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TOPOLOGICAL CHARGE AND TOPOLOGICAL SUSCEPTIBILITY IN CONNECTION WITH TRANSLATION AND GAUGE INVARIANCE

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It is shown that the evaluation of the expectation value (EV) of topological charge density over θ vacuum is reduced to investigation of the Chern–Simons term EV. An equation for this quantity is established and solved. EV of the topological charge density at an arbitrary θ occurs to be equal to zero at least in the pure Yang–Mills (YM) and QCD in the chiral limit (QCD_χ) theories. As a consequence, topological susceptibilities of both YM and (known result) QCD_χ vacua defined in a Wick sense are equal to zero, whereas, when defined in a Dyson sense, they can differ from zero by the quantities proportional to the respective condensates of the chromomagnetic field. Thus, the usual Witten–Veneziano formula for the η'-meson mass is modified.

Показано, что вычисление величины ожидания топологического заряда по θ-вакууму сводится к исследованию ожидания по θ-вакууму черн-саймоновского члена. Уравнение для этой величины получено и решено. Ожидание по θ-вакууму плотности топологического заряда при произвольном значении параметра θ оказывается равным нулю, по крайней мере, в чисто калибровочной теории Янга–Миллса и КХД в киральном пределе. Вследствие этого топологические восприимчивости как вакуума теории Янга–Миллса, так и вакуума (известный результат) КХД в киральном пределе оказываются равными нулю, если они определены в «виковском» смысле, в то время как соответствующие восприимчивости, определенные в «дайсоновском» смысле, могут отличаться от нуля вкладами, пропорциональными соответствующим конденсатам хромомагнитного поля. Таким образом, обычная формула Виттена–Венециано для массы η'-мезона модифицируется.

The effects connected with the nontrivial topological configurations of the gauge fields attract a great attention in modern physics. In this respect the QCD topological susceptibility $\chi_{\mathrm{QCD}} = \int d^4x \langle Tq(x)q(0)\rangle$ is the quantity of a special importance because it enters as a key object in a lot of physical tasks, in particular, in such important puzzles as a famous $U(1)$ problem [1–5] (see [6] for a recent review) and the «spin crisis» [7]. In the equation for χ, $q(x)$ is the topological charge density $q(x) = (g^2/32\pi^2)F^a_{\mu\nu}(x)\tilde{F}^{\mu\nu}_a(x)$ related with the Chern–Simons current, $K_\mu(x)$ by $q(x) = \partial^\mu K_\mu(x)$, where $K_\mu = (g^2/32\pi^2)\epsilon^{\mu\nu\rho\sigma}A^a_\nu(F^a_{\rho\sigma} - (g/3)f_{abc}A^b_\rho A^c_\sigma)$.

It is well known that topological susceptibility χ_{QCD} is equal to zero in all orders of perturbation theory and, also, that this quantity is just zero in the presence of even one massless quark (Crewther theorem [2]).

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In this paper the consideration based on the fundamental translation and gauge symmetries will be performed which will allow us to draw some unexpected conclusions about the topological charge and susceptibility.

Let us prove the following statement.

EV of the topological charge density $(\theta|q(0)|\theta) = (1/VT)(\theta|Q|\theta)$ over θ vacuum with an arbitrary θ is equal to zero if EV of operator $K_\mu(0)$ over θ vacuum exists, i. e., $|(\theta|K_\mu(0)|\theta)| < \infty$, where symbol $|\theta)$ denotes the θ-vacuum state normalized to unity:

$$(\theta|\theta) = 1. \tag{1}$$

This statement directly follows from translation invariance of θ vacuum:

$$(\theta|q(0)|\theta) = (\theta|\partial^\mu K_\mu(0)|\theta) = \\ = -i(\theta|[\hat{P}^\mu, K_\mu(0)]|\theta) = -i(P^\mu_\theta - P^\mu_\theta)(\theta|K_\mu(0)|\theta) = 0. \tag{2}$$

