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Observation of short range three-particle correlations in e^+e^- annihilations at LEP energies

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Abstract

Measurements are presented of short range three-particle correlations in e^+e^- annihilations at LEP using data collected by the DELPHI detector. At small values of the four-momentum difference, strong three-particle correlations are observed for like-sign $(+++$ and $---)$ and for unlike-sign $(++-$ and $+--)$ pion combinations which are not a consequence of two-particle correlations. A possible explanation of the observed effects in like-sign combinations is the existence of higher order Bose-Einstein interference, which significantly changes the particle distributions in jets.

1. Introduction

Studies of the properties of hadronic events in e^+e^- annihilations and tests of QCD generators are usually performed using global event shape variables and single particle distributions. As the multiplicity of particles increases with increasing initial energy, correlations between particles change their distributions in jets and an understanding of these correlations becomes important. Studies of correlations at high energy are thus related to the composition of jets and tests of QCD models. A clear example of the influence of correlations on particle distributions is the distortion of the Breit-Wigner shape for oppositely charged pions from the decay of broad resonances by residual Bose-Einstein (BE) correlations, which was observed at LEP energies [1] (see also [2]). Recently, it was also shown that the observed W mass at LEP200 is likely to be affected by BE correlations [3]. A better understanding of these correlations, as well as comparisons of BE algorithms with experimental data at high energies, becomes necessary for high precision measurements of the W mass.

Two-particle correlations have been studied extensively in hadron-hadron interactions [4,5] and in e^+e^- annihilations at LEP energies [6]. However, very few experimental results are available on three-particle correlations which can be generated either by two-particle correlations or by genuine three-particle correlations. Identifying the latter is complicated because of the need for high statistics of large multiplicity events and the problem of subtracting the direct consequences of two-particle correlations. Obvious $\pi^+\pi^+\pi^-$ and $\pi^+\pi^-\pi^-$ correlations are generated in the decays of many resonances. The most prominent example at low masses is the η' decay into $\eta\pi^+\pi^-$ with subsequent decay of the η into $\pi^+\pi^-\pi^0$ or $\pi^+\pi^-\gamma$. At higher three-pion masses there are many contributions from the decays of charm and beauty particles.

Short-range rapidity correlations are observed for $(+ + -)$ and $(+ - -)$ combinations in several experiments [7–10]. In contrast to the unlike-sign combinations, no genuine correlations were found for the like-sign $(+++)$ and $(---)$ configuration. Furthermore, in the three-particle Bose-Einstein effect studied in [8,10,11], no evidence was found for three-particle correlations beyond those generated from pairs in the triplet. Only one hadron-proton experiment recently reported the observation of positive short-range higher order like-sign correlations [12].

Experimental evidence for genuine like-sign three-particle correlations has still to be reported; it constitutes an important theoretical issue for the understanding of BE correlations [13].

In this paper, the first evidence is presented for genuine three particle correlations at LEP using data collected by the DELPHI detector. We discuss like-sign and unlike-sign three-particle correlations after subtraction of the two-particle ones. It is demonstrated that positive short range correlations exist for both like-sign and unlike-sign configurations. Both these correlations can be understood, at least partly, as following from higher order Bose-Einstein correlations, which change the event shape at small relative momenta. Experimental data are compared with various modifications of the JETSET PS model [14].

2. Analysis

The single-particle density is defined as

$$\rho_1(q_a) = \frac{E_a}{N_{ev}} \frac{d^3n}{d^3p_a}, \tag{1}$$

the two-particle density as

$$\rho_2(q_a, q_b) = \frac{E_aE_b}{N_{ev}} \frac{d^6n}{d^3p_a d^3p_b} \tag{2}$$

and the three-particle density as

$$\rho_3(q_a, q_b, q_c) = \frac{E_aE_bE_c}{N_{ev}} \frac{d^9n}{d^3p_a d^3p_b d^3p_c}, \tag{3}$$

where q_i and p_i are the four vector and three vector momentum, respectively, and E_i the energy of the particle; n is the number of particles (in Eq. (1)), of doublets (in Eq. (2)) or triplets (in Eq. (3)) and N_{ev} is the number of events. For n identical particles, these densities are normalized to $< n >$, $< n(n-1) >$ and $< n(n-1)(n-2) >$, respectively. We study the normalized three-body correlation function written as [15]

$$R_3 = R_3'' - R_3' + 1 \tag{4}$$

where the quantity

$$R_3'' = [\rho_3(q_a, q_b, q_c) - \rho_1(q_a)\rho_1(q_b)\rho_1(q_c)] / \\ \rho_1(q_a)\rho_1(q_b)\rho_1(q_c) \tag{5}$$

