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NONPERTURBATIVE EXPANSION TECHNIQUE IN QCD AND ITS APPLICATIONS

H.F. JONES

Physics Department, Imperial College, London SW7 2BZ, United Kingdom

A.N. SISSAKIAN and I.L. SOLOVTSOV

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region, 141980, Russia

A nonperturbative technique in QCD based on a new small expansion parameter is constructed and the connection between the perturbative and nonperturbative regimes is investigated. We argue that a renormalon representation is obtained as a particular renormalization group improvement of the lowest order radiative corrections which takes into account both the analytic properties and the structure of the operator product expansion. Application to the inclusive semi-leptonic decay of the τ is considered.

1 Introduction

There are many problems in QCD requiring a nonperturbative approach. Here we consider a method based on the ideas of the δ expansion and variational perturbation theory. The method leads to the so-called "floating" series, the convergence properties of which can be controlled by special parameters. The idea of constructing such a series in quantum theories was suggested and applied to the anharmonic oscillator in Refs. [1,2]. Within this approach, a certain variational principle is combined with the possibility of systematically calculating higher-order, thus allowing one to assess the validity of the principal contribution and the region of applicability of the results obtained. At present, this idea has found many applications in developing various approaches, which all go beyond perturbation theory. Among these are the Gaussian effective potential method [3], the optimized δ-expansion [4], and variational perturbation theory.[5,6] In certain cases, there is a rigorous proof of the convergence of such an expansion.[7,8] The generalization of the method to the QCD case has been suggested in Ref. [10]. Within the method, the quantity under consideration, for example a Green function, can be approximated by a series different from the perturbative expansion and which can be used to go beyond the weak-coupling regime, thus allowing one to deal with considerably lower energies than in the case of perturbation theory.

2 Small expansion parameter in QCD

Consider the QCD action

$$S(A,q,\varphi) = S_2(A) + S_2(q) + S_2(\varphi) + g\,S_3(A,q,\varphi) + g^2\,S_4(A)\,, \quad (1)$$

where $S_2(A)$, $S_2(q)$ $S_2(\varphi)$ are the free action functionals of the gluon, quark, and ghost fields, respectively; the term $S_2(A)$ also contains a term fixing the covariant α_G–gauge. The term $S_3(A,q,\varphi)$ describes the Yukawa interaction of gluons, gluons with quarks, and gluons with ghosts

$$S_3(A,q,\varphi) = S_3(A) + S_3(A,q) + S_3(A,\varphi)\,. \quad (2)$$

The terms $S_3(A)$, $S_3(A,q)$ and $S_3(A,\varphi)$ generate the three–line vertices (AAA), $(\bar{q}Aq)$ and $(\varphi A\varphi)$ respectively; whereas the term $S_4(A)$ in (1) generates the four-gluon vertex $(AAAA)$. We will transform the latter term by introducing auxiliary χ-fields.[9] After making the χ-transformation, the diagrams for the Green functions will consist only of diagrams of Yukawa type. In addition to the usual three-line vertices of QCD, vertices of the type $A\chi A$ will appear. Thus, any Green function of QCD can be represented in the following functional integral form

$$G(\cdots) = \int D\chi D_{\rm QCD}(\cdots)\exp i\Big[S(A,\chi) + S_2(q) + S_2(\varphi) + S_2(\chi) + gS_3(A,q,\varphi)\Big], \quad (3)$$

where

$$S(A,\chi)=\frac{1}{2}\int dxdy\,A(x)D^{-1}(x,y|\chi)A(y) \quad (4)$$

with the gluon propagator $D(x,y|\chi)$ in the χ-field, and the term $(\cdots)$ is a set of ν gluon, quark and ghost fields.

