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**A M Baldin, V V Burov and L P Kaptari**

*JINR, Dubna, USSR*

# ASYMPTOTIC MULTIPLICITIES: PHENOMENOLOGY, EXPERIMENTAL PERSPECTIVES

J. Manjavidze, A. Sissakian

*Laboratory of Theoretical Physics, Joint Institute for Nuclear Research*

## 1. Introduction

We consider the production processes in which multiplicity of hadrons

$$n \gg \bar{n}(s) \quad (1)$$

-mean multiplicity at given c.m. energy  $\sqrt{s}$ . In such processes

a. Relative energies of created particles

$$\sqrt{s_{ij}} \ll \sqrt{s}, \quad i, j = 1, 2, \dots, n. \quad (2)$$

b. Density of particles in phase space is large. We want to consider what this conditions gives to particle physics.

To our mind the mostly interesting problem of hadron physics is the collective phenomena: color-plasma physics, phase transitions, charge and energy-momentum transport problems. But on experiments we see usually (at  $n \sim \bar{n}$ ) mixture of two processes: (color) parton creations and process of hadron formation and last one screens all interesting affects.

Investigation of the very large multiplicity processes can separate this two fractions: because of (2) large strings can not be create and one can omit the process of parton creations from vacuum. So, neglecting nonperturbative effects we can construct theory on the base of perturbative QCD.

The very high multiplicity processes gives the unique possibility investigate pure quark-gluon systems.

## 2. Phenomenology

Theory of considered processes has characteristic features. For this purpose it is useful introduce the density matrix  $\rho(\beta, z)$  where  $\beta$  is inverse temperature in the created particle system and  $z$  is activity ( $\frac{1}{\beta} \ln z$  is chemical potential). This quantities are defined by the (state) equations:

$$-\sqrt{s} = \frac{\partial}{\partial \beta} \ln \rho(\beta, z) \quad (3)$$

$$n = z \frac{\partial}{\partial z} \ln \rho(\beta, z) \quad (4)$$

If system is in equilibrium then solutions of this equations  $\bar{z}, \bar{\beta}$  completely describes the system. In another, nonequilibrium, case it should be taken into account (Gauss) fluctuations:

$$\sigma_n(s) = \frac{1}{2\pi i} \oint \frac{dz}{z^{n+1}} \frac{1}{2\pi i} \frac{1}{\sqrt{s}} \int_{\text{Re}\beta > 0} \frac{d\beta}{2\pi} \beta^2 I_1(\beta\sqrt{s}) \rho(\beta, z)$$

Strictly speaking considered system can not be in the equilibrium state because energy  $\sqrt{s}$  is finite (and not all degrees of freedom are excited). But in the region (1) it is some change that system is in equilibrium (density of particles is large). Then one can use asymptotic estimation:

$$\sigma_n^{(0)} \propto e^{-n \ln \bar{z}(n, s)} \quad (5)$$

To avoid influence of phase space boundary we propose that energy  $\sqrt{s}$  is asymptotically large. From estimation (5) we see that asymptotic of  $\sigma_n$  controlled [1] by the minimal solution of eq.(4). Using (5) we can distinguish three case: with increasing  $n$

$$\begin{array}{ll} a. & \bar{z}(n, s) \rightarrow 0, \\ b. & \bar{z}(n, s) \rightarrow \text{const}, \\ c. & \bar{z}(n, s) \rightarrow \infty. \end{array} \quad (6)$$

Asymptotic (6a) describes decay of unstable (with respect to particles production) state. This decay produces of clusters and if size of cluster is larger than a critical one, cluster's size infinitely increases. During this motion the cluster wall "accelerate", i.e. the larger number of particles forming a cluster, the smaller energy is needed to add particle into a cluster. Just this phenomenon is observed in the decrease of  $\bar{z}(n, s)$  with increasing  $n$ . It is evident that in the case (6a) we observe first order phase transition.

In the case (6c) no collective phenomena can occur: in this case  $\bar{z}(n, s)$  increase with  $n$ .

The intermediate case (6b) must be considered separately. In this case  $\rho(\bar{\beta}, z) \equiv \bar{\rho}(z, s)$  singular at finite  $z$  (it simply follows from (5) and (6b)):

$$\bar{\rho}(z, s) \sim (z_c(s) - z)^{-\gamma}, \quad \gamma > 0 \quad (7)$$

Using normalization condition:  $\frac{\partial \ln \bar{\rho}}{\partial z} \Big|_{z=1} = \bar{n}(s)$  one can finds:

$$z_c = 1 + \frac{\gamma}{1 + \frac{\gamma}{\bar{n}(s)}} \quad (8)$$

Then

$$\bar{z}(n, s) = \frac{z_c(s)}{1 + \frac{\gamma}{\bar{n}}} \quad (9)$$

and

$$\sigma_n(s) \propto e^{-\gamma \frac{n}{\bar{n}(s)}} \quad (10)$$

has KNO-form. It is well known that branching processes leads to the singular at finite  $z$  partition functions  $\bar{\rho}(z, s)$ . It should be noted that with increasing singularity (7) moves to the left. Then, according to the energy-momentum conservation laws the production to a particle in the decay of one jet will dominate in the asymptotic in  $n$ . Indeed, the production of particles in the decay of two jets  $\sim (z_c(s/4) - z)^{-2\gamma}$  and this contribution generate in the  $z$  plane singularity

$$1 + \frac{\gamma}{\bar{n}(s/4)} > 1 + \frac{\gamma}{\bar{n}(s)}.$$

Assuming that the correlations between particles produced in the decay of one jet differ for those between particles in the decay of various jets the afore-side implies the presence of a "phase transition" which is reflected in the change of the nature of correlations among particles with increasing  $n$ . However, this transition is smooth, without sharp changes. Therefore, it is better called the "structure phase transition". Using (7) one can find the ratio of dispersion  $D$  to  $\bar{n}$  with regard to the production of two jets:

$$\frac{D^2}{\bar{n}} = 1 - a \frac{\bar{n}(s/4)}{\bar{n}(s)}, \quad (11)$$

where the positive constant  $a$  takes into account a relative weight of the production of two clusters. We see that the ratio  $D/\bar{n}$  increases with energy - the well known result.