is the full three-particle correlation function and

$$R_3' = [\rho_2(q_a, q_b)\rho_1(q_c) + \rho_2(q_a, q_c)\rho_1(q_b) \\ + \rho_2(q_b, q_c)\rho_1(q_a) - 3\rho_1(q_a)\rho_1(q_b)\rho_1(q_c)] / \\ \rho_1(q_a)\rho_1(q_b)\rho_1(q_c) \tag{6}$$

is the three-particle correlation arising from correlations in the pairs of the triplet. The $\rho_1(q_a)\rho_1(q_b) \times \rho_1(q_c)$ term represents uncorrelated production of the three particles, each to be taken with its density defined in Eq. (1). The $\rho_2(q_a, q_b)\rho_1(q_c)$ term, as well as the next two terms in Eq. (6), contain the two-body correlations. The genuine three-particle correlation function R_3 is then given by

$$R_3 = [\rho_3 + 3\rho_1\rho_1\rho_1 - 3\rho_2\rho_1]/\rho_1\rho_1\rho_1 \tag{7}$$

where the q-variables have been omitted for clarity and $3\rho_2\rho_1$ is the sum of the three $\rho_2\rho_1$ terms in Eq. (6). The quantity R_3 therefore does not contain three-particle correlations which arise from two-body correlations nor is it sensitive to possible bias in two-particle combinations due to Dalitz pairs, γ-conversions, or particle or resonance decays.

The kinematical variable used in this analysis is

$$Q^2 = q_{12}^2 + q_{23}^2 + q_{31}^2 \tag{8}$$

with $q_{ij}^2 = -(q_i - q_j)^2$. For three-pion systems, $Q^2 = M_{123}^2 - 9m_\pi^2$, with M_{123} the invariant mass of the pion triplet. It has been shown [5] that this Lorentz invariant variable is more sensitive to correlations than the rapidity variable.

3. Experimental procedure

This study is based on a sample of about 1.5 million hadronic events collected with the DELPHI detector during the 1992 and 1993 running periods, at a center of mass energy at and around the Z^0 peak. The detector has been described in [16]. The analysis uses the same methodology as in [17]. All charged particles were considered to be pions and only charged particles satisfying the following requirements were used:

- polar angle θ with respect to the beam axis between 25° and 155°;
- momentum greater than 0.2 GeV/c and less than 50 GeV/c;
- measured track length in the TPC, the main tracking chamber, greater than 50 cm;
- measured impact parameter with respect to the event vertex less than 5 cm in the transverse plane and 10 cm along the beam direction.

Hadronic events were selected by requiring that:

- there were at least 5 charged particles in the event;
- the total energy of the charged particles exceeded 3 GeV in each of the two hemispheres defined with respect to the beam direction;
- the total energy of all charged particles was larger than 15 GeV;
- the total momentum imbalance was less than 30 GeV/c;
- the polar angle of the thrust axis θ_{th}, defined with respect to the beam direction, satisfied $|\cos\theta_{\text{th}}| < 0.75$.

The contamination from events due to beam-gas scattering, $\gamma\gamma$ interactions and $\tau^+\tau^-$ events was estimated to be less than 0.3% of the selected events. Only charged particles with an impact parameter less than 0.1 cm in the transverse plane and 1 cm in the beam direction were used in the calculation of the Q distribution. Thus only particles originating from the primary vertex were included. These strict cuts on both transverse and longitudinal impact parameters removed most tracks from K_S^0 and Λ decays. To select clean two-jet events, a cut on the thrust variable $T > 0.97$ was imposed. A sample of 327,162 events was available after all cuts.

In order to calculate the terms $\rho_2\rho_1$ and $\rho_1\rho_1\rho_1$ in Eqs. (5)-(7), the following mixing technique was used. A pool of particle four-vectors with momentum components calculated with respect to the thrust axis, was constructed from a large number of events. To calculate $\rho_1\rho_1\rho_1$, three tracks were randomly drawn from the pool and the term was calculated in the same way as in real events; to calculate $\rho_2\rho_1$, two particles were taken from the same event and the third one from another event.

