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SOME REGULARITIES IN THE PROCESSES OF HIGH-ENERGY MULTI-PARTICLE PRODUCTION

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1. Introduction

At the present time a good deal of theoretical and experimental papers are devoted to the problem of multi-particle production in high-energy hadron collisions. In spite of the intense study of a rather large energy interval (10^1-10^3 GeV) no reasons so far exist to consider the present description of various phenomena in hadron interactions to be clear and complete. A number of regularities and specific properties, mainly new, are nevertheless established for such interactions.

Let us note, that the prediction by G. Vatagin[/1/] is confirmed. This prediction concerns the growth of the relative number of inelastic channels at high energies. The data from ISR give $\frac{\sigma_{el.}}{\sigma_{tot}} \sim 0.18$[/2/]. So, the hadron-hadron collisions are mainly inelastic, and the elastic ones obviously show themselves as a shadow of nonelastic channels (see, e.g., the papers on the quasipotential approach[/3/]).

The experiments made at Serpukhov, Batavia and CERN[/4/] also indicate the apparent scale invariance or automodelity of processes involving strong interactions. This fundamental regularity allows us to understand a great number of empirical facts on the dynamics of hadron-hadron collisions.

Another important property of these collisions at high-energies is the smallness of the momentum transfers, $q_\perp^\ell$, which is closely related to the existence of leading particles.

The deviation, found at energies higher than 25 GeV, between the multiplicity distribution and usual Poisson distribution makes it possible to assume that the secondaries are produced not through the independent production of individual particles but through that of whole clusters.

The factors known at present about hadron-hadron collisions are not limited to the above-listed properties. However these properties reflect the basic characteristics of multiple production in strong interaction. It is just these properties that are constantly exploited by theoreticians, no matter, which way they go. Those advocating the phenomenological schemes and empirical formulae seek to use these properties in constructing their models. Others who keep to field-theoretic approaches, verify consistency of these properties with basic axioms of quantum field theory and develop approximations adequate for the general properties found.

It may be hoped that these two approaches, studying the same phenomena from various directions, after being united, will provide a closer description of high-energy multi-particle production.

In this lecture we will give two examples for constructing models of multi-particle production. The first part concerns the results obtained for multi-particle production within the framework of quantum field theory. Here we shall speak about the straight-line path approximation, which realizes in a field-theoretical language the idea of a leading particle as well as giving rise to the generalized Poisson distribution over multiplicities and to the automodel behaviour of cross sections. In the second example, we attempt to build up a concrete phenomenological scheme for describing experimental data, through the use of some results of the straight-line path approximation. The model is compared with the experiments performed at Serpukhov with the two-meter propane chamber when it was irradiated by 40 GeV negative pions.

These examples are based on articles performed at Dubna.

Ref. to 1. Introduction

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II. Multiple Particle Production in the Straight-Line Path Approximation

Let us consider some results concerning multi-particle production in the framework of field-theoretic models, using the Feynman-Bogolubov functional integration method [1]. The straight-line path approximation is used as the main approximation which makes it possible to extract the necessary information from expressions containing functional integrals. As is known, this approximation has been suggested and dveloped by the Dubna group (Tavkhelidze, Barbashov, Matveev et al. [2]) for high energies and fixed momentum transfers, and represents a generalization of the approximation $k_i k_j = 0$ formulated by Fradkin and Barbashov [3] in their papers on the infrared asymptotics of the Green's functions.

The concept of the straight-line path approximation is the following. At high energies and fixed momentum transfers the main contribution to the amplitude of the process written in the form of a Feynman integral over the particle paths is assumed to come from trajectories which are nearly straight lines having the same direction as the momentum vectors of the leading particles before and after interaction.

Such paths were taken into account in expressions containing functional integrals because of the procedure of averaging over the functional variable

$$\int[\delta\nu]e^{F(\nu)} \to e^{\int[\delta\nu]F(\nu)}, \tag{2.1}$$

where $\int[\delta v]$ means the integration over the Gaussian measure.

We note that averaging according to the rule

$$\int[\delta\nu]e^{F(\nu)} \to e^{F(\nu=0)} \tag{2.2}$$

would be to take into account the classical particle trajectory.

Calculations performed in the straight-line path approximation at high energies and fixed momentum transfers as well as corrections to this approximation[4] show that the application of this method is well justified, at least in models which make use of a nonsingular effective quasipotential.

This statement is in agreement with the situation in quantum mechanics and confirms the importance of the hypothesis about the smoothness of the local quasipotential suggested by Blokhintsev and Logunov and co-workers[5] and investigated in Refs.[6].

