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MATHEMATICAL METHODS

VARIATIONAL APPROACH TO QCD AND ITS APPLICATIONS

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Joint Institute for Nuclear Research, Dubna, Russia

A nonperturbative expansion technique in quantum field theory, variational perturbation theory, and some its applications are reviewed. Within this method a quantity under consideration is represented by the so-called variational or "floating" series, the convergence properties of which can be controlled by special parameters. It is shown that in the case of QCD a new small expansion parameter is appeared and obeyed an equation whose solutions are always smaller than unity for any value of the initial coupling constant. The method is applied to the inclusive semileptonic decay of the τ-lepton.

1. INTRODUCTION

The perturbative expansion is a powerful tool for performing calculations in quantum chromodynamics. Perturbation theory with renormalization group improvement has been widely applied to the description of various processes. As is well-known, perturbative series for many interesting models including realistic models are not convergent. Nevertheless, at small values of the coupling constant these series provide a useful information. However, even in the theories with a small coupling constant there exist problems which cannot be solved by perturbative methods. Also, many problems of quantum chromodynamics require nonperturbative approaches.

A lot of approaches have been devoted to the development of nonperturbative methods. There have been approaches that are not directly based on the perturbative series. Many of nonperturbative approaches make use of a variational procedure for finding the leading contribution. However, in this case there is no always an algorithm of calculating corrections to the value found by a variational procedure, and this makes difficult to answer the question how adequate is the so-called main contribution to the object under investigation and what is the range of applicability of the obtained estimations.

Therefore, useful approaches to the study of the nonperturbative structure of quantum field theory are the methods that combine an expansion of a given quantity in a series that defines the algorithm of calculating the correction with an optimizing procedure. The nonperturbation Gaussian effective potential for a quantum system has been constructed by an approach of that sort in [1–4]. There exist the various optimizing procedures. In [5, 6], for example, the principle of minimal sensitivity has been applied to the third-order calculation of $R_{e^+e^-}$. Different ways of constructing the variational procedures for scalar models of quantum field theories are discussed in [7–9]. However, even if the algorithm of calculating corrections, i.e. terms of a certain approximating series, exists, it is not still sufficient. Here of fundamental importance are the properties of convergence of a series. Indeed, unlike the case when even a divergent perturbative series in the weak coupling constant approximates a given object as an asymptotic series, the approximating series in the absence of a small parameter should obey more strict requirements. Reliable information in this case may be obtained only on the basis of convergent series.

We shall consider the method of a series construction with the aid of a variational procedure of the harmonic type. It has been observed empirically in [10] that the results seem to converge if the variational parameter is chosen, in each order, according to the principle of minimal sensitivity. This "induced convergence phenomenon" is discussed in detail in [11]. In [12] the proof of convergence of an optimized δ-expansion is given in the cases of zero and one dimensions. The proof of convergence of variational series in the case of anharmonic procedure is given in [8]. Here, we discuss a method which allows one to systematically determine the low-energy structure in quantum chromodynamics. We shall construct the expansion which is based on a new small parameter and apply this method to the nonperturbative renormalization group analysis in quantum chromodynamics. Applications to the definition of the QCD running coupling in the time-like domain and to the semileptonic decay of the τ-lepton will be considered. The main results concerning the method of variational perturbation theory (VPT) and some its applications can be found in the papers [1, 6–9] and [13–18] (see also references therein).

2. VARIATIONAL PERTURBATION THEORY IN QCD

Let us write down the QCD action functional in the form

$$S(A, q, \varphi) = S_2(A) + S_2(q) + S_2(\varphi) + \\ + gS_3(A, q, \varphi) + g^2 S_4(A), \quad (1)$$

where $S_2(A)$, $S_2(q)$, $S_2(\varphi)$ are free action functionals of the gluon, quark, and ghost fields, respectively; the term $S_2(A)$ also contains a term fixing the covariant α_G-gauge. The term $S_3(A, q, \varphi)$ describes the Yukawa interaction of gluons, gluons with quarks, and gluons with ghosts