The key point here is the condition $|(\theta|K_\mu(0)|\theta)| < \infty$ which, as we will see below, in $A_0 = 0$ gauge is equivalent to the condition

$$|(\theta|W_{\rm CS}(0)|\theta)| < \infty, \tag{3}$$

where $W_{\rm CS}(t) \equiv \int d^3x K_0(x)$ is the Chern–Simons operator (see [8] for review). However, as we will see, within the conventional formulation of θ-vacuum theory a rather amazing situation arises. On the one hand, the condition (3) is not satisfied due to the gauge noninvariance of the operator $W_{\rm CS}$ with respect to the «large», topologically nontrivial gauge transformations. Nevertheless, despite EV $\langle\theta'|W_{\rm CS}(0)|\theta\rangle$ is more singular function than $\delta(\theta' - \theta)$ at $\theta' \to \theta$ (namely, it behaves as $\delta'(\theta' - \theta)$ in this limit), the EV of the topological charge density over θ vacuum is just zero again.

Since we deal with the gauge-invariant quantity (EV of the topological charge), let us choose the Weyl gauge $A_0 = 0$, which allows us to essentially simplify a consideration. Choosing the periodic boundary conditions in the space directions (topology of a hypercylinder oriented along the time axis), one has

$$\int d^3x \; \partial^i K_i(t, \mathbf{x}) = 0, \tag{4}$$

and the expression for the topological charge $Q \equiv \int d^4x q(x) = \int d^4x \partial^\mu K_\mu$ becomes (see, for example, [8, 9]) $Q = W_{\rm CS}(t = \infty) - W_{\rm CS}(t = -\infty)$.

EV $(\theta|\hat{O}|\theta)$ of an arbitrary operator $\hat{O}$ over θ vacuum is defined as (see, for example, [8, 9])

$$(\theta|\hat{O}|\theta) = \langle\theta|\hat{O}|\theta\rangle / \langle\theta|\theta\rangle, \tag{5}$$

where $|\theta\rangle$ is, simultaneously, the eigenfunction of the full QCD Hamiltonian H and of the unitary operator T_ν of the large gauge transformations with a winding number ν:

$$H|\theta\rangle = E_\theta|\theta\rangle, \tag{6}$$

$$T_\nu|\theta\rangle = \mathrm{e}^{-i\theta\nu}|\theta\rangle; \tag{7}$$

i. e., the state $|\theta\rangle$ is, up to a phase multiplier, gauge-invariant against the large gauge transformations. Notice also that on the contrary to (1) the states $|\theta\rangle$ are normalized as

$$\langle\theta'|\theta\rangle = \delta(\theta' - \theta), \tag{8}$$

so that the prescription (5) reads

$$\langle\theta'|\hat{O}|\theta\rangle = (\theta|\hat{O}|\theta)\delta(\theta' - \theta); \tag{9}$$

i. e., $(\theta|\hat{O}|\theta)$ is just the limit at $\theta' \to \theta$ of the multiplier at δ function in the expression for $\langle\theta'|\hat{O}|\theta\rangle$.

Since we are interested in the quantity

$$(\theta|q(0)|\theta) = (VT)^{-1}(\theta|Q|\theta), \tag{10}$$

we will keep the normalization factor $(VT)^{-1}$. Using (4) and the Heisenberg equations, one easily gets

$$\begin{aligned}(VT)^{-1}\langle\theta'|Q|\theta\rangle = (VT)^{-1}\int dt\, \mathrm{e}^{i(E_{\theta'}-E_\theta)t}\langle\theta'|\int d^3x q(0,\mathbf{x})|\theta\rangle = \\ = 2\pi(VT)^{-1}\delta(E_{\theta'} - E_\theta)\langle\theta'|\dot{W}_{\mathrm{CS}}(0)|\theta'\rangle = \\ = 2\pi(VT)^{-1}\delta(E_{\theta'} - E_\theta)\langle\theta'| - i[W_{\mathrm{CS}}(0), H]|\theta\rangle = \\ = 2\pi i(VT)^{-1}\delta(E_{\theta'} - E_\theta)(E_{\theta'} - E_\theta)\langle\theta'|W_{\mathrm{CS}}(0)|\theta\rangle,\end{aligned} \tag{11}$$

and, thus, the task now is to evaluate EV $\langle\theta'|W_{\mathrm{CS}}(0)|\theta\rangle$.

