

Some notes about multiplicity distribution at hadron colliders

E.A.Kuraev, J.Manjavidze*, A.Sissakian
JINR, Dubna, Russia

April 13, 2007

Abstract

The idea that the hard processes are dominate at the very high multiplicity (VHM) final states creation is considered. For this purpose quantitative realization of the Pomeron, DIS and large-angle annihilation (LAA) mechanism combinations are considered in the pQCD frame. The phase transition (condensation) in the soft pions system is described as the alternative to above mechanism. It is shown that in compared to QCD prediction last one predicts enhancement in the VHM distribution tail.

1

The estimations of an expected multiplicities distribution tails at LHC energies are offered as possible physical program. Investigation of the multiplicity distributions was popular since seventies [1]. The very high multiplicity (VHM) processes as the attempt to get beyond standard multiperipheral hadron physics was considered in [2]. The hadron theory based on the local QCD Lagrangians [3] and the experimental consequences was given in the review papers [4].

We begin with general analysis. Let $\sigma_n(s)$ be the cross section of n particles creation at the CM energy \sqrt{s} . We introduce the generating function:

$$T(s, z) = \sum z^n \sigma_n, s = (p_1 + p_2)^2 \gg m^2. \quad (1.1)$$

So, the total cross section and the averaged multiplicity will be:

$$\sigma_{tot} = T(s, 1) = \sum \sigma_n, \sigma_{tot} \bar{n} = \sum n \sigma_n = \frac{d}{dz} T(s, z)_{z=1}. \quad (1.2)$$

*Permanent address: Inst.of Phys., Tbilisi, Georgia

At the same time

$$\sigma_n = \int \frac{dz}{2\pi i z^{n+1}} T(s, z) = \int \frac{dz}{2\pi i} e^{-(n+1) \ln z + \ln T(s, z)}. \quad (1.3)$$

Applying the steepest descent method we may determine the asymptotical behavior of σ_n at large n . It was shown in paper of T.D.Lee and C.N.Yang [5] that the singularities z_s of $T(z, s)$ in the z plane may be located at $|z| \geq 1$ only. We may distinguish following possibilities at $n \rightarrow \infty$:

- 1) $z_s = 1$: $\sigma_n > O(e^{-n})$;
- 2) $z_s = \infty$: $\sigma_n < O(e^{-n})$;
- 3) $z_s = z_c, 1 < z_c < \infty$: $\sigma_n = O(e^{-n})$.

The second type belong to the multiperipheral processes kinematics: created particles form jets moving with different velocities along the CM incoming particles.

Another information is included in

$$\ln T(s, z) = \sum \frac{(z-1)^m}{m!} c_m. \quad (1.4)$$

For instance, if $c_m = 0, m > 1$ we have the Poisson distribution: $\sigma_n = \sigma_{tot} \frac{(\bar{n})^n}{n!} \exp(-\bar{n})$. If $c_m = \gamma_m (c_1)^m$, i.e. γ_m is the some restricted function of m , than the so called KNO scaling take place: $\sigma_n \sim \sigma_{tot} f(n/\bar{n})$. One of the mostly interesting question: is the KNO scaling really takes place?

It was found in seventieth that the multiperipheral kinematics dominates inclusive cross sections $f(s, p_c)$. Moreover, the created particles spectra do not depend on s at high energies in the multiperipheral region:

$$f(s, p_c) = 2E_c \frac{d\sigma}{d^3 p_c} = \int \frac{dt_1 dt_2 s_1 s_2 \phi_1(t_1) \phi_2(t_2)}{(2\pi)^2 s (t_1 - m^2)^2 (t_2 - m^2)^2}, \quad s_1 s_2 (-p_{c\perp}^2) = s t_1 t_2.$$

Here $s_1 = (p_a + p_c)^2, s_2 = (p_b + p_c)^2, p_c = \alpha_c p_a + \beta_c p_b + p_{c\perp}$ and $\phi_i(t_i)$ are the impact factors of hadrons. So the particle c forgot the details of its creation. It was found experimentally that the ratio

$$\frac{f(\pi^+ p \rightarrow \pi_- + \dots)}{\sigma(\pi^+ p)} = \frac{f(K^+ p \rightarrow \pi_- + \dots)}{\sigma(K_+ + p)} = \frac{f(pp \rightarrow \pi_- + \dots)}{\sigma(pp)} \quad (1.5)$$

is universal [6]. This take place due to the two Pomeron multiperipheral exchange providing the nonvanishing contribution in the s asymptotics to the cross section. It was implied that the Pomeron intercept is exactly equal to one. Just this kinematics leads to the KNO-scaling [7].

