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Production of heavy quarkonia and new Higgs physics at hadron colliders

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Abstract
Based on the two Higgs doublet model, we study the effect of Higgs–boson exchange on the heavy quarkonium $\bar{Q}Q$ which induces a strong attractive force between a heavy quark Q and an antiquark $\bar{Q}$. An interesting application is the decay of heavy quarkonia $\bar{Q}Q$ to a Higgs boson associated with gauge bosons. The criterion for making the $\bar{Q}Q$ bound state is studied. We also show that nonperturbative effects due to gluonic field fluctuations are rather small in such a heavy quark sector. Possible enhancement for production and decays of $\bar{Q}Q$ bound states coming from the fourth generation quark Q is discussed for $\bar{p}p$ (at the Tevatron) and pp (at the LHC) collisions.

1. Introduction

A study on quarkonia $T(\bar{Q}Q)$ composed of a heavy quark Q and an antiquark $\bar{Q}$ (a possible and interesting candidate for Q is the up(U)- and/or down(D)-quarks in the fourth generation family) is required in current particle physics for testing the standard model (SM) and/or searching for signals for physics beyond the SM. It is nowadays one of the most interesting subjects since it can be studied, with high priority, in the experiments at high energy hadron colliders, i.e. the Tevatron and the forthcoming LHC. In particular, no theoretical arguments are seen to rule out the heavy quarks and the heavy quarkonium states with masses around a hundred GeV or even a few TeV (see, e.g., [1–4]). A motivation for exciting interest to explore and study the fermions (mostly the quarks) of the fourth generation can be obtained from the review [2] by Frampton *et al*. Namely this motivation is based on the grand unification theory (GUT) extended models view, the CP-violation problem, gauge-mediated supersymmetry breaking models view, the higher-dimension reasons at TeV scale where vector-like heavy fermions may occur, the point of view where the fourth generations can have successful unification of the gauge couplings at the unification scale. It was shown [3, 4] that according to the latest data third and fourth SM family cases have similar status. In addition, within the vacuum stability reasons the upper bounds of fermion (quark) masses can be obtained from the requirement that fermionic (quark) corrections to the effective potential do not destabilize the SM vacuum.

As a recent example, in one of the extended models, the little Higgs model [5], there could be an additional heavy quark $\tilde{q}$ with a mass of the order of O(1 TeV) to promote the quark doublet q to quark triplet under the global $SU(3)$, $Q = (q, \tilde{q})$. By preserving the global symmetry of the coupling, the one-loop quadratic divergence to the top quark is removed. In fact, the SM and its extensions, e.g., the minimal supersymmetric standard model (MSSM), do not explain the family (generation) structure of the quark masses. Each quark has an arbitrary Yukawa coupling and hence is independent of the family to which it belongs. It is required to explain the family structure and the Cabibbo–Kobayashi–Maskawa (CKM) matrix for the quark sector in any extension of the SM or even in the SM.

It is known from the history of particle physics that the first signals for c- and b-quarks in hadronic collisions were leptonic decays of their $J/\psi(\bar{c}c)$ or $\Upsilon(\bar{b}b)$ bound states. Can new heavy quarks also be first discovered through the decay of their quark–antiquark bound states into lepton pairs? It seems the answer is apparently not transparent, because one of the main properties of very heavy quarkonia is concerned with the appearance of new decay modes into weak bosons and even Higgs bosons in the final states. Once the Higgs boson H is discovered, one needs to measure its couplings to other particles. The value of H-boson couplings can be extracted by measuring a variety of Higgs boson production and its decay modes. Thus, it is important to find the H-boson in as many channels as possible, including its production coming from the decay of very heavy quarkonia. Such a Higgs boson can be looked for in the following new decay modes; $T(\bar{Q}Q) \to HZ, T(\bar{Q}_1 Q_1) \to HT(\bar{Q}_2 Q_2), T(\bar{Q}Q) \to \gamma H$ and $T(\bar{Q}Q) \to ggH, \gamma\gamma H$, where $T(\bar{Q}Q)$ carries quantum numbers of $J^{PC} = 1^{--}, 0^{-+}$. The importance of the search for heavy vector and pseudoscalar quarkonia decays into a Z and a neutral Higgs boson has been attached in [6–8]. Relevant to the vector states it was a complement to the Wilczek radiative decay of the spin-1 particle with an emission of a Higgs boson [9]. In the papers [6, 7], based on the quantum chromodynamics (QCD)-inspired potentials, it was shown that bound states composed of fourth-generation quarks give the substantial signals in pp-collisions at high energies. In particular, the decays of these bound states could lead to the identification of both the Higgs boson and a fourth-generation quarkonium for a wide range of their masses.

Since heavy quarkonia provide an ideal set of observables to probe the properties of low-energy QCD there was a wealth of theoretical advances in physics of heavy quarkonia based on the paper [10] by Caswell and Lepage and the review [11] by Bodwin *et al* (see, for example, the papers [12, 13] and the references therein). In general aspects the theory of heavy quarkonia decays is provided by non-relativistic QCD (NRQCD) which is obtained from QCD by integrating out the degrees of freedom of the typical energy M associated with the heavy quark mass. The effective field theory obtained by subsequent matching from QCD, where only the lightest degrees of freedom of the energy Mv^2 (v is the relative heavy quark velocity) are left dynamical, is the potential NRQCD [14, 15]. Many theoretical approaches (see, for example, the papers [2, 16] and the references therein) were used to deal with heavy quarkonia physics and relied on the separation between long- and short-distance physics in accordance with the main lines on NRQCD [10, 11]. A number of papers were devoted to the indirect manifestations of the fourth family, e.g. in B-meson decays [17, 18]; however, there are many SM extensions which lead to similar consequences [2].

Quark–antiquark bound states with masses <1 TeV would be produced via gluon fusion with substantial cross sections at hadron colliders with a centre-of-mass energy $\sqrt{s} \sim \mathcal{O}$ (10 TeV). Due to the small mixing of the fourth SM family quarks with those of the first three ones, the fourth family constituents can form corresponding quarkonia which will lead to spectacular signature at hadron colliders [19]. Production of heavy quarkonia such as $\bar{b}b$ and $\bar{t}t$ associated with a Higgs boson emission in the decay of extra gauge bosons Z' was studied in [20, 21].

The decays of ultra-heavy quarks into light quarks and W- and Z-bosons or Higgs bosons were discussed in [22]. The direct decay of the top quark, $t \to Wb$ [23–25], and the flavour changing top quark decays, $t \to c\gamma(cg, cZ)$ and $t \to cH$ [26], in both the SM and two-Higgs doublet model (2HDM), have been studied intensively for the last decade. The SM predictions of the branching ratios (BR) for those decays $t \to c\gamma, t \to cg, t \to cZ$ and $t \to cH$ are significantly small, being BR $\sim 5 \times 10^{-13}, 4 \times 10^{-11}, 1.3 \times 10^{-13}$ and 10^{-14}–10^{-13}, respectively. Here we are interested in the case in which the new decay modes mentioned above become dominant and hence the branching ratio for a single heavy quark decay accompanying the real weak boson emission $Q \to qW$ (q is a lighter quark), leaving q as a spectator, is small. Of course, it is necessary to examine whether the spectator mode can be dominant or not. Direct search for the fourth generation quarks U and/or D is an ongoing process at the Fermilab Tevatron. The detection of the U quark would then depend on the decay properties of the D quark and the mass difference $M_U - M_D$. One can refer to [27] and [28] for a discussion of this subject, where in particular Gunion *et al* [27] find the only way that $\bar{U}U$ events can evade being included in the CDF and D0 Tevatron data sample is if $M_U - M_D$ is sufficiently small so that the $W^\star$ in $U \to DW^\star$ is virtual and the jets and leptons from the two $W^\star$ are soft. Obviously, the rare decays are not of interest here, since they are very difficult to observe in hadron colliders even at the highest luminosity and thus we neglect them in this work. Current experiments at Tevatron Run II or future experiments at forthcoming LHC closely approach the rate required for ruling out the Higgs boson production or discovering it through the decay of very heavy quarkonia mentioned above. In this connection, precise theoretical estimates of the rates are required for an unambiguous interpretation of experimental upper limits.

