ЯДЕРНАЯ ФИЗИКА, 1998, том 61, № 11, с. 2074–2079

QUANTUM HALL EFFECT

CHERN–SIMONS TERM AT FINITE DENSITY AND TEMPERATURE

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The Chern–Simons topological term dynamical generation in the effective action is obtained at arbitrary finite density and temperature. By using a few different approaches it is shown that at zero temperature $\mu^2 = m^2$ is the crucial point for Chern–Simons. So when $\mu^2 < m^2$, μ-influence disappears and we get the usual Chern–Simons term. On the other hand, when $\mu^2 > m^2$, the Chern–Simons term vanishes because of non-zero density of background fermions. In particular, for massless case parity anomaly is absent at any finite density or temperature. This result holds in any odd dimension as in abelian so as in nonabelian case.

Since introducing the Chern–Simons (CS) topological term [1] and by now the great number of papers devoted to it appeared. Such interest is explained by variety of significant physical effects caused by CS secondary characteristic class. These are, for example, gauge particle mass appearance in quantum field theory, applications to condensed matter physics such as the fractional quantum Hall effect and high-T_c superconductivity, possibility of free metric tensor theory construction and so on.

It was shown [2–4] in a conventional zero-density gauge theory that the CS term is generated in the Euler–Heisenberg effective action by quantum corrections. The main goal of this paper is to explore the parity anomalous CS term generation at finite density. In the excellent paper by Niemi [5] it was emphasized that the charge density at $\mu \neq 0$ becomes nontopological object, i.e. contains both topological and nontopological parts. The charge density at $\mu \neq 0$ (nontopological, neither parity-odd nor parity-even object)[3)] in QED_3 at finite density was calculated and exploited in [6]. It must be emphasized that in [6] charge density contains as well parity-odd part corresponding to CS term so as parity-even part, which cannot be covariantized and do not contribute to the mass of the gauge field. Here we are interested in the finite-density influence on the covariant parity-odd form in action leading to the gauge field mass generation – CS topological term. Deep insight into this phenomena at small densities was done in [5, 7]. The result for CS term coefficient in QED_3 is $\left[\text{th}\frac{1}{2}\beta(m-\mu) + \text{th}\frac{1}{2}\beta(m+\mu)\right]$ (see [7], formulas (10.18)). However, to get this result, it was heuristicaly supposed that at small densities the index theorem could still be used and only the parity-odd term in the energy part of spectral density is responsible for parity nonconserving effect. Because of this in [7] it had been stressed that the result holds for small μ. However, as we shall see below, this result holds for any values of chemical potential. Thus, to obtain trustful result at any values of μ, one has to use transparent and free of any restrictions on μ procedure, which would allow us to perform calculations with arbitrary nonabelian background gauge fields.

Since the chemical potential term $\mu\bar{\psi}\gamma^0\psi$ is odd under charge conjugation, we can expect that it would contribute to P and CP nonconserving quantity – CS term. As we will see, this expectation is completely justified.

The zero-density approach is usually a good quantum field approximation when the chemical potential is small as compared with characteristic energy scale of physical processes. Nevertheless, for investigation of topological effects, it is not the case. As we will see below, even a small density could lead to principal effects.

Introduction of a chemical potential μ in a theory corresponds to the presence of a nonvanishing background charge density. So, if $\mu > 0$, then the number of particles exceeds that of antiparticles and vice versa. It must be emphasized that the formal addition of a chemical potential looks like a simple gauge transformation with the gauge function μt. However, it does not only shift the time component of a vector potential but also gives a corresponding prescription for handling Green's function poles. The correct introduction of a chemical potential redefines the ground state (Fermi energy), which leads to a new spinor propagator with the correct ϵ-prescription for poles. So, for the free spinor propagator we have (see, for example, [8, 9])

$$G(p;\mu) = \frac{\tilde{\not p} + m}{(\tilde{p}_0 + i\epsilon\,\text{sgn}\,p_0)^2 - \vec{p}^{\,2} - m^2}, \qquad (1)$$