$$S_3(A, q, \varphi) = S_3(A) + S_3(A, q) + S_3(A, \varphi). \quad (2)$$

The terms $S_3(A)$, $S_3(A, q)$ and $S_3(A, \varphi)$ generate, respectively, three-line vertices, (AAA), $(\bar{q}Aq)$ and $(\varphi A\varphi)$; whereas the term $S_4(A)$ in (1), four-gluon vertices $(AAAA)$. We will transform the latter term by introducing auxiliary fields $\chi^a_{\mu\nu}$ [14]. Upon the χ-transformation, the diagrams of the Green functions will consist only of diagrams of the Yukawa type. In addition to the usual three-line vertices of QCD vertices of the type $A\chi A$ will appear. Thus, a certain Green function of QCD can be represented in the following functional integral form

$$G(\ldots) = \int D\chi D_{\mathrm{QCD}}(\ldots)\exp\{i[S(A, \chi) + \\ + S_2(q) + S_2(\varphi) + S_2(\chi) + gS_3(A, q, \varphi)]\}, \quad (3)$$

where

$$S(A, \chi) = \frac{1}{2}\int dx dy A^a_\mu(x)[D^{-1}(x, y|\chi)]^{ab}_{\mu\nu}A^b_\nu(y) \quad (4)$$

with the gluon propagator $D(x, y|\chi)$ in the χ-field

$$[D^{-1}(x, y|\chi)]^{ab}_{\mu\nu} = [(-g_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu)\delta^{ab} + \\ + g\sqrt{2}f^{abc}\chi^c_{\mu\nu} + \text{gauge terms}]\delta(x-y) \quad (5)$$

and the term $(\ldots)$ is a set of ν gluon, quark and ghost fields. Integration measure D_{QCD} in (3) defines standard integrations over gluon, quark, and ghost fields.

Following the ideas of the VPT method, we introduce auxiliary parameters ζ and ξ and rewrite the action in (3) in the form

$$S(A, q, \varphi, \chi) = S'_0(A, q, \varphi, \chi) + S'_I(A, q, \varphi, \chi), \quad (6)$$

where

$$S'_0(A, q, \varphi, \chi) = \zeta^{-1}[S(A, \chi) + S_2(q) + S_2(\varphi)] + \\ + \xi^{-1}S_2(\chi), \quad (7)$$

$$S'_I(A, q, \varphi, \chi) = gS_3(A, q, \varphi) - (\zeta^{-1} - 1)\times \\ \times[S(A, \chi) + S_2(q) + S_2(\varphi)] - (\xi^{-1} - 1)S_2(\chi). \quad (8)$$

The exact value of the quantity under consideration, for instance, the Green function does not depend on the parameters ζ and ξ. However, the approximation of that quantity with a finite number of terms of the VPT series, that results from the expansion in powers of the action $S'_I(A, q, \varphi, \chi)$, does depend on those parameters. We can employ the freedom in the choice of the parameters ζ and ξ for our aim, construction of a new small parameter of the expansion.

It is more convenient to rewrite $S'_I(A, q, \varphi, \chi)$ in (7) by replacing ζ^{-1} by $[1 + \kappa(\zeta^{-1} - 1)]$ and ξ^{-1} to $[1 + \kappa(\xi^{-1} - 1)]$ and putting $\kappa = 1$ at the end of calculations. In this case, any power of the expression $(\zeta^{-1} - 1)[S(A, \chi) + S_2(q) + S_2(\varphi)] + (\xi^{-1} - 1)S_2(\chi)$ appearing in the factor of the exponential upon expanding the Green function in powers of (8), can be obtained by differentiating with respect to the parameter κ as many times as required. Then, the integrand in the factor of the exponential will contain only the powers of the action $gS_3(A, q, \varphi)$ that generate the QCD Yukawa diagrams with modified propagators defined by appropriate quadratic forms in the new "free" action S'_0. The VPT series for the Green function is given by