One cannot exclude the possibility of the new strong interactions which primarily control the dynamics of very heavy quarks such as the fourth generation up(U) and/or down(D) quarks. In one of the ‘top-colour’ models [29] with 1 TeV scale, there is the following ‘top-colour’ gauge structure,

$$SU(3)_4 \times SU(3)_h \times SU(3)_l \times U(1)_{Y_4} \times U(1)_{Y_h} \times U(1)_{Y_l} \times SU(2)_L \\ \to SU(3)_{QCD} \times U(1)_{EM}, \tag{1}$$

where $SU(3)_4 \times U(1)_{Y_4}$, $SU(3)_h \times U(1)_{Y_h}$ and $SU(3)_l \times U(1)_{Y_l}$ are generally coupled to the fourth, third and first two generations, respectively. The $U(1)_{Y_i}$ are just rescaled versions of the electroweak $U(1)_Y$ into the strongly interacting world. In this model, below the symmetry-breaking scale μ_{SB}, the spectrum includes massive ‘top-gluons’ which mediate vectorial colour-octet interactions among heavy quarks Q (=t, U, D)

$$-\left(4\pi\kappa/\mu_{SB}^2\right)\left(\bar{Q}\gamma_\mu\frac{\lambda^a}{2}Q\right)^2. \tag{2}$$

If the coupling κ lies above some critical value $\kappa_{\rm crit}$, the heavy quark condensate $\langle\bar{Q}Q\rangle$ can be formed. The strong ‘top-colour’ dynamics can bind $\bar{Q}$ and Q into a set of ‘heavy-pions’ $(\bar{Q}Q)$. The criterion for existence of a heavy quarkonium is that the binding energy ϵ_B should be larger at least than the total decay width $\Gamma_{\rm tot}$ of its quarkonium, namely $c = (\Gamma_{\rm tot}/\epsilon_B) < 1$.

Since such a heavy quark–antiquark bound state is considered to be a nonrelativistic system, the quark potential model should be applicable to the analysis. Creation of $\bar{Q}Q$ out of vacuum may be resulting also in a screening of quark colour charges at large distances. The breaking of colour flux tubes or the splitting of quarks may occur. Beyond the splitting energy the same interaction with sea quarks can give rise to hadronization. The implementation of an interaction between $\bar{Q}Q$ with effective one gluon and/or boson exchanges, or other effective interactions, turns out to be fruitful in the construction of a quark potential model that provides a precise description of heavy quarkonia properties. Hence, as pointed out in [30–32], one cannot exclude the possibility of the Higgs–boson interaction which dominates significantly over the one-gluon exchange $\sim -(4/3)\alpha_s(m_Q)/r$ for the very heavy quarkonium, where α_s is the strong coupling constant depending on the quark mass m_Q and r is a distance between a quark and an antiquark. We show in section 2 that the strong binding force due to the Higgs–boson exchange gives rise to a necessary condition $c < 1$ for enabling the heavy quarkonium to exist and leading to observation of its resonance.

On the other hand, in the physics of interplay among quarks, it is well known that the exact QCD vacuum should contain fluctuations of gluonic fields at large scales [33]. These nonperturbative fluctuations cause the distortion of interactions between quarks and antiquarks. We consider heavy quarks as external objects allocated in the gluonic vacuum. Here, we study these nonperturbative fluctuation effects of the gluon field on the decay of a quarkonium $T(\bar{U}U)$ into the Higgs- and Z-bosons. Our result is based on the well-known statement (see, e.g., [33]) that the nonperturbative effect on dynamics of heavy quark systems is expressed in terms of vacuum expectation values of the local operators constructed from gluonic field operators. The leading effect is proportional to a matrix element of the form $\langle {G^a_{\mu\nu}}^2(0)\rangle_0$, where $G^a_{\mu\nu}(x)$ is the standard gluonic field strength tensor with colour indices $a = 1, 2, \ldots, 8$. The lowest level of the heavy quarkonium is determined by the colour-singlet Yukawa-type attractive force mediated by the 'light' scalar χ-boson. We were interested in the corrections due to non-perturbative gluonic fluctuations in the exact QCD vacuum. In this paper, we show that in a heavy quarkonium decay such as $T(\bar{U}U) \to hZ$ (h means the lightest CP-even Higgs–boson in 2HDM), the nonperturbative fluctuation effect of the gluonic field can be calculated, to some extent, without a detailed knowledge of the vacuum structure and, furthermore, it gives a negligible result.

The outline of this work is organized as follows. In section 2, we discuss an effective potential mediated by a Higgs boson. An effective model for the lower-energy theorem will be discussed in section 3. Finally, in section 4, we give our conclusion and discussion.

2. Effective potential via Higgs–boson exchange

Let us consider a production (in pp or $\bar{p}p$) of heavy quarkonium $T(\bar{Q}Q)$ followed by the decay process $T(\bar{Q}Q) \to hZ$ with $Q = t, U, D$, being assumed to be the dominant decay process of $T(\bar{Q}Q)$. Then, it is supposed that this dominant mechanism at high transverse momentum involves, e.g. the production of a gluon that produces a colour-octet $\bar{Q}$ and Q pair which then fragments into a colour-singlet bound state $T(\bar{Q}Q)$ by emitting two or more soft gluons. This $T(\bar{Q}Q)$-state should be transversely polarized at high transverse momentum, since it is emanated from a gluon which has only transverse polarization states.

In the lowest bound state $\bar{Q}Q$, the quark Q and the antiquark $\bar{Q}$ are assumed to be located at a distance

$$r \sim [m_Q \lambda(m_Q)]^{-1} \tag{3}$$

which is small compared to the scale of strong interactions; $\lambda(m_Q)$ is the strength of the interaction between a quark and an antiquark. The wavefunction of the lowest bound state is proportional to $\exp(-\mu r)$ with $\mu \sim m_Q\lambda(m_Q)$. Note that for $r_0 = \mu^{-1}$ being smaller than in the typical size of fluctuations, our approach becomes applicable since the potential dumps exponentially for distances $r \gg \mu^{-1}$.

Since the relative momentum in a heavy quark–antiquark bound state is supposed to be small enough there is a common belief to admit the nonrelativistic description and hence to use the potential formalism. In addition to the one-gluon exchange one can take into account the other contributions, e.g., those generated by a scalar boson exchange leading to a stronger effective $\bar{Q}Q$ potential. Let us consider the well-known expression for the effective potential between a quark Q and an antiquark $\bar{Q}$ characterized by their momenta k_μ and k'_μ, respectively at short distances (or large $q^2 = (\vec{k} - \vec{k}')^2$) in the form of a one-dimensional integral (the integration over the angles is performed already)

$$V(r) \sim \int_0^\infty \mathrm{d}q\, \alpha_s(q^2) \frac{\sin(qr)}{qr},$$

where $\alpha_s(Q^2) = 4\pi/[\beta_0 \ln(Q^2/\Lambda^2)]$ at sufficiently large Feynman four-momentum transfer $Q^2 = (k_\mu - k'_\mu)^2$, Λ sets the scale of the hadronic parameter, $\beta_0 = 11n_c - 2n_f$ (n_c and n_f are numbers of colour and flavour, respectively). Dealing with heavy $(\bar{Q}Q)$-bound states we are in the range of large q^2, and thus we need to seek the appropriate mechanism to avoid the formal divergence like $\ln\ln q$ at the upper limit. The simple way is to introduce the screening function (see also [34])

$$Y(q^2, m^2) = \frac{1}{1 + q^2/m^2}$$

containing the screening parameter m with the asymptotic condition $Y(q^2, m^2) \to 1$ at $m^2 \to \infty$, and thus one can get

$$V(r) \to V(r) \sim \int_\Lambda^\infty \mathrm{d}q\, \alpha_s(q^2) Y(q^2, m^2) \frac{\sin(qr)}{qr}.$$