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[3)] For abbreviation, speaking about parity invariance properties of local objects, we will keep in mind symmetries of the corresponding action parts.

where $\tilde{p} = (p_0 + \mu, \vec{p})$. Thus, when $\mu = 0$, one gets at once the usual ϵ-prescription because of the positivity of $p_0 \mathrm{sgn} p_0$. In the presence of a background Yang–Mills field we consequently have for the Green function operator

$$\hat{G} = (\gamma\tilde{\pi} - m)\frac{1}{(\gamma\tilde{\pi})^2 - m^2 + i\epsilon(p_0 + \mu)\mathrm{sgn}(p_0)}, \quad (2)$$

where $\tilde{\pi}_\nu = \pi_\nu + \mu\delta_{\nu 0}$, $\pi_\nu = p_\nu - gA_\nu(x)$.

Let us first consider a (2 + 1)-dimensional abelian theory. Here we shall use constant magnetic background. We shall evaluate fermion density by performing the direct summation over Landau levels. As a starting point, we shall use the formula for fermion number at finite density and temperature [5]

$$N = -\frac{1}{2}\sum_n \mathrm{sgn}(\lambda_n) + \sum_n \left[\frac{\theta(\lambda_n)}{\exp(-\beta(\mu - \lambda_n)) + 1} - \frac{\theta(-\lambda_n)}{\exp(-\beta(\lambda_n - \mu)) + 1}\right] = \frac{1}{2}\sum_n \mathrm{th}\frac{1}{2}\beta(\mu - \lambda_n) \xrightarrow{\beta\to\infty} \frac{1}{2}\sum_n \mathrm{sgn}(\mu - \lambda_n). \quad (3)$$

Landau levels in the constant magnetic field have the form [10]

$$\lambda_0 = -m\mathrm{sgn}(eB), \quad \lambda_n = \pm\sqrt{2n|eB| + m^2}, \quad (4)$$

where $n = 1, 2, \ldots$. It is also necessary to take into account in (3) the degeneracy of Landau levels. Namely, the number of degenerate states for each Landau level is $|eB|/2\pi$ per unit area. Even now we can see only zero modes (because of sgn(eB)) could contribute to the parity-odd quantity. So, for zero temperature, by using the identity

$$\mathrm{sgn}(a - b) + \mathrm{sgn}(a + b) = 2\mathrm{sgn}(a)\theta(|a| - |b|),$$

one gets for zero modes

$$\frac{|eB|}{4\pi}\mathrm{sgn}(\mu + m\mathrm{sgn}(eB)) = \frac{|eB|}{4\pi}\mathrm{sgn}(\mu)\theta(|\mu| - |m|) + \frac{|eB|}{4\pi}\mathrm{sgn}(eB)\mathrm{sgn}(m)\theta(|m| - |\mu|), \quad (5)$$

and for nonzero modes

$$\frac{1}{2}\frac{|eB|}{2\pi}\sum_{n=1}^{\infty}\mathrm{sgn}(\mu - \sqrt{2n|eB| + m^2}) + \mathrm{sgn}(\mu + \sqrt{2n|eB| + m^2}) = \frac{|eB|}{2\pi}\mathrm{sgn}(\mu)\sum_{n=1}^{\infty}\theta(|\mu| - \sqrt{2n|eB| + m^2}). \quad (6)$$

Combining contributions of all modes, we get for fermion density

$$\rho = \frac{|eB|}{2\pi}\mathrm{sgn}(\mu)\sum_{n=1}^{\infty}\theta(|\mu| - \sqrt{2n|eB| + m^2}) + \frac{1}{2}\frac{|eB|}{2\pi}\mathrm{sgn}(\mu)\theta(|\mu| - |m|) + \frac{1}{2}\frac{eB}{2\pi}\mathrm{sgn}(m)\theta(|m| - |\mu|) = \frac{|eB|}{2\pi}\mathrm{sgn}(\mu)\left(\mathrm{Int}\left[\frac{\mu^2 - m^2}{2|eB|}\right] + \frac{1}{2}\right)\theta(|\mu| - |m|) + \frac{eB}{4\pi}\mathrm{sgn}(m)\theta(|m| - |\mu|). \quad (7)$$

