QUANTUM CHROMODYNAMICS IN HIGH TEMPERATURE LIMIT

A.N.Sissakian [a], O.Yu.Shevchenko [b, 1]

[a] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region 141980, Russia*

[b] *Laboratory of Nuclear Problems, Joint Institute for Nuclear Research, Dubna, Moscow region 141980, Russia*

Abstract

By use of nonperturbative procedure it is shown that effect of dimensional reduction from four-dimensions to three dimensions arises in quantum field theory at high temperature. High temperature limit of QCD is considered. It is shown that the effective theory is not just Euclidean QCD_3 with a new coupling constant $g\sqrt{T}$, but the term with chemical potential $\mu = 2T$, corresponding to the large fermion density, arises in the effective action. Due to this gluon acquires the "electric" mass proportional to the chemical potential and the phenomenon of quark deconfinement occurs.

One of the most remarkable and amazing future of finite-temperature field theory (FTFT) discovered long time ago [1-5] is its so-called "dimensional reduction" in high (infinite) temperature limit into 3-dimensional Euclidean field theory. The usual, pure perturbative, consideration gives rise to conclusion that at $T \to \infty$ fermions completely decouple and only bosons with nontrivial selfinteraction survive composing the three-dimensional theory. Let us remind these arguments.

In the imaginary time formalism of FTFT, the four-momentum p_μ is replaced by $(\omega_n,\ \vec{p})$, where $\omega_n = (\pi/-i\beta)\times$ even (odd) integer for bosons (fermions). The zero temperature integration measure $\int d^4k/(2\pi)^4$is replaced by $\frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}$. The usual basis for the assertion that $T\to\infty$ limit of a field theory is equivalent to a dimensional reduction is the following: a one-loop graph with a boson in the loop is proportional to

$$ig^2T\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}\frac{i}{-4\pi^2n^2T^2-\vec{p}^2-m^2},$$

which is suppressed at high temperature unless we pick $n=0$ mode in the sum. The last expression then becomes

$$g^2T\int\frac{d^3k}{(2\pi)^3}\frac{1}{\vec{p}^2+m^2},$$

and with the definition $\lambda^2 = g^2T$, we obtain effectively the integral we would have written for an Euclidean theory in three dimensions with a dimensionful coupling constant $g\sqrt{T}$. The case of fermions, however, is different in that no mode in the sum survives at large T. Thus, we make conclusion that fermions decouple. In the case of gauge fields, the temporal component A_0 acquires an (electric) mass [3] g^2T which screens it from the low

[1] E-mail address: shevch@nusun.jinr.dubna.su

energy sector. Thus the $T \to \infty$ limit of a $4D$ theory of fermions interacting with gauge fields appears to be a $3D$ theory of gauge fields only:

$$S_{eff} = \frac{1}{2} \int d^3x \mathrm{tr} F^{ij} F_{ij}.$$

While all these arguments are quick and simple they may be physically inadequate. Indeed, such "naive" diagram into diagram reduction does not account for initial four-dimensional divergences at all[2] and it is absolutely unclear how initial renormalizable theory transforms into superrenormalizable at dimensional reduction. The attempts to start with renormalized up to some order theory and account for appropriate counterterms [6] destroy this simple picture of dimensional reduction and make it vague and obscure. On the other hand, there exist indications that fermions do not entirely decouple and contribute in the effective action if we consider essentially nonperturbative by their nature effects such as, for example, topological Chern-Simons term generation at finite density and temperature [7,8].

All these circumstances inspire us to look for some trustful nonperturbative procedure to study such subtle phenomenon as dimensional reduction.

Let us first consider the simplest example of a thermal system described by a field theory–thermodynamics of a scalar field theory with the Hamiltonian

$$H = \int d^3x [\frac{1}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{m^2}{2}\varphi^2 + V(\varphi)]. \tag{1}$$

Consider the partition function which, by definition, is

$$Z = \mathrm{tr}\, \mathrm{e}^{-\beta H}. \tag{2}$$

To calculate trace we choose as the basis states the eigenfunctions of Schrödinder-picture field operator $\hat{\varphi}(\vec{x}) \equiv \hat{\varphi}(\vec{x}, 0) : \hat{\varphi}(\vec{x})|\varphi\rangle = \varphi(\vec{x})|\varphi\rangle$. Then the partition function looks as

$$Z = \int \prod_{\vec{x}} d\varphi(\vec{x}) \langle\varphi|\, \mathrm{e}^{-\beta \hat{H}} |\varphi\rangle, \tag{3}$$

where the integration measure $\prod_{\vec{x}} d\varphi(\vec{x})$ is a product[3] over the fields φ at all spatial coordinates $\vec{x}$.