$$G(\ldots) = \sum_n \sum_{k=3D0}^{n} \frac{1}{(n-k)!}\left(-\frac{\partial}{\partial\kappa}\right)^{n-k}\frac{i^k}{k!}\times \\ \times\int D\chi D_{\mathrm{QCD}}(\ldots)[gS_3(A, q, \varphi)]^k\times \\ \times\exp[iS'_0(A, q, \varphi, \chi)] \quad (9)$$

with the above replacement in $S'_0(A, q, \varphi, \chi)$. Further, it is convenient to rescale the fields

$$(A, q, \varphi) \Rightarrow \frac{(A, q, \varphi)}{\sqrt{1 + \kappa(\zeta^{-1} - 1)}}, \\ \chi \Rightarrow \frac{\chi}{\sqrt{1 + \kappa(\xi^{-1} - 1)}}. \quad (10)$$

As a result, the propagators acquire the standard form and only the diagram vertices get modified. Integrating then over the field χ we obtain for the Green function

$$G(\ldots) = \sum_n \sum_{k=3D0}^{n} \frac{1}{(n-k)!}\left(-\frac{\partial}{\partial\kappa}\right)^{n-k}\frac{i^k}{k!}\times \\ \times\frac{1}{[1 + \kappa(\zeta^{-1} - 1)]^{\nu/2}}\int D_{\mathrm{QCD}}(\ldots)[g_3S_3(A, q, \varphi)]^k\times \\ \times\exp\{i[S_0(A, q, \varphi) + g_4^2S_4(A)]\}. \quad (11)$$

Here $S_0(A, q, \varphi)$ no longer contains the term describing the field χ and represents a usual functional of the QCD

free action, whereas g_3 and g_4 in the Yukawa and four-gluon vertices are defined as follows:

$$g_3 = \frac{g}{[1+\kappa(\zeta^{-1}-1)]^{3/2}}, \qquad g_4 = \frac{g}{[1+\kappa(\xi^{-1}-1)]^{1/2}}. \tag{12}$$

Analysis of the structure of the VPT series shows [14, 15] that we will succeed in constructing the small expansion parameter if we put $\xi = \zeta^3$ and if the parameter ζ is connected with the coupling constant by the equation

$$\lambda = \frac{g^2}{(4\pi)^2} = \frac{1}{C}\frac{a^2}{(1-a)^3}, \quad a = 1-\zeta, \tag{13}$$

where C is a positive constant. As follows from (13), at any values of the coupling constant g, the new expansion parameter a obeys the inequality $0 \le a < 1$.

We present the result of the VPT expansion for the Green functions with an accuracy of $O(a^7)$ that allows us to carry out calculations at the two-loop level in this approach. Writing the Green functions in the form

$$G(\ldots) = \int D_{QCD}(\ldots)V(A, q, \varphi)\exp(iS_0) \tag{14}$$

and using equations (11)–(13), we obtain

$$\begin{aligned} V = {} & 1 + aA_3 + a^2\left[\frac{1}{2}A_3^2 + A_4 + \frac{3}{2}A_3\right] + \\ & + a^3\left[\frac{1}{6}A_3^3 + \frac{3}{2}A_3^2 + A_3A_4 + 3A_4 + \frac{15}{8}A_3\right] + \\ & + a^4\left[\frac{1}{24}A_3^4 + \frac{1}{2}A_4^2 + \frac{1}{2}A_3^2A_4 + \frac{3}{4}A_3^3 + \frac{9}{2}A_3A_4 + \right. \\ & \left. + 3A_3^2 + 6A_4 + \frac{35}{16}A_3\right] + \\ & + a^5\left[\frac{1}{120}A_3^5 + \frac{1}{6}A_3^3A_4 + \frac{1}{2}A_3A_4^2 + \frac{1}{4}A_3^4 + 3A_3^2A_4 + \right. \\ & \left. + 3A_4^2 + \frac{33}{16}A_3^3 + \frac{99}{8}A_3A_4 + 5A_3^2 + 10A_4 + \frac{315}{128}A_3\right] + \\ & + a^6\left[\frac{1}{720}A_3^6 + \frac{1}{24}A_3^4A_4 + \frac{1}{4}A_3^2A_4^2 + \frac{1}{6}A_4^3 + \frac{1}{16}A_3^5 + \right. \\ & + \frac{5}{4}A_3^3A_4 + \frac{15}{4}A_3A_4^2 + \frac{7}{8}A_4^4 + \frac{21}{2}A_3^2A_4 + \frac{21}{2}A_4^2 + \frac{143}{32}A_3^3 + \\ & \left. + \frac{429}{16}A_3A_4 + \frac{15}{2}A_3^2 + 15A_4 + \frac{693}{256}A_3\right] + O(a^7), \end{aligned} \tag{15}$$

