Soft partons production in deep inelastic kinematics

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Abstract

It is argued considering multiple production in the deep inelastic scattering kinematics that the very high multiplicity events are extremely sensitive to the low-x partons density.

1 Introduction

The role of soft color partons in the high energy hadron interactions is the mostly intriguing modern problem of particles physics. So, the collective phenomena and symmetry breaking in the non-Abelian gauge theories, confinement of colored charges and the infrared divergences of the pQCD are the phenomena just of the soft color particles domain.

It seems natural that the very high multiplicity (VHM) hadron interaction, where the energy of created particles is small, should be sensitive to the soft color particles densities. Indeed, the aim of this paper is to show that even in the hard by definition deep inelastic scattering (DIS) the soft color particles role becomes important in the VHM region.

The standard (mostly popular) hadron theory considers perturbative QCD (pQCD) at small distances (in the scale of $\Lambda \simeq 0.2$ *Gev*) as the exact theory. This statement is confirmed by a number of experiments, e.g. the DIS data, the QCD jets observation. But one should have in mind that the pQCD has finite range of validity since the non-perturbative effects should be taken into account at distances larger then $1/\Lambda$.

It is natural to assume building the complete theory that at large distances the non-perturbative effects exceed the perturbative ones[1]. In result the pQCD contributions becomes *screened* by the non-perturbative interactions.

But exist another, more natural, possibility. It should be noted here that the pQCD running coupling constant $\alpha_s(q^2) = 1/b\ln(q^2/\Lambda^2)$ becomes infinite at $q^2 = \Lambda^2$ and we do not know what happens with pQCD if $q^2 < \Lambda^2$. There is a suspicion [2] that at $q^2 \sim \Lambda^2$ the properties of theory changed so drastically (being defined on new vacuum) that even the notions of pQCD is *disappeared*. This means that pQCD should be truncated from below on the 'fundamental' scale Λ. It seems evident that this infrared cut-off should influence on the soft hadrons emission in the VHM domain.

So, it is important try to rise predictability of pQCD in the 'forbidden' area of large distances. For this purpose one should split experimentally the perturbative and

[1] The corresponding formalism was described e.g. in [1].

non-perturbative effects at the large distances to check out quantitatively the role of soft color partons. One must realize for this purpose highly unusual condition that the non-perturbative effects must be negligible even if the distance among color charges is high.

The non-perturbative effects lead to the strong polarization of QCD vacuum and, in result, to the color charge confinement. This vacuum is unstable [3] against creation of real quark-antiquark ($q\bar{q}$) pairs if the distance among charges became large. This pairs are captured into the colorless hadrons and just emission of this 'vacuum' hadrons is the mostly important non-perturbative effect. Therefore, generally, the number of hadrons n, produced even in the hard DIS process, $n \neq n_p$, where n_p is the number of $\bar{q}q$ pairs created 'perturbatively'.

Notice, if the kinetic energy of colored partons is small, i.e. is comparable with hadron masses, creation of 'vacuum' hadrons should be negligible. Just this is the VHM process kinematics: because of the energy-momentum conversation law, produced (final-state) partons can not have high relative momentum and, if they was created at small distances, the production of 'vacuum' hadrons will be negligible (or did not play important role). Therefore, if the 'vacuum' channel is negligible, the pQCD contributions only should be considered [4].

The aim of this article is to show that $n \to \infty$ unavoidably leads to 'low-x' domain.

2 DIS kinematics in the VHM domain

To describe the hadron production in pQCD terms the parton-hadron duality is assumed. This means that the multiplicity, momentum etc. distributions of hadron and colored partons are the same. This reduce the problem practically on the level of QED.

Let us consider now n particles (gluons) creation the DIS. We would like to calculate $D_{ab}(x,q^2;n)$, where

$$\sum_n D_{ab}(x,q^2;n) = D_{ab}(x,q^2). \tag{2.1}$$

As usual, let $D_{ab}(x,q^2)$ be the probability to find parton b with virtuality $q^2<0$ in the parton a of $\sim \lambda$ virtuality, $\lambda >> \Lambda$ and $\alpha_s(\lambda) << 1$. We always may chose q^2 and x so that the leading logarithm approximation (LLA) will be acceptable. One should assume also that $(1/x) >> 1$ to have the phase space, into which the particles are produced, sufficiently large.