It is important to note that the straight-line path approximation as applied to the study of processes of multiple particle production leads to a partial neglect of the nucleon recoil when emitting secondary particles. In this sense the suggested model is close to multiple particle production models constructed by analogy to the "bremsstrahlung" model in electrodynamics.

These problems were already studied by Heisenberg[7], who considered a special field-theory model using the Bloch-Nordsik method, as well as by Lewis, Oppenheimer, and Wouthuysen, who suggested the "shaking" model[8]. Among more recent studies we should note a series of papers on the "bremsstrahlung" model by Kastrup[9].

Note should also be made that the straight-line path approximation as stated here rests on the assumption of a leading particle. This is an inherent feature of all the models considered by means of the above approximations. However, it has been shown in one of the recent papers of the Dubna group /4/ that for the straight-line path method in fact the assumption is essential on "inertia of large momentum in the interaction process". In this sense the approximation considered is adequate for the effect of the leading particle (i.e. the particle conserving its high-energy individuality) and is a particular example of the application of this rather general method.

A. The main results of the model

We performed a detailed study of field-theoretic models with the following interaction Lagrangians

$$L_{int} = g : \psi\psi^*\Phi : , \tag{A}$$

$$L_{int} = g : \psi \, i\overleftrightarrow{\partial}_a \psi \, A_a : + g^2 : A_a^2 \psi^* \psi : . \tag{B}$$

In applying the straight-line path approximation we first considered the possibility of obtaining the eikonal representation by summing directly a certain class of Feynman graphs. As is well known, the summing of the s-channel ladder graphs with a multiparticle exchange yields an eikonal formula for the scattering amplitude /10/. It is also important to note that by taking into account radiative corrections, we obtain a smooth or non-singular effective quasipotential /11/.

Inelastic process amplitudes describing the production of a certain number of quanta of the field A in the collision of two nucleons may be found by means of the generating function $f(p_1,p_2;\ q_1,q_2\ A^{ext})$, (we are considering model B)/1/. The quantity $f(p_1,p_2;\ q_1,q_2\ A^{ext})$ denotes the scattering amplitude for two nucleons in the presence of the external field. As an example, we give expressions for the generating function in the presence of the external field A^{ext}. In the framework of the straight-line path approximation it has the following form:

$$if(p_1,p_2;q_1,q_2|A^{ext})=g^2\int d^4y\, e^{iy(p_1-q_1)}\, d^4x\, e^{ix(p_2-q_2)}$$

$$\Delta(x-y;p_1,q_1:p_2,q_2)\exp\{ig\int d^4\ell A^{ext}_{\alpha}(\ell)$$

$$|j^{(1)}_{\alpha}(\ell;p_1,q_1)e^{i\ell x}+j^{(2)}_{\alpha}(\ell;p_2,q_2)e^{i\ell y}|\} \tag{2.3}$$

$$\int_0^1 d\lambda\exp\{\frac{ig^2}{2}\int d^4k D_{\gamma\xi}(k)|e^{ik(x-y)}\ 2\lambda j^{(1)}_{\gamma}(k;p_1,q_1)$$

$$j^{(2)}_{\zeta}(-k;p_2,q_2)+\sum_{i=1}^{2}j^{(i)}_{\gamma}(k;p_i,q_i)j^{(i)}_{\xi}(-k;p_i,q_i)|\}$$

where j are the nucleon currents averaged over the functional variables,

$$\Delta(x)=\int d^4k\, e^{ikx}D_{\alpha\beta}(k)(k+p_1+q_1)_{\alpha}(-k+p_2+q_2)_{\beta}\ . \tag{2.4}$$

The production amplitude for N quanta of the field A is determined by means of variational derivatives with respect to the field A^{ext}. If we require the components of the produced mesons to obey the following conditions in the c.m.s.

$$\frac{1}{\sqrt{s}} \sum_{i=1}^{N} k_{0i} \ll 1, \quad |\sum_{i=1}^{N} \vec{k}_{i\perp}| \ll |\vec{p}_{\ell\perp} - \vec{q}_{\ell\perp}|, \quad = 1,2, \tag{2.5}$$

then the *N* meson production amplitude is factorized

$$f_{inel}(N) = f(q_1, q_2; p_1, p_2) \times \prod_{i=1}^{N} g E_\alpha^*(k_i) | j_\alpha^{(1)}(k_i, p_1, q_1) + j_\alpha^{(2)}(k_i, p_2, q_2)|,$$

$$j_\alpha^{(\ell)}(k; p_\ell, q_\ell) = \left(\frac{2p_{\ell\alpha} + k_\alpha}{2p_\ell k - \mu^2} - \frac{2q_{\ell\alpha} + k_\alpha}{2q_\ell k + \mu^2 u} \right), \quad (\ell = 1,2). \tag{2.6}$$

This fact may be considered as the first important result for the models studied.