### 3. Predictions

Now we shall formulate the main predictions for experimentalists:

1. The asymptotic  $\sigma_n > 0$  ( $e^{-n}$ ) is associated with the first order phase transition (among quark-gluon phase and hadron phase?).
2. The asymptotic  $\sigma_n < 0$  ( $e^{-n}$ ) are typical for multiperipheral models.
3. The asymptotic  $\sigma_n = 0$  ( $e^{-n}$ ) associated with the "structure" phase transitions.

In this case:

- a)  $\sigma_n$  has KNO-scaling form, but multiplicity scaled by the mean multiplicity in jet;
- b) production process is hard and mean transfer momentum increase with increasing  $n$ ;

- c) because  $z_c$  decrease when  $s$  increase there is duality among asymptotic of  $n$  and of  $s$  ;  
 d) fractal dimension will be non zero.

#### 4. Experiments

We understand that experimental investigation of very high multiplicity processes at high energies is very complicated problem: we discuss processes in final state of which is some hundred (even Thousand) particles and cross sections are very small (some  $nb$  ). It is too hard construct trigger for this processes. But from previous decision follows that it is enough distinguish asymptotic  $\sigma_n = 0(e^{-n})$  from another asymptotic.

Consider [2] following formulation of experiment (which is naive but can be considered as the starting point). If energies of produced particles

$$\varepsilon_i \leq \varepsilon_0, \quad i = 1, 2, \dots, n, \quad \varepsilon_i = \sqrt{p_i^2 + m^2}, \quad (12)$$

where  $\varepsilon_0$  fixed on experiment then using energy conservation law number of particles can not be arbitrary small:

$$n \geq n_{min} \equiv \frac{E}{\varepsilon_0} \quad (13)$$

With this restriction experiment gives the value of

$$T(\varepsilon_0) = \sum_{n=n_{min}}^{\infty} \int \prod_{i=1}^n \frac{d\varepsilon_i}{2\varepsilon_i} \omega_n(\varepsilon_1, \dots, \varepsilon_n) \quad (14)$$

where  $\omega_n$  is nonnormalized  $n$  particle production probability. If in integral are essential  $\varepsilon_i \ll \varepsilon_n$  then

$$T(\varepsilon_0) = \sum_{n=n_{min}} \sigma_n(s) \quad (15)$$

with accuracy which is  $\sim \bar{n}_{\varepsilon_0}$  - mean number of particles with energies  $\varepsilon_i > \varepsilon_0$ . Introducing  $\varepsilon_n = \frac{E}{n}$  we see that

$$\sigma_n = T(\varepsilon_n) - T(\varepsilon_{n+1}), \quad (16)$$

if  $\bar{n}_{\varepsilon_0} \ll 1$ .

Lets adopt this idea and consider possibility of using calorimeter to fix  $\varepsilon_0$ .

1. Calorimeter measure energy with some accuracy. Then, instead of (14) measured following quantity:

$$T_\sigma(\varepsilon_0) = \sum_{n=n_{min}}^{\infty} \int \prod_{i=1}^n \frac{dz_i \theta_\sigma(\varepsilon_0 - \varepsilon_i)}{2\varepsilon_i} \omega_n(\varepsilon_1, \dots, \varepsilon_n) \quad (17)$$

where  $\sigma$  denotes half-width. One can note that  $n_{min}$  determined only by the energy conservation law. Again if  $\bar{n}_{\epsilon_0} \ll 1$  one can use eq.(15).

2. In each cell of calorimeter can get  $n_i \geq 1$  particles,  $\sum n_i = n$ . Then, instead of (12) we have conditions:

$$\bar{\epsilon}_i \leq \epsilon_0, \quad i = 1, 2, \dots, k, \quad (18)$$

where  $k$  is number of cells. Then on experiment measured

$$\bar{T}_\sigma(\epsilon_0) = \sum_{k=k_{min}} \int \prod \frac{d\bar{\epsilon}_i \theta_\sigma(\epsilon_0 - \epsilon_i)}{2\bar{\epsilon}_i} \bar{\omega}_k(\bar{\epsilon}_1, \dots, \bar{\epsilon}_n) \quad (19)$$

where  $\bar{\omega}_k$  is probability that  $n$  created particles distributed on  $k$  cells and

$$k_{min} = \frac{E}{\epsilon_0} \quad (20)$$

It is evident that

$$n \geq k \quad (21)$$

and choosing  $\epsilon_0$  small get again into asymptotic region of  $n$ .

Considered effect can lead to the "smoothing" of density fluctuations and can lead to the unphysical result when intermittancy is zero. This difficulty demand further considerations.

## References

- [1] I.Manjavidze, A.Sissakian. JINR Rapid Com. 5 [31]-88, 5.
- [2] I.Manjavidze, A.Sissakian. JINR Rapid Com. 2 [28]-88, 13.