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Gaussian effective potential in variational perturbation theory

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The nonperturbative Gaussian effective potential is derived in the $\lambda\varphi^4$-theory as the first nontrivial order of variational perturbation theory. In the framework of the proposed approach the algorithm of calculating corrections is determined from the very beginning. Various ways of constructing the variational procedure for the action functional and questions about the convergence of the variational perturbation theory series are discussed. The series convergence for the anharmonic variational perturbation theory functional is proved.

The method of the Gaussian effective potential (GEP) belongs to the most wide-spread nonperturbative methods [1–4]. Like other nonperturbative approaches, within the GEP method the important question arises of the stability of the obtained results. In other words, it is not at all always obvious in nonperturbative approaches to what extent the main contribution calculated, for example, by using the variational procedure, adequately describes a searched quantity. It is also not obvious what the region of reliability of the obtained results is. The difficulty is that, as a rule, the small initial parameter is absent in nonperturbative tasks. Thus there is not a parameter in powers of which the explored quantity could be expanded. Hence, no judgement about the stability of the results can be made. Moreover, in the majority of nonperturbative approaches finding the algorithm for calculating corrections is obstructed in principle. In view of this the GEP method is advantageously distinguished [5,6]. It should be noted, however, that only the possibility of calculating corrections is still not enough to conclude about stability. Of special importance are here the properties of convergence of the series. Indeed, if a small parameter, the coupling constant, is present in the theory, then even divergent perturbative series regarded as asymptotic can give useful information concerning the region of small coupling constant. Quite a different picture arises when such a small parameter is absent from the very beginning and does not emerge in a certain effective way. Here we may hope to derive reliable results only when we deal with convergent series. Thus, in nonperturbative approaches the tasks of calculating corrections to the main contribution and analyzing the properties of series convergence have to accompany each other.

In the present work we shall consider the nonperturbative method – variational perturbation theory (VPT) [7–9] in the $\lambda\varphi^4$-theory in n-dimensional space. Within this approach the investigated quantity from the beginning is written in the form of a series, which determines the algorithm of calculating corrections up to any order. It

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is important technically that one manages here to construct the VPT series so that the nth order of the VPT approximation uses only the Feynman diagrams which compose the same Nth order of the standard perturbation theory. Thus only the form of the propagator and the structure of the series alter. The presence within the method of free parameters allows one, through their choice, to influence the VPT series convergence.

The present work will be performed in view of the connection between the VPT and GEP methods. A few procedures of obtaining GEP from VPT will be proposed. Under all these procedures the GEP emerges as the first nontrivial order of VPT. The series corresponding to different variants display, however, essentially different convergence properties. The essential point of this article is to show that there exists a VPT procedure (we call it "anharmonic") that gives rise to series convergence and hence provides the stability property of the VPT series. Thus, only within this procedure may truncation of the series be well founded for any value of the coupling constant.

Let us first implement the VPT method for obtaining the variational correction to the quasiclassical approximation. We consider the φ^4-theory in n-dimensional space with the pseudo-Euclidean signature. The action functional reads

$$S[\varphi] = S_0[\varphi] - \tfrac{1}{2}m^2 S_2[\varphi] - \lambda S_4[\varphi], \tag{1}$$

where

$$S_0[\varphi] = \frac{1}{2}\int \mathrm{d}x\,(\partial\varphi)^2, \tag{2}$$

$$S_p[\varphi] = \int \mathrm{d}x\,\varphi^p. \tag{3}$$

In the following, we shall have in mind dimensional regularization setting $n = d - 2\varepsilon$, where d is an integer number. We separate the classical contribution in the generating functional of the Green functions $W[J]$ by writing

$$W[J] = \int \mathrm{D}\varphi\,\exp\{\mathrm{i}[S[\varphi] + \langle J\varphi\rangle]\} = \exp\{\mathrm{i}[S[\varphi_c] + \langle J\varphi_c\rangle]\}D[J], \tag{4}$$

where

$$D[J] = \int \mathrm{D}\varphi\,\exp(-\mathrm{i}P[\varphi]), \tag{5}$$

$$P[\varphi] = \int \mathrm{d}x\,\left[\tfrac{1}{2}\varphi\,(\partial^2 + m^2 + 12\lambda\varphi_c^2)\,\varphi + 4\lambda\varphi_c\varphi^3 + \lambda\varphi^4\right] \tag{6}$$

and the function φ_c satisfies a classical equation of motion $\delta S/\delta\varphi_c = -J$.