If β is not restricted and takes arbitrary values we would fulfill familiar procedure of splitting interval $[0, \beta]$ into $N (N \to \infty)$ small intervals β/N and then, performing the famous "Feynman trick" (see, for example [9]) would derive the usual path integral representation for the partition function

$$Z = \int_{\varphi(0)=\varphi(\beta)} \prod_{t,\vec{x}} [d\varphi(t, \vec{x})] d\pi(t, \vec{x})\, \mathrm{e}^{-\int_0^\beta dt[\pi\dot{\varphi} - H(\pi,\varphi)]}, \tag{4}$$

[2]Let us remind that temperature (as well as chemical potential) does not influence the divergences and they are the same in FTFT as in the ordinary four-dimensional field theory.

[3]As usual, we correctly define with product on a finite spatial lattice considered as a regularization. Then this product involves a finite number of spatial coordinates and mathematically well defined.

which, after integration over momentum and analytic continuation of fields φ from interval $0 \leq t \leq \beta : \varphi(0) = \varphi(\beta)$ to periodic in "time" t functions $\varphi(t+n\beta, \vec{x}) = \varphi(t, \vec{x})$, reads

$$Z = N(\beta) \int\limits_{periodic} \prod_{t,\vec{x}} [d\varphi(t,\vec{x})] \, e^{-\int\limits_0^\beta dt \int d^3x \, L_E(t,\vec{x})}, \tag{5}$$

where L_E is four-dimensional Euclidean lagrangian

$$L_E(t,\vec{x}) = \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi + \frac{m^2}{2}\varphi^2 + V[\varphi], \tag{6}$$

$N(\beta)$ is β-dependent normalization factor which is relevant only for ideal gas contribution [3,9].

Namely Eq.(5) which already contains the whole four-dimensional divergences and requires the respective renormalizations is the starting point for described above standard perturbative exploring of dimensional reduction phenomenon at high temperature. However, remember that the basic definition of partition function is Eq.(2), not (5) ! To obtain secondary Eq.(5) we have to split interval $[0,\beta]$, regarding as an arbitrary one, into the set of infinitely small intervals and then to perform the Feyuman trick. The question arises what for we must do it when the value β we are interested about is infinitely small from the very beginning? Indeed, it seems to be rather illogical to construct at first the functional integral over "time" variable, containing needless for our high temperature task information and then, when we already unlucky have earned on the way the divergences (which, as we will see below, just absent in original, pure three-dimensional by its nature, high temperature task) to descent back to the small β interval and to struggle with that needless divergences. Thus, our main statement is that in the case when $\beta = 1/T$ is the (infinitely) small value we need not split interval $[0,\beta]$ at all, regarding (2), (3) as the starting point. One gets immediately[4]

$$\langle\pi| \, e^{-\beta H(\hat{\pi},\hat{\varphi})}|\varphi\rangle = \langle\pi|1 - \beta H(\hat{\pi},\hat{\varphi}) + O(\beta^2)|\varphi\rangle = e^{\beta H(\pi,\varphi) + i\int d^3x\pi(\vec{x})\varphi(\vec{x})} + O(\beta^2). \tag{7}$$

To derive (7) we have used the expression

$$\langle\varphi|\pi\rangle = \exp\left[i\int d^3x\pi(\vec{x})\varphi(\vec{x})\right] \tag{8}$$

for the overlap of eigenstates $|\pi\rangle$ and $|\varphi\rangle$

$$\hat{\varphi}(\vec{x})|\varphi\rangle = \varphi(x)|\varphi\rangle; \quad \hat{\pi}(\vec{x})|\pi\rangle = \pi(\vec{x})|\pi\rangle, \tag{9}$$

where the Schrödinger-picture field operator $\hat{\varphi}(\vec{x})$ and its conjugate momentum $\hat{\pi}(\vec{x})$ satisfy the (equal to zero time) commutation relations

$$[\hat{\varphi}(\vec{x}), \hat{\pi}(\vec{x}')] = i\delta^{(3)}(\vec{x} - \vec{x}').$$

