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ON STATISTICAL DESCRIPTION OF VERY HIGH MULTIPLICITY EVENTS

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Abstract
Possibility to use the statistical methods for multiple production phenomena is investigated. For this purpose, we derive the new interpretation of 'correlations cancellation' condition proposed by N.N.Bogolyubov. It is argued that this condition may be satisfied in the very high multiplicity region.
Key-words: multiple production, nonequilibrium statistical physics.

1 Introduction

The topic of my talk is a possibility of using statistical methods to describe multiple production phenomenon.

Of course, everybody understands that "the main road" of particle physics development is now connected with the "Standard Model" (beyond the SM may be SUSY). But there are other roads, less important at first glance, which also allow to see other new aspects of Nature. The multiple production phenomenon is one of them.

We would like to note at the very beginning that the description of multiple production events in the ordinary terms is noneffective because of a very large number of involved degrees of freedom (particles).

But let us remember here that particle physics always felt the influence of statistical physics: the notions of vacuum, of phase transition, etc. Besides, we also know that statistical physics deals very well with an enormous number of particles. We wish to engage this rich experience to describe the multiple production phenomena.

The main attention will be concentrated on the "equilibrium state". It will be considered as a phenomenon, which can be examined experimentally in the particle collisions and may be predicted theoretically.

We will conclude with a few questions to the experiment.

2 Why the Very High Multiplicities?

We will start with the explanation why the very high multiplicity (VHM) events are interesting.

The intuitive feeling that the hadron matter should be essentially perturbative in the high energy extremely inelastic hadron collisions was the main reason of our efforts to consider the VHM domain.

There was a hope to observe a new dynamical phenomenon in this region.

Particularly, we will argue an idea that in the VHM domain all degrees of freedom would be excited and due to this reason, the system comes to equilibrium.

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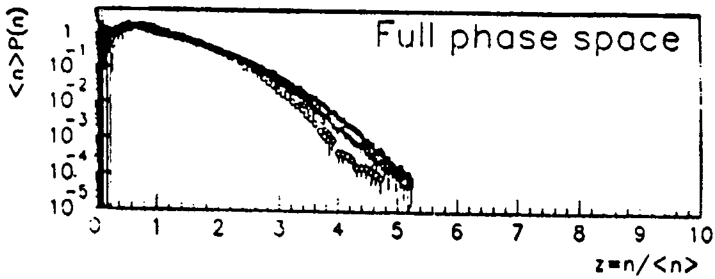

Figure 1: Multiplicity distribution

Notice also that the energies of produced particles can not be high in the VHM ever One can say that the system becomes calm. This is one more argument why we expect equilibrium in the VHM domain.

3 What was done in the VHM field?

The phenomenology and an idea of rough (statistical) description of the VHM proces was formulated in our first publications [1]. Later on, we accumulated our main publi tions on the VHM theory in the review paper [2]. The definite connection with idea N.N.Bogolyubov concerning transition to the equilibrium was mentioned in [3]

We understand that the multiple production process may contain a number of su processes, and corresponding formalism of the semi-inclusive approach developed in [4] w used.

It should be stressed that no experimental information in the VHM domain has be known till now.

4 Where is VHM domain?

The multiple production is a process of kinetic energy dissipation of colliding particles in the mass of produced particles.

Let ε_{max} be the energy of the fastest particle in the given frame and let E be the tot incident energy in the same frame. Then the difference $(E - \varepsilon_{max})$ is the energy spent f production of less energetic particles. It is useful to consider the inelasticity coefficient

$$\kappa = \frac{E - \varepsilon_{max}}{E} = 1 - \frac{\varepsilon_{max}}{E} \leq 1. \qquad ($$

We wish to consider processes with

$$\frac{\varepsilon_{max}}{E} = 1 - \kappa << 1. \qquad (:$$

Using thermodynamical terminology, we will investigate the production and properties of comparatively 'cold' multi-hadron state.