In the standard classical approximation one would retain only terms quadric in the fields in expression (6) for the quantity $P[\varphi]$. In this case the functional integral for $D[J]$ becomes Gaussian and for $W[J]$ the ordinary one-loop representation arises.

To calculate $D[J]$ by the VPT method we shall first apply [#1] the *harmonic variational procedure*. We represent the functional $P[\varphi]$ in the form

$$P[\varphi] = \int \mathrm{d}x\,\left[\tfrac{1}{2}\varphi\,(\partial^2 + z^2)\varphi + \lambda\left(4\varphi_c\varphi^3 + \varphi^4 - \tfrac{1}{2}\chi^2\varphi^2\right)\right], \tag{7}$$

where $z^2 = m^2 + 12\lambda\varphi_c^2 + \lambda\chi^2$. Then the VPT series is written as

[#1] A similar procedure for GEP construction was explored in refs. [5,6].

$$D[J] = \left(\det \frac{\partial^2 + z^2}{\partial^2}\right)^{-1/2} \sum_{n=0}^{\infty} \frac{(-\mathrm{i}\lambda)^n}{n!} \left[\int \mathrm{d}x \left(4\varphi_c \hat{\varphi}^3 + \hat{\varphi}^4 - \tfrac{1}{2}\chi^2 \hat{\varphi}^2\right)\right]^n \exp\left[-\tfrac{1}{2}\mathrm{i}\langle j\Delta j\rangle\right]_{j=0}, \tag{8}$$

where $\Delta(p) = (p^2 - z^2 + \mathrm{i}0)^{-1}$, $\hat{\varphi}(x) = \mathrm{i}\,\delta/\delta j(x)$. Let us restrict ourselves to the first two terms in (8). Their contributions to the effective potential equal, respectively,

$$V_0 = (1/n)\, z^2 \Delta_0(z^2), \tag{9}$$

$$V_1 = \lambda\left[3\Delta_0^2(z^2) - \tfrac{1}{2}\chi^2 \Delta_0(z^2)\right], \tag{10}$$

where

$$\Delta_0(z^2) = \mu^{2\varepsilon} \frac{\Gamma(1 - n/2)}{(4\pi)^{n/2}} (z^2)^{n/2-1} \tag{11}$$

is the Euclidean propagator $\Delta(x = 0, z^2)$ written with the help of dimensional regularization. The optimization condition $\mathrm{d}(V_0 + V_1)/\mathrm{d}z^2 = 0$ gives rise to the equation for the variational parameter z^2,

$$z^2 = m^2 + 12\lambda\varphi^2 + 12\lambda\Delta_0(z^2). \tag{12}$$

Making use of (12) for the effective potential in the considered VPT order we find the expression

$$V_{\mathrm{eff}}(\varphi) = V_{\mathrm{cl}} + V_0 + V_1 = \tfrac{1}{2}m^2\varphi^2 + \lambda\varphi^4 + (1/n)z^2\Delta_0(z^2) + \tfrac{1}{2}(m^2 - z^2)\Delta_0(z^2) + \lambda\left[3\Delta_0^2(z^2) + 6\varphi^2\Delta_0(z^2)\right]. \tag{13}$$

It is easy to see that expression (13) coincides with GEP in n-dimensional space [10], if one takes into account the optimization condition (12).

Let us now calculate the quantity $D[J]$ by using the *anharmonic variation* of the action functional. We choose the VPT functional in the form $R^2[\varphi]$, where $R[\varphi] = (\chi/2\Omega^{1/2}) \int \mathrm{d}x\, \varphi^2(x)$. The space volume Ω appears here because V_{eff} is derived from the effective action by using the constant-field configurations. Thus, the parameter χ, optimizing the VPT series, does not depend on Ω.