Now, inserting once the full set of states $|\pi\rangle$

$$\int \prod_{\vec{x}} [d\pi(\vec{x})] |\pi\rangle\langle\pi| = 1 \tag{10}$$

[4] This is the main element in the Feynman trick leading to (5), where infinitely small value $\varepsilon = \beta/N$ $(N \to \infty)$ plays the role of β in our case.

in (3) and using (8)-(10)we easily find in the leading in β order

$$Z = \int \prod_{\vec{x}} [d\varphi(\vec{x}) d\pi(\vec{x})] \, e^{-\beta H[\pi,\varphi]}, \tag{11}$$

where Hamiltonian $H[\pi, \varphi]$ is given by Eq.(1), in which operators $\hat{\pi}$ and $\hat{\varphi}$ are replaced with c-number variables of integration.

To proceed let as note that majority of physical measurable quantities are expressed in terms of thermal expectation values $\langle O \rangle = Tr \; e^{-\beta \hat{H}} \hat{O} / \; \mathrm{Tr} \; e^{-\beta H}$, where $\hat{O}$ is any operator and therefore do not depend upon the extra measure factors because they are canceled between numerator and denominator. On the other hand, nontrivial, arising from interaction part of thermodynamic potential, does not depend on these measure factors too. Indeed, if we divide thermodynamic potential $\Omega = -(1/\beta V) \ln Z$ in two parts $\Omega = \Omega_0 + \Omega_I$, where Ω_0 is an ideal gas contribution and Ω_I is a part arising due to interaction, then the expression for interaction part reads $e^{-\beta V \Omega_I} = Z/Z_0 = \mathrm{tr}(e^{-\beta H}) / \mathrm{tr}(e^{-\beta H_0})$ and does not depend on measure factors (like $N(\beta)$ in (5)). So, now and then we will not pay any attention to such measure factors and will use, where it is necessary, symbol $\simeq$ instead of exact equality symbol, implying equality up to unessential normalization multiple.

Returning to Eq.(11) we choose, for definiteness, potential $V[\varphi]$ in the form[5]

$$V[\varphi] = g\varphi^N. \tag{12}$$

Performing Gaussian integration over momentum in (11) and by rescaling field $\varphi(\vec{x})$:

$$\varphi(\vec{x}) \to \varphi^{-1/2} \varphi(\vec{x}) \tag{13}$$

we get eventually

$$Z \simeq \int \prod_{\vec{x}} d\varphi(\vec{x}) \, e^{-S_E^{(d=3)}} [\varphi], \tag{14}$$

where

$$S_E^{(d=3)}[\varphi] = \int d^3x [\frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{m^2}{2}\varphi^2 + (g\beta^{1-N/2})\varphi^N] \tag{15}$$

is three-dimensional Euclidean action for $\lambda\varphi^N$-theory with a new coupling constant $\lambda = g\beta^{1-N/2}$. In particular, when we study high temperature limit of $g\varphi^4$-theory with dimensionless coupling constant g we have $\lambda = g\sqrt{T}$ which coincide with [6] prediction of "naive" perturbative arguments given above. However, our nonperturbative analysis does not suffer of questions which arise in perturbative consideration, in particular, what to do with four-dimensional divergences. Our procedure clearly shows that high-temperature task is three-dimensional from the very beginning.

Let us now include the fermions into consideration. Most field theories with fermions contain both fermions and bosons. Though our procedure is suitable for any such

[5] $N = 3, 4, 6$ for the most popular scalar models.

[6] Note that in the contrary to our approach, the perturbative analysis makes sense only in this case, when dimensionless expansion parameter exists in the theory.

theory,[7] we concentrate our attention on Quantum Chromodynamics for its great physical significance. Lagrangian has standard form

$$L = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + i\bar{\psi}(\gamma^\mu\partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu)\psi - M\bar{\psi}\psi.$$

The partition function [8]

$$Z = \text{Tr}\, e^{-\beta\hat{H}}$$

may be written in the form

$$Z = \text{sp}\left\{ \text{tr}\left[e^{-\beta\left(\hat{H}_{YM}(t=0)+\hat{H}_{ext}(t=0)\right)}\right]_{physical\ states}\right\}, \tag{16}$$

where symbol tr denotes the trace operation over *physical* gauge field (gluon) states, symbol sp denotes the trace over fermion (quark) states, $\hat{H}_{ext}$ is the part of the whole Hamiltonian corresponding to the quarks in external gluon field and $\hat{H}_{YM}$ is the part of H containing only the gluon field operators and their conjugate momenta.

