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ON THE STATUS OF VERY HIGH MULTIPLICITY PHYSICS

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The paper contains the description of the main trends in the very high multiplicity physics. The incident energy dissipation into the secondaries is considered as the thermalization phenomenon. The experimental fact that the particle production process is stopped at such an early stage that the mean multiplicity is nothing but the logarithm of incident energy. This phenomenon is considered as the indication of the absence of complete thermalization in the mostly probable inelastic processes. The quantitative definition of thermalization phenomenon is offered and the very high multiplicity domain where the thermalization must occur is discussed. The physical consequences and model predictions of the thermalization effect are considered. The short review of the last publications on the very high multiplicity physics is also offered.

1. INTRODUCTION

The interest in the very high multiplicity (VHM) hadron processes becomes so many-sided that it is time to give a general description of the situation in this field. The trends can be divided into the three sectors. They are pure theoretical, experimental, and intermediate where the theoretical efforts are directed to the VHM experiment.

The paper is based mainly on the talks presented at the VHM Physics Workshops held in Dubna during 2000–2002 years [1]. It must be mentioned from the very beginning that there is not any experimental information concerning VHM high energy hadron reactions till now. Moreover, there are not either theory predictions for such processes even on the model level. For this reasons the spectrum of efforts presented in [1] is wide.

We would like to start from the well known experimental fact that the high energy hadron collisions are, for the most part, inelastic, see Fig. 1, where the experimental value of total and elastic cross sections are shown.

Having the multiparticle state the idea to introduce thermodynamical methods for hadron interactions description seems fruitful. It is important to notice here that the thermalization means the possibility to use the equilibrium thermodynamics phenomenology. Actually one must be careful with the above mentioned idea. The reason why this is not trivial will be the main subject of the discussion.

Attempts to introduce the thermodynamical notions into the multiple production physics have a long history. The first so-called "thermodynamical model" was offered by Fermi and Landau in 50th of the preceding century [2, 3]. It was assumed that particles production may be considered as the process of cooling of the incident high-temperature state. The reason of cooling is a tendency to equilibrium with the environment. Indeed, the considered process takes place in the *"zero-temperature"* vacuum and, therefore, the results of cooling should be the state with zero-momentum particles. In this case the hadron mean multiplicity would be approximately equal to the incident total energy and, therefore, the multiplicity would have culminated its maximal value $n_{\max} = \sqrt{s}/m$, where $\sqrt{s}$ is the total CM energy and $m \simeq 0.2$ GeV is the characteristic hadron mass.

But we know that the mean hadron multiplicity is only a logarithm or the second power of the logarithm of incident total energy, see Fig. 2, where the best fit of the power dependence, $\bar{n}(s) \sim \sqrt{s}$, is shown for comparison.

Therefore, something prevents the dissipation of the incident energy into the produced particle masses and, for this reason, the thermalization does not taken place. Here the term "thermalization" means the *uniform* distribution of perturbation over *all* degrees of freedom. At the same time, fluctuations must have the Gaussian character.

In other words, we would like to offer for discussion the most important question of hadron dynamics from our point of view: *why is the process of incident energy dissipation stopped at such an early stage* that the mean multiplicity is comparably small and, for this reason, the complete thermalization does not occur?

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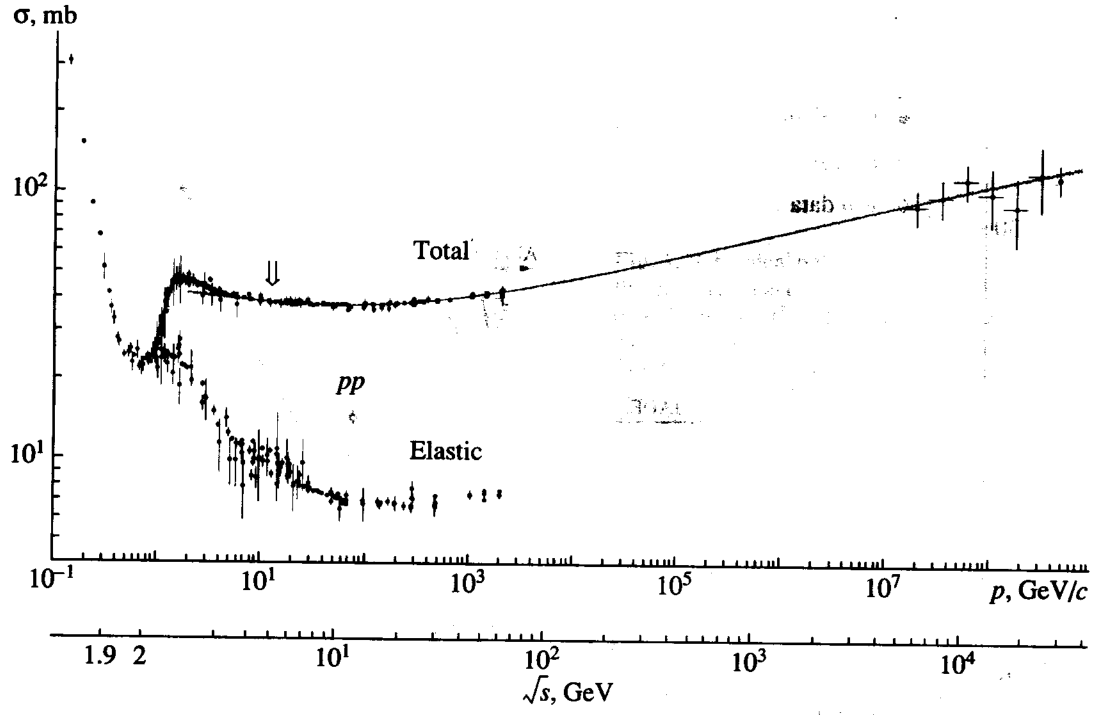