As a result, we get

$$D[J] = \sum_{n=0}^{\infty} \frac{(-\mathrm{i})^n}{n!} \int \mathrm{D}\varphi \left[\lambda \int \mathrm{d}x \left(\varphi^4 + 4\varphi_c\varphi^3\right) - R^2[\varphi]\right]^n$$
$$\times \exp\left\{-\mathrm{i}\left[\frac{1}{2}\int \mathrm{d}x\, \varphi\left(\partial^2 + m^2 + 12\lambda\varphi_c^2\right)\varphi + R^2[\varphi]\right]\right\}. \tag{14}$$

Any power of $R^2[\varphi]$ in (14) can be obtained by the corresponding number of differentiations of the expression $\exp(-\mathrm{i}\varepsilon R^2[\varphi])$ with respect to ε, putting $\varepsilon = 1$ at the end. As to the term $R^2[\varphi]$ in the exponential, giving rise to a non-Gaussian form of the functional integral, the problem is easily solved by implementing the Fourier transformation, due to which only the first power of $R[\varphi]$ emerges in the exponential.

As a result, the VPT series takes the form

$$D[J] = \sum_{n=0}^{\infty} (-1)^n \sum_{k=0}^{n} \frac{(-\mathrm{i})^{n-k}}{(n-k)!\,n!} \left(\frac{\mathrm{d}}{\mathrm{d}\varepsilon}\right)^{n-k} \sqrt{\Omega} \int_{-\infty}^{\infty} \frac{\mathrm{d}v}{2\sqrt{\pi}} \exp\left(\tfrac{1}{4}\mathrm{i}\Omega v^2 - \tfrac{1}{4}\mathrm{i}\pi\right) \left(\det \frac{\partial^2 + M^2}{\partial^2}\right)^{-1/2}$$
$$\times \left[\lambda \int \mathrm{d}x \left(4\varphi_c\hat{\varphi}^3 + \hat{\varphi}^4\right)\right]^k \exp\left(-\tfrac{1}{2}\mathrm{i}\langle j\Delta j\rangle\right)_{j=0}, \tag{15}$$

where $M^2 = m^2 + 12\lambda\varphi_c^2 + \sqrt{\varepsilon}\chi v$. The integral over v in (15) contains the large parameter Ω and, hence, can be evaluated by using the stationary phase method. Then, the effective potential in the first nontrivial VPT order looks as

$$V_{\rm eff} = V_{\rm cl} + V_0 + V_1, \qquad V_0 = \frac{1}{n} M^2 \Delta_0 - \tfrac{1}{4}\chi^2 \Delta_0^2, \qquad V_1 = -\tfrac{1}{4}\chi^2 \Delta_0^2 + 3\lambda \Delta_0^2. \tag{16}$$

Here M^2 is the massive parameter taken at $\varepsilon = 1$ and $v = v_0$, where v_0 is the stationary phase point in the integral (15). The corresponding equation reads

$$M^2 = m^2 + 12\lambda\varphi^2 + \chi^2 \Delta_0(M^2). \tag{17}$$

One can apply now the following optimization versions (see refs. [7,8]): (i) The requirement min$|V_1|$ (here there exists a solution to the equation $V_1 = 0$); (ii) $\partial V_{\rm eff}/\partial\chi^2 = 0$. It is easy to find out that these different versions give rise to the same optimal value of the parameter χ^2: $\chi^2 = 12\lambda$. As a result, the effective potential (16) with the condition (17) yields the GEP.