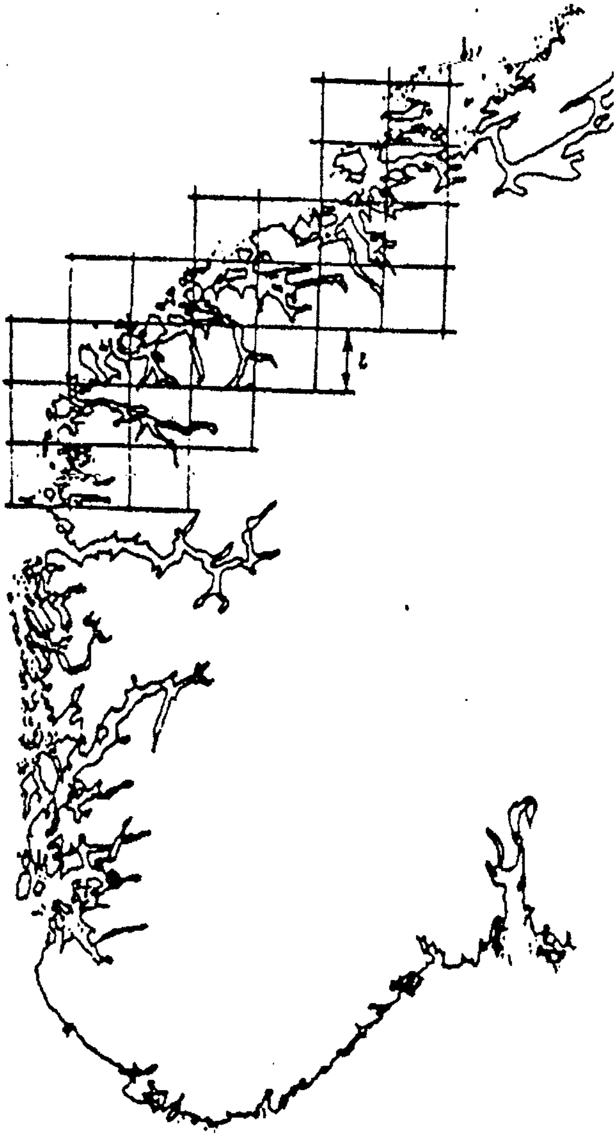

Figure 2: Norway shore

Using the energy conservation law ($n\varepsilon_{max} > E$),

$$n\frac{\varepsilon_{max}}{E} = n(1-\kappa) > 1. \tag{3}$$

As follows from (2), the multiplicity

$$n > \frac{1}{1-\kappa} >> 1 \tag{4}$$

This is an 'ordinary' definition of the considered 'VHM processes'.

But we will adopt a more acceptable condition from the experimental point of view (2).

5 When can the statistical method be used?

The multiple production cross section $\sigma_n(s)$ falls down rapidly in the discussed very high multiplicity (VHM) domain. Therefore, the precise experiments with high statistics cannot be carried out in the VHM region, see Fig.1.

At the LHC energy

$$\bar{n}(s) \simeq 100$$

is valid and we will assume that $n \sim \bar{n}(s)^2 \simeq 10\ 000$ is just the discussed VHM region ($n_{max} \simeq 100\ 000$ at the LHC energy). It is easy to explain why

$$n \sim \bar{n}(s)^2 \tag{5}$$

may be chosen to define the VHM region. Indeed, if the transverse momenta of the produced particles are in the restricted quantity, hadron interaction radii are $\sim \sqrt{\ln s}$. So, the disk contains $\bar{n}(s)$ ~(disk area)~ $\ln s$ partons. Then, each parton in its turn may form the disk of $\sim \ln s$ area. Therefore, at $n > \bar{n}(s)^2$ we get out of the standard hadron kinematics. So, we will consider the interval

$$10\ 000 \leq n << 100\ 000$$

Generally, investigating the multiple production phenomena, we face the problem:

(i) Investigated system is rather non-regular and for its *complete* description all parameters should be used.

(ii) Number of parameters $N = 3n - 4$ is enormously large ($\sim$ 30 000!)

The experiments and the fractal analysis have shown that the particle density fluctuation is unexpectedly large and the fractal dimension D_f is not equal to zero.

We know that if the fractal dimension is non-trivial, the system is extremely 'non-regular'. So, $D_f \simeq$1+0.3 for the perimeter of the Great Britain and $D_f \simeq$1+0.5 for Norway. The discrepancy is connected with the fact that the shore of Norway is much more broken than of the Great Britain, see Fig.2.

The multifractal (Reney) dimension [5] is

$$D_q = 1 - d_q,$$

where q is the number of factorial moment that can be investigated. It is seen from Fig.3 that d_q increases with q, i.e. D_q would decrease with multiplicity. This may mean tendency to 'calmness' with rising multiplicity.