Fig. 1. Total cross section. It is remarkable that σ_{tot} is approximately constant in the interval of 10–100 GeV. The Froissart constrain: $\sigma_{tot} \lesssim \ln^2 s$.

1.1. Role of Symmetry Constrains

We know, at least qualitatively, the answer to this question: the reason why the mean hadron multiplicity is much smaller than n_{max} is hidden in the symmetry constrains. Namely, one may hope that this is an effect of underlying non-Abelian gauge symmetry recorded in the Yang–Mills field theory.

So, the purpose of the present paper is to discuss the most intriguing question of the hadron physics: the dynamical consequence of the non-Abelian gauge symmetry of Yang–Mills field theory. One of the known consequences of this symmetry is the colour charge confinement. The other one is the incomplete thermalization and, as a result, smallness of the total hadron multiplicity.

In the most inelastic hadron processes the thermalization is not produced and it is impossible to use the methods of thermodynamics for them. But, it can be proved that at the VHM the final state is completely thermalized. So, we would like to provide for the condition where the Fermi–Landau model works. It is evident that such condition is realized in Nature extremely rarely.

This also means that in the VHM region one may use the "rough" thermodynamical parameters, the "temperature", "chemical potential", etc., for the *complete* description of the system. For instance, in this case one may completely describe the energy distribution of the system knowing only the mean energy of secondaries [4, 5].

Therefore, the confinement forces, as the symmetry constrains, should not act in the VHM region. This conclusion is crucial in our further considerations since considerably simplifies the multiple production picture.

It is interesting to note that, on the other hand, the system of colour charges must be considered as the plasma state in the case of absence of confinement forces.

Moreover, the thermalized state is *calm*. This means that the kinetic forces are not important in comparison with the potential ones. This situation is the best to observe the collective phenomena.

1.2. References

The phenomenology of the VHM events was formulated in the papers published in [6]. It includes two basic ideas. The first one gives the classification of asymptotics over multiplicity and the physical interpretation of the classes. The offered interpretation excludes from consideration the final-state interactions, for instance, the Bose–Einstein correlation [7]. The papers [8, 9] fill this deficiency. It was shown [9] that the final-state interaction can cardinally change the multiplicity distribution tail. The experimental