The remarkable property of the Chern–Simons term is its transformation law under topologically nontrivial (often called «large» [8]) gauge transformations $A_i \to A_i^{\Omega_\nu} = \Omega_\nu A_i \Omega_\nu^{-1} + \partial_i \Omega_\nu \Omega_\nu^{-1}$ $(i = 1, 2, 3;\ \Omega = \Omega(\mathbf{x}))$, with topological index (winding number) ν. Namely, the quantity $W_{\mathrm{CS}}[A]$ is not gauge-invariant under such transformations but transforms as

$$W_{\mathrm{CS}}[A] \to W_{\mathrm{CS}}[A^{\Omega_\nu}] = W_{\mathrm{CS}}[A] + \nu; \tag{12}$$

i. e., it only shifts by the winding number ν of the respective gauge transformation.

The compatibility of the quantum

$$W_{\mathrm{CS}}[A^{\Omega_\nu}] = T_\nu W_{\mathrm{CS}}[A]T_\nu^+ = W_{\mathrm{CS}}(A) + [T_\nu, W_{\mathrm{CS}}(A)]T_\nu^{-1} \tag{13}$$

and classical (12) gauge transformation laws of the Chern–Simons term gives rise to the commutation law [1]

$$[T_\nu, W_{\mathrm{CS}}(t)] = [T_\nu, W_{\mathrm{CS}}(0)] = \nu T_\nu. \tag{14}$$

Now one already can evaluate $\langle\theta'|W_{\mathrm{CS}}(0)|\theta\rangle$. Indeed, due to the unitarity of the operator T_ν and Eq. (7), one has

$$\langle\theta'|[T_\nu, W_{\mathrm{CS}}(0)]|\theta\rangle = \left(\mathrm{e}^{-i\nu\theta'} - \mathrm{e}^{-i\nu\theta}\right)\langle\theta'|W_{\mathrm{CS}}(0)|\theta\rangle. \tag{15}$$

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[1] Here one uses $[T_\nu, H] = 0$.

On the other hand, the commutation law (14) together with Eqs. (7), (8) give

$$\langle\theta'|[T_\nu, W_{\rm CS}(0)]|\theta\rangle = \nu\,{\rm e}^{-i\nu\theta}\delta(\theta-\theta'). \tag{16}$$

Comparing (15) and (16), one gets the basic for what follows equation

$$\frac{1}{\nu}\left({\rm e}^{-i\nu(\theta'-\theta)}-1\right)\langle\theta'|W_{\rm CS}(0)|\theta\rangle = \delta(\theta'-\theta). \tag{17}$$

So, one has to solve the equation

$$\frac{1}{\nu}\left({\rm e}^{-i\nu z}-1\right)f(z,\theta) = \delta_{2\pi}(z), \tag{18}$$

where $z\equiv\theta'-\theta$; $f(z,\theta)\equiv\langle\theta'|W_{\rm CS}(0)|\theta\rangle$, and $\delta_{2\pi}(z)\equiv\delta(\theta'-\theta)$ is 2π-periodic δ function. Expanding $f(z,\theta)$ and $\delta_{2\pi}(z)$ in the Fourier series:

$$f(z,\theta) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\tilde{f}_n(\theta)\,{\rm e}^{inz}, \quad \delta_{2\pi}(z) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}{\rm e}^{inz}, \tag{19}$$

one easily obtains instead of (18) the difference equation for Fourier image $\tilde{f}$

$$\tilde{f}_{n+\nu}(\theta)-\tilde{f}_n(\theta) = \nu \tag{20}$$

with the solution

$$\tilde{f}_n(\theta) = n + C(\theta), \tag{21}$$

where $C(\theta)$ is some arbitrary function of θ.

Thus, the general solution [1] of Eq. (17) reads [2]

$$\langle\theta'|W_{\rm CS}(0)|\theta\rangle = -i\delta'(\theta'-\theta)+C(\theta)\delta(\theta'-\theta), \tag{22}$$

where $\delta'(\theta'-\theta) = (i/2\pi)\sum n\,{\rm e}^{in(\theta'-\theta)}$.