We shall now derive the GEP by using another approach that, instead of utilizing the representation (4), directly operates with the initial functional $W[j]$. We consider the *two-parameter, anharmonic-type* VPT functional:

$$\Omega[\varphi] = (a^2/\Omega) S_2^2[\varphi] + (b^4/\Omega^3) S_1^4[\varphi]. \tag{18}$$

The VPT series for the generating functional of Green functions looks as follows:

$$W[j] = \sum_{n=0}^{\infty} \frac{{\rm i}^n}{n!} \int {\rm D}\varphi\, (\Omega - \lambda S_4)^n \exp\left[{\rm i}\left(S_0 - m^2 S_2 - \varepsilon \frac{a^2}{\Omega} S_2^2 - \theta \frac{b^4}{\Omega^3} S_1^4 + \langle j\varphi\rangle\right)\right]. \tag{19}$$

The parameters ε and θ are introduced here to allow us to get in a factor before the exponential, the terms connected with S_1 and S_2, by differentiation with respect to these parameters (they are to be set to 1 at the end). Then only the interaction action λS_4 remains in a factor in front of the exponential in the functional integral. The expression in the exponential in (19) is reduced to a form quadric in the fields by using Fourier transformation. As a result, (19) is rewritten in the form

$$W[j] = \Omega^2 \int_{-\infty}^{\infty} {\rm d}x \frac{{\rm d}p}{2\pi} \int_{-\infty}^{\infty} {\rm d}y \frac{{\rm d}q}{2\pi} \exp[{\rm i}\Omega(px - qy - p^2 - q^4)]$$
$$\times \sum_{n=0}^{\infty}\sum_{k=0}^{n}\sum_{m=0}^{n-k} \frac{{\rm i}^{n-k}}{m!(n-k-m)!} \left({\rm i}\frac{\partial}{\partial\varepsilon}\right)^m \left({\rm i}\frac{\partial}{\partial\theta}\right)^{n-k-m} \left(\det \frac{\partial^2 + M^2}{\partial^2}\right)^{-1/2} w_k[J, M^2], \tag{20}$$

where $M^2 = m^2 + \sqrt{\varepsilon}\, a\, x$, $J = j + \theta^{1/4} b\, y$, and $w_k[J, M^2]$ are the ordinary perturbative expansion coefficients for the generating functional of Green functions $W[j]$. To evaluate w_k, the standard Feynman rules with a massive parameter M^2 in the propagator can be used.

In the first nontrivial order we get for the generating functional of connected Green functions $Z[j] = ({\rm i}\Omega)^{-1} \ln W[j]$ the expression:

$$Z^{(1)} = jx - \tfrac{1}{2}m^2x^2 + (\tfrac{1}{2} - 1/n)m^2 y \Delta_0(m^2 y) - \tfrac{1}{2}m^2 \Delta_0(m^2 y) - \lambda\left[3\Delta_0^2(m^2 y) + 6\Delta_0(m^2 y)x^2 + x^4\right], \tag{21}$$

where $x = J/M^2$, $y = M^2/m^2$. The optimization condition $\partial Z^{(1)}/\partial x = 0$ gives rise to the equation

$$m^2 x + 4\lambda x(3\Delta_0 + x^2) = j. \tag{22}$$

By analogy, requiring $\partial Z^{(1)}/\partial y = 0$ we obtain

$$m^2(y-1) = 12\lambda(\Delta_0 + x^2). \tag{23}$$

Making use of (22) and (23) we find $\varphi = {\rm d}Z^{(1)}/{\rm d}j = \partial Z^{(1)}/\partial j = x$. For the effective potential $V_{\rm eff} = j\varphi - Z$ we get

$$V_{\text{eff}} = \tfrac{1}{2}m^2\varphi^2 + (1/n - \tfrac{1}{2})M^2\Delta_0(M^2) + \tfrac{1}{2}m^2\Delta_0(M^2) + \lambda[3\Delta_0^2(M^2) + 6\Delta_0(M^2)\varphi^2 + \varphi^4]. \tag{24}$$

It is easy to show that (24) coincides with the GEP.

The above-considered procedures of constructing the VPT series were chosen so that the first nontrivial order should lead to the GEP. However, despite the same result in the first order, other properties of the series are different. Let us exemplify this statement by considering the Euclidean vacuum [#2] functional

$$W[0] = \int D\varphi \, \exp\{-[S_0[\varphi] + \lambda S_4[\varphi]]\}. \tag{25}$$

The terms of the VPT series look as follows:

$$W_k = \frac{1}{k!}\int D\varphi \, (\tilde{S}[\varphi] - \lambda S_4[\varphi])^k \exp\{-(S_0[\varphi] + \tilde{S}[\varphi])\}, \tag{26}$$

where $\tilde{S}[\varphi]$ is a certain VPT functional. The asymptotic behaviour of remote terms of the VPT series can be found by using the k-saddle-point method [10,12]. Then the main contribution to the functional integral comes from the field configurations proportional to a positive power of the large saddle-point parameter k.