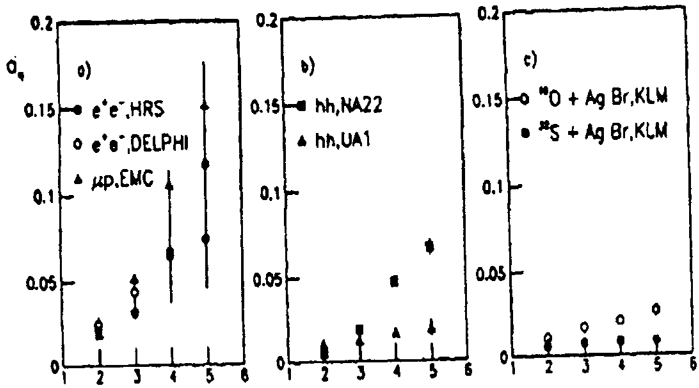

Figure 3: Reney fractal dimension

Generally speaking, having the state of a large number of particles, it is reasonable to depart from an exact definition of the final state kinematics.

For instance, we suppose that nothing will happen if n is measured with $\Delta n \neq 0$ accuracy since $(\Delta n/n) << 1$ is easily attainable in the VHM region.

So, it is important to understand under what conditions the restricted set of the dynamical variables allows to describe the process (state) completely.

The same problem was solved in statistical physics, where the 'rough' description by a restricted number of (thermodynamical) parameters is a basis of its success.

The multiplicity n measures the entropy S: in the VHM domain the entropy should tend to its maximum, i.e. the system 'calms down'. To describe calm systems, a small number of parameters is necessary.

For this reason, we will discuss a possibility of the 'rough' description introducing the temperature.

So, we will find the criteria when only the mean value of produced particles energy may completely define the energy distribution in the VHM final state.

On the same ground one can consider the mean value of charge, spin, etc. distributions for 'rough' description.

6 'Rough' variable for energy spectrum

By definition,

$$\sigma_n^{ab}(s) = \int d\omega_n(q)\delta(q_a + q_b - \sum_{i=1}^{n} q_i)|A_n^{ab}|^2, \tag{6}$$

where A_n^{ab} is the amplitude of n creation at interaction of particles a and b.

Considering Fourier transform of energy-momentum conservation δ-function, one can introduce the generating function ρ_n. We may find in the result that σ_n is defined by

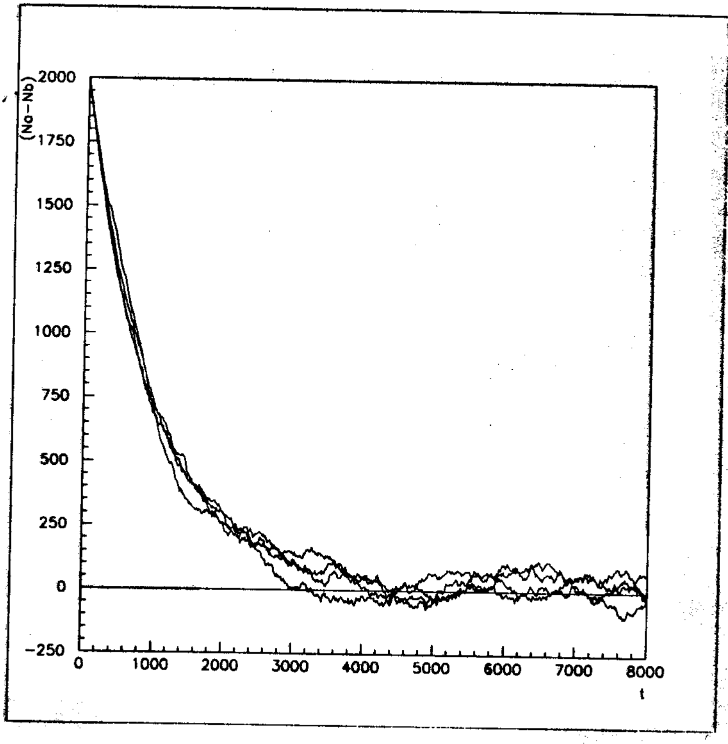

Figure 4: Particles production in the Markovian process. $N_a = N_b$ corresponds to equilibrium

equality:

$$\sigma_n(s) = \int_{-i\infty}^{+i\infty} \frac{d\beta}{2\pi} e^{\beta\sqrt{s}} \rho_n(\beta), \tag{7}$$

where

$$\rho_n(\beta) = \int \left\{ \prod_{i=1}^{n} \frac{d^3 q_i e^{-\beta\epsilon(q_i)}}{(2\pi)^3 2\epsilon(q_i)} \right\} |A_n^{ab}|^2. \tag{8}$$

The most probable value β_c in this integral is defined by the equation of state:

$$\sqrt{s} = -\frac{\partial}{\partial\beta} \ln \rho_n(\beta). \tag{9}$$

The solution of this equation will be $\beta_c(s, n)$.