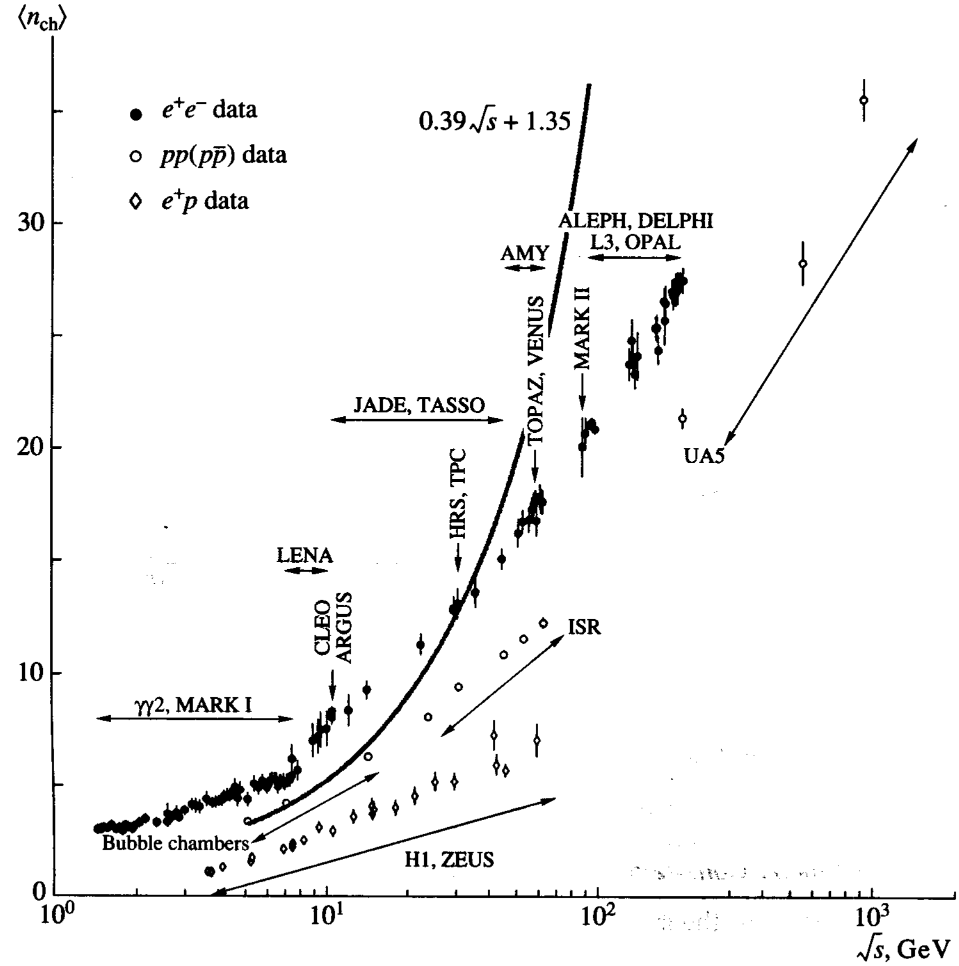

Fig. 2. Mean multiplicity. The mean multiplicity in QCD jet $\bar{n}(s)_j$ and in the e^+e^- annihilation processes is relatively high: $\ln \bar{n}(s)_j \sim \sqrt{\ln s}$.

investigation of this prediction will be performed during the experiment at U-70 (Protvino), see [10]. We would also stress the efforts toward the experiment, published in [11].

An idea was offered [12] that the measurements in the VHM region may be performed "roughly", see also [13]. For instance, it is quite possible to have the multiplicity with some, but definite, error. One may also generalize the inclusive approach, combining particles into the groups and considering the group as a "particle", and so on. The effectiveness of such formulation of the experimental programme was shown in [14].

The paper [4] contains a qualitative feature of the VHM physics. The main question is: how much confidence may predictions of the perturbative QCD and existing multiperipheral models in the VHM region have. We have found that pQCD can not be used even if the VHM production process is hard. The experimental investigation of the range of validity of pQCD predictions has been performed in [15]. The point is that it is hard to use the leading logarithm approximation (LLA) ideology [16] in the VHM region. That is why a new perturbation theory has been built [17–20].

One can hope that the VHM domain is much "simpler" from theoretical point of view than the traditional domain of $n \sim \bar{n}(s)$ [4]. Nevertheless, the attempts to find new characteristics of inelastic collisions are extremely important. The "wavelet" analysis [21] presents the corresponding example.

2. DEFINITION OF THE VHM REGION

It is natural that just the mean multiplicity defines the scale of multiplicities. Then, generally, we wish to consider the processes with multiplicity

$$n \gg \bar{n}(s),$$

where $\bar{n}_s$ is the mean multiplicity. The VHM domain can be more specified while considering the details of production processes.

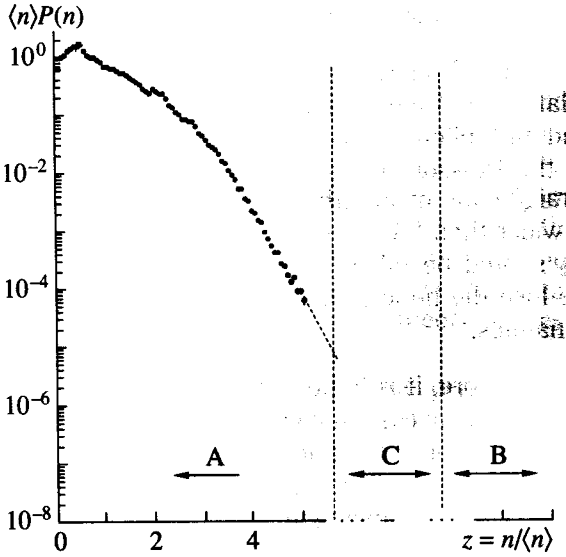

Fig. 3. Multiplicity distribution in terms of KNO variables. A — the domain of miltiperipheral models, B — the deep asymptotics over multiplicity, C — the very high multiplicity domain.

One may also introduce the inelasticity coefficient

$$\frac{E-\epsilon_{\max}}{E},$$

where E — the total energy in the CM frame, $\epsilon_{\max}$ — the energy of the fastest particle in the same frame. Then, VHM events mean

$$1-\frac{E-\epsilon_{\max}}{E}\ll 1.$$

So, the produced particles momentum would be comparatively small.

At the same time, we would like to exclude the influence of the phase space boundaries. For this reason, we would assume that the multiplicity can not be too large:

$$n \ll n_{\max} = E/m, \qquad m \approx 0.2\ \mathrm{GeV}.$$

From the experimental point of view VHM domain includes extremely rare processes, see Fig. 3. For this reason the B range of multiplicity in Fig. 3 seems to be not attainable.

The fact that A is the multiperipheral dynamics domain and C presents the deep asymptotics over multiplicity must be noted. We will call the domain C as the VHM domain and it is defined from the condition that the thermalization is attained.

2.1. VHM Phenomenology

The kinematics conditions are changed with multiplicity and it is natural to expect the change of particle productions mechanism [22]. One may consider the result presented in [23] as the experimental confirmation of this idea.