Then $D_{ab}(x,q^2)$ is described by ladder diagrams. From qualitative point of view this means approximation of Markovian process of random walk over coordinate $\ln(1/x)$ and time is $\ln\ln|q^2|$. LLA means that the 'mobility' $\sim \ln(1/x)/\ln\ln|q^2|$ should be large

$$\ln(1/x) >> \ln\ln\left|q^2/\lambda^2\right|. \tag{2.2}$$

But, on other hand [5],

$$\ln(1/x) << \ln\left|q^2/\lambda^2\right|. \tag{2.3}$$

The leading, able to compensate smallness of $\alpha_s(\lambda) << 1$, contributions give integration over wide range $\lambda^2 << k_i^2 << -q^2$, where $k_i^2 > 0$ is the 'mass' of real, i.e.

time-like, gluon. If the time needed to capture the parton into the hadron is $\sim (1/\Lambda)$ then the gluon should decay if $k_i^2 >> \lambda^2$. This leads to creation of (mini)jets. The mean multiplicity $\bar{n}_j$ in the QCD jets is high if the gluon 'mass' $|k|$ is high: $\ln \bar{n}_j \simeq \sqrt{\ln(k^2/\lambda^2)}$.

Rising multiplicity may (i) rise number of (mini)jets ν and/or (ii) rise the mean value mass of (mini)jets $< |k_i| >$. We will see that the mechanism (ii) would be favorable. This is consequence of the Markovian character of considered process.

But rising mean value of gluon masses $|k_i|$ decrease the range of integrability over k_i, i.e. violate the condition (2.2) for fixed x. One can remain the LLA taking $x \to 0$. But this may contradict to (2.3), i.e. in any case the LLA becomes invalid in the VHM domain and the next to leading order corrections should be taken into account.

Noting that the LLA gives main contribution, that the rising multiplicity leads to the infrared domain, where the soft gluons creation becomes dominant.

3 Preliminary notes

First of all, neglecting the vacuum effects, we introduce definite uncertainty to the formalism. It is reasonable to define the level of strictness of our computations. Let us introduce for this purpose the generating function $T_{ab}(x,q^2;z)$:

$$D_{ab}(x,q^2;n) = \frac{1}{2\pi i}\oint \frac{dz}{z^{n+1}} T_{ab}(x,q^2;z). \qquad (3.1)$$

Having large n the integral may be calculated by the saddle point method. The smallest solution z_c of the equation

$$n = z\frac{\partial}{\partial z} \ln T_{ab}(x,q^2;z) \qquad (3.2)$$

defines the asymptotic over n behavior:

$$D_{ab}(x,q^2;n) \propto e^{-n\ln z_c(x,q^2;n)}. \qquad (3.3)$$

Using the statistical interpretation of z_c as the fugacity it is natural to write:

$$\ln z_c(x,q^2;n) = \frac{C_{ab}(x,q^2;n)}{\bar{n}_{ab}(x,q^2)}. \qquad (3.4)$$

Notice that the solution of eq.(3.2) $z_c(x,q^2;n)$ should be the increasing function of n. At first glance this follows from positivity of all $D_{ab}(x,q^2;n)$. But actually this assumes that $T_{ab}(x,q^2;z)$ is a regular function of z at $z = 1$. This is a natural assumption considering just the pQCD predictions.

Therefore,

$$D_{ab}(x,q^2;n) \propto e^{-\frac{n}{\bar{n}_{ab}(x,q^2)} C_{ab}(x,q^2;n)}. \qquad (3.5)$$

This form of $D_{ab}(x,q^2;n)$ is useful since usually $C_{ab}(x,q^2;n)$ is the slowly varying function of n. So, for Poisson distribution $C_{ab}(x,q^2;n) \sim \ln n$. For KNO scaling we have $C_{ab}(x,q^2;n) = const.$ over n.

We would like to note, that neglecting effects of vacuum polarization we introduce into the exponent so high uncertainty assuming $n \simeq n_p$ that it is reasonable perform the calculations with the exponential accuracy. So, we would calculate

$$-\bar{\mu}_{ab}(x,q^2;n) = \ln \frac{D_{ab}(x,q^2;n)}{D_{ab}(x,q^2)} = \frac{n}{\bar{n}_{ab}(x,q^2)} C_{ab}(x,q^2;n)(1+O(1/n)) \tag{3.6}$$

The n dependence of $C_{ab}(x,q^2;n)$ defines the asymptotic behavior of $\bar{\mu}_{ab}(x,q^2;n)$ and calculation of its explicit form would be our aim.