Here $\alpha_s(q^2)Y(q^2, m^2)\sin(qr)/(qr)$ is a well-defined function, and for large enough q^2 the constant $\alpha_s(q^2) \to \alpha_s(q_0^2)$ has a very weak q^2-dependence in comparison with the rapid oscillated function $\alpha_s(q^2)\sin(qr)/(qr)$. Performing the formal integration in the limit $\Lambda^2/Q^2 \to 0$ one can get an additional contribution to that of the one-gluon exchange one, namely

$$V(r) \to \alpha_s(q_0^2)\left[-\frac{1}{r} + \frac{\exp(-mr)}{r}\right].$$

Here, the second term is nothing but the additional contribution due to scalar (Higgs) boson exchange with the screening mass m.

Let us consider a simple model (model I) where the dominant effective potential for a $\bar{Q}$ and Q system at a small distance looks like [31, 32]

$$V_{\mathrm{eff}}(r) \sim -\frac{C_F}{r}\alpha_s(m_Q) - \frac{\lambda(m_Q, \xi_{\chi Q})}{r}\exp(-m_\chi r), \tag{4}$$

with

$$\lambda(m_Q, \xi_{\chi Q}) = \frac{m_Q^2}{4\pi v^2}\xi_{\chi Q}^2, \tag{5}$$

and $\xi_{\chi Q}$ reflects the model 'flavour' in the strength of the interaction between the scalar χ-boson and a heavy quark Q ($\xi_{\chi Q} = 1$ in the minimal SM, otherwise $\xi_{\chi Q} > 1$); v is

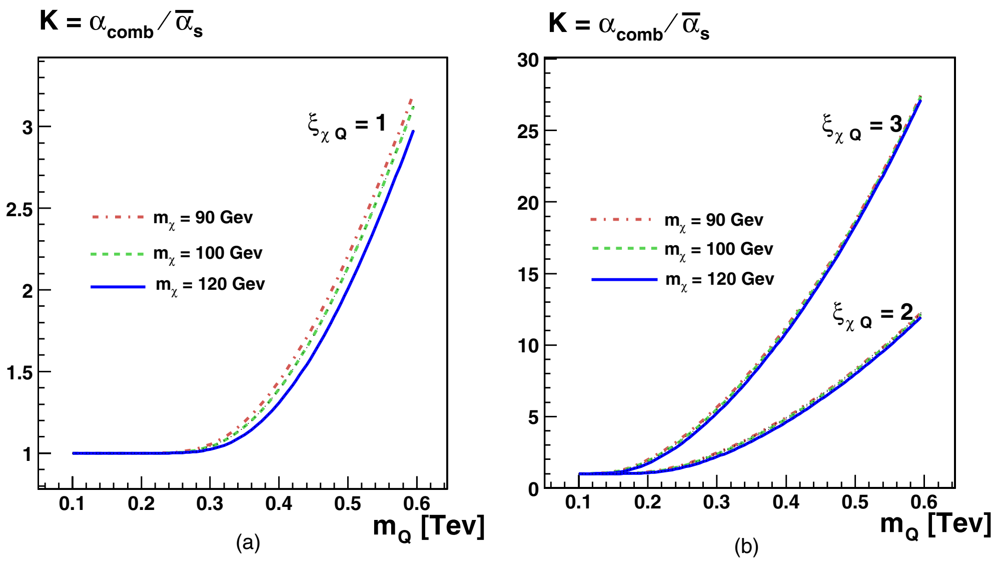

Figure 1. Ratio of the combined coupling α_{comb} to the pure QCD coupling $\bar{\alpha}_s$ as a function of a heavy quark mass for (a) $\xi_{\chi Q} = 1$ and (b) $\xi_{\chi Q} = 2, 3$. The curves are presented for χ-boson masses $m_\chi = 90$, 100 and 120 GeV.

the vacuum expectation value of Higgs boson, $v = 246$ GeV (in 2HDM, $v = \sqrt{v_1^2 + v_2^2}$, v_1 and v_2 are two neutral Higgs field vacuum expectation values) and C_F is the colour factor, $C_F = 4/3$ for the colour $SU(3)$ group. In figure 1, we show the ratio of the combined coupling $\alpha_{\text{comb}} = (4/3)\alpha_s(m_Q) + \lambda(m_Q, \xi_{\chi Q}) \exp(-m_\chi r)$ (see (4)) to the pure QCD coupling $\bar{\alpha}_s = (4/3)\alpha_s$, as a function of a heavy quark mass m_Q for different values of $\xi_{\chi Q}$ and m_χ. The second term in α_{comb} is appropriate for $r \sim [m_Q \lambda(m_Q)]^{-1}$. The ratio becomes somewhat bigger for smaller Higgs–boson masses.

Because of our demand, $\lambda(m_Q, \xi_{Q\chi}) > C_F \alpha_s(m_Q)$, for a relevance of the χ-boson interaction, the lower bound on m_Q is given as

$$m_Q > \frac{v}{\xi_{\chi Q}} (4\pi C_F \alpha_s)^{1/2}, \tag{6}$$

which leads to $m_Q \geqslant m_t$ even if $\xi_{\chi Q} = 2$ (see figure 2).

The requirement of the positivity of the variational parameter

$$\mu \simeq \frac{\lambda m_Q}{2} \frac{(\lambda m_Q)^2 - m_\chi^2}{(\lambda m_Q)^2 + 2m_\chi^2} \tag{7}$$

entering in both the bound state wavefunction $\Psi(r) = 2\mu^{-3/2} \exp(-\mu r)$ and the binding energy (see, for details, [32]),

$$\epsilon_B = 2\lambda \frac{\mu^3 (2\mu - m_\chi)}{(2\mu + m_\chi)^3} \tag{8}$$

leads to an upper limit on m_χ (see figure 3).

In model I, the ratio $c = (\Gamma_{\text{tot}}/\epsilon_B) < 1$ could be guaranteed for $T(\bar{U}U)$ quarkonia to be formed by a strong attractive force via scalar Higgs–boson exchange with a sufficiently 'hard' Yukawa coupling $\lambda(m_Q, \xi_{\chi Q})$. The total decay width Γ_{tot} is given by a sum of two terms

$$\Gamma_{\text{tot}} = \Gamma_T + \Gamma_U, \tag{9}$$

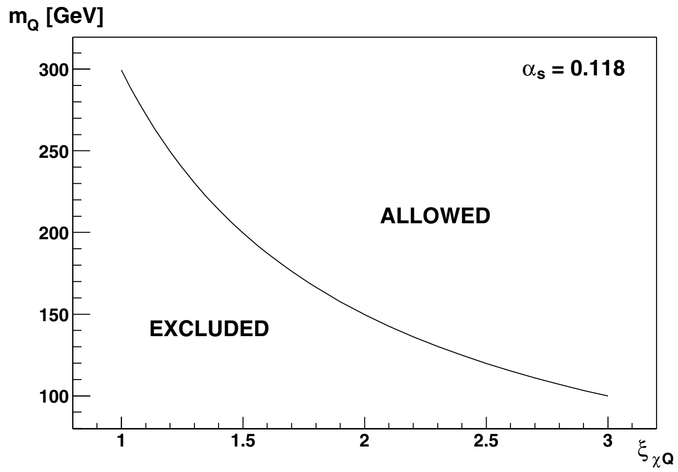

Figure 2. Lower bound on heavy quark masses as a function of $\xi_{\chi Q}$.