To proceed we must choose any appropriate gauge condition. The most convenient for us occures the ghost free axial gauge

$$A^a_3 = 0. \tag{17}$$

Since the Gauss's law constraint

$$D^{ab}_i \pi^b_i - \rho_a = 0 \quad (i = 1,2,3), \tag{18}$$

where $D^{ab}_\mu = \delta_{ab}\partial_\mu - f_{abc}A^c_\mu$, $\rho_a \equiv J^a_0 = g\bar{\psi}\gamma_0\frac{\lambda^a}{2}\psi$, has the solution

$$\pi^a_3 = \frac{1}{\partial_3}\left(-D^{ab}_\perp \pi^b_\perp + \rho^a\right) \quad (\perp = 1,2), \tag{19}$$

the Hamiltonian, under gauge condition (17), may be expressed entirely in terms of independent dynamical variables $A_\perp,\pi_\perp$ as

$$\hat{H}_{phys} = \hat{H}^{phys}_{YM} + \hat{H}^{phys}_{ext}, \tag{20}$$

where

$$\hat{H}^{phys}_{ext} \equiv H_{ext}[\hat{\psi}^+, \hat{\psi}; \hat{A}_\perp] = \int d^3x\, \hat{\psi}^+ h(\hat{A}_\perp)\hat{\psi}, \tag{21}$$

with the notation

$$h(\hat{A}_\perp) = -i\gamma_0\gamma_\perp(\partial_\perp - ig\frac{\lambda^a}{2}\hat{A}^a_\perp) + M\gamma_0 \tag{22}$$

[7]The only restriction is that the system must be isolated, i.e. its Hamiltonian must be independent of time and one could regard field operators in the Hamiltonian as the Schrödinger picture operators. Thus, we cannot, for example, just consider fermions in external classical gauge field as an independent task and must deal with a full system.

[8]The proper space lattice regularization for the Schrödinger picture fermion and gauge field operators is asumed.

and

$$\hat{H}_{YM}^{phys} \equiv H_{YM}[\hat{A}_\perp, \hat{\pi}_\perp] = \int d^3\vec{x} \left[\frac{1}{2}\hat{\pi}_i^a\hat{\pi}_i^a + \frac{1}{4}\hat{F}_{ij}^a\hat{F}_{ij}^a\right]_{\pi_3^a=\partial_3^{-1}\left(-D_\perp^{ab}\pi_\perp^b+\rho^a\right)} . \tag{23}$$

We can now to perform tr operation in (16). By use of the eigenstates $|A_\perp^a(\vec{x})\rangle \equiv |A_\perp\rangle$ of Schrödinger-picture gauge field operators $\hat{A}_\perp^a(\vec{x})$ as the basis states, we get

$$Z = \mathrm{sp}\left\{\int \prod_{\vec{x},a} dA_\perp^a(\vec{x})\langle A_\perp|\, \mathrm{e}^{-\beta\left[H_{YM}(\hat{A}_\perp,\hat{\pi}_\perp)+H_{ext}[\hat{\psi}^+,\hat{\psi};\hat{A}_\perp]\right]}|A_\perp\rangle\right\}. \tag{24}$$

Operating now as well as in the considered above scalar case [9] we easily obtain

$$Z = \int \prod_{\vec{x},a} dA_\perp^a(\vec{x})d\pi_\perp^a(\vec{x})\, \mathrm{e}^{-\beta H_{YM}(A_\perp,\pi_\perp)}\, \mathrm{sp}\left[\mathrm{e}^{-\beta H_{ext}(\hat{\psi}^+,\hat{\psi};A_\perp)}\right]. \tag{25}$$

The only remaining procedure we have to do now is to evaluate the trace over fermion states and one can see that we deal just with fermions in the stationary external gauge field $A_\perp^a(\vec{x})$.