Here we see that zero modes contribute to parity-odd as well as to parity-even part, while nonzero modes contribute to the parity-even part only (note that under parity transformation $B \longrightarrow -B$). Thus, fermion density contains as parity odd part leading to CS term in action after covariantization, so as parity even part. It is straightforward to generalize the calculations on finite temperature case. Substituting zero modes into (3), one gets

$$N_0 = \frac{|eB|}{2\pi}\frac{1}{2}\mathrm{th}\left[\frac{1}{2}\beta(\mu + m\mathrm{sgn}(eB))\right] = \frac{|eB|}{4\pi}\left[\frac{\mathrm{sh}(\beta\mu)}{\mathrm{ch}(\beta\mu) + \mathrm{ch}(\beta m)} + \mathrm{sgn}(eB)\frac{\mathrm{sh}(\beta m)}{\mathrm{ch}(\beta\mu) + \mathrm{ch}(\beta m)}\right], \quad (8)$$

so, extracting parity-odd part, one gets for CS at finite temperature and density

$$N_{\mathrm{CS}} = \frac{eB}{4\pi}\frac{\mathrm{sh}(\beta m)}{\mathrm{ch}(\beta\mu) + \mathrm{ch}(\beta m)} = \frac{eB}{4\pi}\mathrm{th}(\beta m)\frac{1}{1 + \mathrm{ch}(\beta\mu)/\mathrm{ch}(\beta m)}. \quad (9)$$

So, the result coincides with result for CS term coefficient by Niemi [7] obtained for small $\mu\left[\mathrm{th}\frac{1}{2}\beta(m - \mu) + \mathrm{th}\frac{1}{2}\beta(m + \mu)\right]$. It is obvious how to go to the limit of zero temperature. The lack of this method is that it works only for abelian and constant field case.

This result at zero temperature can be obtained by use of Schwinger proper-time method. Consider (2 + + 1)-dimensional theory in the abelian case and choose background field in the form $A^\mu = \frac{1}{2}x_\nu F^{\nu\mu}$, $F^{\nu\mu} = \mathrm{const}$. To obtain the CS term in this case, it is necessary to consider the background current $\langle J^\mu \rangle = \delta S_{\mathrm{eff}}/\delta A_\mu$, rather than the effective action itself. This is because the CS term formally vanishes for such choice of A^μ but its variation with respect to A^μ produces a nonvanishing current. So, consider

$$\langle J^\mu \rangle = -ig\mathrm{tr}[\gamma^\mu G(x, x')]_{x\to x'}, \quad (10)$$

where

$$G(x, x') = \exp\left(-ig\int_{x'}^{x} d\zeta_\mu A^\mu(\zeta)\right)\langle x|\hat{G}|x'\rangle. \quad (11)$$

Let us rewrite Green function (2) in a more appropriate form

$$\hat{G} = (\gamma\tilde{\pi} - m)\left[\frac{\theta((p_0+\mu)\mathrm{sgn}(p_0))}{(\gamma\tilde{\pi})^2 - m^2 + i\epsilon} + \frac{\theta(-(p_0+\mu)\mathrm{sgn}(p_0))}{(\gamma\tilde{\pi})^2 - m^2 - i\epsilon}\right]. \quad (12)$$

