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Дубна

E2 - 4983

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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THE ACCOUNT OF RADIATION CORRECTIONS FOR THE EIKONAL SCATTERING AMPLITUDE IN QUANTUM FIELD THEORY MODEL

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THE ACCOUNT OF RADIATION CORRECTIONS FOR THE EIKONAL SCATTERING AMPLITUDE IN QUANTUM FIELD THEORY MODEL

I. In the authors' paper /1/ a method has been proposed for the derivation of a closed relativistically invariant and cross-symmetrical expression for the scattering amplitude of two high-energy particles in the scalar model $\mathcal{L}_{int} = g : \psi^2(x)\,\phi(x) :$ in the framework of the functional integration method. In the limit of asymptotically high energies $s \to \infty$ and fixed momentum transfers t this expression has the form of the Glauber representation/1/ with the eikonal function, which corresponds to the Yukawa potential for particle interaction. Besides, for simplicity, the diagrams with the radiation corrections to the "nucleon" (field ψ) lines and also the closed loops of the field $\psi(x)$ were neglected.

In the present note we give the estimation of the radiation corrections in the approximation in which the terms $k_i k_j$ $(i \neq j)$ are neglected in the "nucleon" propagators. This approximation for the region $\frac{t}{s} \to 0$ is discussed in detail in papers/1-4/.

II. The scattering amplitude of two scalar "nucleon" can be represented in the following form/1/:

$$f(q_1 q_2 | p_1 p_2) = \frac{(ig)^2}{(2\pi)^4} \int dx\, D(x)\, e^{-ix(p_1 - q_1)} \int_0^1 d\lambda\, S_\lambda (q_1 q_2 | p_1 p_2) + (p_1 \leftrightarrow p_2), \qquad (1)$$

where

$$S_\lambda(q_1q_2|p_1p_2) = C_\nu \int\delta\nu_1 \int\delta\nu_2 \exp\{-i\int\nu_1^2(\eta)d\eta - i\int\nu_2^2(\eta)d\eta\}\times$$

$$\times \exp\{ig^2\lambda\int d\xi_1\int d\xi_2\, D[-x+2\xi_1(p_1\theta(\xi_1)+q_2\theta(-\xi_1)) -$$

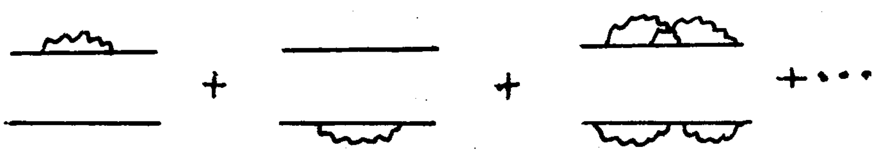
$$- 2\xi_2(p_2\theta(\xi_2)-q_2\theta(-\xi_2)) - 2\int_{-\xi_1}^{0}\nu_1(\eta)d\eta + 2\int_{-\xi_2}^{0}\nu_2(\eta)d\eta]\}\times$$

$$\times\exp\{\frac{ig^2}{2}\int d\xi_1\int d\xi_2\, D[-2p_1(\xi_1-\xi_2)+2(p_1-q_1)\int_{\xi_2}^{\xi_1}\theta(\eta)d\eta + \qquad (2)$$

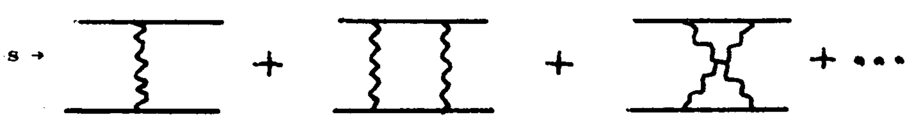
$$+ 2\int_{\xi_2}^{\xi_1}\nu_2(\eta)d\eta]\}\exp\{\frac{ig^2}{2}\int d\xi_1\int d\xi_2\, D[-2p_2(\xi_2-\xi_1)-2(p_1-q_1)\int_{\xi_2}^{\xi_1}\theta(\eta)d\eta +$$

$$+2\int_{\xi_2}^{\xi_1}\nu_1(\eta)d\eta]\} - i\delta m^2(A\to\infty)\}$$? (в статье ТМФ 3,3)