4 Correlation functions

Considering particles creation in the DIS processes one should distinguish correlation of particles in the (mini)jets and the correlations between (mini)jets. We will describe the jet correlations. One should introduce the ν jets creation inclusive cross section $\Phi_\nu^{(r)_\nu}(k_1,k_2,....,k_\nu;q^2,x)$, where $k_i,\ i=1,2,...,n$ are the jets 4-momentum. Having Φ_ν we can find the correlations function $N_\nu^{(r)}(k_1,k_2,....,k_\nu;q^2,x)$, where $(r)=r_1,...,r_\nu$ and $r_i=(q,g)$ defines the sort of created color particle. It is useful introduce the generating functional

$$F^{ab}(q^2,x;w) = \sum_n \int d\Omega_n(k) \prod_{i=1}^{n} w^{r_i}(k_i) \left| a_n^{ab}(k_1,k_2,....,k_n;q^2,x)\right|^2, \tag{4.1}$$

where a_n^{ab} is the amplitude, $d\Omega_n(k)$ is the phase space volume and $w^{r_i}(k_i)$ are the arbitrary functions. It is evident,

$$F^{ab}(q^2,x;w)\mid_{w=1} = D^{ab}(q^2,x). \tag{4.2}$$

The inclusive cross sections

$$\Phi_\nu^{(r)}(k_1,k_2,....,k_\nu;q^2,x) = \prod_{1=1}^{\nu} \frac{\delta}{\delta w^{r_i}(k_i)} F^{ab}(q^2,x;w)\mid_{w=1}. \tag{4.3}$$

The correlation function

$$N_\nu^{(r)}(k_1,k_2,....,k_\nu;q^2,x) = \prod_{1=1}^{\nu} \frac{\delta}{\delta w^{r_i}(k_i)} \ln F^{ab}(q^2,x;w)\mid_{w=1}. \tag{4.4}$$

We will use this definitions to find the partial structure functions $D^{ab}(q^2,x;n)$.

It will be useful to introduce the Laplace transform over variable $\ln(1/x)$:

$$F^{ab}(q^2,x;w) = \int_{\mathrm{Rej}<0} \frac{dj}{2\pi i} (\frac{1}{x})^j f^{ab}(q^2,j;w) \tag{4.5}$$

The expansion parameter of our problem $\alpha_s \ln(-q^2/\lambda^2) \sim 1$. By this reason one should take into account all possible cuttings of the ladder diagrams. So, calculating

$D^{ab}(q^2,x)$in the LLA all possible cuttings of sceleton ladder diagrams is defined by the factor [5]:

$$\frac{1}{\pi}\left\{\Gamma^{ab}_r G_r \Gamma^{ab}_r\right\}, \tag{4.6}$$

i.e. the cutting line may get not only through the exact Green function $G_r(k_i^2)$ but through the exact vertex functions $\Gamma^{ab}_r(q_i,q_{i+1},k_i)$ also (q_i^2, q_{i+1}^2 are negative). We have in the LLA that

$$\lambda^2 << -q_i^2 << -q_{i+1}^2 << -q^2$$

and

$$x \le x_{i+1} \le x_i \le 1.$$

Following to our approximation, see previous section, we would not distinguish the way as cut line go through the Born amplitude

$$a^{ab}_r) = \left\{(\Gamma^{ab}_r)^2 G_r\right\}.$$

We will simply associate $w^r \mathrm{Ima^{ab}_r})$ to each rung of the ladder.

Considering the asymptotics over n the time-like partons virtuality $k_i \simeq -q_i^2/y_i$ should be maximal. Here y_i is the fraction of the longitudinal momentum of the jet. Then, slightly limiting the jets phase space,

$$\ln k_i^2 = \ln|q_{i+1}|^2 \left(1 + O(\ln(1/x)/|q_{i+1}|^2)\right). \tag{4.7}$$

In result, introducing useful in the LLA variable $\tau_i = \ln(q_i^2/\Lambda^2)$, where $\alpha_s(q^2) = 1/\beta\tau$, $\beta = (11N/3) - (2n_f/3)$, we can find following set of equations:

$$\tau\frac{\partial}{\partial\tau} f_{ab}(q^2,j;w) = \sum_{c,r} \varphi^r_{ac}(j) w^r(\tau) f_{ab}(q^2,x;w), \tag{4.8}$$

where

$$\varphi^r_{ac}(j) = \varphi_{ac}(j) = \int_0^1 \frac{dx}{x} x^j P_{ac}(x) \tag{4.9}$$

and $P_{ac}(x)$ is the regular kernel of the Bethe-Salpeter equation [5]. At $w=1$ this equation is the ordinary one for $D^{ab}(q^2,x)$.