Now, we use the well-known integral representation of denominators

$$\frac{1}{\alpha \pm i0} = \mp i\int_0^\infty ds e^{\pm i\alpha s},$$

which corresponds to introducing the "proper-time" s into the calculation of the Eulier–Heisenberg lagrangian by the Schwinger method [11]. We obtain

$$\hat{G} = (\gamma\tilde{\pi} - m)\Bigg[-i\int_0^\infty ds \exp(is[(\gamma\tilde{\pi})^2 - m^2 + i\epsilon])\times \times\theta((p_0+\mu)\mathrm{sgn}(p_0)) + + i\int_0^\infty ds\exp(-is[(\gamma\tilde{\pi})^2 - m^2 - i\epsilon])\times \times\theta(-(p_0+\mu)\mathrm{sgn}(p_0))\Bigg]. \quad (13)$$

Here, we restrict ourselves only to the magnetic field case, where $A_0 = 0$, $[\tilde{\pi}_0, \tilde{\pi}_\mu] = 0$. Then we easily can factorize the time-dependent part of Green function. Further we use the obvious relation

$$(\gamma\tilde{\pi})^2 = (p_0+\mu)^2 - \vec{\pi}^2 + \frac{1}{2}g\sigma_{\mu\nu}F^{\mu\nu}. \quad (14)$$

In the calculation of the current the following trace arises:

$$\mathrm{tr}[\gamma^\mu(\gamma\tilde{\pi} - m)e^{isg\sigma F/2}] = 2\pi^\nu g^{\nu\mu}\cos(g|{*F}|s) + + 2\frac{\pi^\nu F^{\nu\mu}}{|{*F}|}\sin(g|{*F}|s) - 2im\frac{{*F}^\mu}{|{*F}|}\sin(g|{*F}|s), \quad (15)$$

where ${*F}^\mu = e^{\mu\alpha\beta}F_{\alpha\beta}/2$ and $|{*F}| = \sqrt{B^2 - E^2}$. Since we are interested in calculation of the parity-odd part (CS term), it is enough to consider only terms proportional to the dual strength tensor ${*F}^\mu$. On the other hand, the term $2\pi^\nu g^{\nu\mu}\cos(g|{*F}|s)$ at $\nu = 0$ (see expression for the trace, we bear in mind that here there is only magnetic field) also gives nonzero contribution to the current J^0 [6]

$$J^0_{\mathrm{even}} = g\frac{|gB|}{2\pi}\left(\mathrm{Int}\left[\frac{\mu^2 - m^2}{2|gB|}\right] + \frac{1}{2}\right)\theta(|\mu| - |m|). \quad (16)$$

This part of current is parity invariant under $B \longrightarrow -B$. It is clear that this parity-even object contributes neither to the parity anomaly nor to the mass of the gauge field. Moreover, this term has magnetic field in the argument's denominator of the cumbersome function – integer part. So, the parity-even term seems to be "noncovariantizable", i.e. it cannot be converted in covariant form in effective action. Since we explore the parity anomalous topological CS term, we will not consider this parity-even term. So, only the term proportional to the dual strength tensor ${*F}^\mu$ gives rise to CS. The relevant part of the current after spatial momentum integration reads

$$J^\mu_{\mathrm{CS}} = \frac{g^2}{4\pi^2}m{*F}^\mu\int_{-\infty}^{+\infty} dp_0\int_0^\infty ds[e^{is(\tilde{p}_0^2 - m^2)} - - \theta(-\tilde{p}_0\mathrm{sgn}(p_0))(e^{is(\tilde{p}_0^2 - m^2)} + e^{-is(\tilde{p}_0^2 - m^2)})]. \quad (17)$$

Thus, we get besides the usual CS part [3], also the μ-dependent one. It is easy to calculate it by use of the formula

$$\int_0^\infty ds e^{is(x^2 - m^2)} = \pi\left(\delta(x^2 - m^2) + \frac{i}{\pi}\mathcal{P}\frac{1}{x^2 - m^2}\right)$$

and we get eventually

$$J^\mu_{\mathrm{CS}} = \frac{m}{|m|}\frac{g^2}{4\pi}{*F}^\mu[1 - \theta(-(m+\mu)\mathrm{sgn}(m)) - - \theta(-(m-\mu)\mathrm{sgn}(m))] = \frac{m}{|m|}\theta(m^2 - \mu^2)\frac{g^2}{4\pi}{*F}^\mu. \quad (18)$$