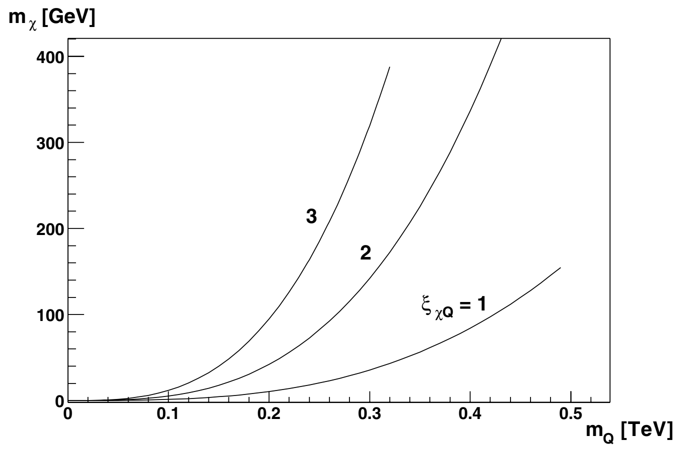

Figure 3. Upper limit on the scalar χ-boson mass as a function of m_Q for different values of $\xi_{\chi Q}$. The regions above the corresponding curves are excluded.

where the width Γ_T is defined by the following decay channels:

$$T(\bar{U}U) \to hZ, \gamma Z, \gamma h, W^+W^-, \bar{b}b, \bar{t}t, \tau^+\tau^-, \mu^+\mu^-, ggg,$$

and the single quark decay width Γ_U is a sum of the following contributions: $U \to DW^+, bW^+, bH^+$ (H^+ is the charged Higgs–boson in 2HDM). In the decays of $T(\bar{U}U)$ presented above, the radiative channel is suppressed by the coupling constant α, the ggg channel gives a small contribution due to the α_s^3 factor, and furthermore, the productions of the pairs of quarks and antiquarks or leptons and antileptons are also small because they follow via two-loop diagrams, where the amplitude is very small because of the presence of α (intermediate photons) or the Fermi constant G_F (virtual $W^{\pm}$-bosons).

We do not consider the contributions from the decay $U \to bH^+$ because it is expected to have a rather small probability to be observed as expected from the following consideration:

the Higgs–boson mass sum rule

$$m_{H^+}^2 \simeq \left(m_A^2 + m_W^2\right)(1+\delta) \tag{10}$$

with the one-loop correction $\delta < 10\%$ [35] does not allow the production of charged Higgs–bosons in the decay of top quark and, perhaps, also of U quark due to the kinematical reason in the decoupling limit $\left(m_W^2/m_A^2\right) \ll 1$ (m_A is the CP-odd Higgs–boson in 2HDM). In addition, the experimental data at the Tevatron do not yet clarify the status of the $t \to H^+ b$ decay. The CDF results in the direct search for τ-lepton emission from top quark decays give an upper limit on the branching ratio BR$(t \to H^+ b) \sim 0.5$–0.6 at 95% CL in the range 60 GeV $< m_{H^+} <$ 160 GeV, assuming BR$(H^+ \to \tau\nu_\tau) = 1$ [36]. Furthermore, the D0 Collaboration excludes BR$(t \to H^+ b) > 0.36$ at 95% C.L. in the region $0.3 < \tan\beta < 150$ and $m_{H^+} <$ 160 GeV [37]. Assuming $\sum_{X=H^+,W^+} \mathrm{BR}(t \to bX) = 1$, the decay width of the H^+-boson channel in U- and t-quark decays is small and hence this channel is not of interest.

Therefore, for the case of U quarks, one can expect that $\Gamma_{\rm tot}$ is given by

$$\Gamma_{\rm tot} = \Gamma_T(T(\bar{U}U) \to hZ, W^+W^-) + \Gamma_U(U \to DW^+, bW^+). \tag{11}$$

The main contribution to Γ_T arises from the channel $T(\bar{U}U) \to hZ$ whose decay width for $T(\bar{U}U)(1^{--})$ is given by the following expression (see, e.g., [32]),

$$\Gamma(T(\bar{U}U) \to hZ) = \frac{\lambda^3 m_U}{16}\eta_{hU}^2 \left[\lambda\alpha_z v_U^2 + \frac{1}{2}\alpha_W^2 \left(\frac{m_U}{m_W}\right)^4\right] f^3 \Phi, \tag{12}$$

with

$$\alpha_W = \alpha/s_W^2, \qquad \alpha_z = \alpha_W/c_W^2 \ (s_W \equiv \sin\theta_W,\ c_W \equiv \cos\theta_W),$$

$$v_U = \left(1 - \frac{8}{3}s_W^2\right), \qquad \Phi = \left(1 - \frac{4m_h^2}{M_T^2}\right)^{1/2},$$

$$f \equiv f(\lambda, m_U, m_\chi) = \frac{(\lambda m_U)^2 - m_\chi^2}{(\lambda m_U)^2 + 2m_\chi^2}, \qquad M_T \simeq 2m_U.$$

m_U is the mass of an up(U)-quark of the fourth generation. We suppose that the couplings of the Higgs–boson h and the U-quark have the same form as those for the couplings of the Higgs–boson h and the top-quark (see the review [38])

$$\eta_{hU} \simeq 1 + \frac{m_Z^2}{m_A^2} s_{2\beta} c_{2\beta} \tan^{-1}\beta, \tag{13}$$

where $s_{2\beta}(c_{2\beta}) \equiv \sin 2\beta(\cos 2\beta)$ and $\tan\beta$ is the standard ratio between two vacuum expectation values for two Higgs doublets in 2HDM.

Using the standard formulae [23, 24], the single U-quark decays are given for the $U \to DW^+$ channel by

$$\Gamma_U(U \to DW^+) = \frac{G_F m_U^3}{8\sqrt{2}\pi}|V_{UD}|^2 \left[\left(1 - \frac{m_D^2}{m_U^2} - \frac{m_W^2}{m_U^2}\right)^2 - 4\frac{m_D^2 m_W^2}{m_U^4}\right]^{1/2}$$
$$\times \left[\left(1 - \frac{m_D^2}{m_U^2}\right)^2 + \left(1 + \frac{m_D^2}{m_U^2}\right)\frac{m_W^2}{m_U^2} - 2\frac{m_W^4}{m_U^4}\right], \tag{14}$$

and for $U \to bW^+$ decay by

$$\Gamma_U(U \to bW^+) = \Gamma_0(U \to bW^+)(1 - \Delta), \tag{15}$$

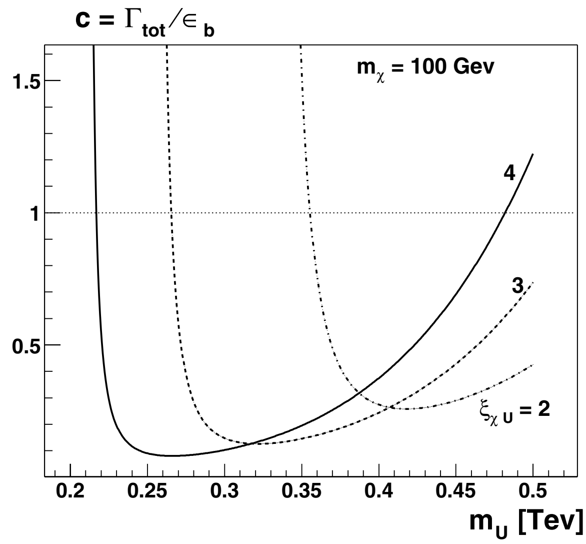

Figure 4. Ratio $c = \Gamma_{\rm tot}/\epsilon_B$ being responsible for occurrence of the bound state $T(\bar{U}U)$ as a function of the quark mass m_U at different values of $\xi_{\chi U}$ for $m_\chi = 100$ GeV. The region below the value $c = 1$ is allowed.