Then β_c may be considered as a 'rough' variable: instead of the n energies

$$\epsilon(q_1),\ \epsilon(q_2), \ldots, \epsilon(q_n)$$

we introduce one variable β_c in such a way that, as follows from (9), $1/\beta_c$ is the mean energy and the fluctuations of energies near $1/\beta_c$ are defined by the Boltzmann factor $e^{-\beta_c\epsilon}$, see (8).

It is important to note that the equation(9) has unique real rising with n and decreasing with s solution $\beta_c(s,n)$.

To find the physical meaning of β_c, one may consider the example of noninteracting particles, when $A_n = const$. The direct calculation gives

$$\rho_n(\beta) = |A_n|^2 \{2\pi m K_1(\beta m)/\beta\}^n,$$

where K_1 is the Bessel function. Inserting this expression into (9) we can find that in the nonrelativistic case ($n \simeq n_{max}$)

$$\beta_c^0 = \frac{3}{2}\frac{(n-1)}{(\sqrt{s}-nm)}.$$

This means that

$$E_{kin} = \frac{3}{2}T, \tag{10}$$

where $E_{kin} = (\sqrt{s}-nm)$ is the kinetic energy and T is the temperature. The eq.(10) is obvious for the 'ideal gas' approximation.

7 Relaxation of correlations

The expansion of integral (7) near $\beta_c(s,n)$ unavoidably gives asymptotic series with zero convergence radii since $\rho_n(\beta)$ is an essentially nonlinear function of β. From the physical point of view this means that fluctuations in the vicinity of $\beta_c(s,n)$ may be arbitrarily high and in this case $\beta_c(s,n)$ has not any physical sense.

But if fluctuations are small, $\rho_n(\beta)$ should coincide with partition function of n particles and $\beta_c(s,n)$ may be interpreted as the inverse temperature.

Let us define now the conditions when the fluctuations are small. Firstly, we should expand $\ln\rho_n(\beta+\beta_c)$ over β:

$$\ln\rho_n(\beta+\beta_c) = \ln\rho_n(\beta_c) - \sqrt{s}\beta + \frac{1}{2!}\beta^2\frac{\partial^2}{\partial\beta_c^2}\ln\rho_n(\beta_c) - \\ -\frac{1}{3!}\beta^3\frac{\partial^3}{\partial\beta_c^3}\ln\rho_n(\beta_c) + \ldots \tag{11}$$

and, secondly, expand the exponent in the integral, for instance, over

$$\partial^3\ln\rho_n(\beta_c)/\partial\beta_c^3$$

neglecting higher decomposition terms in (11). As a result, k-th term of the perturbation series is

$$\rho_{n,k} \sim \left\{\frac{\partial^3\ln\rho_n(\beta_c)/\partial\beta_c^3}{(\partial^2\ln\rho_n(\beta_c)/\partial\beta_c^2)^{3/2}}\right\}^k \Gamma\left(\frac{3k+1}{2}\right). \tag{12}$$

Therefore, one should assume that

$$\partial^3\ln\rho_n(\beta_c)/\partial\beta_c^3 << (\partial^2\ln\rho_n(\beta_c)/\partial\beta_c^2)^{3/2}. \tag{13}$$