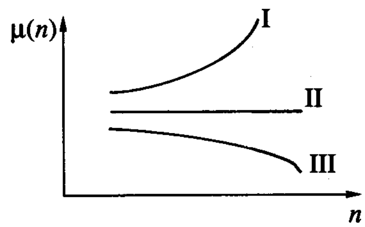

Fig. 4. "Chemical potential" $\mu = \langle\varepsilon\rangle \ln(\sigma_n/\sigma_{\rm tot})$ vs. multiplicity. The case I corresponds to the multiperipheral model; the case II is predicted by the QCD jet; III is a case when the vacuum is unstable against particle production. The latter case may include a situation with final-state interactions. To distinguish this possibility, one should investigate the analytical properties of μ over n.

It can be shown that only three classes of asymptotics can be realized:

The cross section falls down faster than any power of the exponent of $(-n)$:

$$\text{(I)}\ \ \textit{multiperipheral interactions}: \sigma_n < O(e^{-n});$$

The cross section falls down as the exponent:

$$\text{(II)}\ \ \textit{hard processes}: \sigma_n = O(e^{-n});$$

The cross section falls down slower than any power of the exponent:

$$\text{(III)}\ \ \textit{vacuum instability}: \sigma_n > O(e^{-n}).$$

Therefore, one may neglect the factors in front of $\exp\{-n\}$ and, since the cross section is extremely small at $n \gg \bar{n}(s)$, it is natural to estimate cross sections only with logarithmic accuracy. In other words, we offer to measure the following quantity

$$\mu(n) = -\langle\varepsilon\rangle \frac{1}{n} \ln\frac{\sigma_n}{\sigma_{\rm tot}},$$

where $\langle\varepsilon\rangle$ is the mean energy of secondaries. Then, one may distinguish the following possibilities in the region of large multiplicities, see Fig. 4,

$$\text{(I)}: \frac{\partial}{\partial n}\mu(n) > 0, \quad \text{(II)}: \frac{\partial}{\partial n}\mu(n) = 0,$$

$$\text{(III)}: \frac{\partial}{\partial n}\mu(n) < 0.$$

The case (I) corresponds to the multiperipheral model; the case (II) is predicted by the QCD jet; (III) is the case when the vacuum is unstable against particle production.

The latter may include the situation with final-state interactions. To distinguish this possibility, one should investigate the analytical properties of μ over n.

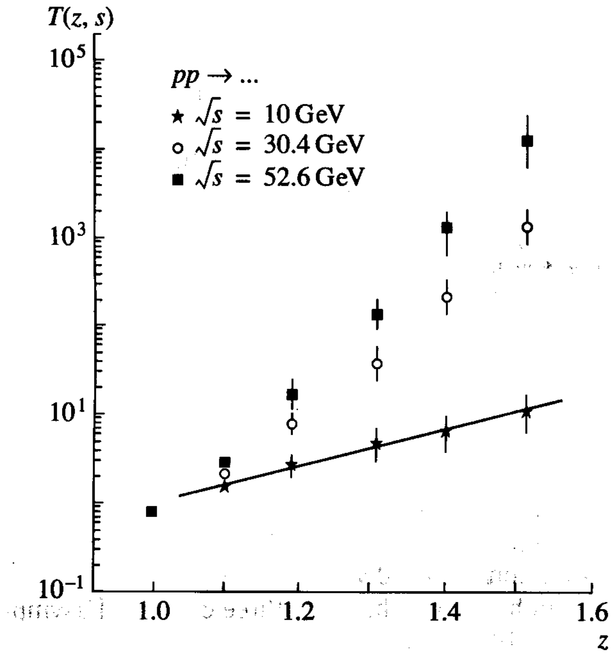

Fig. 5. Chemical potential vs. activity z. Straight line means the Poisson distribution.

3. THERMODYNAMICS

The multiple production amplitude, generally, is a function of $(3n-4)$ variables. This number is too high even if $n \sim \bar{n}(s)$ since at LHC energies $\bar{n}(s) \simeq 100$.

One may write:

$$T(z,s) = \sum_n z^n \sigma_n(s)/\sigma_{\text{tot}}(s) = e^{C(z,s)}.$$

Then exist the decomposition:

$$C(z,s) = \sum_k (z-1)^k C_k(s)/k!,$$

where C_k is the binomial moment, $C_1(s) = \bar{n}(s)$.

Figure 5 shows that the higher multiparticle correlators become important with rising the energy and multiplicity. Indeed, the straight line corresponds to the Poisson distribution. The deviation from the straight line means that the multiplicity distribution is wider than the Poissonian. In this case the higher C_k should be taken into account and, accordingly, we lose the hope to describe completely the hadron dynamics.

Therefore, it is important to find conditions when the number of essential variables is sufficiently small. This condition will define the "thermalization" region. Now, one of the most important results of our investigations is the following: if

$$|K_l(E,n)| \ll |K_2(E,n)|^{l/2}, \quad l = 3,4,\ldots, \tag{1}$$

where K_l is the ordinary l-particle energy correlator then the energy spectrum of produced particles is defined by one parameter $1/\beta$, which is the mean energy of produced particles. It is important that (1) is the criterium of thermalization. In practice this means that without checking this condition it is *impossible* even to discuss the thermodynamical property of the system.