where

$$\Gamma_0(U \to bW^+) = \frac{G_F m_U^3}{8\sqrt{2}\pi} |V_{Ub}|^2 \beta_W^4 \left(3 - 2\beta_W^2\right) \tag{16}$$

with $\beta_W = \sqrt{1 - m_W^2 / m_U^2}$. V_{UD} and V_{Ub} are the generalized CKM matrix elements and the $O(\alpha_s)$ QCD correction [24] in (15) is

$$\Delta = \frac{C_F \alpha_s}{2\pi} \left(\frac{2\pi^2}{3} - \frac{5}{2} \right). \tag{17}$$

As an example, for a typical set of parameters: $m_U = 400$ GeV, $m_D = 300$ GeV, $m_\chi \simeq m_h = 100$ GeV, $|V_{UD}| \simeq 1$, $\xi_{\chi U} = 2$ ($\eta_{hU} \simeq 1$) we give the numerical results for those decay widths

$$\Gamma(T(\bar{U}U) \to hZ) \simeq 2.2\eta_{hU}^2 \text{ GeV},$$
$$\Gamma(U \to DW^+) \simeq 1.21 \text{ GeV}, \qquad \Gamma(U \to bW^+) \simeq 0.0021 \text{ GeV}, \tag{18}$$

where $c = \Gamma_{\rm tot}/\epsilon_B = 0.27$ at the calculated value $\epsilon_B = 12.65$ GeV.

The contribution of the decay $U \to bW^+$ is negligible due to the rather small value of $|V_{Ub}| \sim 10^{-2}$. We have checked the possibility of the existence of bound states $T(\bar{Q}Q)$ composed of U (and D)-quarks, starting at the lowest value of $\xi_{\chi Q} \geqslant 1$, where the increasing $\xi_{\chi Q}$ gives rise to an effect on $c^{-1} > 1$ (see figure 4).

The expected event topology of the decays $U \to bW^+$ and $U \to DW^+$ is similar to that of $t \to bW^+$. However, for the down-type heavy quark D, the dominant decay mode could be $D \to tW^-$ and thus, in the case of $\bar{D}D$-pair production, the final state can consist of two pairs of leptons and neutrinos (originated from decays of the t quark and W boson) with different flavours, in general. Now, by taking, as an example, the following parameters: $m_D = 400$ GeV, $m_\chi \simeq m_h = 100$ GeV, $\xi_{\chi D} = 2$, $|V_{Dt}| \simeq 0.012$, we can obtain the following decay widths:

$$\Gamma(T(\bar{D}D) \to hZ) \simeq 2.25\eta_{hD}^2 \text{ GeV}, \qquad \Gamma(D \to tW) \simeq 1.47 \text{ MeV}. \tag{19}$$

One can see that the decay width $\Gamma(T(\bar{D}D) \to hZ)$ can be enhanced by the Yukawa-flavour factor (within 2HDM) $\eta_{hD} = -\sin\alpha/\cos\beta$ at large values of $\tan\beta$.

For comparison, let us consider an instructive example, i.e. the decay of $T(\bar{U}U)(0^{-+})$ states into h- and Z-boson where the decay width is given by the following expression (see, e.g., [32]):

$$\Gamma(T(\bar{U}U) \to hZ) = \frac{3\lambda^3 m_U}{32}\eta_{hU}^2\alpha_z^2\left(\frac{m_U}{m_Z}\right)^4 f^3\Phi. \tag{20}$$

The numerical estimation gives $\Gamma(T(\bar{U}U) \to hZ) \simeq 6.80\,\eta_{hU}^2$ GeV which is roughly three times larger than that in the case of $T(\bar{U}U)(1^{--}) \to hZ$ decay mode given in (18) and hence leads to a larger production rate of $T(\bar{U}U)(0^{-+})$ than $T(\bar{U}U)(1^{--})$. This simple example confirms our belief that the most promising candidate for the (super)heavy quarkonium which could be searched at the LHC should be the pseudoscalar state $T(\bar{U}U)(0^{-+})$.

It is interesting to estimate the effect of the scalar χ-boson exchange on the $(\bar{Q}Q)$ production cross-section at different m_χ as a function of m_Q.

If the Q-quark is relatively long-lived, the peak of the cross-section at a $\bar{Q}Q$ resonance due to the one-gluon exchange is given by

$$\sigma_c \sim \alpha_s^3\left(\frac{m_Q}{\Gamma_Q}\right)\frac{1}{s} \tag{21}$$

for a given centre-of-mass energy s. Cross-section (21) has a strong sensitivity to α_s and decreases sharply with increasing m_Q because of the rapid growth of Γ_Q (see equations (14) and (15)). To leading order, the scalar χ-boson exchange effect is taken into account simply by making the replacement

$$\alpha_s \to \alpha_s + \tilde{\alpha}(m_Q, m_\chi, \xi_{\chi Q}), \tag{22}$$

where

$$\tilde{\alpha}(m_Q, \xi_{\chi Q}) = \frac{3m_Q^2}{16\pi v^2}\xi_{\chi Q}^2\exp(-\epsilon) \tag{23}$$

with $\epsilon = m_\chi/(m_Q\lambda) \ll 1$. Here, we see a simple increase in the coupling strength between $\bar{Q}$ and Q. We find a relative enhancement of the effective cross-section σ_{eff}

$$\sigma_{\text{eff}} = \sigma_c\left(1 + 3\tilde{\alpha}\alpha_s^{-1} + 3\tilde{\alpha}^2\alpha_s^{-2} + \tilde{\alpha}^3\alpha_s^{-3}\right) \tag{24}$$

due to the χ-boson exchange effect. In figure 5 we show the ratio of the effective cross-section (24) to the cross-section (21) as a function of m_Q. The curve in figure 6 is appropriate for a massless χ-boson.

At the end of this section we briefly give the results of the decays $\Phi \to T(\bar{Q}Q)+\gamma$ where Higgs–boson Φ ($\Phi = h$ or $\Phi = H$) with the 4-momentum q_μ and mass m_Φ decays in a heavy vector (1^{--}) quarkonium $T(\bar{Q}Q)$ ($Q = b$-quarks, t-quarks, etc), carrying the momentum $P_\mu = 2p_\mu$ $\left(P^2 = m_T^2\right)$, and a photon with the momentum squared $k^2 = 0$.

