)-V_{C_\pm(t_{\mathrm{in}})}(\Phi_\pm)$, and the Mills time contour

$$C_\pm(t_{\mathrm{in}}):\ t\to\pm i\epsilon,\ \epsilon\to+0,\ t_{\mathrm{in}}\leqslant t\leqslant+\infty,\ t_{\mathrm{in}}\to-\infty\qquad\text{(C.3)}$$

was introduced. We should use the closed-path boundary condition (41):

$$\Phi_\pm(\sigma_{t_{\mathrm{in}}})=\Phi(\sigma_{t_{\mathrm{in}}}),\qquad\text{(C.4)}$$

where the hypersurface $\sigma_{t_{\mathrm{in}}}$ crosses the point t_{in}. It should be stressed that the integration over the «turning-point» field $\Phi(\sigma_{t_{\mathrm{in}}})$ must be performed. Notice that the closed-path boundary condition only allows at finite t_{in} to be saved from the unlike «surface terms».

The representation (C.2) contains the vertices

$$V'_{C_-(t_{\mathrm{in}})}(\Phi_\pm;Y\pm y/2)=\frac{\delta}{\delta\Phi_\pm(Y\pm y/2)}V_{C_-(t_{\mathrm{in}})}(\Phi_\pm).\qquad\text{(C.5)}$$

Notice that there is no any necessity to cut the integral over y at $y=t_{\mathrm{in}}$ since the action of the operators $\hat{\phi}_j(Y\pm y/2)$ ends at the time $(Y\pm y/2)_0=t_{\mathrm{in}}$. Therefore, $W_1(\beta;Y,q)$ exist for the time interval $Y_0<t_{\mathrm{in}}$. The t_{in} dependence of $D\Phi_\pm$ on t_{in} is not important since we always can add (infinite) integration over

$$\prod_{t=-\infty}^{t_{\mathrm{in}}}d\Phi(t),$$

assuming that this infinity may be cancelled by normalization factor.

It is known that the double functional integral (C.2) is defined on the δ-like Dirac measure [1, 17]. The result looks as follows:

$$W_1(\beta;Y,q)=\int dy\,\mathrm{e}^{iqy}\,\mathrm{e}^{-\beta_k(Y)\varepsilon(q)}\,\mathrm{e}^{-i\mathbf{K}(\mathbf{je})}\int DM(\Phi)V'_{C_+(t_{\mathrm{in}})}\times$$
$$\times\,(\Phi+e;Y+y/2)V'_{C_-(t_{\mathrm{in}})}(\Phi-e;Y-y/2)\,\mathrm{e}^{-iU_C(\Phi;e)}.\quad\text{(C.6)}$$

Expanding $\exp\{-i\mathbf{K}(je)\}$ over the operator

$$2\mathbf{K}(je) = \int dx \frac{\delta}{\delta j(x)} \frac{\delta}{\delta e(x)}, \tag{C.7}$$

we will obtain the ordinary perturbation theory. Notice that the operator $\mathbf{K}(je)$ is $t_{\rm in}$-independent.

The functional integral (C.6) is defined on the Dirac measure:

$$DM(\Phi) = \prod_x{}' d\Phi(x)\delta\left(\frac{\delta S(\Phi)}{\delta\Phi(x)} - j(x)\right), \tag{C.8}$$

where the prime means that the functional δ function does not include the time end point $x_0 = t_{\rm in}$.

At the end of definitions, the functional $U_{C_+}(\Phi;\phi,e)$ describes interactions [1]. The explicit form of it is not important for us.

Deriving the Liouville equation, the dynamics should be described in the phase space. It is easy to see that the measure (C.8) has the following form in the phase space:

$$DM(\Phi) = \prod_x d\Phi(x)dP(x)\delta\left(\dot\Phi - \frac{\delta H_j(\Phi,P)}{\delta P(x)}\right)\delta\left(\dot P + \frac{\delta H_j(\Phi,P)}{\delta\Phi(x)}\right), \tag{C.9}$$

where the Hamiltonian

$$H_j(\Phi,P) = \int d^3x\left\{\frac{1}{2}P^2 + \frac{1}{2}(\nabla\Phi)^2 + v(\Phi) - j\Phi\right\} \tag{C.10}$$

explicitly depends on the produced quantum perturbation force $j(x)$, and $v(\Phi)$ is the potential term. Transition from (C.8) to (C.9) may raise a doubt caused by a possible symmetry of the problem under consideration. To avoid this ambiguity, one may consider this transition as the introduction of the first order formalism.

Notice now that the equalities:

$$\dot\Phi = \frac{\delta H_j(\Phi,P)}{\delta P(x)}, \quad \dot P = -\frac{\delta H_j(\Phi,P)}{\delta\Phi(x)} \tag{C.11}$$

cannot fix the boundary values Φ_0 and P_0. For this reason one may omit the prime in the measure (C.9), i.e., including the boundary value of $t = t_{\rm in}$ in the Dirac measure.

Now it is important to note that, following our definition,

$$\int\prod_x d\Phi(x)\delta(\dot\Phi(x)) = \int d\Phi(t_{\rm in}) = \int d\Phi_0.$$

Therefore, the boundary values (Φ_0, P_0) stay undefined by our functional δ functions and, generally speaking, integration over them is assumed:

$$W_1(\beta; Y, q, t_{\rm in}) = \int d\Phi_0 dP_0 W_1(\beta; Y, q, \Phi_0, P_0). \tag{C.12}$$

The Liouville equation exists just for $W_1(\beta; Y, q, \Phi_0, P_0)$.

Indeed, let us calculate the total derivative over $t_{\rm in}$:

$$\frac{d}{dt_{in}} W_1(\beta; Y, q, \Phi_0, P_0) = \\ = \frac{\partial W_1(\beta; Y, q, \Phi_0, P_0)}{\partial \Phi_0}\ \dot{\Phi}_0 + \frac{\partial W_1(\beta; Y, q, \Phi_0, P_0)}{\partial P_0}\ \dot{P}_0. \tag{C.13}$$

But having the measure (C.9), one may write that

$$\frac{d}{dt_{\rm in}} W_1(\beta; Y, q, \Phi_0, P_0) = \\ = {\rm e}^{-i\mathbf{K}(je)} {\rm e}^{-iU_C(\Phi;0,e)} \left\{ W_1(\beta; Y, q, \Phi_0, P_0), H_j(\Phi_0, P_0) \right\}, \tag{C.14}$$

where the Poisson bracket

$$\{W_1, H_j\} = \frac{\partial W_1}{\partial \Phi}\ \frac{\partial H_j}{\partial P} - \frac{\partial W_1}{\partial P}\ \frac{\partial H_j}{\partial \Phi}.$$

Notice the quantum character of this equation: r.h.s. contains the operator of quantum perturbations $\mathbf{K}(je)$. It acts on j in the Hamiltonian $H_j = H_j(\Phi_0, P_0)$ and e in the interaction functional $U_C(\Phi; 0, e)$. Notice, all quantities are defined at the time moment $t_{\rm in}$.

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