Following the ideas of the VPT method, we introduce auxiliary parameters ζ and ξ and rewrite the action in Eq. 3 in the form

$$S(A,q,\varphi,\chi)=S_0'(A,q,\varphi,\chi)+S_I'(A,q,\varphi,\chi) \quad (5)$$

with

$$\begin{aligned} S_0'(A,q,\varphi,\chi) &= \zeta^{-1}[S(A,\chi)+S_2(q) \\ &+ S_2(\varphi)]+\xi^{-1}S_2(\chi), \\ S_I'(A,q,\varphi,\chi) &= g\,S_3(A,q,\varphi) \quad (6)\\ &- (\zeta^{-1}-1)[S(A,\chi)+S_2(q) \\ &+ S_2(\varphi)]-(\xi^{-1}-1)\,S_2(\chi). \end{aligned}$$

The exact value of the quantity under consideration does not depend on the parameters ζ and ξ. However, the approximation of that quantity with a finite number of terms of the VPT series resulting from the expansion in powers of the action $S_I'(A,q,\varphi,\chi)$, does so depend. One can employ the freedom in the choice of the parameters ζ and ξ for our aim, the construction of a new small expansion parameter.

It is convenient to rewrite $S_0'(A,q,\varphi,\chi)$ by replacing ζ^{-1} by $[1+\kappa(\zeta^{-1}-1)]$ and ξ^{-1} by $[1+\kappa(\xi^{-1}-1)]$ and setting $\kappa=1$ at the end of the calculation. In this case, any power of the expression $(\zeta^{-1}-1)[S(A,\chi)+S_2(q)+S_2(\varphi)]+(\xi^{-1}-1)S_2(\chi)$ appearing in the expansion of the corresponding exponential can be obtained by differentiating with respect to the parameter κ. Then, the integrand will contain only powers of the action $g\,S_3(A,q,\varphi)$, which generate the QCD Yukawa diagrams with modified propagators defined by appropriate quadratic forms in the new "free" action S_0'. After rescaling of fields and the Gaussian integration over the χ, the VPT series for the Green function is given by

$$\begin{aligned} G(\cdots)' &= \sum_n\sum_{k=0}^{n}\frac{1}{(n-k)!}\Big(-\frac{\partial}{\partial\kappa}\Big)^{n-k}\frac{i^k}{k!} \\ &\times \frac{1}{[1+\kappa(\zeta^{-1}-1)]^{\nu/2}}\int D_{\rm QCD}(\cdots) \\ &\times [g_3S_3]^k\exp\{i[S_0+g_4^2S_4]\}. \quad (7) \end{aligned}$$

Here $S_0(A,q,\varphi)$ no longer contains the term describing the field χ and represents the usual functional of the QCD free action, whereas g_3 and g_4 in the Yukawa and four–gluon vertices are defined as follows: $g_3=g[1+\kappa(\zeta^{-1}-1)]^{-3/2}$, $g_4=g[1+\kappa(\xi^{-1}-1)]^{-1/2}$.

Analysis of the structure of the VPT series shows [9] that we will succeed in constructing the small expansion parameter if we set $\xi=\zeta^3$ and if the parameter ζ is connected with the coupling constant by the equation

$$\lambda=\frac{g^2}{(4\pi)^2}=\frac{1}{C}\frac{a^2}{(1-a)^3}\,, \quad a=1-\zeta\,, \quad (8)$$

where C is a positive constant. As follows from Eq. 8, for any values of the coupling constant g, the new expansion parameter a obeys the inequality $0\le a<1$.

Consider the connection between the perturbative and non-perturbative regimes of the running coupling constant $\alpha_s(Q^2)$. To fix the parameter C we will use non-perturbative information from meson spectroscopy and derive $\alpha_s(Q^2)$ in the perturbative region at large Q^2. In other words, we will find the connection between the universal tension σ in the linear part of the quark-antiquark static potential $V_{\rm lin}(r)=\sigma r$, which can be determined from meson spectroscopy, and the description of high energy physics. If, as usual, we assume that the quark potential in momentum space can be written as $V(q^2)=-16\pi\alpha_s(q^2)/3q^2$, where $\alpha_s(q^2)$ describes both large and small momentum, and that $\alpha_s(q^2)$ has the singular infrared asymptotics $\alpha_s(q^2)\sim q^{-2}$, we obtain, by taking the three-dimensional Fourier transform, the large-distance linear potential in coordinate space. The corresponding singular infrared behaviour of $\lambda=\alpha_s/(4\pi)$ conforms to the asymptotics of the Gell-Mann–Low function: $\beta(\lambda)\to-\lambda$ for a large coupling constant.