The analysis was limited to $Q > 0.15$ GeV/c in order to avoid the region where the three tracks could be geometrically confused. This value for triplets corresponds, on average, to $q_{ij} > 0.09$ GeV/c for pairs. Moreover, if false two-body correlations are introduced they are subtracted in the quantity R_3. The correction for geometrical acceptance, kinematical cuts, particle interactions with the detector material and other detector imperfections, was made by using the Monte Carlo simulation program DELSIM [18]. A sample of Z^0 hadronic events was generated with JETSET and all tracks were followed through the full detector. After application of the same cuts as on the data, including the thrust cut, about 1,000,000 events

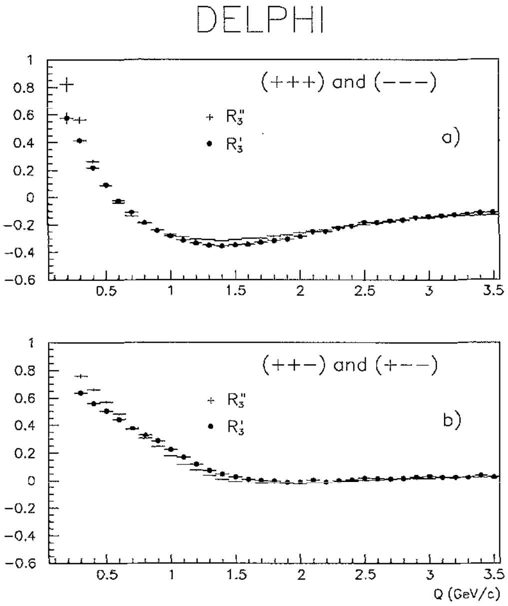

Fig. 1. Q-dependence of R_3'', the total three-particle correlation, and R_3', the three-particle correlation arising from two-body correlations, (a) for like-sign and (b) for unlike-sign triplets.

remained. Bin by bin correction factors were calculated for ρ_3, $\rho_2\rho_1$, and $\rho_1\rho_1\rho_1$. In the following, only corrected distributions will be presented. However, it was checked that the same features are present at the level of the uncorrected distributions.

4. Results and discussion

Fig. 1a shows R_3'', the full three-particle correlation function for like-sign triplets using the normalization of the density functions as given above. A strong correlation is observed at small Q-values. However, R_3', the background three-particle correlation arising from two-particle correlations between particles in the triplet, also shows a strong rise towards $Q = 0$. Fig. 1a demonstrates that most of the three-particle correlations are due to two-particle correlations, but also that some excess of R_3'' over R_3' is present at small Q. The three-particle correlation functions R_3'' and R_3' for unlike-sign triplets are presented in Fig. 1b. A difference between the R_3'' and R_3' functions at small Q is also present for unlike-sign triplets.

The genuine three-particle correlations for like-sign triplets, R_3, are shown in Fig. 2a. A clear signal is observed at $Q < 0.5$ GeV/c, corresponding to a short-range three-particle correlation. A reference sample such as the one of unlike-sign pion combinations is inadequate because of the presence of correlations in this sample. This is clearly seen from Fig. 2b where the function $R_3(Q)$ is plotted for unlike-sign triplets.

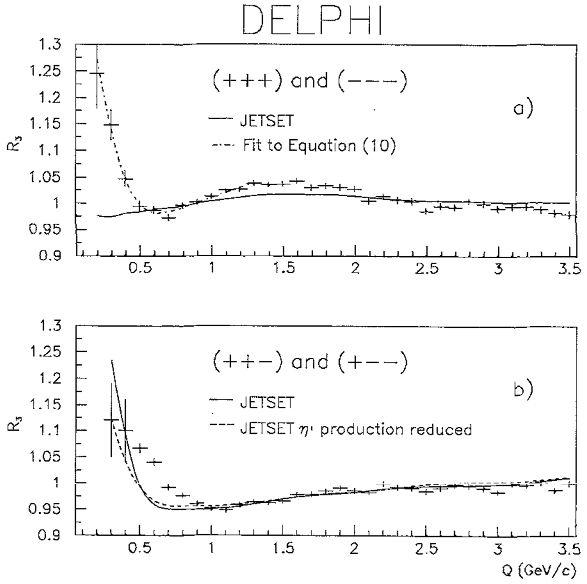

Fig. 2. The three-particle correlation function R_3 for (a) like-sign triplets and (b) for unlike-sign triplets, compared to the JETSET prediction with no Bose-Einstein correlations included. The dot-dashed curve in (a) is the result of a fit with Eq. (10).