We will search the correlation functions from eq.(4.8) in terms of Laplace transform $n^{(r)_\nu}_{ab}(k_1,k_2,....,k_\nu;q^2,j)$. Let us write:

$$f_{ab}(q^2,j;w) = d_{ab}(q^2,j)\exp\left\{\sum_\nu \frac{1}{\nu!}\int\prod_{i=1}^{\nu}\left(\frac{d\tau_i}{\tau_i}(w^{r_i}(\tau_i)-1)\right) n^{(r)_\nu}_{ab}(k_1,k_2,....,k_\nu;q^2,j)\right\} \tag{4.10}$$

Inserting (4.10) into (4.8) and expanding over $(w-1)$ we find the sequence of coupled equation.

Omitting the cumbersome calculations, we write in the LLA that

$$\phi^{(r)_\nu}_{ab}(\tau_1,\tau_2,....,\tau_\nu;q^2,j) = d_{ac_1}(j,\tau_1)\varphi^{r_1}_{c_1c_2}(j) d_{c_2c_3}(j,\tau_2)\cdots\varphi^{r_\nu}_{c_\nu c_{\nu+1}}(j) d_{c_{\nu+1}b}(j,\tau_{\nu+1}). \tag{4.11}$$

One should take into account the conservation laws:

$$\tau_1 \cdot \tau_2 \cdots \tau_{\nu+1} = \tau,\ \tau_1 < \tau_2 < \ldots < \tau_{\nu+1} < \tau. \tag{4.12}$$

Computing the Laplace transform of this expression we find $\Phi_{ab}^{(r)_\nu}(\tau_1, \tau_2, ..., \tau_\nu; q^2, x)$.

The kernel $d_{ab}(j,\tau)$ was introduced in (4.11). Let us write it in the form:

$$d_{ab}(j,\tau) = \sum_{\sigma=\pm} \sigma \frac{d_{ab}(j)}{\nu_+ - \nu_-} \tau^{\nu_\sigma(j)}, \tag{4.13}$$

where

$$d_{qq}^\sigma = \nu_s - \varphi_{gg},\ d_{gg}^\sigma = \nu_s - \varphi_{qq},\ d_{qg}^\sigma = \varphi_{gq},\ d_{gq}^\sigma = \varphi_{qg} \tag{4.14}$$

and

$$\nu_\sigma = \frac{1}{2}\left\{\varphi_{qq} + \varphi_{gg} + \sigma\left[(\varphi_{qq} - \varphi_{gg})^2 - 4n_f\varphi_{qg}\varphi_{gq}\right]^{1/2}\right\}. \tag{4.15}$$

If $x << 1$, then $(j-1) << 1$ are essential. In this case [5],

$$\varphi_{gg} \sim \varphi_{gq} >> \varphi_{qg} \sim \varphi_{qq} = O(1). \tag{4.16}$$

This means the gluon jets dominance and

$$n_{gg}^g = \varphi_{gg} + O(1). \tag{4.17}$$

One can find following estimation of the two-jet correlation function:

$$n_{ab}^{r_1 r_2}(\tau_1, \tau_2; \mathrm{j}, \tau) = O\left(\max\{(\tau_1/\tau)^{\varphi_{gg}}, (\tau_2/\tau)^{\varphi_{gg}}, (\tau_1/\tau_2)^{\varphi_{gg}}\}\right). \tag{4.18}$$

This correlation function is small since in the LLA $\tau_1 < \tau_2 < \tau$. This means that the jet correlations becomes high iff the mass of correlated jets are comparable. But this condition shrinks the range of integration over τ and by this reason one may neglect the 'short-range' correlations among jets. Therefore, as follows from (4.10),

$$f_{ab}(q^2, j; w) = d_{gg}(\tau, j) \exp\left\{\varphi_{gg} \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'} w^g(\tau')\right\} \tag{4.19}$$

We will use this expression to find the multiplicity distribution in the DIS domain.

5 Generating function in the DIS kinematics

To describe particles production one should replace $w^r \mathrm{Ima_r^{ab}})$ on $w_n^r \mathrm{Ima_r^{ab}})$, where w_n^r is the *probability* of n particles production,

$$\sum_n w_n^r = 1. \tag{5.1}$$

Having ν jets one should take into account the conservation condition $n_1 + n_2 + ... + n_\nu = n$. By this reason the generating functions formalism is useful. In result one can find that if we take (4.19)

$$w^g = w^g(\tau, z),\ w^g(\tau, z)\mid_{z=1} = 1, \tag{5.2}$$

then $f_{ab}(q^2, j; w)$ defined by (4.19) is the generating functional of the multiplicity distribution in the 'j representation'. In this expression $w^g(\tau, z)$ is the generating function of the multiplicity distribution in the jet of mass $|k| = \lambda e^{\tau/2}$.