So, we deal with the Hamiltonian (21), in which the Schrödinger picture fermion field operators ψ, ψ^+ satisfy the anticommutation relations

$$\begin{aligned} [\psi_\alpha(\vec{x}), \psi_\beta(\vec{y})] &= 0 \\ [\psi_\alpha(\vec{x}), \psi_\beta^+(\vec{y})] &= \delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \end{aligned} \tag{26}$$

where α denotes the full set of quark quantum numbers (coloures, flavours and spinor indices).

We can, certainly, determine sp operation in (25) in the basis of the quark (antiquark) occupation numbers:

$$\mathrm{sp}\,\mathrm{e}^{-\beta\hat{H}_{ext}} = \sum_{\{n\}}\langle\{n\}|\,\mathrm{e}^{-\beta\hat{H}_{ext}}|\{n\}\rangle. \tag{27}$$

However, the occupation numbers basis represenation is not convenient for the practical use. Much more convenient and natural in the fermion case is the *coherent states* representation [9,10]. The coherent states are labeled by anticommuting[10] c numbers η and are defined as

$$|\{\eta\}\rangle = \prod_{\vec{x},\alpha} \mathrm{e}^{-\eta_\alpha(\vec{x})\hat{\psi}_\alpha^+(\vec{x})}|0\rangle, \quad \langle\{\eta^+\}| = \langle 0|\prod_{\vec{x},\alpha} \mathrm{e}^{-\hat{\psi}_\alpha(\vec{x})\eta_\alpha^+(\vec{x})}. \tag{28}$$

The basic properties of the coherent states are

$$\hat{\psi}_\alpha(\vec{x})|\{\eta\}\rangle = \eta_\alpha(\vec{x})|\{\eta\}\rangle, \quad \langle\{\eta^+\}|\hat{\psi}_\alpha^+(\vec{x}) = \langle\{\eta^+\}|\eta_\alpha^+(\vec{x}); \tag{29}$$

[9] What concerns the term $\frac{1}{2}\pi_3^a\pi_3^a$ with π_3^a given by (19) in the Hamiltonian considered as the quantum operator, we, as usual, must resolve uncertainty in the ordering of quanum operators at quantization procedure and take this term in "normal form" (i.e. when the whole coordinate operators are placed on the right to their conjugate momenta).

[10] They anticommute also with quark fields operators.

$$\langle\{\eta_i^+\}|\{\eta_j\}\rangle = e^{-\eta_j\eta_i^+} \tag{30}$$

and the completeness relation

$$\int \vec{D}\eta^+\vec{D}\eta\, e^{-\eta^+\eta}|\{\eta\}\rangle\langle\{\eta^+\}| = 1, \tag{31}$$

where we have used the short notation

$$\sum_\alpha \int d^3x\, \eta_\alpha(\vec{x})\eta_\alpha^+(\vec{x}) \equiv \eta\eta^+, \quad \prod_{\vec{x},\alpha} d\eta_\alpha(\vec{x}) \equiv \vec{D}\eta. \tag{32}$$

We need also in two auxiliary identities[11]:

$$\langle\{n\}|\{\eta\}\rangle\langle\{\bar{\eta}^+\}|\{n\}\rangle \equiv \langle\{-\bar{\eta}^+\}|\{n\}\rangle\langle\{n\}|\{\eta\}\rangle \equiv \langle\{\bar{\eta}^+\}|\{n\}\rangle\langle\{n\}|\{-\eta\}\rangle \tag{33}$$

and

$$\int \vec{D}\bar{\eta}^+\vec{D}\bar{\eta}\langle\{\bar{\eta}^+\}|\{-\eta\}\rangle\, e^{\bar{\eta}\bar{\eta}^+} F\left(\{\eta^+\},\{\bar{\eta}\}\right) \equiv F\left(\{\eta^+\},\{-\eta\}\right). \tag{34}$$