Let us now discuss the non-abelian case. Then $A^\mu = T_a A^\mu_a$ and

$$\langle J^\mu_a \rangle = -ig\mathrm{tr}[\gamma^\mu T_a G(x, x')]_{x\to x'}.$$

It is well-known [3, 12] that there exist only two types of the constant background fields. The first is the "abelian" type (it is easy to see that the self-interaction $f^{abc}A^\mu_b A^\mu_c$ disappears under that choice of the background field)

$$A^{\mu}_{a} = \eta_a \frac{1}{2} x_\nu F^{\nu\mu}, \qquad (19)$$

where η_a is an arbitrary constant vector in the color space, $F^{\nu\mu}$ = const. The second is the pure "non-abelian" type

$$A^{\mu} = \text{const}. \qquad (20)$$

Here the derivative terms (abelian part) vanish from the strength tensor and it contains only the self-interaction part $F^{\mu\nu}_{a} = gf^{abc}A^{\mu}_{b}A^{\nu}_{c}$. It is clear that to catch abelian part of the CS term, we should consider the background field (19), whereas for the nonabelian (derivative non-containing, cubic in A) part we have to use the case (20).

Calculation in the "abelian" case reduces to the previous analysis, except for the trivial adding of the color indices in the formula (18):

$$J^{\mu}_{a} = \frac{m}{|m|}\theta(m^2-\mu^2)\frac{g^2}{4\pi}{}^*F^{\mu}_{a}. \qquad (21)$$

In the case (20) all calculations are similar. The only difference is that the origin of term $\sigma_{\mu\nu}F^{\mu\nu}$ in (14) is not the linearity of A in x (as in abelian case) but the pure non-abelian A^{μ} = const. Here term $\sigma_{\mu\nu}F^{\mu\nu}$ in (14) becomes quadratic in A and we have

$$J^{\mu}_{a} = \frac{m}{|m|}\theta(m^2-\mu^2)\frac{g^3}{4\pi}e^{\mu\alpha\beta}\mathrm{tr}[T_a A^{\alpha}A^{\beta}]. \qquad (22)$$

Combining formulas (21) and (22) and integrating over field A^{μ}_{a} we obtain eventually

$$S^{\mathrm{CS}}_{\mathrm{eff}} = \frac{m}{|m|}\theta(m^2-\mu^2)\pi W[A], \qquad (23)$$

where W[A] is the CS term

$$W[A] = \frac{g^2}{8\pi^2}\int d^3x\varepsilon^{\mu\nu\alpha}\mathrm{tr}\Big(F_{\mu\nu}A_{\alpha} - \frac{2}{3}gA_{\mu}A_{\nu}A_{\alpha}\Big).$$

In conclusion note, that it may seem that covariant notation used here is rather artificial. However, the covariant notation is useful here because it helps us to extract Levi–Chivita tensor corresponding to parity anomalous CS term.

This result can also be obtained with an arbitrary initial field configuration by use of the perturbative expansion. Here we work immediately in the nonabelian case.

Let us first consider nonabelian 3-dimensional gauge theory. The only graphs whose P-odd parts contribute to the parity anomalous CS term are shown in Fig. 1.

So, the part of effective action containing the CS term looks as

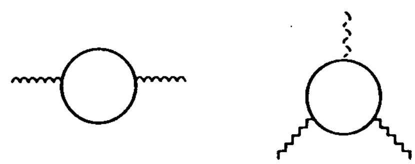

Fig. 1. Graphs whose P-odd parts contribute to the CS term in nonabelian 3D gauge theory.

