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ON THE NUMERICAL CALCULATION OF PHASE SPACE INTEGRAL OF VERY HIGH MULTIPLICITY PROCESSES

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ABSTRACT. Two methods of numerical calculation phase space integral of very high multiplicity processes are compared.

1. INTRODUCTION

It is well known that the existing generators of event badly describes the very high multiplicity hadron processes. In the framework of creation more efficient generator there are very important the knowledge of phase space integral. Note, that the program of numerical calculation should be the fast enough.

The problem is reduced to the calculation of following type of integral:

$$Z_n = \int \left\{ \prod_{i=1}^{n} \frac{d^3 k_i}{2\sqrt{k_i^2 + m^2}} \right\} \delta^4 \left(P - \sum k_i \right) f(k_1, \ldots, k_n) \tag{1.1}$$

where, for instance:

$$f(k_1, \ldots, k_n) = \prod_{i=1}^{n} \exp\left(-r_0^2 \, k_{t,i}^2\right) \tag{1.2}$$

and $P \equiv (E, 0, 0, 0)$; $k_{t,i} = \sqrt{k_{i,x}^2 + k_{i,y}^2}$ is the transverse momentum;
r_0 is the phenomenological constant (r_0=0.2GeV/c)

The theory of calculation of integrals of this type have a long history. Some methods [1-5] used statistical interpretation of transformed integrals which can be expressed in terms of the central limit theorem of probability theory. In [6] calculation consist in taking the Laplace transform of (1.1), inverting it and evaluating the inverted form approximately by saddle point. In [7] recursive equation for numerical calculation are obtained. Some methods [8-10] used different modification of Monte Carlo simulation.

Considering the very high multiplicity, it is important to have an universal method of calculation of the phase space integrals. It can be useful in a wide range of multiplicity. In our work we compare two numerical methods of integration. First is the ordinary Monte Carlo method, but we connect accounting the energy-momentum conservation laws. Second uses the Fourie transformation of the energy-momentum conservation low δ function.

2. MONTE CARLO METHOD

From the energy conservation low:

$$\delta\left(E - \sqrt{k_1^2 + m^2} - \ldots - \sqrt{k_n^2 + m^2} \right) \tag{2.1}$$

we have the condition for the maximum value of the momentum

$$E - \sqrt{k_{max}^2 + m^2} - (n-1)m = 0 \tag{2.2}$$

Solving this equation we find it:

$$k_{max} = E\sqrt{u_{max}(u_{max} + 2/n_{max})} \tag{2.3}$$

where

$$u_{max} = 1 - n/n_{max}; \quad n_{max} = E/m \tag{2.4}$$

Then we introduce variables $u_{i,\nu}$ defined in the interval $u_{i,\nu} \in [-1/g, 1/g]$:

$$k_{i,\nu} = k_{max}\ g\ u_{i,\nu} \tag{2.5}$$

As a result :

$$\begin{aligned} Z_n = {} & E^{2n-4}\left[u_{max}(u_{max} + 2/n_{max})\right]^{(3n-3)/2} g^{3n-3}\ 2^{-n} \times \\ & \times \int \left\{ \prod_{i=1}^{n} d^3 u_i \frac{\exp(-r_0^2\ k_{max}^2\ g^2\ u_{t,i}^2)}{\sqrt{u_{max}(u_{max} + 2/n_{max})\ g^2\ u_i^2 + 1/n_{max}^2}} \right\} \delta^4 \end{aligned} \tag{2.6}$$

where δ^4 is :

$$\begin{aligned} \delta^4 = {} & \delta\left(1 - \sum_{i=1}^{n} \sqrt{u_{max}(u_{max} + 2/n_{max})\ g^2\ u_i^2 + 1/n_{max}^2}\ \right) \times \\ & \times\ \delta(u_{1,x} + \ldots + u_{n,x})\ \delta(u_{1,y} + \ldots + u_{n,y})\ \delta(u_{1,z} + \ldots + u_{n,z}) \end{aligned} \tag{2.7}$$

Our calculating scheme have following form. In the interval $[-1/g, 1/g]$ we randomly select $u_{i,\nu}$ $i = 1 \div n - 1$.