Since

$$\lim_{\theta'\to\theta}\left(\frac{2\pi}{T}\delta\left(E_{\theta'}-E_\theta\right)\right)(E_{\theta'}-E_\theta) = (E_\theta-E_\theta) = 0, \tag{23}$$

the term $C(\theta)\delta(\theta'-\theta)$ in the solution (22) does not contribute to the coefficient at $\delta(\theta'-\theta)$ in the r. h. s. of (11) and, thus,

$$(VT)^{-1}\langle\theta'|Q|\theta\rangle = 2\pi(VT)^{-1}\delta'(\theta'-\theta)\left[(E_{\theta'}-E_\theta)\,\delta(E_{\theta'}-E_\theta)\right]. \tag{24}$$

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[1] It is of importance that (22) is the solution of (17) only if the winding number ν is an arbitrary integer number (the existence of the fractional winding numbers was advocated in [2]). Otherwise, one cannot perform the necessary change $n-\nu\to n$ in the sum $\sum n\,{\rm e}^{i(n-\nu)z}$.

[2] Notice that for any gauge-invariant operator $[T_\nu, O_{\rm g.inv}] = 0$ and, therefore, Eq. (17) is replaced by $[\exp(-i\nu\theta')-\exp(-i\nu\theta)]\langle\theta'|O_{\rm g.inv}|\theta\rangle = 0$ with the general solution $\langle\theta'|O_{\rm g.inv}|\theta\rangle = C(\theta)\delta(\theta'-\theta)$.

Considering θ' as a variable whereas θ is kept fixed, one gets

$$(VT)^{-1}\langle\theta'|Q|\theta\rangle = -2\pi(VT)^{-1}\delta(\theta'-\theta)\frac{\partial E(\theta')}{\partial\theta'}\times \\ \times\left[\delta'(E_{\theta'}-E_\theta)(E_{\theta'}-E_\theta)+\delta\left(E_{\theta'}-E_\theta\right)\right], \quad (25)$$

At first sight, the expression in the square brackets is equal to zero, since usually $x\delta'(x) = -\delta(x)$. However, this is not correct conclusion and one has to properly work with the generalized function $(E_{\theta'}-E_\theta)\delta'(E_{\theta'}-E_\theta)$ when one takes the limit $\theta'\to\theta$, i. e., $(E_{\theta'}-E_\theta)\to 0$. Indeed, let us consider the generalized function

$$\Delta(x) \equiv x\delta'(x). \quad (26)$$

Then

$$\Delta(0) = \int_{-\infty}^{\infty} dx\delta(x)\Delta(x) \equiv \int_{-\infty}^{\infty} dx\delta(x)[x\delta'(x)] = -\int_{\infty}^{\infty} dx\delta(x)[\delta(x)+x\delta'(x)] = \\ = -\int_{\infty}^{\infty} dx\delta(x)\Delta(x) - \int_{\infty}^{\infty} dx\delta(x)\delta(x) = -\Delta(0)-\delta(0),$$

and, thus [1],

$$\Delta(0) \equiv [x\delta'(x)]\Big|_{x=0} = -\frac{1}{2}\delta(0). \quad (27)$$

In particular,

$$\lim_{\theta'\to\theta}[(E_{\theta'}-E_\theta)\delta'(E_{\theta'}-E_\theta)] = -\frac{1}{2}\delta(E=0) = -\frac{1}{2}\frac{T}{2\pi}. \quad (28)$$

In accordance with the general prescription (9) and Eqs. (10), (25), (28), one obtains

$$(\theta|q(0)|\theta) = -\frac{\partial E_\theta}{\partial\theta}\frac{2\pi}{VT}\left[-\frac{1}{2}\frac{T}{2\pi}+\frac{T}{2\pi}\right] = -\frac{1}{2}\frac{\partial\epsilon(\theta)}{\partial\theta}, \quad (29)$$

where

$$\epsilon(\theta) \equiv E_\theta/V \quad (30)$$

is the energy density of θ vacuum.