The asymptotics 1) assumes phase transition [8]. The signal of creation of exotic state of pions say in the isotopic state with $I_z = 0$ (production of anomalous number of π_0) in the region of space of pions Compton wavelength order – so called pion condensate – may lead to the observable effect in multiplicity distributions. One may expect considerable deviation from the regime $O(e^{-n})$ mentioned above.

Let us demonstrate this reason using almost hand-waving arguments. The effective pion's lagrangian of S.Weinberg [9] $L_{eff} \sim (1 - \vec{\pi}^2/f_\pi^2)^{-1}$, $f_\pi = 140MeV$, regards the current algebra theorems, describe rather satisfactory the soft pions interaction. Using the functional integral approach we may consider the n pion's correlator

$$\int D\pi f_\pi^{-2n} \pi^{i_1}(x_1) \dots \pi^{i_{2n}}(x_{2n}) e^{-L_{eff}(\pi)}.$$

The associated probability of creation of $2n$ pions may estimated, assuming that the kinetic part of Lagrangian is negligible,

$$p_{2n} = \int_0^1 dx x^n \frac{1}{1-x} e^{[-\frac{1}{1-x}]} \sim e^{(-2\sqrt{n})}, \quad n \rightarrow \infty. \quad (1.6)$$

Note, the $\pi^2 \approx 1 - \sqrt{f_\pi/n} \rightarrow 1$ is essential in this integral. This means that the potential part of Lagrangian is $\sim \sqrt{n/f_\pi} \rightarrow \infty$ and thus the semiclassical approximation is valid.

2

The Pomeron is treated as a (infinite) set of particles emitted close to the CM beams direction (within the small angles of order $\theta_i \sim 2m_h/\sqrt{s} \ll 1$). We expect that these type of particles will not be detected by the detectors since they are move into the beams pipe. The collider experiment detectors locate at finite angles $\theta_D \sim 1$ and will measure the products only of particle c decay.

What will happened when instead of one particle a set of particles with invariant mass square s_t is created at large angles? Then the cross section will acquire the type suppression factor $(m^2/s_t)F(\alpha_s \ln^2(s_t/s_0))$ with the function

$$\begin{aligned} \sigma_n &= \frac{\alpha_s^2}{s_t} N F_n(\alpha_s^2 \ln^2(\frac{s}{s_0})), \quad N = (\frac{s}{s_0})^\Delta, \\ \Delta &= \alpha_P - 1 = \frac{12 \ln 2}{\pi} \alpha_s \approx 0.55, \quad \alpha_s = 0.2 \end{aligned} \quad (2.1)$$

Radiative corrections to the intercept was calculated [10] in recent time. The resulting value is $\Delta \approx 0.2$.

The way to obtain detected large multiplicity is to organize DIS-like experiments, expecting the large-angle scattered hadrons in the detectors. Large transfers momenta will be decreasing by ordinary evolution mechanism to the value of order m_π and then the Pomeron mechanism of peripheral scattering of the created hadrons from the pionization region will start.

What the characteristic multiplicities expected from Pomeron mechanism with the intercept exceeding unity, $\Delta \sim 0.2$? It is the quantity of order $(s/m_\pi^2)^\Delta \approx 200$ for $\sqrt{s} = 14TeV$. This rather rough estimation is in agreement with the phenomenological analysis of A.Kaidalov [1], based on multi-pomeron exchange in the scattering channel.

3

Let now construct the relevant cross sections. It is convenient to separate them to the classes

- a) Pomeron regime (P);
- b) Evolution regime (DIS);
- c) Double logarithmic regime (DL);
- d) DIS+P regime;
- e) P+DL+P regime.

The description of every regime may be performed in terms of effective ladder-type Feynman diagrams (The set of relevant FD depends on the gauge chosen and include much more number of them).

For the pure Pomeron regime [3] the estimated cross section have the form: ($y = \frac{\alpha_s}{16\pi^2}$, $m^2 \sim s_0 \sim m_\pi^2$)

$$\begin{aligned}
 d\sigma_{2 \rightarrow 2+n} &= \frac{1}{64\pi^2} \int_{m^2/s}^1 \frac{d\beta_n}{\beta_n} \int_{m^2/s}^{\beta_n} \frac{d\beta_{n-1}}{\beta_{n-1}} \dots \\
 &\times \int_{m^2/s}^{\beta_2} \frac{d\beta_1}{\beta_1} dZ_n \int \frac{d^2 q_{n+1}}{(q_{n+1}^2 - m^2)^2} (\Gamma_1 \Gamma_2)^2, \\
 dZ_n &= y^n \prod_{i=1}^{i=n} \int d^2 q_i \prod_{i=1}^{i=n} \frac{(s_i/s_0)^{\alpha(q_i)} \gamma_{i,i+1}^2}{(q_i^2 - m^2)^2}.
 \end{aligned} \tag{3.1}$$