By use of (31),(33) and the completeness relation

$$\sum_{\{n\}} |\{n\}\rangle\langle\{n\}| = 1$$

we easily obtain instead of (27) the equation

$$\mathrm{sp}\, e^{-\beta H_{ext}} = \int \vec{D}\eta^+\vec{D}\eta\, e^{\eta\eta^+}\left\{\int \vec{D}\bar{\eta}^+\vec{D}\bar{\eta}\langle\{\bar{\eta}^+\}|\{-\eta\}\rangle\, e^{\bar{\eta}\bar{\eta}^+}\langle\{\eta^+\}|\, e^{-\beta\hat{H}_{ext}}|\{\bar{\eta}\}\rangle\right\}. \tag{35}$$

Using now the identity (34) we get eventually the expression for $\mathrm{sp}\, e^{-\beta\hat{H}_{ext}}$ in the coherent states representation

$$\mathrm{sp}\, e^{-\beta\hat{H}_{ext}} = \int \vec{D}\eta^+\vec{D}\eta\, e^{\eta\eta^+}\langle\{\eta^+\}|\, e^{-\beta\hat{H}_{ext}}|\{-\bar{\eta}\}\rangle. \tag{36}$$

Notice that the appearance of minus sign in r.h.s. of Eq.(36) is the crucial circumstance. Namely this gives rise to the antiperiodicity in imaginary time of the fermion variables in the standard functional integral for an arbitrary temperatures [9,10]. As we will see below, the appearance of $e^{\eta\eta^+}$ in r.h.s. of (36) is also very important for us.

Now we are ready to repeat the trick we have performed in the case of the scalar field theory. Expanding $e^{-\beta\hat{H}_{ext}}$ in powers of β and using Eqs. (21), (29) and (30) we obtain in the leading order in β

$$\begin{aligned}\langle\{\eta^+\}|\, e^{-\beta H_{ext}[\hat{\psi}^+,\hat{\psi};A_\perp]}|\{-\eta\}\rangle &= \langle\{\eta^+\}|[1-\beta\int d^3x\,\hat{\psi}^+h_{A_\perp}\hat{\psi}+O(\beta^2)]|\{-\eta\}\rangle \\ &= e^{\beta H_{ext}[\eta^+,\eta;A_\perp]-\eta^+\eta},\end{aligned} \tag{37}$$

Substituting this in Eq.(36) we get the final expression for $\mathrm{sp}\, e^{-\beta\hat{H}_{ext}}$:

$$\mathrm{sp}\, e^{-\beta\hat{H}_{ext}} = \int \vec{D}\eta^+\vec{D}\eta\, e^{\beta H_{ext}[\eta^+,\eta;A_\perp]+2\eta\eta^+}. \tag{38}$$

[11]The detailed proof of Eqs.(29)-(34) one can find in review article [9]

Returning now to Eq.(25) for the partition function, we can rewrite it as

$$Z = \int [\vec{D}\psi^+ \vec{D}\psi][\vec{D}A_\perp \vec{D}\pi_\perp]$$

$$\exp\left\{-\beta\left[H_{YM}(A_\perp,\pi_\perp) - H_{ext}(\psi^+,\psi;\,A_\perp) - 2\beta^{-1}\int d^3x\,\psi^+\psi\right]\right\}, \qquad (39)$$

where $H_{YM}(A_\perp,\pi_\perp)$ and$H_{ext}(\psi^+,\psi;\,A_\perp)$ are given by Eqs. (21), (23) with the replacement of the operators by the respective c numbers and anticommuting c numbers.

By use of the functional δ-functions: $\delta[A_3^a] \equiv \prod_{\vec{x},a}\delta(A_3^a(\vec{x}))$, $\delta[\pi_3^a - \partial_3^{-1}(-D_\perp^{ab}\pi_\perp^b + \rho_a)]$ corresponding to the axial gauge (17) and Gauss's law constraint (18), using the identity

$$\delta[\pi_3^a - \partial_3^{-1}(-D_\perp^{ab}\pi_\perp^b + \rho_a)]\Big|_{A_3^a=0} \equiv \det\left\|\frac{\delta(D_i^{ac}\pi_i^c - \rho_a)(\vec{x})}{\delta\pi_3^b(\vec{y})}\right\|_{A_3^a=0} \delta[D_i^{ab}\pi_i^b - \rho_a]\Big|_{A_3^a=0}$$

and omitting the irrelevant constant

$$\det\left\|\frac{\delta(D_i^{ac}\pi_i^c - \rho_a)(\vec{x})}{\delta\pi_3^b(\vec{y})}\right\|_{A_3^a=0} = \det\left\|\delta_{ab}\frac{\partial}{\partial x_3}\delta^{(3)}(\vec{x}-\vec{y})\right\|,$$