On the other hand, it is well known that the energy density $\epsilon(\theta)$ is defined via the functional integral as (see, for example, [5, 9])

$$\epsilon(\theta) = i(VT)^{-1}\ln W_\theta, \quad (31)$$

[1] The quantity $\Delta(x)$ is equal to $-\delta(x)$ as a generalized function only in the convolution with a function $F(x)$, satisfying the condition $xF'(x)|_{x=0} = 0$. However, this is just not the case for the choice $F(x) = \delta(x)$.

where

$$W(\theta) \equiv \int \mathcal{D}A\mathcal{D}[\bar{\psi}, \psi] \exp i \left[S_{\rm QCD} + \theta Q\right]. \tag{32}$$

In this picture, one has

$$(\theta|q(0)|\theta) = (VT)^{-1}(\theta|Q|\theta) = -i(VT)^{-1}\frac{\partial W[\theta]}{\partial\theta}W^{-1}[\theta] = -\frac{\partial\epsilon(\theta)}{\partial\theta}, \tag{33}$$

whereas the second derivative of $\epsilon(\theta)$ with respect to θ just produces the topological susceptibility — the connected part of the two-point correlator of the topological charge densities at zero momentum:

$$\chi_\theta = \int d^4x(\theta|Tq(x)q(0)|\theta)_{\rm conn} = -(VT)^{-1}\frac{\partial^2 \ln W_\theta}{\partial\theta^2} = i\frac{\partial^2\epsilon(\theta)}{\partial\theta^2}, \tag{34}$$

and $\chi_{\rm QCD} = \chi_0$.

It is easy to see now that the only way to reconcile Eqs. (29) and (33) is to put

$$(\theta|q(0)|\theta) = -\frac{\partial\epsilon(\theta)}{\partial\theta} = 0. \tag{35}$$

Then one can see that the topological susceptibility defined by (34) is also equal to zero:

$$\chi_\theta = \chi_0 = 0. \tag{36}$$

Let us now attempt to realize the obtained result. At first glance, Eq. (36) is in a severe contradiction with a standard point of view [2–6] that the quantity χ must differ from zero because it is directly connected with the solution of $U(1)$ problem and mass of η' meson is explicitly expressed via topological susceptibility. However, the situation perhaps is not so bad because of two important circumstances.

First point is that there exists the principal difference between purely gauge YM and QCD with quarks theories. At first sight, nothing changes in the derivation of Eq. (35) if one considers the full QCD case. However, one must be careful here because only in YM theory the topological charge density q is renormalization group (RG) invariant: $q_B = q_R \equiv q$ [10]. On the contrary [10] (see also [4] and references therein), the topological charge density operator in QCD with quarks is not RG-invariant and mixes under renormalizations with the divergence of the flavor singlet anomalous current $J_{\mu 5}$. So, one cannot at once conclude that Eq. (35) holds in QCD with massive quarks, and one can with confidence use here only the information given by the unrenormalizable anomalous Ward identities. In particular, these identities predict the well-known result that [1] the topological charge density EV over θ vacuum is just zero in massless QCD. So, one arrives at the important conclusion. Eq. (35) and, as an immediate consequence, Eq. (36) are not necessarily valid in the full QCD case but are valid at least in the pure gauge YM theory (where topological charge is RG-invariant operator

[1] The simplest way to see this is to notice that, if even one quark mass is equal to zero, then the all θ-dependence in the functional integral representation of all Green functions can be just removed by the simple chiral transformation of the fermionic variables.

and there are no chiral Ward identities), and, also, in the chiral limit QCD (QCD_χ), where all θ-dependence is trivial and just removed, performing the respective chiral transformations, so that[1] $\langle\theta|q(0)|\theta\rangle\Big|_{\mathrm{QCD}_\chi} = \langle 0|q(0)|0\rangle\Big|_{\mathrm{QCD}_\chi} = 0$.

The second point is that there exist two different topological susceptibilities in accordance with the different sense of the time-ordering operation in the respective correlators.