Performing β -integration,

$$\int_{m^2/s}^1 \frac{d\beta_n}{\beta_n} \int_{m^2/s}^{\beta_n} \frac{d\beta_{n-1}}{\beta_{n-1}} \dots \int_{m^2/s}^{\beta_2} \frac{d\beta_1}{\beta_1} = L^n n!, \quad L = \ln \frac{s}{m^2}. \tag{3.2}$$

Here $q_i = \alpha_i p_2 + \beta_i p_1 + q_{i\perp}$ is the 4-momentum of the virtual gluon joining the emitted particles with 4-momenta k_i, k_{i+1} , $s_i = (k_i + k_{i+1})^2$ is their invariant mass square. One should use here that

$$\begin{aligned}
 s_1 s_2 \dots s_{n+1} &= s E_{1\perp}^2 \dots E_{n\perp}^2, \quad E_{i\perp}^2 = m_i^2 + (\vec{q}_i - \vec{q}_{i-1})^2, \\
 m^2 &\ll s_i \ll s = 2p_1 p_2,
 \end{aligned} \tag{3.3}$$

where

$$\alpha(q_i) = \frac{\alpha_s (q_i^2 - m^2)}{2\pi^2} \int \frac{d^2 k}{(k^2 - m^2)((q_i - k)^2 - m^2)},$$

is the reggeized gluon trajectory. Here we imply the arrangement on the rapidities of the emitted gluons

$$\frac{m^2}{s} \ll \beta_n \ll \beta_{n-1} < \dots \ll \beta_1 \sim 1. \tag{3.4}$$

The quantity $\Gamma_{1,2}$ may be associated with the formfactors of the initial hadrons (simply we replace them by the coupling constants of g , $g^2 = 4\pi\alpha_s$) whereas the quantities $\gamma_{i,i+1}$ associated with the effective vertices of transition of two gluons to the emitted particle.

For the case of emission of scalar particle we have $\gamma_{i,i+1} = m$. For the case of emission of gluon with momentum $k_i = q_i - q_{i+1}$ we have

$$\gamma_{i,i+1} = g[-(q_i + q_{i+1}) - p_2(\frac{2p_1k_i}{p_1p_2} - \frac{m^2 - q_i^2}{p_2k_i}) + p_1(\frac{2p_2k_i}{p_1p_2} - \frac{m^2 - q_{i+1}^2}{p_1k_i})]. \quad (3.5)$$

4

For the pure deep inelastic case, when one of the initial hadrons is scattered at the angle θ have the energy E' in the cms of beams whereas the another is scattered at small angle and the large transfer momentum $Q = 4EE' \sin^2(\theta/2) \gg m^2$, is distributed to the some number of the emitted particles due to evolution mechanism we have [11](θ is small):

$$d\sigma_n^{DIS} = \frac{4\alpha^2 E'^2}{Q^4 M} dD_n dE' d\cos\theta, \\ dD_n = \left(\frac{\alpha_s}{4\pi}\right)^n \int_{m^2}^{Q^2} \frac{dk_n^2}{k_n^2} \int_{m^2}^{k_n^2} \frac{dk_{n-1}^2}{k_{n-1}^2} \dots \int_{m^2}^{k_2} \frac{dk_1^2}{k_1^2} \int_x^1 d\beta_1 \int_{\beta_1}^1 d\beta_2 \dots \\ \times \int_{\beta_2}^1 d\beta_1 P\left(\frac{\beta_n}{\beta_{n-1}}\right) \dots P(\beta_1), \quad P(z) = 2\frac{1+z^2}{1-z}, \quad (4.1)$$

where the limits of integrals show the intervals of variation and the integrand is the differential cross section. Again the rapidities β_i are rigorously arranged as well as the transverse momenta squared.

5

For the large-angles particles production process the differential cross section (as well as the total one) fall with cms energy \sqrt{s} . We will consider for definiteness the process of annihilation of electron-positron pair to n photons [12]:

$$d\sigma_n^{DL} = \frac{2\pi\alpha^2}{s} dF_n, \\ dF_n = \left(\frac{\alpha}{2\pi}\right)^n \int_0^\rho dy_n \int_0^\rho dx_n \theta(x_n - y_n) \int_0^{y_n} dy_{n-1} \\ \times \int_0^{x_n} dx_{n-1} \theta(x_{n-1} - y_{n-1}) \dots, \quad x_i = \ln \frac{q_i^2}{m^2}, \quad y_n = \ln \frac{1}{\beta_i}, \quad \rho = \ln \frac{s}{m^2}, \quad (5.1)$$

The similar formulae takes place for subprocess of quark-antiquark annihilation into the n large-angle moving gluons. We note that the quantities q_i^2 may vary up to maximal value, s which corresponds to the emission at large angles. The total cross section of annihilation to any number of photons is:

$$\sigma_{tot}(s) = \frac{2(2\pi\alpha)^{3/2}}{s} \frac{2}{x} I_2(x), \quad x^2 = \frac{2\alpha}{\pi} \rho^2. \quad (5.2)$$

We conclude that the differential cross sections of the n particles production may be presented as a product of factors

$$\frac{dq_i^2}{q_i^2} \frac{d\beta_i}{\beta_i}$$

under various assumptions about transverse momentum q_i and rapidity β_i .