The correlation functions are usually defined as follows:

$$K_2(n,E) = \langle \varepsilon^2; n, E\rangle - \langle \varepsilon^1; n, E\rangle,$$
$$K_3(n,E) = \langle \varepsilon^3; n, E\rangle - 3\langle \varepsilon^2; n, E\rangle \times$$
$$\times \langle \varepsilon^1; n, E\rangle + 2\langle \varepsilon^1; n, E\rangle^3,$$

etc., $E = \sqrt{s}$. Here,

$$\langle \varepsilon^l; n, E\rangle = \frac{\int \varepsilon(q_1) d^3q_1 \varepsilon(q_2) d^3q_2 \cdots \varepsilon(q_l) d^3q_l \{d^{3l}\sigma_n(E)/d^3q_1 d^3q_2 \cdots d^3q_l\}}{\int d^3q_1 d^3q_2 \cdots d^3q_l \{d^{3l}\sigma_n(E)/d^3q_1 d^3q_2 \cdots d^3q_l\}}$$

is the l-particle mean energy and

$$d^{3l}\sigma_n(E)/d^3q_1 d^3q_2 \cdots d^3q_l$$

is the corresponding differential cross section.

In conclusion, we can show that if the inequality (1) is held true, then the produced particles system may be described using the formalism of thermodynamics. This important conclusion is general, it weakly depends on details of dynamics. In other words, the proof of this statement is formal for the hadron system and uses only one fact that the fluctuations in the thermalized state must be Gaussian.

Having the quantitative definition of the equilibrium state, one may investigate predictions of the existing theories. First of all, we can prove that the system should be equilibrium in the domain B. It is easy to find what we have in the domain B:

$$\frac{|K_3(E,n)|}{|K_2(E,n)|^{3/2}} \sim \frac{1}{n}.$$

This estimation is the model independent conclusion.

3.1. Theory

Now let us consider the question: can we predict a tendency to the thermalized state? Considering the production as a process of thermalization, let us review the possible mechanisms of the hadron production.

First of all, the most popular in hadron physics is the multiperipheral model. It describes the "soft" channel of hadron production. The phase space domain, corresponding to the "Regge" production mechanism, is shown in Fig. 6. It means that the longitudinal momenta of secondaries are large, but the transverse ones are small. Till now there has been no well established quantitative theory of these processes.

The second model is based on the perturbative QCD and it describes the "hard" channel. The typical kinematics is the same as in deep inelastic scattering (DIS) processes. It means that, opposite to Regge, the DIS kinematics, shown in Fig. 6, assumes that the transverse momenta of secondaries are large and the longitudinal momenta are relatively small.

As it has been explained above, the hard kinematics must dominate in the VHM region. But it is important to note that the VHM region did not overlap either Regge, or DIS regions, see Fig. 6. This means that for Regge and DIS theories one can not apply the ordinary formalism in the VHM domain.

We would like to explain the last conclusion in detail. This would also explain why the thermalization effect is important for hadron physics.

3.2. Multiperipheral Model

The main statement of the Regge multiperipheral model looks as the condition that the transverse momentum of secondaries is restricted.

Then the amplitude in the one-Pomeron "Born approximation":

$$A_{ab}(s,t) = ig_a g_b (s/s_0)^{\alpha(t)-1},$$

$$\alpha(t) = \alpha(0) + \alpha'(0)t, \quad 0 < \alpha(0) - 1 \ll 1,$$
$$\alpha'(0) = 1\,\mathrm{GeV}^{-2}, \quad s_0 = 1\,\mathrm{GeV}^2,$$

well describes the experimental data at intermediate energies.

Multiplicity distribution: In the Pomeron "Born approximation" the topological cross section

$$\sigma_n(s) = \sigma_{\mathrm{tot}} e^{-\bar{n}(s)} (\bar{n}(s))^n / n!$$

This distribution has a sharp maximum at $n \simeq \bar{n}(s) \approx \approx a\ln(s/s_0)$. So, for the production of the $n \gg \bar{n}(s)$ particles, one should consider the exchange of $\nu \sim \sim n/\bar{n}(s)$ Pomeron.

But this production mechanism is restricted. So, the contribution of the diagram with ν Pomeron exchange gives, since the diffraction radii increase with s, the decreasing with ν mean value of the impact

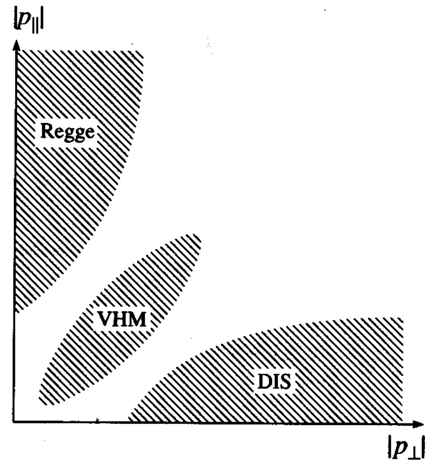

Fig. 6. Produced hadrons phase space.

parameter: $\bar{\mathbf{b}}^2 \simeq 4\alpha' \ln(s/s_0)/\nu \sim \alpha'\bar{n}(s)/\nu$. On the other hand, the number of necessary Pomeron exchanges $\nu \sim n/\bar{n}(s)$. As a result, $\bar{\mathbf{b}}^2(s)/\alpha' \sim \bar{n}^2(s)/n$. Therefore, if the transverse momentum of created particles is a restricted quantity, i.e., $\bar{\mathbf{b}}^2/\alpha' \gtrsim 1$, then the Pomeron mechanism of the particle production is valid if $n \lesssim \bar{n}^2(s)$.