In the case of the *harmonic variational procedure*, when $\tilde{S}[\varphi]$ is a functional quadric in the fields, the expression $\lambda S_4[\varphi]$ dominates in the leading order in k in a factor before the exponential in (26). Thus, a situation analogous to the case of ordinary perturbation theory arises and the VPT series proves to be divergent. The harmonic method, nevertheless, can be used to improve the perturbation theory. It concerns situations when there is a small parameter in the theory, which allows us to regard the VPT series as asymptotic. By consideration of the examples of a zero-dimensional analog and a quantum-mechanical oscillator, it has been shown that VPT with the harmonic variational procedure permits one to extend the region of the stable behaviour of the series sum to some larger values of the coupling constant as compared with perturbation theory. However, we cannot gain sensible advancement into the nonperturbative region by using the harmonic method. This comes about because for large constants even the first terms of the VPT series get sensitive to the asymptotic nature of the series and it becomes problematical to judge about the stability of the results because of the emergence of specific "beats" in the partial sums. This can be clearly seen from the zero-dimensional example

$$Z[g] = \int dx_1 \, dx_2 \, \exp\{-[x_1^2 + x_2^2 - g(x_1^4 + x_2^4)]\},$$

represented in fig. 1 for $g = 1$. In the case of the harmonic variational procedure the situation for $g \gg 1$ becomes even more drastic.

Quite a different situation occurs in the case of the *anharmonic VPT functional*. Here the functionals $\tilde{S}[\varphi]$ and $S_4[\varphi]$ have the same powers in the fields φ and hence both of them exert equal influence on the remote terms of the series. Thus, the hope arises that there is a region of variational parameters where the VPT series proves to be convergent. So, let us choose the VPT functional of the *anharmonic type*:

$$\tilde{S}[\varphi] = \lambda A^2[\varphi], \qquad A[\varphi] = \theta S_0[\varphi] + \tfrac{1}{2}\chi S_2[\varphi]. \tag{27}$$

After the change $\varphi \to k^{1/4}\varphi$ the term of the VPT series W_k is written as

$$W_k = \lambda^k \frac{k^k}{k!}\int D\varphi \, \exp\left(-kS_{\text{eff}}[\varphi] - k^{1/2}S_0[\varphi]\right), \tag{28}$$

where

[#2] The introduction of a source and also a mass does not alter the arguments concerning the convergence.

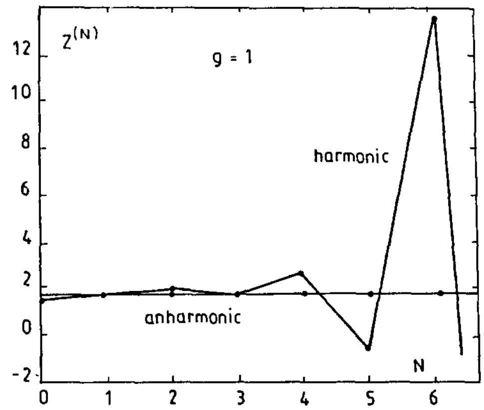

Fig. 1. Zero-dimensional example. N-dependence of the Nth partial sum (sum of the first N terms of the VPT series) for the cases of the harmonic and anharmonic variational procedures for $g = 1$.

$$S_{\text{eff}}[\varphi] = \lambda A^2[\varphi] - \ln D[\varphi], \qquad D[\varphi] = A^2[\varphi] - S_4[\varphi]. \tag{29}$$

The main contribution to the integral (28) comes from the field configurations $\varphi_0(x)$ which minimize the effective action functional S_{eff}. The corresponding equation reads

$$-\partial^2\varphi_0 + a\varphi_0 - b\varphi_0^3 = 0, \tag{30}$$

where

$$a = \chi/\theta, \qquad b = 2\left[\theta A[\varphi_0]\left(1 - \lambda D[\varphi_0]\right)\right]^{-1}. \tag{31}$$