$$I^{\mathrm{CS}}_{\mathrm{eff}} = \frac{1}{2}\int_x A_{\mu}(x)\int_p e^{-ixp}A_{\nu}(p)\Pi^{\mu\nu}(p) + \\ + \frac{1}{3}\int_x A_{\mu}(x)\int_{p,r} e^{-ix(p+r)}A_{\nu}(p)A_{\alpha}(r)\Pi^{\mu\nu\alpha}(p,r), \qquad (24)$$

where polarization operator and vertices have a standard form

$$\Pi^{\mu\nu}(p) = g^2\int_k \mathrm{tr}[\gamma^{\mu}S(p+k;\mu)\gamma^{\nu}S(k;\mu)], \\ \Pi^{\mu\nu\alpha}(p,r) = \qquad (25) \\ = g^3\int_k \mathrm{tr}[\gamma^{\mu}S(p+r+k;\mu)\gamma^{\nu}S(r+k;\mu)\gamma^{\alpha}S(k;\mu)].$$

Here the following notation is used $\int_x = \int_0^{\beta}dx_0\int d\vec{x}$ and $\int_k = \frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d\vec{k}}{(2\pi)^2}$. First consider the second-order term (Fig. 1a). It is well-known that the only object giving us the possibility to construct P and T odd form in action is Levi–Chivita tensor[4]. Thus, we will drop all terms noncontaining Levi–Chivita tensor. Signal for the mass generation (CS term) is $\Pi^{\mu\nu}$ $(p^2 = 0) \neq 0$. So we get

$$\Pi^{\mu\nu} = g^2\int_k(-i2me^{\mu\nu\alpha}p_{\alpha})\frac{1}{(\bar{k}^2+m^2)^2}. \qquad (26)$$

After some simple algebra one obtains

$$\Pi^{\mu\nu} = -i2mg^2e^{\mu\nu\alpha}p_{\alpha}\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{1}{(\bar{k}^2+m^2)^2} = \\ = -i2mg^2e^{\mu\nu\alpha}p_{\alpha}\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\frac{i1}{4\pi\omega_n^2+m^2}, \qquad (27)$$

where $\omega_n = (2n+1)\pi/\beta + i\mu$. Performing summation,

[4] In three dimensions it arises as a trace of three γ-matrices (Pauli matrices).

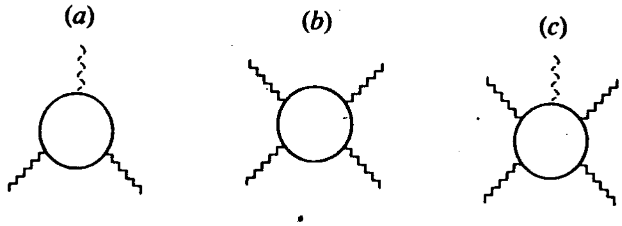

Fig. 2. Graphs whose P-odd parts contribute to the CS term in nonabelian $5D$ theory.

we get

$$\Pi^{\mu\nu} = i\frac{g^2}{4\pi}e^{\mu\nu\alpha}p_\alpha \operatorname{th}(\beta m)\frac{1}{1+\operatorname{ch}(\beta\mu)/\operatorname{ch}(\beta m)}. \quad (28)$$

It is easily seen that at $\beta \longrightarrow \infty$ limit we shall get zero-temperature result [13]

$$\Pi^{\mu\nu} = i\frac{m}{|m|}\frac{g^2}{4\pi}e^{\mu\nu\alpha}p_\alpha\theta(m^2-\mu^2). \quad (29)$$

In the same manner handling the third-order contribution (Fig. 1b), one gets

$$\Pi^{\mu\nu\alpha} = -2g^3 i e^{\mu\nu\alpha}\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k\, m(\tilde{k}^2+m^2)}{(2\pi)^2(\tilde{k}^2+m^2)^3} = \\ = -i2mg^3e^{\mu\nu\alpha}\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{1}{(\tilde{k}^2+m^2)^2}, \quad (30)$$

and further all calculations are identical to the second order

$$\Pi^{\mu\nu\alpha} = i\frac{g^3}{4\pi}e^{\mu\nu\alpha}\operatorname{th}(\beta m)\frac{1}{1+\operatorname{ch}(\beta\mu)/\operatorname{ch}(\beta m)}. \quad (31)$$