In the frame of the multiperipheral model the cross section must sharply fall down at

$$n > \bar{n}^2(s)$$

since there is no interaction at $\bar{\mathbf{b}}^2 < \alpha'$ in this model.

Therefore, the multiperipheral picture is applicable if, and only if, the multiplicity is smaller than the square of the mean multiplicity. From the outside of this region the cross section must fall down rapidly.

3.3. Perturbative QCD

Now let us consider n particles (gluons) creation in the DIS kinematics. We calculate $D_{ab}(x,q^2;n)$, where

$$\sum_n D_{ab}(x,q^2;n) = D_{ab}(x,q^2)$$

and $D_{ab}(x,q^2)$ is the probability to find parton b with virtuality $q^2 < 0$ in the parton a of $\sim\lambda$ virtuality, $\lambda \gg \gg \Lambda$ and $\alpha_s(\lambda) \ll 1$. One also should assume that x is sufficiently small, $(1/x) \gg 1$, to have the produced particles phase space sufficiently large. Then $D_{ab}(x,q^2)$ is described by ladder diagrams of Fig. 7.

One can conclude that the LLA is applicable in the VHM domain till the diffusion time τ is sufficiently large:

$$\tau = \ln(-q^2/\lambda) \gg \ln(1/x) \gg \omega(\tau, z), \qquad (2)$$

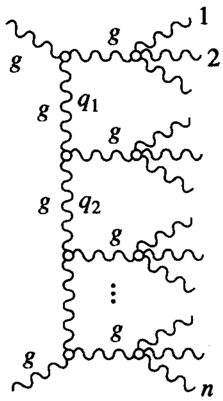

Fig. 7. Feynman ladder diagram: $-q_1^1 \gg -q_2^2 \gg \ldots$

where

$$\omega(\tau, z) = \sum_n z^n \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'} w_n^g(\tau'), \quad \omega(\tau, 1) = \ln(\tau/\tau_0),$$

is the generating function of the gluon jet multiplicity distribution. It is important to note here that all $w_n^g(\tau)$ are positive. It is a function with sharp maximum at $\ln n \simeq \ln \bar{n}_j(\tau) \sim \sqrt{\tau}$. Therefore, to have $n \gg \bar{n}_j(\tau)$ — jet mean multiplicity, one should choose $z > 1$.

For this reason the inequality (2) can not be satisfied in the VHM region. Indeed, one always may find such large $n \gg \bar{n}_j(\tau)$ that, for fixed q^2, the inequality (2) becomes failed.

The reason why this DIS kinematics, based on the LLA assumption, can not be considered in the VHM region leads to the evident conclusion that in order to have the thermalized state the mean values of the transverse and longitudinal momenta must be close to each other. Just this dynamical condition is in contradiction with LLA ideology.

4. SCENARIO FOR VHM PROCESS

Our multiple productions scenario is the following:

The multiperipheral model is applicable for n less than n_s. The *rough* estimation gives that $n_s \sim \bar{n}^2(s)$. One may expect that at high energies n_s is smaller than $\bar{n}^2(s)$.

When the multiplicity exceeds n_s (Fig. 8) one should take into account the hard processes. This means the production of (mini)jets and dominance of Fig. 7 diagrams. So, this contribution may be described in the frame of LLA.

But, in the VHM region, when the multiplicity exceeds $n_h > n_s$, the LLA can not be used since all components of the particle momentum are comparable to each other in this domain.

We will define the VHM region as the multiplicity domain where one may use the thermodynamical description.

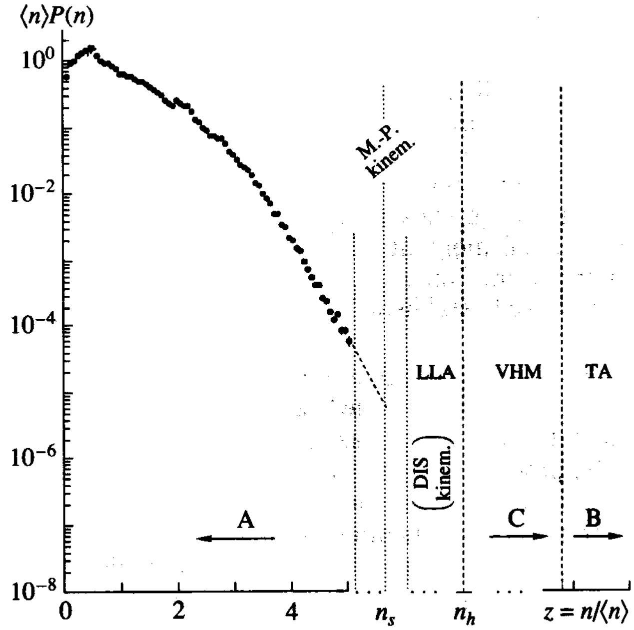

Fig. 8. (A): the range of multiperipheral models, $n < n_s$; (B): the range of deep asymptotics over n, where particles momentum $|p| \ll m$; (C): the VHM domain; $n_s < n < n_h$ the transition region into the VHM domain.