It is convenient to pass to the function $f(x)$ that satisfies the equation $[-\partial^2 + 1]f(x) - f^3(x) = 0$ and is connected with the function $\varphi_0(x)$ as $\varphi_0(x) = \sqrt{a/b}\, f(\sqrt{a}x)$. We define a constant

$$C_n = \int \mathrm{d}x\, f^4(x), \tag{32}$$

which depends on the space dimension [12]. Here, however, we do not care for the numerical value of C_n. With the help of (32) the functionals $S_4[\varphi_0]$ and $A^2[\varphi_0]$ are rewritten as $S_4[\varphi_0] = \alpha/b^2$, $A^2[\varphi_0] = \alpha^2\tau/b^2$. Here we have defined the parameters $\alpha = C_n a^{2-n/2}$, $\tau = \theta^2/4$. As follows from (31), the three parameters α, b and τ are connected by the relation $\alpha\tau\left(1 - \lambda D[\varphi_0]\right) = 1$, where $D[\varphi_0] = \alpha(\alpha\tau - 1)/b^2$. In the leading order in k we get for the integral (28) the relation

$$W_k \sim k^{-1/2}\lambda^k D^k[\varphi_0]\exp\left\{-k\left[\lambda A^2[\varphi_0] - 1\right]\right\}. \tag{33}$$

The range of values of the parameters at which the VPT series is convergent is determined by the inequality $|\lambda D[\varphi_0]| < \exp(\lambda A^2[\varphi_0] - 1)$. The best choice of parameters, at which the contribution of the remote terms of the VPT series is minimal (the so-called *asymptotic optimization* of the series) implies the condition $D[\varphi_0] = 0$ giving rise to the following relation between the parameters: $\alpha\tau = 1$. Thus, only one independent parameter remains which can also be fixed by optimizing the first few terms of the VPT series which we deal with in practice. The asymptotic optimization condition for the initial parameters θ and χ reads

$$\chi = \left(16/\theta^n C_n^2\right)^{1/(4-n)}. \tag{34}$$

In particular, in the one-dimensional case $C_1 = 16/3$ and the condition (34) transforms into the optimization condition for the anharmonic oscillator [7].

Thus, in theories without a small parameter it is preferable to calculate corrections by using the VPT series where the VPT functional behaves at large fields like the initial interaction action. This again can be illustrated through the zero-dimensional case (fig. 1). We see that with the anharmonic variational procedure, contrary to the harmonic one, we obtain stable results for corrections of any order (in this connection see ref. [9]).

Let us give a brief *resumé*.

One should realize that in order to estimate a physical quantity it is not at all enough to construct a procedure just giving a leading contribution and some algorithm of correction calculation. Of fundamental importance is the question of the possibility to perform a well-founded series truncation. A well known example is the quasiclassical loop expansion. The direct physical interpretation of the minima of the one-loop effective potential is in contradiction with the higher order corrections. This is a clean example where the "pro forma" improvement obtained by including the next order corrections can lead to wrong physical conclusions (the two-loop potential has no minimum but the three-loop potential has the same minimum as the one-loop potential and so on).

Let us stress that the results obtained in $0+1$ dimensional space cannot be directly relevant for the quantum field-theoretical case due to the renormalization effect. In $3+1$ dimensions the renormalization effects become essential. In this case the remarkable result has been obtained [14,15] that the simple one-loop effective potential and GEP are equivalent up to some uninteresting renormalization of the bare parameters which does not affect the relations between the physical quantities in the limit of infinite cutoff. It has been argued [15] that the massless φ^4-theory can exhibit asymptotic freedom and a new treatment of the "triviality" of the φ^4-model has been given. In this connection the investigation of the corrections to GEP becomes important. We have shown that there exist a set of VPT procedures giving the GEP as a leading contribution. However, the problems of stability and VPT series truncation arise again and one should prefer the anharmonic VPT procedure, which leads to a convergent series.

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