Then we determinate $u_{n,\nu}$ from equations:

$$\begin{aligned} u_{n,x} &= -u_{1,x} - \ldots - u_{n-1,x} \\ u_{n,y} &= -u_{1,y} - \ldots - u_{n-1,y} \\ u_{n,z} &= -u_{1,z} - \ldots - u_{n-1,z} \end{aligned} \tag{2.8}$$

After this we solving the equation:

$$1 - \sum_{i=1}^{n} \sqrt{u_{max}(u_{max} + 2/n_{max})\ g^2\ u_i^2 + 1/n_{max}^2} = 0 \tag{2.9}$$

for g. Thus we satisfy all conditions for momenta. In such a way, we may calculate "one" point in Monte Carlo method $f(j)$ and consequently:

$$Z_n = \frac{1}{N}\sum_{j=1}^{N} f(j) \tag{2.10}$$

where

$$\begin{aligned} f(j) = {} & E^{2n-4}\left[u_{max}(u_{max} + 2/n_{max})\right]^{(3n-3)/2} g_0^{-3}\ 2^{2n} \times \\ & \times \prod_{i=1}^{n} \frac{\exp(-r_0^2\ k_{max}^2\ g_0^2\ u_{t,i}^2(j))}{\sqrt{u_{max}(u_{max} + 2/n_{max})\ g_0^2\ u_i^2(j) + 1/n_{max}^2}} \end{aligned} \tag{2.11}$$

Calculation realized on the Alpha-Workstation in Dubna. $N = 10^5$ is chosen. The time of calculation for $n = 10 \div 600$ and step equal to 10 use 4 minutes.

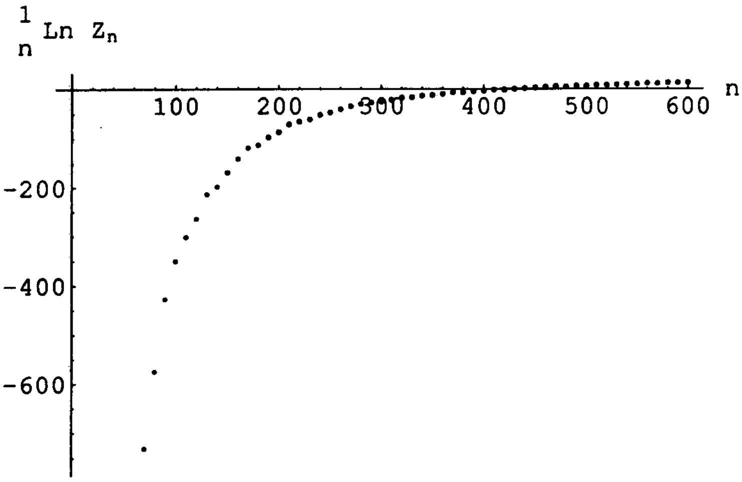

FIGURE 1. Results of calculation by Monte Carlo method

3. Fourie Transformation Method

For Dirac Delta Function we use the integral representation:

$$\delta^4\left(P-\sum k_i\right)=\int\frac{d^4\alpha}{(2\pi)^4}\exp\left[-i\alpha\left(P-\sum k_i\right)\right] \tag{3.1}$$

where $\alpha \equiv (\alpha_0, \vec{\alpha})$.

After simple transformation we get the representation:

$$\begin{aligned} Z_n &= \int \frac{d^4\alpha}{(2\pi)^4}\exp[-i\alpha_0 E]\times \\ &\times\left\{\int\frac{d^3k}{2\sqrt{k^2+m^2}}\exp[-r_0^2\,k_t^2+i\alpha_0\sqrt{k^2+m^2}-\vec{\alpha}\vec{k}\,]\right\}^n= \\ &=\int\frac{d^4\alpha}{(2\pi)^4}\exp[-i\alpha_0E]\Psi(\alpha) \end{aligned} \tag{3.2}$$

Our basic assumption is hypothesis of factorization of $\Psi(\alpha)$:

$$\Psi(\alpha)=\Phi(\alpha_0)\chi(\vec{\alpha}) \tag{3.3}$$

Consequently:

$$\int\frac{d\vec{\alpha}}{(2\pi)^3}\chi(\vec{\alpha})=C \quad \text{is a constant}$$

and we may choose $\vec{\alpha}=0$. We must to note that many authors uses this hypothesis [11].

Then we go to the spherical coordinates and after integration on the angle variables receive:

$$Z_n=\int\left\{\prod_{i=1}^{n}\frac{k_i\,d^3k_i}{r_0\sqrt{k_i^2+m^2}}\,(2\pi)\,F(r_0\,k_i)\right\}\,\delta\left(E-\sum_{j=1}^{n}\sqrt{k_j^2+m^2}\right) \tag{3.4}$$

where F is the Dawson's integral:

$$F(x)=\exp(-x^2)\int_0^x\exp(-t^2)dt \tag{3.5}$$

Now we can use the variables u_i from the equation:

$$\sqrt{k_i^2 + m^2} = E(u_i + 1/n_{max}) \tag{3.6}$$

and after substitution we come to:

$$Z_n = E^{n-1} \int \left\{ \prod_{i=1}^{n} \frac{2\pi}{r_0} \, du \, F(r_0 \, E \, \sqrt{u(u + 2/n_{max})} \,) \right\} \delta \left(u_{max} - \sum u_i \right) \tag{3.7}$$