Let us remind that the functional integral representation (31)–(34) for a correlator means that the respective T product in this correlator must be realized as the Wick (T_W) time-ordering operation, in which all the derivatives are applied after the calculation of the field convolutions (see [5] for the excellent review on this question). On the contrary, the Dyson T-ordering of two arbitrary operators $A(x)$ and $B(y)$ (no matter composite or not) simply looks as

$$T_D[A(x)B(y)] = \theta(x_0 - y_0)A(x)B(y) + \theta(y_0 - x_0)B(y)A(x). \tag{37}$$

So, actually Eq. (36) has to be read as[2]

$$\chi_\theta^W\Big|_{\mathrm{YM,QCD}_\chi} = \chi_0^W\Big|_{\mathrm{YM,QCD}_\chi} = 0, \tag{38}$$

where

$$\chi_\theta^W \equiv \int d^4x[(\theta|T_W[q(x)q(0)]|\theta)_{\mathrm{conn}} = i\frac{\partial^2\epsilon}{\partial\theta^2}, \tag{39}$$

but it does not mean at all that the Dyson topological susceptibilities in these theories

$$\chi_\theta^D = \int d^4x[(\theta|T_D q(x)q(0)|\theta) - (\theta|q(0)|\theta)^2] = \int d^4x(\theta|T_D q(x)q(0)|\theta)_{\mathrm{conn}} \tag{40}$$

are also equal to zero.

Indeed, the connection between Wick and Dyson susceptibilities was found in [5], using the stationary perturbation theory in powers of θ, with a result

$$\chi_0^W = i\frac{\partial^2\epsilon}{\partial\theta^2}\Big|_{\theta=0} = \chi_0^D + i(0|\left(\frac{g^2}{8\pi^2}\mathbf{B}_a\right)^2|0); \tag{41}$$

i. e., these susceptibilities differ by the condensate of the chromomagnetic field $\mathbf{B}_a$. Thus, even despite that, in accordance with (38), the quantities χ_0^W are equal to zero, the Dyson topological susceptibilities χ_0^D can differ from zero by the nonzero values of the chromomagnetic condensates:

$$\chi_0^D\Big|_{\mathrm{YM,QCD}_\chi} = -i(0|\left(\frac{g^2}{8\pi^2}\mathbf{B}_a\right)^2|0)\Big|_{\mathrm{YM,QCD}_\chi}. \tag{42}$$

[1] The fact that VEV of $q(0)$ over nonperturbative but topologically trivial, CP-invariant vacuum $|0)$ is just zero is obvious, since $q(x)$ is a pseudoscalar.

[2] The result $\chi_0^W\Big|_{\mathrm{QCD}_\chi} = 0$ is well known and has been intensely exploited (see, for example, [2–7]).

On the other hand, it is well known that Dyson T product of two arbitrary operators can be represented as the sum over intermediate states with a result

$$\int d^4x\, \mathrm{e}^{ikx} (0|T_D(A(x)B(0))|0)_{\mathrm{conn}} = \sum_n (0|A(0)|n(\mathbf{k}))(n(\mathbf{k})|B(0)|0)\frac{i}{k^2 - m_n^2} + \\ + \text{two particle contributions} \quad (k^2 = k_0^2 - \mathbf{k}^2). \quad (43)$$

As was shown by Witten [3] (see [10], Sec. 5.1, for review), only the one particle contributions survive in the sum over intermediate states in the large N_c limit. So,

$$\chi_0^D\big|_{\mathrm{QCD}} = \int d^4x\; (0|T_D q(x)q(0)|0)_{\mathrm{conn}}\big|_{\mathrm{QCD}} = \\ = \lim_{k\to 0}\left[\sum_{n=\mathrm{mesons}} |(0|q(0)|n(\mathbf{k}))|^2 \frac{i}{k^2 - m^2} + \sum_{l=\mathrm{glueballs}} |(0|q(0)|l(\mathbf{k}))|^2 \frac{i}{k^2 - m^2}\right] = \\ = \chi_0^D\big|_{\mathrm{YM}} + \sum_{n=\mathrm{mesons}} |(0|q(0)|n)|^2 \frac{-i}{m_n^2}, \quad (44)$$