In the straight-line path approximation the differential cross section for meson production in the collission of two nucleons is also factorized.

$$(d\sigma)_{n_1, n_2} \to \frac{1}{2s} \frac{d^4\Delta}{(2\pi)^4} |f_{el}|^2 W_{n_1}(p_1, \Delta) W_{n_2}(p_2, -\Delta),$$
$$s \to \infty, \quad t = \Delta^2 \text{ - fixed,} \tag{2.7}$$

where

$$W_{n_1}(p_1, \Delta) = \frac{2\pi}{n!} \int \frac{d\vec{q}_1}{q_{10}} \delta(p_1 - q_1 - \sum_{i=1}^{n_1} k_i + \Delta) \times$$
$$\times \prod_{i=1}^{n_1} \frac{d\vec{k}_i}{k_{0i}} \frac{(-g^2)}{(2\pi)^3} |j_\alpha(p_1, q_1, k_i, k_i^2 = \mu^2)|^2. \tag{2.8}$$

The differential cross section for production of secondary particles the momentum components of which satisfy the conditions (5) in the straight-line path approximation has the form[12]

$$\left(\frac{d\sigma}{dt}\right)_{n_1,n_2} \to \left|\frac{d\sigma}{dt}\right|_0^2 \tilde{W}_{n_1}(s,t)\tilde{W}_{n_2}(s,t), \tag{2.9}$$

where $\tilde{W}_n$ are functions of the Poisson type

$$\tilde{W}_n(s,t)=\frac{1}{n!}e^{At}\,[\bar{n}(s,t)]^n. \tag{2.10}$$

Note that the results (9) and (10) for the differential cross sections are found under the assumption on fixed momentum transfers. For observable values σ_{n_1,n_2}, with momentum transfers, t, given in definite interval, this result leads naturally to a multiplicity distribution in the form of the superposition of Poisson functions:

$$\sigma_n \to \int_{\Delta r} P_n(\bar{n},r)dr \tag{2.11}$$

It should be noticed that such superpositions appeared from so called geometrical models where superposition is considered in terms of the impact parameters (see Bialas, Koba, Minakata, Lam et al.[13]).

The average multiplicity (in eq. 10) is determined as

$$\bar{n}(s,t) = +\frac{g^2}{(2\pi)^3} \int \frac{d\vec{k}}{2k_0} |j^{(\ell)}(k, p_\ell, q_\ell)|^2 , \qquad (2.12)$$

and e^{At} is the radiative correction factor.

For $t \ll m^2$, i.e., of the differaction region

$$\bar{n}(s,t) = -Bt, \qquad (2.13)$$

a linear dependence of the average multiplicity upon t is observed. In this case, in a certain domain of secondary particle momenta

$$B = A \qquad (2.14)$$

The total differential cross section obtained by summing over the number of all emitted mesons is found to be independent of t

$$\frac{d\sigma^{tot}}{dt} = \left(\frac{d\sigma^{el}}{dt}\right)_0 = \text{const}. \qquad (2.15)$$

This is, in a certain sense, analogous to the point-like or automodel behaviour of the cross sections for deeply inelastic hadron-lepton processes[14].

These consequences of the model are in qualitative analogy with the predictions of the coherent state model[15], which is a realization of the concept that hadrons are complicated systems with many internal degrees of freedom.

B. On the relationship of multiplicity with the slope of the diffraction peak and the total cross section

We return to formula (14), which may be viewed as a consequence of the straight-line path approximation and the coherent state model. It is obvious that this relation is meaningful only for momentum transfers restricted by the domain of the diffraction peak.

The real content of the result (14) consists in the fact that the total differential cross section can change noticeably only by changing $\Delta t \sim t_{eff}$, which greatly exceeds the sizes of the diffraction domain.