Substituting (28), (31) in the effective action (24), we get eventually

$$I_{\text{eff}}^{\text{CS}} = \operatorname{th}(\beta m)\frac{1}{1+\operatorname{ch}(\beta\mu)/\operatorname{ch}(\beta m)}\frac{g^2}{8\pi}\times \\ \times\int d^3x\, e^{\mu\nu\alpha}\operatorname{tr}\left(A_\mu\partial_\nu A_\alpha - \frac{2}{3}gA_\mu A_\nu A_\alpha\right). \quad (32)$$

Thus, we get CS term with temperature and density dependent coefficient.

Let us now consider 5-dimensional gauge theory. Here the Levi–Chivita tensor is 5-dimensional $e^{\mu\nu\alpha\beta\gamma}$ and the relevant graphs are shown in Fig. 2.

The part of effective action containing CS term reads

$$I_{\text{eff}}^{\text{CS}} = \frac{1}{3}\int_x A_\mu(x)\int_{p,r}e^{-ix(p+r)}A_\nu(p)A_a(r)\Pi^{\mu\nu\alpha}(p,r) + \\ + \frac{1}{4}\int_x A_\mu(x)\int_{p,r}e^{-ix(p+r+s)}\times \\ \times A_\nu(p)A_\alpha(r)A_\beta(s)\Pi^{\mu\nu\alpha\beta}(p,r,s) + \\ + \frac{1}{5}\int_x A_\mu(x)\int_{p,r}e^{-ix(p+r+s+q)}A_\nu(p)A_\alpha(r)\times \\ \times A_\beta(s)A_\gamma(q)\Pi^{\mu\nu\alpha\beta\gamma}(p,r,s,q). \quad (33)$$

All calculations are similar to 3-dimensional case. First consider third-order contribution (Fig. 2a)

$$\Pi^{\mu\nu\alpha}(p,r) = g^3\int_k \operatorname{tr}[\gamma^\mu S(p+r+k;\mu)\times \\ \times\gamma^\nu S(r+k;\mu)\gamma^\alpha S(k;\mu)]. \quad (34)$$

Taking into account that trace of five γ-matrices in 5-dimensions is

$$\operatorname{tr}[\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\gamma^\rho] = 4ie^{\mu\nu\alpha\beta\rho},$$

we extract the parity-odd part of the vertice

$$\Pi^{\mu\nu\alpha} = \\ = g^3\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}(i4me^{\mu\nu\alpha\beta\sigma}p_\beta r_\sigma)\frac{1}{(\tilde{k}^2+m^2)^3}, \quad (35)$$

or, in more transparent way,

$$\Pi^{\mu\nu\alpha} = i4mg^3e^{\mu\nu\alpha\beta\sigma}p_\alpha r_\sigma\times \\ \times\frac{i}{\beta}\sum_{n=-\infty}^{+\infty}\int\frac{d^4k}{(2\pi)^4}\frac{1}{\left(\omega_n^2+\vec{k}^2+m^2\right)^3} = \\ = i4mg^3e^{\mu\nu\alpha\beta\sigma}p_\alpha r_\sigma\frac{i}{\beta}\sum_{n=-\infty}^{+\infty}\frac{-i}{64\pi^2}\frac{1}{\omega_n^2+m^2}. \quad (36)$$

Evaluating summation, one comes to

$$\Pi^{\mu\nu\alpha} = i\operatorname{th}(\beta m)\times \\ \times\frac{1}{1+\operatorname{ch}(\beta\mu)/\operatorname{ch}(\beta m)}\frac{g^3}{16\pi^2}e^{\mu\nu\alpha\beta\sigma}p_\alpha r_\sigma. \quad (37)$$