The amplitude of the transition $\Phi \to T\gamma$ is [39]

$$A(\Phi \to T\gamma) = -2\sqrt{4\pi\alpha}g_V e_Q\frac{1}{v}\eta_{\Phi Q}\frac{1}{1-(m_\Phi/m_Q)^2}\epsilon^\alpha k^\beta(P_\alpha\phi_\beta - P_\beta\phi_\alpha), \tag{25}$$

where e_Q is the charge of the quark Q, ϕ_μ is the polarization vector of $T(\bar{Q}Q)$ and g_V is defined in the standard manner

$$\langle T(\bar{Q}Q)|\bar{Q}\gamma_\mu Q|0\rangle = m_T^2 g_V\phi_\mu$$

and can be estimated from the leptonic decay width $T(\bar{Q}Q) \to \bar{l}l$ ($l = e, \mu, \tau$):

$$\Gamma(T(\bar{Q}Q) \to \bar{l}l) = \tfrac{4}{3}\pi(\alpha e_Q g_V)^2 m_T.$$

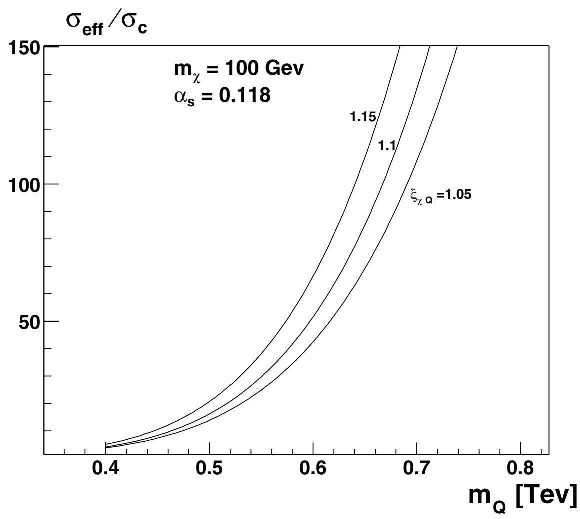

Figure 5. Ratio of the effective cross-section σ_{eff} (24) and the cross-section σ_c at the 1S resonance $\bar{Q}Q$ as a function of m_Q. The curves are for $\xi_{\chi Q} = 1.05$, 1.10, 1.15 and $m_\chi = 100$ GeV at $\alpha_s = 0.118$.

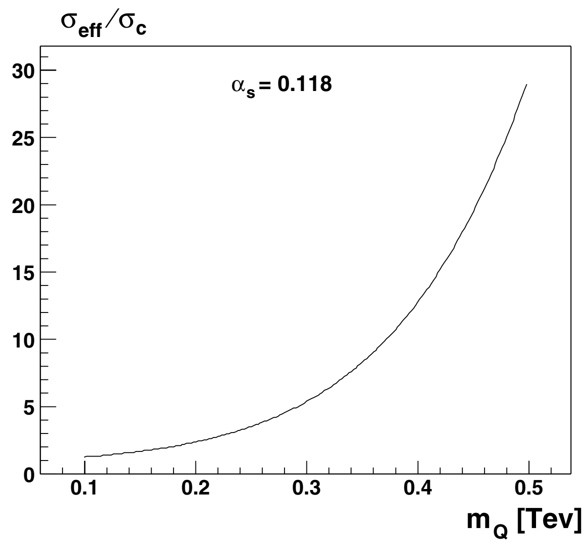

Figure 6. Ratio of the effective cross-section σ_{eff} (24) and the cross-section σ_c at the 1S resonance $\bar{Q}Q$ as a function of m_Q. The curve is for the massless χ-boson.

At arbitrary large values of $y = (m_\Phi/2m_Q)^2 > 1$ there are corrections to the amplitude (25) due to the one-loop $O(\alpha_s)$ gluon contributions

$$1 - F(y)C_F\alpha_s\left(m_\Phi^2 - 4m_Q^2\right),$$

where the function $F > 0$ was calculated in [39]. For the decay processes $h \to \Upsilon(\bar{b}b)\gamma$, $H \to \Upsilon(\bar{b}b)\gamma$ and $H \to T(\bar{t}t)\gamma$ considered here as the promising channels in a wide range of the Higgs–boson mass we expect the relative decay width compared to the final state of a

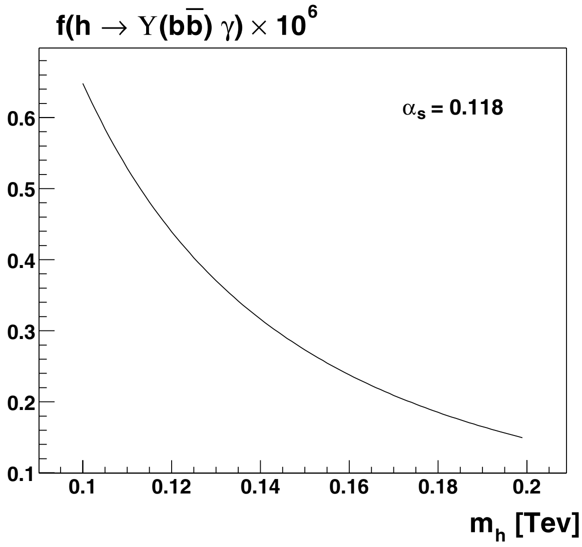

Figure 7. The relative decay width $\Gamma(h \to \Upsilon(\bar{b}b)\gamma)$ as a function of h-boson mass m_h compared to the $h \to \bar{b}b$ channel.

heavy quark and an antiquark $\bar{Q}Q$ given by [39]

$$f \equiv \frac{\mathrm{BR}(\Phi \to T(\bar{Q}Q)\gamma)}{\mathrm{BR}(\Phi \to \bar{Q}Q)} = \frac{\Gamma(\Phi \to T(\bar{Q}Q)\gamma)}{\Gamma(\Phi \to \bar{Q}Q)}$$
$$= \frac{64\pi\alpha e_Q^2}{3} g_V^2 \left(\frac{m_Q}{m_\Phi}\right)^2 K^2(y, \alpha_s) \left[1 - \left(\frac{m_T}{m_\Phi}\right)^2\right]^{-1/2}, \quad (26)$$

where

$$K(y, \alpha_s) \simeq 1 - \frac{\alpha_s\left(m_\Phi^2 - m_T^2\right)}{\pi} C_F \ln 2 \ln(4y)$$

will be important for the processes $h \to \Upsilon(\bar{b}b)\gamma$, $H \to \Upsilon(\bar{b}b)\gamma$, where $(m_b/m_\Phi)^2 \to 0$. In figures 7 and 8 we plot the relative widths of the $h \to \Upsilon(\bar{b}b)\gamma$ and $H \to \Upsilon(\bar{b}b)\gamma$ decays, respectively, compared to the $\bar{b}b$ final state, versus m_h and m_H, respectively, and the $H \to T(\bar{t}t)\gamma$ decay (see figure 9) compared to the $\bar{t}t$-state as a function of m_H.

The observation of the quark–antiquark bound state can be done through the resonance structure having a specific signal, e.g., the final leptonic pairs e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$. This resonance is expected to give clear evidence of a bound state production over the QCD background if the production is significantly large.

3. Effective model for lower-energy theorem

Before proceeding to the lower-energy theorem applied to the transition $T \to hZ$ (model II), we give the general expression for the amplitude of the decay mentioned above

$$A(T \to hZ) = \langle\phi_Z|\mathcal{L}_{\mathrm{int}}|\phi_T\rangle$$
$$= -\frac{1}{v}\langle\phi_Z| \sum_{Q=t,U,D} \frac{\alpha_s}{12\pi}\eta_{hQ} G^a_{\mu\nu} G^{a\mu\nu} - \sum_{q=u,d,s,c,b} \eta_{hq} m_q \bar{q}q|\phi_T\rangle. \quad (27)$$

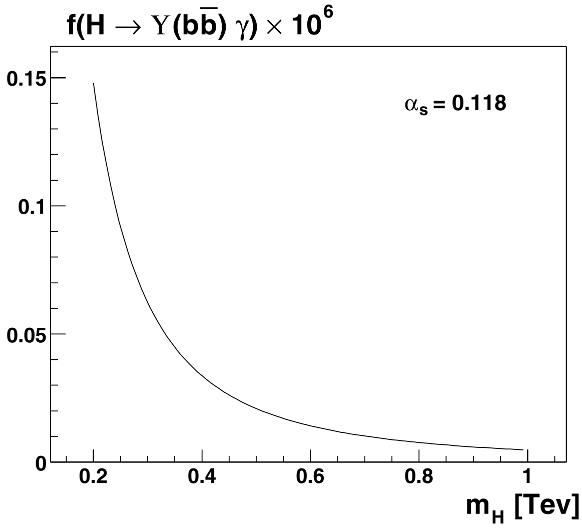

Figure 8. The relative decay width $\Gamma(H \to \Upsilon(\bar{b}b)\gamma)$ as a function of H-boson mass m_H compared to the $H \to \bar{b}b$ channel.