In result, see (4.5),

$$F^{ab}(q^2, x; w) \propto \int_{\mathrm{Rej}<0} \frac{dj}{2\pi i} (1/x)^j e^{\varphi_{gg}\omega(\tau,z)} \tag{5.3}$$

where

$$\omega(\tau, z) = \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'} w^g(\tau', z). \tag{5.4}$$

Noting the normalization condition (5.2),

$$\omega(\tau, z = 1) = \ln \tau. \tag{5.5}$$

The integral (5.3) may be calculated by steepest descent method. It is not hard to see that

$$j \simeq j_c = 1 + \{4N\omega(\tau, z)/\ln(1/x)\}^{1/2} \tag{5.6}$$

is essential. Notice that $j - 1 << 1$ should be essential we find, instead of the constraint (2.2), that

$$\omega(\tau, z) << \ln(1/x). \tag{5.7}$$

In the frame of this constraint,

$$F^{ab}(q^2, x; w) \propto \exp\left\{4\sqrt{N\omega(\tau, z)\ln(1/x)}\right\}. \tag{5.8}$$

Generally speaking, exist such values of z that $j_c - 1 \sim 1$. This is possible if $\omega(\tau, z)$ is a regular function of z at $z = 1$. Then z_c should be the increasing function of n and consequently $\omega(\tau, z_c)$ would be the increasing function of n. Therefore, one may expect that in the VHM domain $j_c - 1 \sim 1$.

Then $j \simeq 1 + \omega(\tau, z)/\ln(1/x)$ would be essential in the integral (5.3). This leads to following estimation:

$$F^{ab}(q^2, x; w) \propto e^{-\omega(\tau,z)}.$$

But this is impossible since $F^{ab}(q^2, x; w)$ should be the increasing function of z. This shows that the estimation (5.8) has a finite range of validity.

Solution of this problem with unitarity is evident. One should take into account correlations among jets considering the expansion (4.10). Indeed, smallness of $n_{ab}^{(r)\nu}$ may be compensated by large values of $\prod_i^{\nu} w^{r_i}(\tau_i, z)$ in the VHM domain.

6 Conclusion

We can conclude that our LLA is applicable in the VHM domain till

$$\omega(\tau, z) << \ln(1/x) << \tau = \ln(-q^2/\lambda). \tag{6.1}$$

The mean multiplicity of gluons created in the DIS kinematics

$$\bar{n}_g(\tau, x) = \frac{\partial}{\partial z} \ln F^{ab}(q^2, x; w) \mid_{z=1} = \omega_1(\tau)\sqrt{4N \ln(1/x)/\ln\tau} >> \omega_1(\tau), \tag{6.2}$$

where

$$\omega_1(\tau) = \int_{\tau_0}^{\tau} \frac{d\tau_1}{\tau_1} \bar{n}_j(\tau) \tag{6.3}$$

and the mean gluon multiplicity in the jet $\bar{n}_j(\tau)$ has following estimation [6]:

$$\ln \bar{n}_j(\tau) \simeq \sqrt{\tau} \tag{6.4}$$

Inserting (6.4) into (6.3),

$$\omega_1(\tau) = \bar{n}_j(\tau)/\sqrt{\tau}.$$

Therefore, noting (2.3),

$$\bar{n}_g(\tau, x) \simeq \bar{n}_j(\tau)\sqrt{4N \ln(1/x)/\tau \ln\tau} << \bar{n}_j(\tau). \tag{6.5}$$

This means that the considered 't-channel' ladder is important in the narrow domain of multiplicities

$$n \sim \bar{n}_g << \bar{n}_j. \tag{6.6}$$

So, in the VHM domain $n >> \bar{n}_g$ one should consider
(i) The ladder diagrams with small number of rungs;
(ii) To take into account the malty-jet correlations assuming that increasing multiplicity leads to the increasing number of rungs in the ladder diagram.
To choose one of this possibilities one should consider the structure of $\omega(\tau, z)$ much more carefully. This will be done in the consequent paper.

We can conclude that the VHM domain multiplicities production unavoidably destroy the ladder LLA. To conserve this leading approximation one should choose $x \to 0$ and, in result, to get to the multi-ladder diagrams, since in this case $\alpha_s \ln(-q^2/\lambda^2) \sim 1$ and $\alpha_s \ln(1/x) \sim 1$. Such theory was considered in [7].

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