we can rewrite Eq.(39) in the form

$$Z = \int [\vec{D}\psi^+\vec{D}\psi][\vec{D}A_\perp\vec{D}A_3\vec{D}\pi_\perp\vec{D}\pi_3]\delta[D_i^{ab}\pi_i^b - \rho_a]\delta[A_3^a]\exp\Bigg\{-\beta\int d^3\vec{x}\,\Bigg[\frac{1}{2}\pi_i^a\pi_i^a$$
$$+\frac{1}{4}F_{ij}^aF_{ij}^a + \psi^+(i\gamma_0\gamma_i(\partial_i - ig\frac{\lambda^a}{2}A_i^a) - M\gamma_0)\psi - 2\beta^{-1}\psi^+\psi\Bigg]\Bigg\}. \qquad (40)$$

We now represent $\delta[D_i^{ab}\pi_i^b - \rho_a]$ in the functional integral form

$$\delta[D_i^{ab}\pi_i^b - \rho_a] = \int \vec{D}A_0 \exp\left[-i\int d^3x\,A_0^a\left(D_i^{ab}\pi_i^b - \rho_a\right)\right]$$

and perform the Gaussian integration over momenta. Then, upon identifying Euclidean gauge variables as $A_0^{a(E)} = -iA_0^{a(M)}$, $A_m^{a(E)} = -A_m^{a(M)}\,(m = 1,2,3)$ and the Euclidean γ matrixes as $\gamma_0^E = i\gamma_0^M$, so that

$$\bar{\psi} = \psi^+\gamma_0^M = -i\psi^+\gamma_0^E \equiv -i\psi^+\gamma_0,$$

we easily obtain

$$\begin{aligned} Z &= \int \vec{D}\bar{\psi}\vec{D}\psi\vec{D}A\,\delta[A_3^a]\exp\Bigg\{-\beta\int d^3\vec{x}\,\Bigg[\frac{1}{2}(\partial_i A_0^a(\vec{x}) + gf_{abc}A_i^0(\vec{x})A_\nu^c(\vec{x}))^2 \\ &+ \frac{1}{4}(\partial_i A_j^a(\vec{x}) - \partial_j A_i^a(\vec{x}) + gf_{abc}A_i^b(\vec{x})A_j^c(\vec{x}))^2 + \\ &\bar{\psi}(\vec{x})\left[i\gamma_i(\partial_i - ig\frac{\lambda^a}{2}A_i^a(\vec{x})) - M + g\gamma_0\frac{\lambda^a}{2}A_0^a(\vec{x}) - 2i\beta^{-1}\right]\psi(\vec{x})\Bigg]\Bigg\}, \end{aligned} \qquad (41)$$

where

$$\vec{D}\psi \equiv \prod_{\vec{x},\alpha}\psi_\alpha(\vec{x}),\ \vec{D}A \equiv \prod_{\substack{\vec{x}\\ \mu,a}} A_\mu^a(\vec{x}).$$

By the fields rescailing

$$A^a_\mu(\vec{x}) \to \beta^{-1/2} A^a_\mu(\vec{x}),\ \psi(\vec{x})\,(\bar{\psi}(\vec{x})) \to \beta^{-1/2}\psi(\vec{x})\,(\bar{\psi}(\vec{x}))$$

we get eventually

$$Z = \int \vec{D}\bar{\psi}\vec{D}\psi\vec{D}A\,\delta[A^a_3]\exp\Big\{-\int d^3\vec{x}\,\Big[\frac{1}{4}F^a_{\mu\nu}(\vec{x})F^a_{\mu\nu}(\vec{x}) +$$

$$\bar{\psi}(\vec{x})\Big[i\gamma_\mu(\partial_\mu - i\,\underline{g\beta^{-1/2}}\,\frac{\lambda^a}{2}A^a_i(\vec{x})) - M + g\gamma_0\frac{\lambda^a}{2}A^a_0(\vec{x}) - i\,\underline{2\beta^{-1}}\Big]\psi(\vec{x})\Big]\Big\}, \qquad (42)$$

where

$$F^a_{\mu\nu}(\vec{x}) = \partial_\mu A^a_\nu(\vec{x}) - \partial_\nu A^a_\mu(\vec{x}) + \underline{g\beta^{-1/2}}\,f_{abc}A^b_\mu(\vec{x})A^c_\nu(\vec{x}).$$