and, in the leading order of χPT where [6]

$$(0|q(0)|\eta') = (0|\frac{1}{2N_f}\partial^\mu J_\mu^5|\eta') = \frac{1}{2N_f} f_{\eta'} m_{\eta'}^2, \quad (45)$$

one gets instead of Witten–Veneziano [3, 4] formula [1]

$$m_{\eta'}^2 = -i\frac{4N_f^2}{f_{\eta'}^2}(\chi_0^D\big|_{\mathrm{YM}} - \chi_0^D\big|_{\mathrm{QCD}_\chi}) = \\ = \frac{4N_f^2}{f_{\eta'}^2}\left[(0|\left(\frac{g^2}{8\pi^2}\mathbf{B}_a\right)^2|0)\Big|_{\mathrm{QCD}_\chi} - (0|\left(\frac{g^2}{8\pi^2}\mathbf{B}_a\right)^2|0)\Big|_{\mathrm{YM}}\right]. \quad (46)$$

It is of importance and seems to be a serious argument in support of our model-independent consideration (based on the general principles of translational and gauge invariance) that very similar results concerning η'-meson mass were obtained within the different QCD-inspired models. These are Cheshire cat principle model [12] and, also, squeezed gluon vacuum [13] and monopole vacuum [14] models (compare [2] Eq. (46) with Eq. (14) of Ref. [12] and, especially, with Eqs. (22) and (26) of Refs. [13] and [14], respectively).

Thus, we get a rather unexpected result: topological susceptibilities of both YM and (known result) QCD_χ vacua defined in a Wick sense are equal to zero, whereas the Dyson topological susceptibilities are just proportional to the respective chromomagnetic condensates. The last circumstance allows one to get the mass formula (46) for the η' meson which directly

[1] One can show (it will be published elsewhere) that the second term in Eq. (46) is just equal to zero and only QCD_χ chromomagnetic condensate survives in the mass formula.

[2] Comparing these formulas, one has to use that $\alpha_s = g^2/4\pi$ and $f_{\eta'} = \sqrt{2N_f} f_\pi$.

expresses its mass via the difference of the respective chromomagnetic condensates with and without quark inclusion.

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REFERENCES

1. *'t Hooft G.* // Phys. Rep. 1986. V. 142. P. 357.
2. *Crewther R. J.* // Phys. Lett. B. 1977. V. 70. P. 349; Acta Phys. Austr. Suppl. 1978. V. 19. P. 47; Riv. Nuovo. Cim. 1979. 2N8. P. 63.
3. *Witten E.* // Nucl. Phys. B. 1979. V. 156. P. 269.
4. *Veneziano G.* // Ibid. V. 159. P. 213.
5. *Dyakonov D. I., Eides M. I.* // Sov. Phys. JETP. 1981. V. 54, No. 2. P. 232.
6. *Ioffe B. L.* // Phys. Atom. Nucl. 1999. V. 62. P. 2025.
7. *Narison S., Shore G. M., Veneziano G.* // Nucl. Phys. B. 1995. V. 433. P. 209.
8. *Jackiw R.* Current Algebra, Anomalies / Eds S. Trieman et al. Singapore: World Scientific, 1985.
9. *Rajaraman R.* An Introduction to Solitons and Instantons in Quantum Field Theory. North-Holland, 1982;

   Yndurain F. J. Quantum Chromodynamics. An Introduction to the Theory of Quarks and Gluons. Springer-Verlag, 1983. Sec. 38.
10. *Espriu D., Tarrach R.* // Z. Phys. C. 1982. V. 16. P. 77.
11. *Christos G. A.* // Phys. Rep. 1984. V. 116. P. 251.
12. *Nielsen H. B. et al.* // Phys. Lett. B. 1992. V. 281. P. 345.
13. *Blaschke D. et al.* // Phys. Lett. B. 1997. V. 397. P. 129.
14. *Pervushin V. N.* Dirac Variables in Gauge Theories. hep-th/0109218.

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