To estimate t_{eff} , we may make use of the unitarity condition which yields

$$-t_{eff} \lesssim \frac{const}{\sigma^{tot}} . \qquad (2.16)$$

This value of t_{eff} can be employed for estimating the average number of secondary particles $n_{diffr.}$ produced in the diffraction collisions of hadrons at high energies

$$\bar{n}_{diffr.}(s) = \frac{1}{\sigma^{tot}} \int_0^{t_{eff}} \frac{d\sigma^{tot}}{dt} A(s)\, t\, dt \lesssim \frac{const\, A(s)}{\sigma^{tot}} . \qquad (2.17)$$

Thus, the diffraction or peripheral part of the average multiplicity is defined by the parameters of the elastic zero-angle scattering amplitude. The conclusion about the behaviour of the total particle number $\bar{n}(s)$ can be drawn only under definite assumptions about the contribution of small distances to high-energy multiple production processes. In particular, if the assumption about the disappearance of "pionization" effects at high energies, i.e., the production of secondaries with limited momenta in the c.m.s. of the colliding hadrons, is used, then relation (16) will define the behaviour of the total average multiplicity [16]

$$\bar{n}(s) = \frac{\text{const}\, A(s)}{\sigma^{tot}} + \tilde{\nu}, \qquad (2.18)$$

where $\tilde{\nu}$ is the number of "leading" particles.

From the viewpoint of attempts to connect the regularities observable in multiple productions with the parameters of elastic scattering, this result can be treated as a contribution to the magnitude of the slope of the elastic scattering amplitude (this contribution is due to the diffraction mechanism). It is known that within the model of uncorrelated streams very small values are obtained for the elastic slope, and a mechanism of the multiperipheral type gives very fast narrowing of the slope with increasing energy [17]. In this respect it would be rather interesting to estimate the value of A(s) within the models allowing for the two mechnisms.

Using the well-known restriction on the asymptotic behaviour of the diffraction peak width in quantum field theory [18] from Eqs. (17), we get in the general case

$$\bar{n}(s) \le \frac{\text{const}}{\sigma^{tot}} \ln^2 s\,. \qquad (2.19)$$

This relation is an interesting interpretation of the increase of the strong interaction radius.

Indeed, A(s) is the "visible" hadron size, σ_{tot} defines the minimal distance R_0 for which the automodel behaviour holds.

One can see from Eqn. (17) that

$$A(s) \sim R^2 = \bar{n} R_0^2 \ . \qquad (2.20)$$

Thus, the strong interaction radius increases under the condition of the constant cross section, at the expense of the "swelling out" of hadrons associated with the "clouds" of secondary particles.

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III. Phenomenological model of "two mechanisms" (TMP-model) and the description of experimental data on the π^-N-interaction at E = 40 GeV

At the present time a unique possibility for verifying various assumptions and models of multiparticle production has emerged in connection with putting into operation the latest generation of accelerators. Recent experiments performed at Serpukhov[1] with a two-meter propane chamber irradiated with 40 GeV π^--mesons have shown that the multiplicity distribution of charged particles is of rather crucial importance. If up to an energy of 25 GeV satisfactory agreement with experiment has been obtained by the use of the Poisson formula, the Wang models (I and II), the Chew-Pignotti model, etc.,[2] then at E=40 GeV reasonable agreement[1] is provided only by the Wang model I, the Czyzewski-Rybicki model and also by the model of "two mechanisms" proposed at Dubna (Grishin, Jancso, Kuleshov, Matveev, Sissakian[3]).

The same experiments on π^-p- and π^-n-collisions at the negative pion momentum p=40 GeV/c have manifested undoubtedly a linear dependence of the average number of neutral pions on the number of charged tracks. Experiments have been made on ISR at CERN and also in NAL at Batavia, justifying this regularity with the example of pp-collisions at energies of 1500 and 200 GeV[4,5]. We note here that at lower energies the effect of correlation between neutral and charged pions is considerably weaker.

Here we will present a joint quantitative decsription of the charge distributions and correlation of the neutral pions and charged tracks in the $\pi^- p$- and $\pi^- n$-interactions at p = = 40 GeV/c by using the model of two mechanisms.

The main point of this model is an assumption that there exist two mechanisms for secondary particle production. These are as follows:

i) There exist the leading particles which can dissociate with local conservation of isospin;

ii) In the interaction process hadron clusters are also produced in a statistically independent way, and then decay into pions.

It is quite natural to suppose that the mean number of these hadron clusters does not depend on the type of colliding particles at high energies.