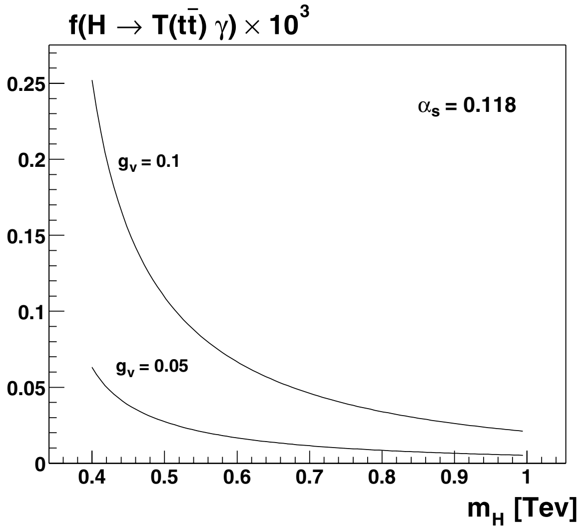

Figure 9. The relative decay width $\Gamma(H \to T(\bar{t}t)\gamma)$ as a function of H-boson mass m_H compared to the $H \to \bar{t}t$ channel for different values of g_V.

Here $|\phi_Z\rangle$ and $|\phi_T\rangle$ are eigenstates responsible for Z-boson and (super)heavy quarkonium $T = T(\bar{Q}Q)$, respectively. The second term in (27) corresponding to the contribution of light quarks comes directly from the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -\frac{h}{v}\Bigg(\sum_l \eta_{hl} m_l \bar{l}l + \sum_{q=u,d,s,c} \eta_{hq} m_q \bar{q}q + \sum_{Q=b,t,U,D} \eta_{hQ} m_Q \bar{Q}Q - 2\eta_{hW} m_W^2 W_\mu^+ W^{\mu-} + \eta_{hZ} m_Z^2 Z_\mu^2\Bigg), \quad (28)$$

while the gluonic contribution given by the operator in the first term of (27) arises from the coupling of the Higgs boson h to t-, U- and D-quarks through the standard loop mechanism (see, e.g., references in [40]) with N_h number of heavy quarks in the loop. The functions η_{hi} in (27) and (28) (i= leptons (l), light quarks (q), Q, $W^{\pm}$, Z) are model-dependent ones describing a deviation from the SM picture where all of η_{hi} are equal to unity. Obviously, considering the fermionic sector, only the terms containing quarks with masses $m_Q \geqslant m_h$ are relevant for giving rise to the amplitude provided by the matrix element of (27) where the only gluonic part survives. Our aim is to calculate the amplitude (27). Fortunately, QCD gives the trace of the energy–momentum tensor $\Theta_{\mu\nu}$ in the following form [40]:

$$\Theta_{\mu\mu} = -\frac{b_6\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu a} + \sum_q m_q \bar{q} q + \sum_Q m_Q \bar{Q} Q, \tag{29}$$

where b_6 is the first coefficient of the $\beta(\alpha_s)$ function in QCD with six flavours of quarks. At the zero-momentum transfer, the matrix element of (29) between any different eigenstates X and Y of the Hamiltonian is vanishing [33]

$$\langle X|\Theta_{\mu\mu}(q^2)|Y\rangle = 0 \tag{30}$$

with $\langle X|\Theta_{00}(q_0^2, \vec{0})|Y\rangle = 0$. Using condition (30), one can obtain the following relation between the matrix elements containing the gluonic part and the heavy quark terms:

$$\langle\phi_Z|\frac{b_6\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu a}|\phi_T\rangle = \langle\phi_Z|m_b\bar{b}b + m_t\bar{t}t|\phi_T\rangle, \tag{31}$$

where the light quark contributions are neglected. Taking into account the gluonic anomaly effect through the replacement of the top-quark mass term [40]

$$m_t\bar{t}t \to -\frac{2}{3}\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu a}, \tag{32}$$

equation (13) transforms into

$$\langle\phi_Z|\frac{b_5\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu a}|\phi_T\rangle = \langle\phi_Z|m_b\bar{b}b|\phi_T\rangle, \tag{33}$$

where $b_5 = b_6 + 2/3$. Let us consider amplitude (27) in the following form,

$$A(T \to hZ) = -\frac{1}{v}\langle\phi_Z|\sum_{Q=t,U,D}\frac{\alpha_s}{12\pi}\eta_{hQ} G^a_{\mu\nu} G^{a\mu\nu} - m_b\bar{b}b|\phi_T\rangle, \tag{34}$$

where the terms containing u, d, s and c quarks were omitted because of their small mass effect on the intermediate quark loop. Comparing equations (34) and (33), we find the amplitude for the decay $T \to hZ$ in the limit of vanishing the 4-momentum of the Higgs–boson h

$$A(T \to hZ) = -\frac{1}{v}\langle\phi_Z|\frac{\alpha_s}{8\pi}\left(b_5 - \frac{2}{3}\sum_{Q=t,U,D}\eta_{hQ}\right) G^a_{\mu\nu} G^{a\mu\nu}|\phi_T\rangle, \tag{35}$$

where the couplings η_{hQ} are [38]

$$\eta_{ht(U)} = \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha), \tag{36}$$

$$\eta_{hD} = -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha). \tag{37}$$

For numerical estimation, we use the decoupling limit where equations (36) and (37) are transformed into the following ones:

$$\eta_{ht(U)} \simeq 1 + z s_{2\beta} c_{2\beta} \tan^{-1}\beta, \tag{38}$$

$$\eta_{hD} \simeq 1 - z s_{2\beta} c_{2\beta} \tan\beta, \tag{39}$$

where $z = (m_Z/m_A)^2$. To calculate amplitude (35), one has to estimate its matrix element as those given by the soft nonperturbative gluonic field. This contribution occurs as the excitation of the nonperturbative gluon condensate in an environment of a pair of a quark and an antiquark bound at the scale

$$r^{-1}_{\text{bound state}} \sim \Lambda_Q = m_Q \alpha_s(\tilde{\mu} \simeq m_Q \alpha_s), \tag{40}$$

which is larger than the scale of strong interactions Λ with $\alpha_s \ll 1$. This nonperturbative effect can be estimated in the transition $T \to Z$ within a minimal point-like source $F\alpha_s \vec{E}^2(x)$ [33] with $\vec{E}$ being the electric component of the gluonic field, and an arbitrary constant F defines the strength of this source. Hence, the only remaining work is to calculate the following two-point function:

$$W(G_{\mu\nu}) = \mathrm{i} \int \mathrm{d}x\, \mathrm{e}^{\mathrm{i}qx} \langle 0| T \left\{ F\alpha_s \vec{E}^2(x), \frac{\beta(\alpha_s)}{4\alpha_s} G^2_{\mu\nu}(0) \right\} |0\rangle \tag{41}$$

embedded in the nonperturbative amplitude A_{NP} of the decay $T \to hZ$

$$A_{NP}(T \to hZ) = \frac{1}{v\Lambda^2} \left(1 - \frac{2}{3}\frac{1}{b_5} \sum_{Q=t,U,D} \eta_{hQ} \right) W(G_{\mu\nu}). \tag{42}$$