where $A_3 = 4\pi(iS_3)/\sqrt{C}$, $A_4 = (4\pi)^2(iS_4)/C$.

Following [15] consider a connection between the perturbative and nonperturbative regimes of the running coupling constant $\alpha_s(Q^2)$. To fix the parameter C we will use nonperturbative information from meson spectroscopy and derive $\alpha_s(Q^2)$ in the perturbative region at large Q^2. In other words, we will find the connection between the universal tension σ in the linear part of the quark–antiquark static potential $V_{lin}(r) = \sigma r$, which can be determined from meson spectroscopy, and the description of high energy physics. If, as usual, we assume that the quark potential in momentum space can be written as $V(q^2) = -16\pi\alpha_s(q^2)/3q^2$, where $\alpha_s(q^2)$ describes both large and small momentum, and that $\alpha_s(q^2)$ has the singular infrared asymptotics $\alpha_s(q^2) \sim q^{-2}$, we obtain, by taking the three-dimensional Fourier transform, the large-distance linear potential in coordinate space. The corresponding singular infrared behavior of $\lambda = \alpha_s/(4\pi)$ conforms to the asymptotics of the β-function: $\beta(\lambda) \longrightarrow -\lambda$ for a large coupling constant.

In the framework of this approach consider the functions $\beta^{(2)}$, $\beta^{(3)}$, $\beta^{(4)}$, and $\beta^{(5)}$ that are obtained if we take into consideration the terms $O(a^2)$, $O(a^3)$, $O(a^4)$, and $O(a^5)$ in the corresponding renormalization constant Z_λ. As has been shown [15], the values of $-\beta^{(k)}(\lambda)/\lambda$ as functions of the coupling constant for parameters $C_2 = 0.977$, $C_3 = 4.1$, $C_4 = 10.4$, and $C_5 = 21.5$ go to 1 at sufficiently large λ. The increase of C_k with the order of the expansion is explained by the necessity to compensate the high-order contribution. A similar situation takes place also in zero- and one-dimensional models. The behavior of the functions $-\beta^{(k)}(\lambda)/\lambda$ gives evidence for the convergence of the results, in accordance with the phenomenon of induced convergence. At large coupling, $-\beta^{(k)}(\lambda)/\lambda \simeq 1$, which corresponds to $\alpha_s(Q^2) \sim Q^{-2}$ at small Q^2.

The value of the coefficient σ in the linear part of the quark–antiquark static potential $V_{lin}(r) = \sigma r$ is $\sigma \simeq$ $\simeq 0.15$–0.20 GeV^2. At a small value of Q^2 the corresponding behavior of $\alpha_s(Q^2)$ is $\alpha_s(Q^2) \simeq 3\sigma/2Q^2$. Here we will use this equation at a certain normalization point Q_0 and the value $\sigma = 0.1768$ GeV^2 which has been obtained in [19]. The renormalization group method gives the following equation for the Q^2-evolution of the expansion parameter a:

$$Q^2 = Q_0^2\exp[\phi(a, N_f) - \phi(a_0, N_f^0)] \tag{16}$$

with

$$\phi(a, N_f) = \int^{\lambda}\frac{d\lambda}{\beta(\lambda)}. \tag{17}$$

In an appropriate region of the momentum, the value of $\sigma(Q^2)$ is almost independent of the choice of Q_0 and lies in the interval 0.15–0.20 GeV^2. This result agrees with the phenomenology of meson spectroscopy. Thus, we have found all the parameters and can now consider the behavior of the effective coupling constant at large Q^2. For example, we find $\alpha_{eff}(m_Z) =$

$= 0.126$. It should be stressed that we have obtained this result by evolution of the effective coupling starting from a very low-energy scale. Taking into account this fact the value of $\alpha_{\rm eff}(m_Z)$ obtained in such a way seems to be quite reasonable.