A comparison of the model with experiment indicates that in order to describe the charge distributions and correlations between neutral and charged particles in the $\pi^- p$- and $\pi^- n$-interactions at p=40 GeV/c, it is sufficient to consider, only the simplest channels of dissociation of the colliding particles and the hadron clusters with isospin I =0. Thus, we consider dissociation of the leading nucleon in the following scheme[x/]:

i) $N \to N$ with probability α_1

ii) $N \to N\pi^0$ -"- α_2

iii) $N \to N'\pi^\pm$ -"- α_3

Here $\sum_{i=1}^{3} \alpha_i = 1$, and, in addition, $\alpha_3 = 2\alpha_2$ by the assumption

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x/ For the sake of simplicity we do not consider strange particle production.

on local isospin conservation.

As another source for secondary particle production let us introduce the σ- and ω- associations which are produced by the Poisson law, with isospin I = 0 and parity G = $\pm$ 1. We confine ourselves to the main schemes for the decay of the σ - and ω-clusters:

i) $\sigma \to \pi^+\pi^-$, $\pi^\circ\pi^\circ$,

ii) $\omega \to \pi^+\pi^-\pi^\circ$.

Keeping in mind the model assumptions one can easily see that the production probability for a number of pairs $n_\pm$, n_o and that of the triplets of pions - n_3, at the given channel "i" of a dissociation of the leading nucleon, is defined as follows:

$$W^{i}_{n_\pm, n_0, n_3} = a_i P_{n_\pm}(a_\pm) P_{n_0}(a_0) P_{n_3}(b), \tag{3.1}$$

where $P_n(\bar{n})$ is the Poisson factor, and $a_\pm$, a_o, b are the average numbers of pion pairs $\pi^+\pi^-$, $\pi^o\pi^o$ and of pion triplets $\pi^+\pi^-\pi^o$ correspondingly. From the condition that the pairs $\pi^+\pi^-$ and $\pi^o\pi^o$ are produced with isospin I=0, $a_\pm = 2a_o \equiv a$ follows.

It is evident that the number of charged particles n_c and neutral pions n_{π^o} can be represented in the following manner:

$$n^i_c = 2n_\pm + 2n_3 + \ell^i_c , \tag{3.2}$$

$$n^i_{\pi^o} = 2n_0 + n_3 + \ell^i_{\pi^o} . \tag{3.3}$$

where l_c^i and $l_{\pi^0}^i$ are the numbers of charged particles and π^0-mesons among the dissociation products of the leading particles in the i-th dissociation channel (see Table 1.).

From the formula (1) for distributions over the number of charged particles it follows:

For the $\pi^- p$-interactions

$$W_{n_c}(\pi^- p) = P_{\frac{n_c-2}{2}}(a'). \qquad (3.4)$$

For the $\pi^- n$-interactions

$$W_{n_c}(\pi^- n) = (1-2a_2)P_{\frac{n_c-1}{2}}(a') + 2a_2 P_{\frac{n_c-3}{2}}(a'), \qquad (3.5)$$

where a' = a+b has the meaning of the average number of pairs $\pi^+\pi^-$ including a contribution from similar combinations among the pion triplets $\pi^+\pi^-\pi^0$.

A comparison of the formulae (4) and (5) with experimental data at p = 40 GeV/c shows good agreement (see Fig. 1 and Table II). We have also found the value of the average number of the $\pi^+\pi^-$ combinations (a' = 1.81 ± 0.02) and the nucleon charge-exchange coefficient ($2a_2$ = 0.36 ± 0.04). Unlike the Wong-I model[6], if one uses a distribution of the type - superposition of the Poisson factors with the same number of parameters - a good joint description for the $\pi^- p$ and $\pi^- n$-collisions with the same average value of the $\pi^+\pi^-$ combinations a' may be given.

As the second step of using the distribution (1), it is not difficult to obtain a formula describing the average number of neutral pions at fixed value of the charged track number n_c.

From (3) it follows that

$$<n_{\pi^o}>_{n_c} = \frac{2<n_0>_{n_c} + <n_3>_{n_c} + <\ell_{\pi^o}>_{n_c}}{W_{n_c}} . \qquad (3.6)$$

After some calculations we find from (6) the following expression for the π^-p-interaction:

$$<n_{\pi^o}>_{n_c} = k_1 + k_2(n_c - \bar{n}_c), \qquad (3.7)$$

where $k_1 = a' + a_2$, $k_2 = \frac{b}{2a'}$, and $n_c = 2a' + 2$ is the average number of charged particles in the π^-p-interactions.