4.1. Prediction of Existing Generators of Events

It is evident that neither in the Regge nor in the DIS kinematics one can see the thermalization phenomena and the VHM state can not be achieved by this production mechanisms. Let us consider the generator of events prediction to demonstrate this conclusion. It is enough to use experience of existing generators of events since they absorb all known information for smaller than n_h multiplicity.

The PYTHIA prediction for ratio K_3 to K_2 is shown in Fig. 9. Notice that it does not predict even tendency to equilibrium. There is no wonder, since PYTHIA is based on the hadron peripheral interactions phenomenology. We can conclude that

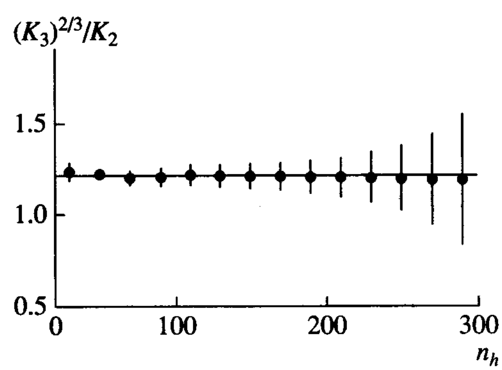

Fig. 9. PYTHIA: K_3/K_2.

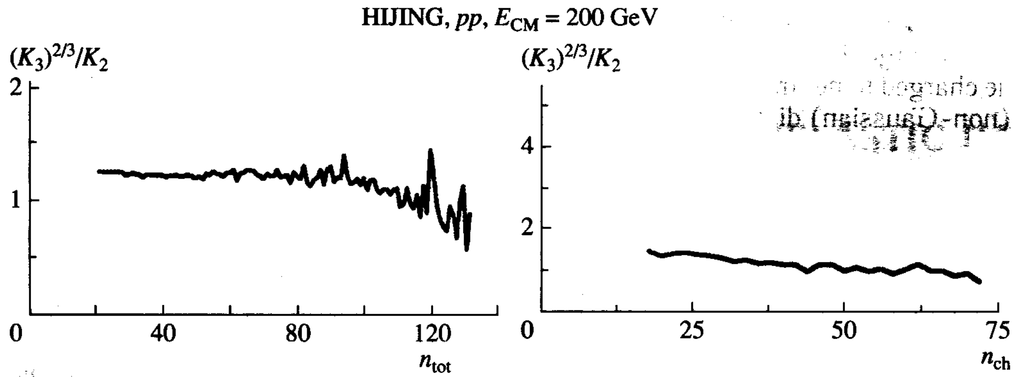

Fig. 10. HIJING: K_3/K_2. The soft tendency to the thermalization is seen from this picture. From this point of view the ion collisions, probably, are interesting.

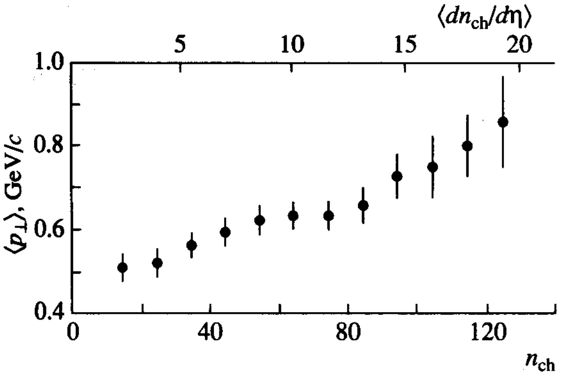

Fig. 11. Transverse momentum vs. multiplicity. The Tevatron data (E-735 Group) [24]. This result is in strong contradiction with the multiperipheral model.

the investigation in the VHM domain will allow to define the range of applicability of the peripheral picture of the hadron interaction.

Figure 10 shows a prediction of the HIJING generator of events for the K_3/K_2 ratio. The slight tendency to equilibrium in this model is explained by the fact that this generator includes rescattering based on the possibility to take into account more accurately the phenomena of scattering on the higher angles.

5. CONCLUSION

At the end we would like to mention the following question.

5.1. Rough Description

We adhere to the position that only "rough measurements" can be performed in the VHM domain. The reasons are as follows:

(i) The VHM cross sections are extremely small.

(ii) The multiplicity is the hardly measurable parameter.

(iii) It is practically impossible to restore the VHM kinematics completely.

This conditions will be taken into account in the VHM Generator of Events.

5.2. Experiment Program

Notice that our prediction that the VHM processes are "hard", i.e., that the VHM dynamics includes the production of highly virtual and extremely slow partons, has an immediate experimental confirmation, see Fig. 11, where the E-735 Group data from Tevatron show that the hadron-production processes become harder with the rising multiplicity.

First of all, it seems important to investigate experimentally the following problems:

(i) The thermalization problem.

(ii) Quantitative definition of the range of validity of the LLA in the VHM domain.

It was mentioned that the VHM problem highlights the most sensitive questions of the hadron physics.

(A) *Phase transition in the colored state.* The VHM gives a good chance for it, since the state is "calm" and "cold". The latter means that the interaction energy is larger than the kinetic one if we have the VHM final state.