3. INCLUSIVE DECAY OF THE τ LEPTON

In this section we will concentrate on a description of the inclusive decay of the τ lepton taking into account renormalon contributions (for details, see [20]). Consider the Adler D-function $D(Q^2) = -Q^2 d\Pi/dQ^2$ corresponding to the vector hadronic correlator in the massless case. The two-loop perturbative approximation is given by $D(t, \lambda) = 1 + 4\lambda(\mu^2)$, where $t = Q^2/\mu^2$. Standard renormalization group improvement leads to the substitution $\lambda(\mu^2) \longrightarrow \bar{\lambda}(t, \lambda)$, which implies a summation of the leading logarithmic contributions. However, due to the ghost pole of the running coupling at $Q^2 = \Lambda^2_{\rm QCD}$ this substitution breaks the analytic properties of the D-function in the complex $q^2 = -Q^2$ plane, namely that the D-function should only have a cut on the positive real q^2 axis. We may correct this feature by noting that the above solution of the renormalization group equation is not unique. The general solution is a function of the running coupling with the asymptotic behavior $1 + 4\lambda$, for small λ. To maintain the analytic properties[1)] of the D-function we can write it as the dispersion integral of $R(s) = (1/\pi)\mathrm{Im}\,\Pi(s + i\epsilon)$, and use RG improvement on the integrand rather than D itself. This method leads to $D(t, \lambda) = 1 + 4\lambda_{\rm eff}(t, \lambda)$. The Borel representation of $\lambda_{\rm eff}(t, \lambda)$ has the form

$$\lambda_{\rm eff}(t, \lambda) = \int_0^\infty db e^{-b/\bar{\lambda}(t,\lambda)} B(b), \qquad (18)$$

with $B(b) = \Gamma(1 + b\beta_0)\Gamma(1 - b\beta_0)$. Here $\beta_0 = 11 - 2/3N_f$ is the first coefficient of the β-function, and N_f is the number of active flavors. Thus, in the Borel plane there are singularities at $b\beta_0 = -1, -2, \ldots$ and $b\beta_0 = 1, 2, \ldots$ corresponding to ultraviolet and infrared (IR) renormalons, respectively.

The first IR singularity at $b\beta_0 = 1$ is probably absent since there is no corresponding operator in the operator product expansion. Although this issue is not currently settled, it seems reasonable to assume that the first IR renormalon occurs at $b = 2/\beta_0$, and we would like to use this property of the operator product expansion as an additional constraint on the choice of solution to the renormalization group equation. This can be simply achieved (by judicious integration by parts), and as a result we obtain the following expression for $\lambda_{\rm eff}$:

$$\lambda_{\rm eff}(t, \lambda) = \int_0^\infty d\tau \omega(\tau) \frac{\bar{\lambda}(kt, \lambda)}{1 + \bar{\lambda}(kt, \lambda)\beta_0 \ln\tau}, \qquad (19)$$

in which the factor k reflects the renormalization scheme ambiguity and the function $\omega(\tau) = 2\tau/(1 + \tau)^3$ describes the distribution of virtuality usually associated with renormalon chains. The function $B(b)$ in the Borel transform of (19) has the form

$$B(b) = \Gamma(1 + b\beta_0)\Gamma(2 - b\beta_0). \qquad (20)$$

Thus in this representation for $\lambda_{\rm eff}$ the positions of all ultraviolet singularities remain unchanged, but the first IR renormalon singularity at $b = 1/\beta_0$ is absent.