For the case of the π^-n-interaction such a dependence is somewhat complicated:[3] however, under the condition that the parameter a_2 is small ($a_2 = 0.18$), from (1) and (6) the linear dependence of the average number of neutral pions $<n_{\pi^o}>_{n_c}$ on the number of the charged tracks n_c also follows. In this case the correlation is defined by (7) with the average number of charged particles in the π^-n-interactions: $\bar{n}_c = 2a' + 2a_2 + 1$.

To compare the formula (7) with experiment an additional parameter Δ has been inserted into (7), which means some correction due to possible charge-exchange process in the π^-p-interactions ($\pi^-p \rightarrow \pi^o n + \ldots$), and also due to other possible dissociation channels of the leading particles[3]: $k_1 \rightarrow k_1 + \Delta_{\pi^o N}$.

At the present time with large statistics (6000 γ-quanta) the data on the dependenc of the average number of π^o-mesons on a number of charged particles has been achieved for the π^-n-interactions at $p = 40$ GeV/c[1]. The results of comparison of the proposed model with these data are shown in Fig. 2.

The parameters are as follows: $b = 0.56 \pm 0.06$, $\Delta_{\pi^- p} = 0.42 \pm 0.05$: and the parameters a and a_2 were taken from the charge distribution fit. Good agreement with experiment ($\chi^2 \equiv 0.5$ on one degree of freedom) confirms the prediction of the model about the linear form of correlation:

$$\langle n_{\pi^0} \rangle_{n_c} = A + B n_c \quad . \tag{3.8}$$

It is worth noting that the slope B coincides with the errors for the $\pi^- p$- and $\pi^- n$-interactions ($B_{\pi^- p} = 0.16 \pm 0.02$; $B_{\pi^- n} = 0.15 \pm 0.02$). This is one of the conclusions of the model. Indeed, the slope is related to a ratio of the average number of hadron dissociations

$$B = \frac{1}{2} \frac{\bar{N}(\omega \to \pi^+ \pi^- \pi^0)}{\bar{N}(\sigma \to \pi^+ \pi^-) + \bar{N}(\omega \to \pi^+ \pi^- \pi^0)} \tag{3.9}$$

which were assumed to be independent of a type of colliding particles.

As it is known, the experimental data obtained at ISR in the pp-collisions also demonstrate a dependence of the type (8), but with the slope $B \approx \frac{1}{2}$ and A is small. Within the framework of the suggested simple model an absence of correlations between $\langle n_{\pi^0} \rangle_{n_c}$ and n_c at $E \approx 20$ GeV and the limiting dependence $\langle n_{\pi^0} \rangle_{n_c} = 1/2 \, n_c$ at $E \approx 2000$ GeV can be naturally explained, if $\bar{N}(\omega) \ll \bar{N}(\sigma)$ at low energies, i.e. $B \approx 0$, is assumed; and with increasing energy a fraction of the σ-associations falls and we have $\bar{N}(\omega) \gg \bar{N}(\sigma)$, i.e. $B = 1/2$.

The scheme presented above can easily be extended to the case of multiple production involving strange particles/7/.

The model gives a distribution over the number of charged particles in the form of the superposition of Poisson functions; and also predicts correlations between K^+ and K^-, as well as between K^o and $\bar{K}^o$. The average number of K^o, Λ^o and Σ^o in the region under consideration does not depend upon the number of charged particles in π^-p-collisions and reaches its constant value at quite a large number of charged particles in π^-n-collisions.

The processing of experimental results from the two-meter propane chamber has produced good agreement of the model with experiment (see fig. 3).

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IV. Conclusion

If can be seen from the above consideration that the idea of joining of two opposite viewpoints on the production mechanism of secondaries (which are as follows: a) independent emission and b) dissociation (or fragmentation) of leading particles) may turn out to be rather fruitful. Such "synthetic" approach is quite attractive just because of its simplicity. The assumption on uncorrelated production of associations (or clusters) allows one to combine the merits of the indepdent-emission models with a possibility to study the correlation dependences within this simple scheme. Taking into account both the secondaries which do not sense the interacting objects and those which are fragments of these objects, - is at present the most suitable method for describing the processes with high multiplicity.

Apart from the approach discussed above still more models are available which use the idea on joining of two mechanisms of secondary production. As the most known among them are the Koba-Nielsen-Olesen model and the two-component model by Fialkowski.

Theoretical and experimental investigations of the possibility of separating the contributions from different mechanism are of considerable interest. Also it is desirable to check experimentally the "clusterization effect".

I think that in the nearest future these questions will be good answered and this will enrich our knowledge about high-energy multi-particle production processes.