Let us now discuss the obtained result. First, one can see that we get in high temperature limit the effective three-dimensional Euclidean theory with a new coupling constant $\lambda = g\sqrt{T}$ and the fermions do not decouple in the partition function.[12] However, this is not just standard Euclidean QCD_3. The new term

$$-i\,(2\beta^{-1})\,\bar{\psi}(\vec{x})\psi(\vec{x}) \equiv -(2T)\,\psi^+(\vec{x})\psi(\vec{x})$$

arises in the effective Lagrangian, which corresponds to the chemical potential $\mu = 2T$ appearance! This is rather amazing result, because we considered the case of zero fermion density from the very beginning and, as it seems at first glance, the temperature and fermion density are not connected with each other at all.

The fact that high temperature transforms into large chemical potential in the effective action leads to a lot of physical consequences. One of them is that we at once may nonperturbatively argue that at sufficiently high temperature QCD definitely loses confinement. Indeed, as is well known, the presence of large fermion density at once [13] gives rise to generation of the "electric" gauge particle mass m_{el} (the nonperurbative proof was first presented by Fradkin in QED [11]) which is proportional to the chemical potential:

$$m_{el} \sim \mu.$$

This immediately leads to the quark deconfinement because the electric mass implies that the correlation function of the timelike component of gluon field behaves at large distances as

$$\langle A_0(\vec{x})A_0(\vec{y})\rangle \sim \mathrm{e}^{-m_{el}|\vec{x}-\vec{y}|}, \quad (|\vec{x}-\vec{y}| \to \infty)$$

that, in turn, means that the heavy quark potential includes exponential

$$V(R) \sim \mathrm{e}^{-m_{el}R} \text{ as } R \to \infty$$

which suppresses any polinomial in R and transforms the force into the shortrange one.

[12]Really, the situation with fermions is more subtle and essentially depends on the quantum field objects under consideration. The fermions do not always survive in the effective theory. So, for example, it is possible to show that the full fermion propagator equals zero in the infinite temperature limit, which is in agreement with the perturbative analysis. These questions will be discussed elsewhere.

[13]In the contrary to the high temperature case, when high loop perturbative calculations of m_{el} are performed in the standard approach which, however, are unreliable when we study such essentially nonperturbative phenomenon as confinement.

References

[1] D.A. Kirzhnits and A.D. Linde, Phys. Lett. B**42** (1972) 471;
S. Weinberg, Phys. Rev. D**9** (1974) 3357.

[2] L. Dolan and R. Jackiw, Phys.Rev. D**9** (1974) 3320.

[3] D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. **53** (1981) 43.

[4] T. Appelquist and R.D. Pisarski, Phys. Rev. D **23** (1981) 2305.

[5] R. Jackiw and G. Amelino-Camelia, "Field Theoretical Background for Thermal Physics" in Banff CAP Workshop on Thermal Field Theory, eds. F. Khanna, R. Kobes, G. Kunstatter and H. Umezawa (World Scientific, Singapore, 1994).

[6] T. Reisz, Z. Phys. C**53** (1992) 169;
P. Lacock, D.E. Miller and T. Reisz, Nucl. Phys. B**369** (1992) 501.

[7] A.N. Redlich and L.C.R. Wijewadhana, Phys. Rev. Lett. **54** (1985) 970;
K. Tsokos, Phys. Lett. B**157** (1985) 413;
V.A. Rubakov, Progr. Theor. Phys. **75** (1986) 366.

[8] A.N. Sissakian, O.Yu. Shevchenko and S.B. Solganik, Phys. Lett. B**403** (1997) 75; Nucl. Phys. B**518** (1998) 455.

[9] L. McLerran, Rev. Mod. Phys. **58** (1986) 1021.

[10] M.B. Halpern, A. Jevicki and P. Senjanovic, Phys. Rev. D**16** (1977) 2476;
D.E. Soper, Phys. Rev. D**18** (1978) 4590.

[11] E.S. Fradkin, Tr. FIAN **29** (1965) 7.