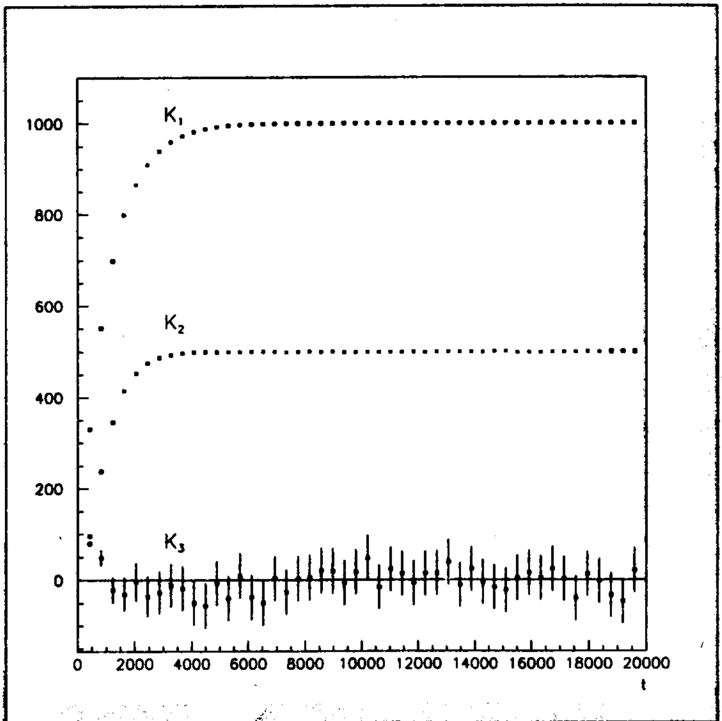

Figure 5: Equilibrium over particles number

to neglect this term. A possible solutions of this condition is

$$\partial^3 \ln \rho_n(\beta_c)/\partial\beta_c^3 \approx 0. \tag{14}$$

If this condition is preserved, the fluctuations are Gaussian.

Now let us consider (14) carefully. We will find computing derivatives that this condition means the following approximate equality:

$$\frac{\rho_n^{(3)}}{\rho_n} - 3\frac{\rho_n^{(2)}\rho_n^{(1)}}{\rho_n^2} + 2\frac{(\rho_n^{(1)})^3}{\rho_n^3} \approx 0, \tag{15}$$

where $\rho_n^{(k)}$ means the k-th derivative. For identical particles (see definition (8)),

$$\begin{aligned}\rho_n^{(k)}(\beta_c) &= n^k(-1)^k \int \left\{\prod_{i=1}^{n} \epsilon(q_i)\frac{d^3q_i e^{-\beta\epsilon(q_i)}}{(2\pi)^3 2\epsilon(q_i)}\right\} |A_n^{ab}|^2 \\ &= \sigma_{tot} n^k \int \left\{\prod_{i=1}^{k} \epsilon(q_i)\frac{d^3q_i e^{-\beta\epsilon(q_i)}}{(2\pi)^3 2\epsilon(q_i)}\right\} \bar{f}_k(q_1, q_2, \ldots, q_k),\end{aligned} \tag{16}$$

where $\bar{f}_k$ is the $(n-k) \geq 0$-point inclusive cross section. It coincides with k-particle distribution function in the n-particle system. Therefore, l.h.s. of (15) is the 3-point correlator K_3:

$$K_3 \equiv \int d\omega_3 \left(< \prod_{i=1}^{3} \epsilon(q_i) >_{\beta_c} -3 < \prod_{i=1}^{2} \epsilon(q_i) >_{\beta_c} < \epsilon(q_3) >_{\beta_c} + \right.$$
$$\left. + 2 \prod_{i=1}^{3} < \epsilon(q_i) >_{\beta_c} \right),$$

where the index means averaging with the Boltzmann factor

$$\exp\{-\beta_c \epsilon(q)\}$$

The Eurenfest-Kac model describes *random* "production" and "absorption" of particles. The particles production in the 'event-by-event' experiments is shown in Fig.4 and Fig.5 shows as the correlations relax in this model.

8 Conclusion: questions to the experiment

As a result, to have all fluctuations in the vicinity of β_c Gaussian, we should have $K_m \approx 0$, $m \geq 3$. But, as it comes from (13), the set of minimal conditions looks as follows:

$$K_m << K_2^{m/2}, \; m \geq 3. \tag{17}$$

If the experiment confirms these conditions, then, independently on the number of particles, the final state may be described by one parameter β_c with a high enough (exponential) accuracy.

Considering β_c as physical (measurable) quantity, we are forced to assume that both the total energy of the system $\sqrt{s} = E$ and conjugated to it variable β_c may be measured.

One more detail. Our consideration has shown the uniqueness of Bogolyubov's solution of the nonequilibrium thermodynamics problem. Indeed, without vanishing of correlations perturbation series in the β_c vicinity, being asymptotic, is divergent.

We would like to stress in conclusion that Bogolyubov's creative works naturally unite particle and statistical physics. As a result, using Bogolyubov's mathematical basis, we have the united scientific space in which both branches of physics, thermodynamics and quantum field theory, supplement each other.

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