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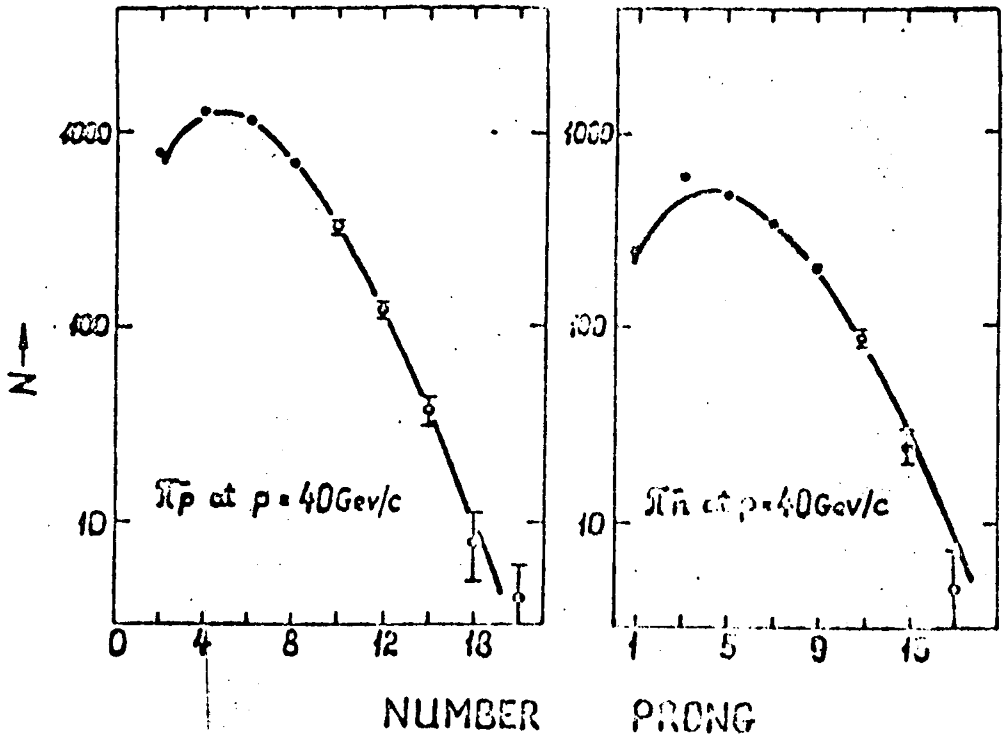
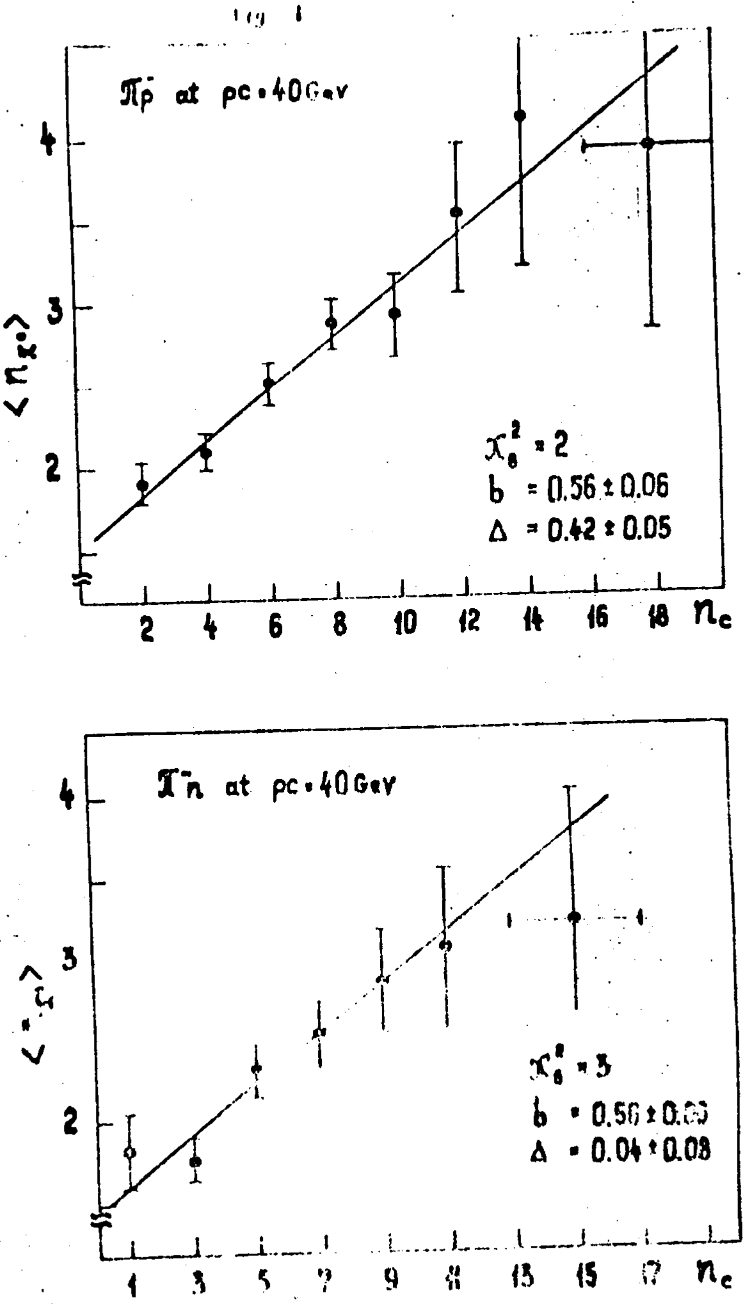