We will suppose that every emitted particle of mass (virtuality) M will decay and create the number of secondary particles (pions) with the probability

$$dW_n(M) = dn \frac{c}{\bar{n}} e^{-\frac{cn}{\bar{n}}}, \bar{n} = \ln \frac{M^2}{m_\pi^2}. \quad (5.3)$$

Construct now the cross sections of combined processes. When the one of the initial particles h_1 is scattered on small but sufficient enough angle to fit the detectors and other is scattered almost forward the combination of DIS and Pomeron regimes take place:

$$d\sigma_{n,m} = d\sigma_n^{DIS} dZ_m, |q_n|^2 \sim m^2 \quad (5.4)$$

provided that the virtuality of the last step of evolution regime of order of hadron mass. For the kinematical case of almost forward scattering of both initial hadrons the situation may be realized with large angles hadron production from the central region:

$$d\sigma_{n,m,k} = dZ_n d\sigma_m^{DL} dZ_k. \quad (5.5)$$

6

The possibility of exponential fall down mechanisms violation of the multiplicity n distribution was discussed. It was mentioned that it take place due to creation of pion condensate.

There is also another possibility [13]. The $k \leq n$ -particle state

$$|k \rangle = \frac{1}{\sqrt{k!}} \sum_{perm} \phi^+(x_1) \cdots \phi^+(x_k) |0 \rangle .$$

If $|x_i - x_j| \geq 1/m_\pi$ then the Bose-correlations are absent and $\langle k|k \rangle = k!/k! = 1$ because of ortho-normalizability of states. But all correlations become essential if $|x_i - x_j| < 1/m_\pi$, then the states are not orthogonal and, in result, $\langle k|k \rangle \sim k!^2/k! = k!$. The probability of such situation rise with rising multiplicity and the exponential fall off will be changed on the (factorial?) growth. If such effect will take place some understanding of the Centauro cosmic event may be reached.

Acknowledgments

We are grateful to V.G.Kadyshevski for interest to discussed in the paper questions.

References

- [1] A.Kaidalov, *Yad. Fiz.*, **61** (1998) 911; I.Dremin R.C.Hwa, *Phys. Rev.*, **D49** (1994) 5805
- [2] J.Manjavidze and A.Sissakian, *JINR Rap. Comm.*, (1999)
- [3] E.Kuraev, L.Lipatov and V.Fadin, *Sov. Phys. JETP*, 44, 443 (1976), *Zh. Eksp. Teor. Fiz.*, 71 (1976) 840; L.Lipatov, *Sov. J. Nucl. Phys.*, **20** (1975) 94, V.N.Gribov and L.Lipatov, *Sov. J. Nucl. Phys.*, **15** (1972) 438, 675, G.Altarelli and G.Parisi, *Nucl. Phys.*, **B126** (1977) 298.
- [4] V.Khoze and W.Ochs, *J. Mod. Phys.*, **A12** (1997) 2949.
- [5] T.D.Lee and C.N.Yang, *Phys. Rev.*, 87 (1952), 404,410.
- [6] M.Ryskin, *VII Winter School*, (LIYaPh, 1972).
- [7] Z.Koba, H.Nielsen and P.Olesen, *Nucl. Phys.*, **B40** (1972) 317.
- [8] M.Kac, G.Uhlenbeck and P.Hemmer, *J. Math. Phys.*, **4** (1963) 216, 229.
- [9] S.Weinberg, *Phys. Rev.*, 166 (1968) 1568.
- [10] V.Fadin and L.Lipatov, *Phys. Lett.*, B429 (1998),127;
M.Giaffaloni and G.Camici, *Phys. Lett.*, B430 (1998),349.
- [11] Yu.Dokshitzer, V.Khoze, A.Mueller and S.Trojan, *Basic of perturbative QCD* (Frontiers, 1991).
- [12] V.G.Gorshkov and L.N.Lipatov, *Yad. Fiz.*, 9 (1969) 818; V.G.Gorshkov, V.N.Gribov, L.N.Lipatov, G.V.Frolov, *Yad. Fiz.*, 6 (1967) 129.
- [13] V.A.Nikitin, *private communication*.