In the framework of this approach consider the functions $\beta^{(2)}$, $\beta^{(3)}$, $\beta^{(4)}$ and $\beta^{(5)}$ that are obtained if we take into consideration the terms $O(a^2)$, $O(a^3)$, $O(a^4)$ and $O(a^5)$ in the corresponding renormalization constant Z_λ. As has been shown [9], the values of $-\beta^{(k)}(\lambda)/\lambda$ as functions of the coupling constant for parameters $C_2=0.977$, $C_3=4.1$, $C_4=10.4$ and $C_5=21.5$ go to 1 at sufficiently large λ. The increase of C_k with the order of the expansion is explained by the necessity to compensate the high order contribution. A similar situation takes place also in zero- and one-dimensional models. The behaviour of the functions $-\beta^{(k)}(\lambda)/\lambda$ gives evidence for the convergence of the results, in accordance with the phenomenon of induced convergence. At large coupling, $-\beta^{(k)}(\lambda)/\lambda \simeq 1$, which corresponds to $\alpha_s(Q^2) \sim Q^{-2}$ at small Q^2.

The value of the coefficient σ in the linear part of the quark-antiquark static potential $V_{\rm lin}(r) = \sigma r$ is $\sigma \simeq 0.15 \div 0.20$ GeV^2. At a small value of Q^2 the corresponding behaviour of $\alpha_s(Q^2)$ is $\alpha_s(Q^2) \simeq 3\sigma/2Q^2$. Here we will use this equation at a certain normalization point Q_0 and the value $\sigma = 0.1768$ GeV^2 which has been obtained in Ref. [10]. The renormalization group method gives the following equation for the Q^2-evolution of the expansion parameter a:

$$Q^2 = Q_0^2 \exp\left[\phi(a, N_f) - \phi(a_0, N_f^0)\right] \quad (9)$$

with

$$\phi(a, N_f) = \int^{\lambda} \frac{d\lambda}{\beta(\lambda)}. \quad (10)$$

In an appropriate region of the momentum, the value of $\sigma(Q^2) = 2/3\, Q^2 \alpha_s(Q^2)$ is almost independent of the choice of Q_0 and lies in the interval $0.15 \div 0.20$ GeV^2. This result agrees with the phenomenology of meson spectroscopy. Thus, we have found all the parameters and can now consider the behaviour of the effective coupling constant at large Q^2. For example, we find $\alpha_{\rm eff}(m_Z) = 0.126$. It should be stressed that we have obtained this result by evolution of the effective coupling starting from a very low energy scale. Taking into account this fact the value of $\alpha_{\rm eff}(m_Z)$ obtained in such a way seems to be quite reasonable.

3 Renormalons and τ decay

In this section we will concentrate on a description of the inclusive decay of the τ lepton taking into account renormalon contributions[a]. Consider the Adler D-function $D(Q^2) = -Q^2 d\Pi/dQ^2$ corresponding to the vector hadronic correlator in the massless case. The two-loop perturbative approximation is given by $D(t,\lambda) = 1 + 4\lambda(\mu^2)$, where $t = Q^2/\mu^2$. Standard renormalization group improvement leads to the substitution $\lambda(\mu^2) \to \overline{\lambda}(t,\lambda)$, which implies a summation of the leading logarithmic contributions. However, due to the ghost pole of the running coupling at $Q^2 = \Lambda_{QCD}^2$ this substitution breaks the analytic properties of the D-function in the complex $q^2 = -Q^2$ plane, namely that the D-function should only have a cut on the positive real q^2 axis. We may correct this feature by noting that the above solution of the renormalization group equation is not unique. The general solution is a function of the running coupling with the asymptotic behaviour $1 + 4\lambda$, for small λ. To maintain the analytic properties[b] of the D-function we can write it as the dispersion integral of $R(s) = (1/\pi)\mathrm{Im}\Pi(s + i\epsilon)$, and use RG improvement on the integrand rather than D itself. This method leads to $D(t,\lambda) = 1 + 4\lambda_{\rm eff}(t,\lambda)$ with $\tau = s/Q^2$. The Borel representation of $\lambda_{\rm eff}(t,\lambda)$ has the form