The predictions of JETSET 7.3 PS [14] with no BE correlations included, are shown in Fig. 2 (full lines). Here and in the following, the JETSET events are subject to the same cuts and contributions to ρ_3 calculated from single events and the other terms with the same mixing technique as the experimental data. The statistical errors on the JETSET are about three times smaller than those on the data points. At small Q-values, strong differences are observed between JETSET and the data for both like-sign and unlike-sign triplets. For like-sign triplets the model does not show any increase of the correlation function for $Q \to 0$, compatible with no like-sign three-particle correlations in the model. A steep increase of the correlation function at small Q-values is observed in the model without BE correlations for $(+ + -)$ and $(+ - -)$ combinations (full line in Fig. 2b). In the model, the

η' production rate was reduced by a factor 4, based on a measurement of η' production at LEP [19] which is about 4 times smaller than the one predicted by the original version of JETSET. It was checked that this modification has little effect on the $(+++)$ and $(---)$ correlations but changes the $(++-)$ and $(+--)$ correlations (dashed line in Fig. 2b). The larger Q value of the first data point for $(++-)$ and $(+--)$ correlations (0.25 GeV/c as compared to 0.15 GeV/c for like-sign triplets) and the larger errors on the the first two data points, are due to the large uncertainty on η' production in JETSET and consequent uncertainties in the correction factors. Simulations based on JETSET show that R_3 for like-sign combinations is only slightly affected if $b\bar{b}$ events are removed from the sample whereas the change in R_3 for unlike-sign combinations is comparable to the effect of reducing η' production. All modifications of JETSET yield expected results, supporting the method used.

Higher-order BE correlations are a natural source of like-sign three-particle correlations. The shape of $R_3(Q)$ for $(+++)$ and $(---)$ combinations is reminiscent of the shape of the two-body BE correlations [20–22]. In analogy to the latter it can be parametrized by [23]

$$R_3(Q) \sim [1 + 2\lambda_3 \exp(-r_3^2 Q^2)]. \qquad (9)$$

The factor two before the λ_3 parameter arises from the presence of two possible diagrams with exchange of identical pions within a triplet [13]. The total three-particle BE correlation is built up from three diagrams with interchange of two particles in the triplet, plus two diagrams of interchange of the three particles. The latter correspond to the quantity R_3 mentioned above, and the former to the "background" term R_3'.

A modification of Eq. (9) is often used to parametrize the experimental data:

$$R_3(Q) = N(1 + \delta Q)[1 + 2\lambda_3 \exp(-r_3^2 Q^2)], \qquad (10)$$

where N is a normalization factor and the term $(1 + \delta Q)$ is an empirical term which takes into account the rise of the correlation function at large Q-values. A fit of Eq. (10) to the data with Q <1.35 GeV/c yielded the parameter values given in Table 1 (χ^2/DF = 5.2/8). The fitted form is shown in Fig. 2a as a dot-dashed curve. The experimental resolution for Q at Q <0.5 GeV/c is less than 25 MeV/c, i.e. four times smaller than the bin size, and was neglected.

Table 1
Results of the fit of the three-particle correlation function $R_3(Q)$ for $(+++)$ and $(---)$ combinations with Eq. (10).

N	0.92 ± 0.01
δ	0.11 ± 0.01 (GeV/c)$^{-1}$
λ_3	0.28 ± 0.05 (stat) ± 0.07 (syst)
r_3	3.33 ± 0.20 (stat) ± 0.16 (syst) (GeV/c)$^{-1}$

Table 2
Values of the parameters λ_3 and r_3 with different analysis methods (see text).

	λ_3	r_3 (GeV/c)$^{-1}$
T> 0.95	0.34±0.05	3.29±0.19
$\theta > 2°$	0.35±0.06	3.48±0.20
Data/JETSET (T>0.97)	0.24±0.04	3.32±0.25
Data/JETSET (T>0.95)	0.26±0.04	3.17±0.22
Uncorrected data	0.25±0.04	4.16±0.30
Data/DELSIM	0.19±0.03	3.55±0.24

To estimate the systematic errors on the parameters λ_3 and r_3:

- the cut on the thrust variable was chosen to be $T >$ 0.95; this increased the data sample by 60%;
- a cut was made of minimum 2° on the opening angle θ between pairs of tracks in the triplet;
- R_3 for the corrected data was divided by R_3 of JETSET in order to reduce possible residual correlations which are not due to BE correlations.

The values obtained are given in Table 2. The maximal deviation of the λ_3 and r_3 values was used as an estimate of the systematic errors given in Table 1. The results were checked by relaxing the impact parameter requirements to 5 and 10 cm respectively. The conclusions of this analysis are not affected by these cuts. In order to check detector influences Eq. (10) was also fit to the uncorrected data of R_3 and to the uncorrected data of R_3 divided by R_3 of simulated events, fully tracked through the detector by means of DELSIM. The values of λ_3 and R_3 obtained with this procedure are presented in Table 2.