(B) *The "preconfinement" VHM state presents the equilibrium coloured plasma.* This means that it can be characterized by a few global parameters. In this sense it will be the "state".

(C) *Measurement of the ratio* $R = \langle p_\parallel \rangle / \langle p_\perp \rangle = \pi/4$. For isotropic case, when the end of produced particle momenta locate on the sphere.

(D) *The process of VHM production must be "fast".* In this case the isotop spin orientation may be frozen randomly. Experimentally it looks like large fluctuations of the charge: if $C = n_{\rm ch}/n_0$ is the ratio of number of the charged to neutral particles, then the "anomalous" (non-Gaussian) distribution over C is expected.

It must be noted at the end that:

(i) There is a definite indication that the thermalization exists in the heavy ion collisions.

(ii) The estimation of the threshold value of multiplicity, n_h exceeds the possibility of LLA of perturbative QCD. Moreover, existing models are unable to find n_h.

(iii) The established S-matrix interpretation of necessary and sufficient condition of thermalization is important from experimental point of view. Besides, it allows to show that the thermalization occur, at least, in the deep asymptotics over n.

(iv) New perturbation theory, the topological QCD, which conserves the topology of the Yang–Mills fields, was builded. It is important that it includes the pQCD as an approximation.

Various experimental approaches to the VHM physics are widely discussed in the ATLAS, CDF, CMS [25], and STAR Collaborations.

The construction of the "fast generator" of the VHM events is the largest problem. There is also necessity to investigate the VHM triggers ideology in details.

It must be noted once more that only in the frame of statistic approach there is a hope to describe completely the inelastic hadron reactions. But, at the same time, we would like to stress that, generally speaking, actually not all many-particle state can be described using the thermodynamical methods. Such attempts may lead to a wrong conclusion: first of all one must include into consideration the necessary and sufficient conditions of thermalization.

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REFERENCES

1. *Proceeding of the First, Second, and Third International Workshops on Very High Multiplicity Physics, Dubna, 2000–2002*; J. Manjavidze and A. N. Sissakian, in *Proceedings of the Bogolyubov Conference on Problems of Theoretical and Mathematical Physics, Moscow–Dubna–Kiev, 27 Sept.–6 Oct., 1999*; in *Proceedings of the XXXI International Symposium on Multiparticle Dynamics, Datong, China, 2001*; in *Proceedings of the XXXII International Symposium on Multiparticle Dynamics, Alushta, Ukraine, 2002*; in *Proceedings of the 31st International Conference on High Energy Physics, Amsterdam, The Netherlands, 2002.*
2. E. Fermi, Prog. Theor. Phys. **4**, 570 (1950); Phys. Rev. **81**, 683 (1951); **92**, 452 (1953).
3. L. D. Landau, Izv. Akad. Nauk SSSR, Ser. Fiz. **17**, 85 (1953).
4. J. Manjavidze and A. N. Sissakian, Phys. Rep. **346**, 1 (2001).
5. J. Manjavidze, this Proceedings; J. Manjavidze and A. N. Sissakian, this Proceedings.
6. J. Manjavidze and A. N. Sissakian, JINR Rapid Commun., No. 5[31]-88, 5 (1988).
7. R. Hanbury-Brown and R. Q. Twiss, Nature **178**, 1046 (1956); G. Goldhaber *et al.*, Phys. Rev. **120**, 300 (1960).
8. G. A. Kozlov, this Proceedings.
9. R. Lednický, this Proceedings.
10. P. F. Ermolov *et al.*, this Proceedings.
11. G. Chelkov, M. Gostkin, J. Manjavidze, A. Sissakian, and S. Tapprogge, JINR Rapid Commun., No. 4[96]-99, 45 (1999); G. Chelkov, J. Manjavidze, and A. Sissakian, JINR Rapid Commun., No. 4[96]-99, 35 (1999); J. Budagov, G. Chelkov, Y. Kulchitsky, *et al.*, *Talk at the Physics ATLAS Week, Lund, 2001.*
12. J. Manjavidze and A. Sissakian, JINR Rapid Commun., No. 2[28], 51 (1988).
13. V. A. Nechitailo, this Proceedings.
14. J. A. Budagov *et al.*, this Proceedings.
15. A. Korytov, this Proceedings.
16. L. N. Lipatov, this Proceedings.
17. J. Manjavidze and A. N. Sissakian, J. Math. Phys. (N. Y.) **41**, 5710 (2000); **42**, 641, 4158 (2001).
18. J. Manjavidze and A. N. Sissakian, Theor. Math. Phys. **123**, 776 (2000); **130**, 153 (2002).
19. J. Manjavidze and A. Sissakian, hep-ph/0201182.
20. J. Manjavidze and V. V. Voronyuk, this Proceedings.
21. I. M. Dremin, this Proceedings.
22. A. N. Sissakian and L. A. Slepchenko, Preprint P2-10651, JINR (Dubna, 1977); Fizika (Zagreb) **10**, 21 (1978).
23. F. Rimondi, this Proceedings.
24. T. Alexopoulos *et al.*, Phys. Rev. Lett. **64**, 991 (1990).
25. O. Kodolova, unpublished.