$$\lambda_{\rm eff}(t,\lambda) = \int_0^\infty db\, e^{-b/\overline{\lambda}(t,\lambda)} B(b)\,, \quad (11)$$

with $B(b) = \Gamma(1 + b\beta_0)\Gamma(1 - b\beta_0)$. Here $\beta_0 = 11 - 2/3 N_f$ is the first coefficient of the β-function, and N_f is the number of active flavours. Thus, in the Borel plane there are singularities at $b\beta_0 = -1, -2, \ldots$ and $b\beta_0 = 1, 2, \ldots$ corresponding to ultraviolet and infrared (IR) renormalons respectively.

The first IR singularity at $b\beta_0 = 1$ is probably absent since there is no corresponding operator in the operator product expansion. Although this issue is not currently settled, it seems reasonable to assume that the first IR renormalon occurs at $b = 2/\beta_0$, and we would like to use this property of the operator product expansion as an additional constraint on the choice of solution to the renormalization group equation. This can be simply achieved (by judicious integration by parts), and as result we obtain the following expression for $\lambda_{\rm eff}$:

$$\lambda_{\rm eff}(t,\lambda) = \int_0^\infty d\tau\, \omega(\tau) \frac{\overline{\lambda}(kt,\lambda)}{1 + \overline{\lambda}(kt,\lambda)\beta_0 \ln\tau}\,, \quad (12)$$

in which the factor k reflects the renormalization scheme ambiguity and the function $\omega(\tau) =$

[a] Some applications of the method have been considered also in Refs. [11-13].

[b] Recently[14], it has been shown that requiring the correct analytic properties for the running coupling is indeed equivalent to the inclusion of non-perturbative power corrections of the form $\exp(-1/(\overline{\lambda}(Q^2)\beta_0))$.

$2\tau/(1+\tau)^3$ describes the distribution of virtuality usually associated with renormalon chains. The function $\omega(\tau)$ coincides with the function used in Ref. [15] and is numerically very close to that found in Ref. [16]. The function $B(b)$ in the Borel transform of (12) has the form

$$B(b) = \Gamma(1 + b\beta_0)\,\Gamma(2 - b\beta_0)\,. \qquad (13)$$

Thus in this representation for $\lambda_{\rm eff}$ the positions of all ultraviolet singularities remain unchanged, but the first IR renormalon singularity at $b = 1/\beta_0$ is absent.

In order to render Eq. (12) integrable we must combine this method with the nonperturbative a-expansion in which from the beginning the running coupling has no ghost pole [c]. Separating the QCD contribution to R_τ-ratio as Δ_τ and writing $R_\tau = R^0_\tau\,(1 + \Delta_\tau)$, where R^0_τ is the wel-known electroweak factor, we obtain the expression

$$\Delta_\tau = 48 \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(\frac{s}{M_\tau^2}\right)^2 \left(1 - \frac{s}{M_\tau^2}\right) \tilde{\lambda}(ks)\,, \qquad (14)$$

in which the factor k again parametrizes the renormalization scheme and $\tilde{\lambda} = a^2(1+3a)/C$. In what follows we shall use the $\overline{MS}$ scheme, in which $k = \exp(-5/3)$. Note that the renormalon representation obtained for the coupling modifies the polynomial in the integral so that the maximum now occurs near $s = (2/3)M_\tau^2$.

Taking as input the experimental value of $R_\tau^{\rm exp} = 3.56 \pm 0.03$ [18], three active quark flavours and the variational parameter $C = 4.1$ as in Refs. [9,13], we find $\alpha_s(M_\tau^2) = 0.339 \pm 0.015$ which differs significantly from that obtained ($\alpha_s(M_\tau^2) = 0.40$ in leading order [13]) without the renormalon-inspired representation for the coupling. The method, applying the matching procedure in the physical s-channel and using standard heavy quark masses, leads to $R_Z = 20.90 \pm 0.03$, which agrees well with experimental data.

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[c] In effect, the representation for the D-function obtained in such a way coincides with a technique explicitly introducing power corrections, and we can, in principle, describe hadronic parameters using, say, the method of QCD sum rules [17].