In order to render equation (19) integrable we must combine this method with the nonperturbative a-expansion in which from the beginning the running coupling has no ghost pole. Separating the QCD contribution to R_τ-ratio as Δ_τ and writting $R_\tau = R^0_\tau(1 + \Delta_\tau)$, where R^0_τ is the well-known electroweak factor, we obtain the expression [20]

$$\Delta_\tau = 48 \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(\frac{s}{M_\tau^2}\right)^2 \left(1 - \frac{s}{M_\tau^2}\right) \tilde{\lambda}(ks), \qquad (21)$$

in which the factor k again parametrizes the renormalization scheme and $\tilde{\lambda} = a^2(1 + 3a)/C$. In what follows we shall use the $\overline{\rm MS}$ scheme, in which $k = \exp(-5/3)$. Note that the renormalon representation obtained for the coupling modifies the polynomial in the integral so that the maximum now occurs near $s = (2/3)M_\tau^2$.

Taking as input the experimental value of $R_\tau^{\rm exp} = 3.56 \pm 0.03$ [23], three active quark flavours and the variational parameter $C = 4.1$, we find $\alpha_s(M_\tau^2) = 0.339 \pm 0.015$ which differs significantly from that obtained ($\alpha_s(M_\tau^2) = 0.40$ in leading order [18]) without the renormalon-inspired representation for the coupling. The method, applying the matching procedure in the physical s-channel and using standard heavy quark masses, leads to $R_Z = 20.90 \pm 0.03$, which agrees well with experimental data [23].

4. CONCLUSION

We have considered an approach to quantum field theory – the method of variational perturbation theory. The original action functional is rewritten using some variational addition and an expansion in the effective interaction is made. Therefore, in contrast to many nonperturbative approaches, in the VPT the quantity under consideration from the very beginning is written in the

[1)] Recently, in [21, 22], it has been shown that requiring the correct analytic properties for the running coupling leads to the nonperturbative power corrections of the form $\exp(-1/(\bar{\lambda}(Q^2)\beta_0))$.

form of a series which makes it possible to calculate the needed corrections. The VPT method thereby allows for the possibility of determining the degree to which the principal contribution found variationally using some variational principle adequately reflects the problem in question and determining the region of applicability of the results obtained.

The possibility of performing calculations using this approach is based on the fact that the VPT, like standard perturbation theory, uses only Gaussian functional quadratures. Here, of course, the VPT series possesses a different structure and, in addition, some of the Feynman rules are modified at the level of the propagators and vertices. The form of diagrams themselves does not change, which is very important technically. The diagrams contributing to the Nth order of the VPT expansion are of the same form as those contributing to the Nth order of ordinary perturbation theory.

The variational parameters arising in the VPT method allow the convergence properties of the VPT series to be controlled. In [4, 8] has been shown that in the case of the anharmonic variational procedure for the scalar φ^4 model there is a finite region of parameter values in which the VPT series converges for all positive values of the coupling constant. For the harmonic variational procedure there are indications that VPT series can also converge in the sense of the so-called induced convergence, by fine-tuning the variational parameters from order to order. Note also, that a possibility of constructing Leibnitz series in field models is interesting, because, in this case, the first few terms of the series can be used to obtain two-sided estimates of the sum of the series, and existence of variational parameters makes it possible to narrow these estimates from the maximum amount in a given order of VPT (see [24]).

Here, we have mainly concentrated upon the application of the method to quantum chromodynamics (see also [25]), where the VPT idea leads to an expansion with a new small expansion parameter. This parameter obeys an equation whose solution is always smaller than unity for any value of the coupling constant. Therefore, while remaining within the limits of applicability of this expansion it is possible to deal with considerably lower energies than in the case of perturbation theory. An important feature of this approach is the fact that for sufficiently small value of the running coupling constant $\bar{\alpha}_s$ it reproduces perturbative predictions. Therefore, all the high-energy physics is preserved in the VPT method. In going to lower energies, where standard perturbation theory ceases to be valid, $\bar{\alpha}_s \simeq 1$, the VPT running expansion parameter $\bar{a}$ remains small and we do not find ourselves outside the region of applicability of the method.

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