N
$\pi^- p$ at p = 40 Gev/c
$\pi^- n$ at p = 40 Gev/c
NUMBER PRONG

Fig. 1



$\pi^- p$ at pc = 40 Gev
$\langle n_{\pi^0} \rangle$
$\chi^2_8 = 2$
b = 0.56 ± 0.06
Δ = 0.42 ± 0.05
n_c
$\pi^- n$ at pc = 40 Gev
n_c

Fig. 2

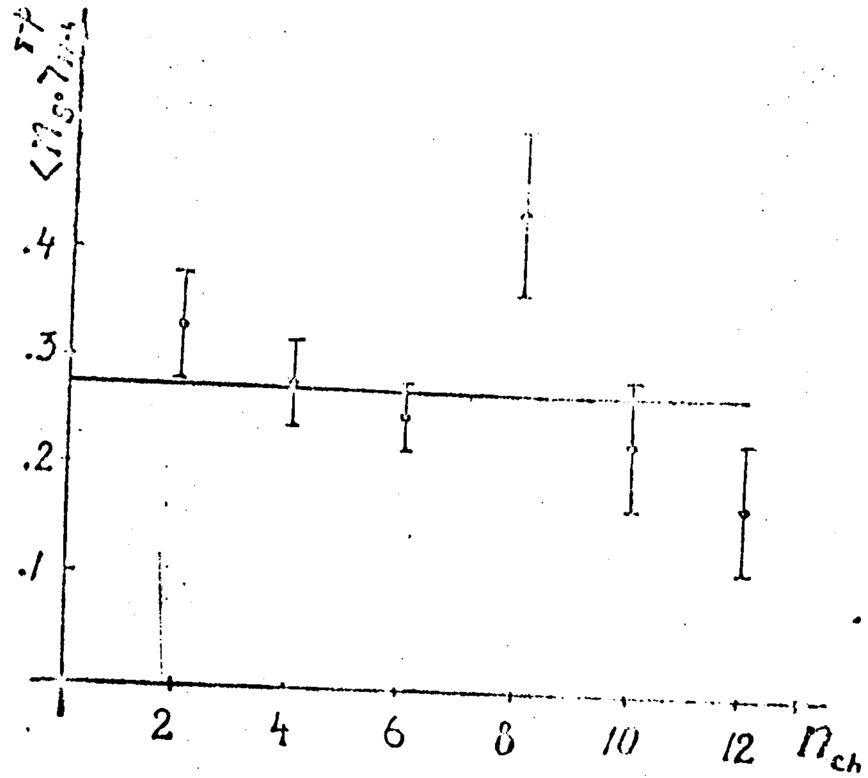


Fig. 3

Table I

	i = 1		i = 2		i = 3	
	$\pi^- p$	$\pi^- n$	$\pi^- p$	$\pi^- n$	$\pi^- p$	$\pi^- n$
ℓ_c	2	1	2	1	2	3
ℓ_{π^0}	0	0	1	1	0	0

Table 2

Type of interaction Тип взаимодействия	Number of events Число событий	$\bar{n}$	$\sqrt{D}$	χ^2 fit by Wang-I model модель	χ^2 fit by suggested model предлагаемая модель	Degrees of freedom Степени свободы
$\pi^- p$	4400	5.62±0.4	2.75	8	8	8
$\pi^- n$	I860	5.32±0.7	2.82	I3	8.5	7