Here $\beta(\alpha_s) = \left(-b_5\alpha_s^2/2\pi\right) + O\left(\alpha_s^3\right)$. Using the tricks followed by the authors in [41, 42], the calculation of amplitude (42) can be achieved in the framework of the QCD low-energy theorem approach in the limit $q^2 \to 0$

$$A_{NP}(T \to hZ) = \frac{1}{v}\frac{\pi F}{\Lambda^2} \left(1 - \frac{2}{3b_5} \sum_{Q=t,U,D} \eta_{hQ} \right) \left\langle \frac{\alpha_s}{\pi} G^2_{\mu\nu} \right\rangle_0, \tag{43}$$

where $\langle(\alpha_s/\pi)G^2_{\mu\nu}\rangle_0$ is the standard gluonic condensate. Using these formulae, now we can estimate the corrections due to non-perturbative fluctuations of the gluonic field in the decay $T(\bar{U}U) \to hZ$

$$\Gamma(T(\bar{U}U) \to hZ) = \Gamma_0(T(\bar{U}U) \to hZ)(1 + \delta_{NP}), \tag{44}$$

where $\Gamma_0(T(\bar{U}U) \to hZ)$ is the decay width (12) while the nonperturbative correction factor δ_{NP} is defined by $\delta_{NP} = \Gamma_{NP}/\Gamma_0$ with

$$\Gamma_{NP}(T(\bar{U}U) \to hZ) = \frac{1}{16\pi m_T} |A_{NP}|^2 \left(1 - \frac{4m_h^2}{m_T^2} \right)^{1/2}. \tag{45}$$

The δ_{NP} correction is obtained at the order of magnitude of 10^{-8} for the strength parameter $F = 1$.

4. Conclusion and discussion

In this paper, we were concerned with the question of existence of heavy quarkonia and their decays with production of a Higgs–boson, e.g., $T(\bar{U}U) \to hZ$. The possible existence of very heavy quarkonium was studied in the framework of a simple Higgs–boson potential model (model I) having phenomenological 'hard' Yukawa couplings $\lambda(m_Q, \xi_{\chi Q})$ between the heavy quark and the Higgs–boson χ. Furthermore, to estimate the decay width of $T(\bar{U}U) \to hZ$ by taking into account the fluctuations of the gluonic field (model II), we used the conformal properties of QCD. We found that the effect of nonperturbative fluctuations of the gluonic field in the total decay width $\Gamma(T(\bar{U}U) \to hZ)$ was rather small; Γ_{NP} is quite sensitive to the strength F of the gluonic point-like source, since it is proportional to the square of this strength. The constant F can be estimated within the approach of the model II through the decay, e.g., $T(\bar{U}U) \to \bar{l}l(l = \mu, \tau)$ with $A_{NP} \sim \langle \bar{l}l|F\alpha_s\vec{E}^2(x)|0\rangle$, if this decay is known. The amplitude of this process is proportional to F, while it is cancelled in the ratio $\Gamma(T(\bar{U}U) \to hZ)/\Gamma(T(\bar{U}U) \to \bar{l}l)$. The conformal properties of QCD are especially important for lighter hadrons, e.g. $(\bar{c}c)$- and $(\bar{b}b)$-bound states where the multipole expansion is more sufficient [40]. In the case of heavy quarkonium transitions, model I yields an enhancement effect compared to the result coming from the model II at $\left\langle(\alpha_s/\pi)G^2_{\mu\nu}\right\rangle_0 \simeq 0.012\ \mathrm{GeV}^4$. Such an enhancement of the decay width $\Gamma(T(\bar{U}U) \to hZ)$ with guaranteed criterion $c = (\Gamma_{\mathrm{tot}}/\epsilon_B) < 1$ could have a quite significant effect on the shapes of bound-state resonances containing heavy quarks in the search for new phenomena at the Tevatron and LHC. Unfortunately, the total decay width Γ_{tot} is unknown. In figure 4, we show the dependence of the ratio of its total decay width Γ_{tot} to the binding energy ϵ_B, i.e. $c = \Gamma_{\mathrm{tot}}/\epsilon_B$ on the real Yukawa couplings $\xi_{\chi Q}$ in the 2HDM at different values of the χ-boson mass. The binding critical ratio c is strongly sensitive to $\xi_{\chi Q}$. Thus, we believe that in some kinematical regions there is a possibility of finding the fourth family quarks at the Tevatron's Run II (the LHC experiments will not miss it). If the scale of the Higgs–boson mass does not exceed 180 GeV, the channels of Higgs decays to gg, $\gamma\gamma$ or $\bar{l}l$ $(\mu^+\mu^-, \tau^+\tau^-)$ may give an enhancement effect which can be interpreted as an indication of heavy quark existence. For example, the decay of the CP-even lightest Higgs–boson $h \to gg(\gamma\gamma)$ with 114 GeV $< m_h <$ 180 GeV could be enhanced (due to the fourth generation quark contribution) by a factor $\rho \simeq 8.9(4.5\text{–}5.5)$ for $m_4 = 200$ GeV and $\rho \simeq 8.5(4.0\text{–}5.0)$ for $m_4 = 600$ GeV. On the other hand, the decays of CP-even heavy Higgs $H \to gg(\gamma\gamma)$ in the mass region 180 GeV $< m_H <$ 800 GeV could give $9 \leqslant \rho \leqslant 13\,(15 \leqslant \rho \leqslant 25)$ for $m_4 = 200$ GeV and $9 \leqslant \rho \leqslant 27\,(15 \leqslant \rho \leqslant 57)$ for $m_4 = 600$ GeV. This effect on the H Higgs–boson decay has a minor dependence of $\tan\beta$. We have shown that a reduction of a single U-quark decay width leads to a significant enhancement of the signal of the $T(\bar{U}U)$ resonance. We investigated also the possible manifestation of the CP-even Higgs–boson states h and H in their rare decays $h \to \Upsilon(\bar{b}b)\gamma$ and $H \to \Upsilon(\bar{b}b)\gamma$ or even the production of heavier bound states $T(\bar{t}t)$ in the rare decays of Higgs bosons H. We obtained that these decays can be detectable at the forthcoming experiments at the LHC.

In the final state, the Higgs bosons h, the gauge bosons $Z/W^{\pm}$ and even charged Higgs–bosons $H^{\pm}$ are on mass-shell. Hence, the masses of heavy quarks can be reconstructed. As a trigger, one can choose semileptonic decays $U \to D\bar{l}\nu_l$, $U \to b\bar{l}\nu_l$, $t \to b\bar{l}\nu_l$ $(l = \mu, \tau)$ and the leptonic one like $W \to l\nu_l$ with different lepton flavours to eliminate the backgrounds $\gamma^{\star}, Z, Z' \to \bar{l}l$. On the other hand, we suppose that the efficiency for observing $\bar{Q}Q$ events may be high enough because of the characteristic kinematics such as $U \to DW$, $U \to bW$ and $t \to bW$. The comparison of the measured decay width mentioned above with theoretically predicted ones can exclude or even confirm the fourth family fermions (quarks). Finally, some comments are in order in the following:

(a) the (super)heavy quarks are considered to be nonrelativistic;
(b) if the χ Higgs–boson mass is $m_\chi \sim \mathcal{O}(100\ \mathrm{GeV})$, then the decoupling limit $(\xi_{\chi Q} \sim \mathcal{O}(1))$ is appropriate only for $m_Q \geqslant 2m_t$;

(c) for the $(\bar{t}t)$-bound state with $\xi_{\chi t} \sim O(1)$, the mass m_χ should be smaller than the lower bound given by the LEP 2 experiments [43];

(d) if the heavy quarkonium mass is of the order of $2m_t$, the χ-boson contribution to the combined potential (4) becomes appreciable only for large values of $\xi_{\chi t} \geqslant 6$.

We conclude that the (super)heavy quarkonia effects calculated here can be significant and should be considered seriously for searching for new physics beyond the SM.

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