Then we use integral representation of Dirac Delta Function:

$$Z_n = E^{n-1} \int_{-\infty}^{+\infty} \frac{d\beta}{2\pi} \exp(-i \, \beta \, u_{max}) \, [\, \varphi_c(\beta) + i \, \varphi_s(\beta) \,]^n \tag{3.8}$$

where

$$\varphi_c(\beta) = \frac{2\pi}{r_0} \int_0^{u_{max}} \cos(\beta u) \, F(r_0 E \sqrt{u(u + 2/n_{max})} \,) \, du \tag{3.9}$$

$$\varphi_s(\beta) = \frac{2\pi}{r_0} \int_0^{u_{max}} \sin(\beta u) \, F(r_0 E \sqrt{u(u + 2/n_{max})} \,) \, du \tag{3.10}$$

And finally we come to the following equation:

$$Z_n = E^{n-1} \int_0^{+\infty} \frac{d\beta}{2\pi} \cos[-\beta u_{max} + n \arctan(\varphi_s/\varphi_c) \,] \, [\, \varphi_c^2 + \varphi_s^2 \,]^{n/2} \tag{3.11}$$

Numerical calculation of this integral is very complicated procedure because integrand is a rapidly oscillating function. After testing some different program, we select the following algorithm. We find all roots of integrand and represent integral as a sum of integrals between neighboring roots.

Calculations realized on the Pentium III 800Mgh. The time of calculation for $n = 500 \div 69500$ with step 500 is about 2 hours.

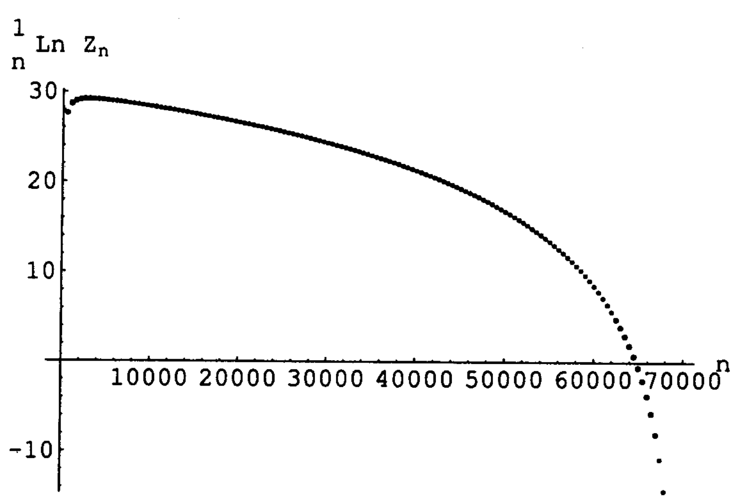

FIGURE 2. Results of calculation by Fourie transformation method

In our selection of parameters ($E = 1400GeV$, $m = 0.2GeV$) $n_{max} = 70000$.

4. Conclusion

We can conclude that the Monte Carlo method is powerful comparable small values of the n and the Fourie method is convenient just for high values of n.

Last one is understandable since for small value of n integrand in the representation is fast oscillating function.

And finally we to give an example behavior of $\frac{1}{n}\ln(Z_n)$ in the intermediate region:

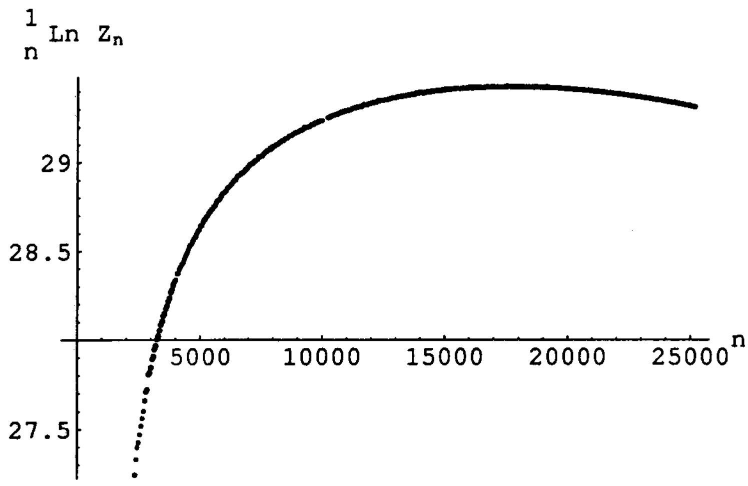

FIGURE 3. $\frac{1}{n}\ln(Z_n)$ in the intermediate region

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