In the same way operating graphs Fig. 2b, 2c, one will obtain

$$\Pi^{\mu\nu\alpha\beta} = i\operatorname{th}(\beta m)\times \\ \times\frac{1}{1+\operatorname{ch}(\beta\mu)/\operatorname{ch}(\beta m)}\frac{g^4}{8\pi^2}e^{\mu\nu\alpha\beta\sigma}s_\sigma \quad (38)$$

and

$$\Pi^{\mu\nu\alpha\beta\gamma} = i\operatorname{th}(\beta m)\times \\ \times\frac{1}{1+\operatorname{ch}(\beta\mu)/\operatorname{ch}(\beta m)}\frac{g^5}{16\pi^2}e^{\mu\nu\alpha\beta\sigma}. \quad (39)$$

Substituting (37)–(39) in the effective action (33), we get the final result for CS in 5-dimensional theory

$$I_{\rm eff}^{\rm CS} = {\rm th}(\beta m)\frac{1}{1+{\rm ch}(\beta\mu)/{\rm ch}(\beta m)}\frac{g^3}{48\pi^2}\int_x e^{\mu\nu\alpha\beta\gamma}\times$$
$$\times{\rm tr}\Big(A_\mu\partial_\nu A_\alpha\partial_\beta A_\gamma+\frac{3}{2}gA_\mu A_\nu A_\alpha\partial_\beta A_\gamma+ \qquad (40)$$
$$+\frac{3}{5}g^2A_\mu A_\nu A_\alpha A_\beta A_\gamma\Big).$$

It is remarkable that all parity-odd contributions are finite in both 3-dimensional and 5-dimensional cases. Thus, all values in the effective action are renormalized in a standard way, i.e. the renormalizations are determined by conventional (parity-even) parts of vertices.

From the above direct calculations it is clearly seen that the chemical potential and temperature-dependent coefficient is the same for all parity-odd parts of diagrams and does not depend on space dimension. So, the influence of finite density and temperature on CS-term generation is the same in any odd dimension:

$$I_{\rm eff}^{\rm CS} = {\rm th}(\beta m)\frac{1}{1+{\rm ch}(\beta\mu)/{\rm ch}(\beta m)}\pi W[A]\xrightarrow{\beta\to\infty}$$
$$\longrightarrow\frac{m}{|m|}\theta(m^2-\mu^2)\pi W[A], \qquad (41)$$

where $W[A]$ is the CS secondary characteristic class in any odd dimension. Since only the lowest orders of perturbative series contribute to CS term at finite density and temperature (the same situation is well-known at zero density), the result obtained by using formally perturbative technique appears to be nonperturbative. Thus, the μ-dependent CS-term coefficient reveals the amazing property of universality. Namely, either it does not depend on dimension of the theory or abelian or nonabelian gauge theory is studied.

The arbitrariness of μ gives us the possibility to see CS coefficient behavior at any masses. It is very interesting that $\mu^2 = m^2$ is the crucial point for CS at zero temperature. Indeed, it is clearly seen from (41) that when $\mu^2 < m^2$, μ-influence disappears and we get the usual CS term $I_{\rm eff}^{\rm CS} = \pi W[A]$. On the other hand, when $\mu^2 > m^2$, the situation is absolutely different. One can see that here the CS term disappears because of non-zero density of background fermions. We would like to emphasize the important massless case $m = 0$ considered in [7]. Then even negligible density or temperature, which always takes place in any physical processes, leads to vanishing of the parity anomaly. Let us stress again that we have nowhere used any restrictions on μ. Thus we not only confirm result in [7] for CS in QED_3 at small density, but also expand it on arbitrary μ, nonabelian case and arbitrary odd dimension.

In conclusion we would like to emphasize that nevertheless there is connection between chiral anomaly and CS term at zero density due to trace identities, at finite density this connection is loosed. That is because of different nature of these objects. The chiral anomaly is an effect of regularization, but the chemical potential does not introduce new divergences in a theory. Thus it does not influence the chiral anomaly. On the other hand, CS term is essentially an effect of the finite part of the theory. So, as we have seen finite density and temperature plays a crucial role in CS term generation.

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