It should be remarked that calculation with JETSET shows that the fraction of pure $\pi\pi\pi$ triplets in all charge combinations is 72% for the interval 0.15< Q <0.5 GeV/c. The values of λ_3 and r_3 in Table 1 are not corrected for the contamination by triplets where (at least) one of the particles is not a pion. Correcting for this effect would increase the value of λ_3 by 39%, in the hypothesis that non-$\pi\pi\pi$ triplets are uncorrelated.

Bose-Einstein correlations can be included in JETSET where they are introduced as a final state interaction. Practically, after the generation of the pion momenta the generated values of the momenta of all identical pions are modified in such a way that their momentum vector differences are reduced with a quantity determined by the chosen form of parametrization and given parameters of correlation strength λ and radius r [14] in order to describe the like-sign two-pion distribution for $R(Q)$. In [17], DELPHI demonstrated that JETSET reproduces well the two-particle correlations if BE correlations are switched on in the model with Gaussian parametrisation for pions which are produced promptly or are decay products of short-lived resonances[1]. The maximal correlation strength $\lambda = 1$ and a radius $r = 0.50$ fm were used. This method of including Bose-Einstein correlations in the model moves *all* direct like-sign pions closer together in momentum space and therefore also yields genuine higher-order correlations with, however, some arbitrary form and correlation strength [3]. Another weakness of the model for BE correlations is that JETSET does not take into account the lifetime of resonances. Consequently, the model can only give a qualitative picture of these final state effects and provide evidence that genuine higher order Bose-Einstein correlations are present at small Q for $R_3(Q)$.

In Fig. 3, the three-particle correlation functions R_3 are shown with the predictions of JETSET with and without BE correlations included. The parameters used to include the BE correlations are the same as in the two-particle correlation study of DELPHI [17]. The model is in reasonable agreement with the data for the $(++-)$ and $(+--)$ configurations and gives an enhancement for the $(+++)$ and $(---)$ correlations. Even if the enhancement in R_3 at low Q values would be partly due to two-particle correlations, arising from the method used, the λ_3 value obtained from the ratio R_3 of the data to R_3 of JETSET with

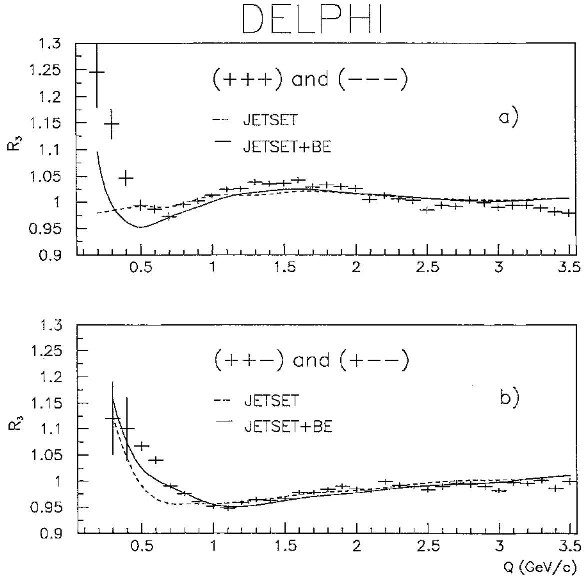

Fig. 3. The function $R_3(Q)$ for (a) like-sign triplets and (b) unlike-sign triplets. The predictions of JETSET without BE (dashed line) and with BE correlations included for direct pions (full line) are also shown, using parameters $\lambda = 1$ and $r = 0.50$ fm in the Gaussian parametrization. In both versions of the model η' production was reduced by a factor four.

BE effects included would give a lower limit of λ_3. The value obtained by a fit to Eq. (10) is 0.14±0.03.

Bose-Einstein interference in JETSET not only changes the distribution of like-sign correlations (Fig. 3a), but also the unlike-sign ones (Fig. 3b) and leads to better agreement with the data. Due to BE correlations, particle distributions and invariant masses of jets are changed [3]. This observation is important for studies at high energies, particularly for the W mass measurement at LEP200.

5. Conclusions

The first observation is reported of genuine three-like-sign particle correlations in e^+e^- annihilations at the Z^0 mass. These correlations can be explained as a higher order Bose-Einstein enhancement. The JETSET model with BE correlations included, with the same λ and r parameters which are used for describing the two-particle correlations, yields reasonable agreement for the $(++-)$ and $(+--)$ correlations and gives an enhancement for $(+++)$ and $(---)$ cor-

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[1] All resonances with lifetime larger than the $K^*(890)$ were considered as long-lived.

relations. One possible explanation of the inadequacy to describe the $(+++)$ and $(---)$ configurations could be an incomplete treatment of Bose-Einstein correlations in the model.

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