Considering the perturbation series it may be established that the two x -independent D -functions in eq. (2) do lead to radiation corrections to each of the two "nucleon" lines and do not lead to a "meson" (field ϕ) exchange between them



The first D -function, the argument of which includes the variable x , is responsible for the interaction between "nucleons" and in perturbation theory gives rise to graphs of the type



From the consideration of the integrals over ξ_1 and ξ_2 it is seen that the D -functions, the argument of which does not include the variable x, resuli in divergent expressions of the type $\delta m^2 x(A\to\infty)$. Therefore to eliminate them the renormalization of the "nucleon" mass is made. The remaining after that expression is already finite since in our approximation in the propagators we do not neglect k^2-terms.

For the calculation of the functional integrals we use an approximate method/5,1/ which corresponds to the neglect of the $k_i k_j (i \neq j)$ terms in the "nucleon" propagators and which permits to keep the k^2-terms.

Thus in the so-called $k_i k_j = 0 (i \neq j)$ approximation we obtain for the scattering amplitude the following expression:

$$T(q_1 q_2 | p_1 p_2) = \frac{(ig)^2}{(2\pi)^4} h(t) \int d^4x \, D(x) \, e^{-ix(p_1 - q_1)} \int_0^1 d\lambda$$

$$\exp\Big\{ -\frac{ig^2\lambda}{(2\pi)^4} \int \frac{d^4k \, e^{-ikx}}{k^2 - \mu^2 + i\epsilon} \Big[\frac{1}{(k^2 - 2kq_1)(k^2 - 2kp_2)} +$$

$$+ \frac{1}{(k^2 + 2kp_1)(k^2 + 2kq_2)} + \frac{1}{(k^2 + 2kp_1)(k^2 - 2kp_2)} + \frac{1}{(k^2 - 2kq_1)(k^2 + 2kq_2)} \Big] \Big\} + \tag{3}$$

$$+ (p_1 \leftrightarrow p_2),$$

where

$$h(t) = \exp\Big\{ \frac{ig^2}{(2\pi)^4 \, 2 \, (?)} \int \frac{d^4k}{k^2 - \mu^2 + i\epsilon} \Big[\frac{1}{(k^2 + 2kq_1)^2} + \frac{1}{(k^2 + 2kq_2)^2} +$$

$$+ \frac{1}{(k^2 + 2kp_1)^2} + \frac{1}{(k^2 + 2kp_2)^2} - \frac{2\,(?)}{(k^2 + 2kp_1)(k^2 + 2kq_1)} - \tag{4}$$

$$- \frac{2\,(?)}{(k^2 + 2kp_2)(k^2 + 2kq_2)} \Big] \Big\}.$$

$$\overline{f^{(1)}(kp_1q_1)}^{\,2} + \overline{f^{(2)}(kp_2q_2)}^{\,2}$$

After sufficently simple but bulky calculation of the integrals over k we get

$$h(t)_{t<0} = \exp\Bigg\{ \frac{g^2}{(4)\cancel{2}(2\pi)^2 m^2} \Bigg[\ln \frac{\cancel{2} m^2}{\mu^2} - \frac{8 m^2}{\sqrt{-t(4m^2-t)}} \Bigg(\ln \frac{m\sqrt{4m^2-t}}{\mu^2} \times$$

$$\times \ln \frac{\sqrt{4m^2-t}+\sqrt{-t}}{\sqrt{4m^2-t}-\sqrt{-t}} + \int\limits_{\frac{\sqrt{4m^2-t}-\sqrt{-t}}{2\sqrt{4m^2-t}}}^{\frac{\sqrt{4m^2-t}+\sqrt{-t}}{2\sqrt{4m^2-t}}} \frac{dx}{x} \ln|1-x| \Bigg) \Bigg] \Bigg\} . \tag{5}$$

At present on the basis of our papers /1/ in the limit $\frac{t}{s} \to 0$ we can write the scattering amplitude in the following manner x/

$$f(s,t) = -\frac{1}{(2\pi)^4} h(t)\, s \int d^2 \vec{x}_\perp e^{-i \vec{x}_\perp \vec{T}_\perp} \left(e^{-\frac{ig^2}{\pi s} K_0 (\mu \sqrt{\vec{x}_\perp^2})} - 1 \right), \tag{6}$$

where

$$T = (p_1 - q_1). \tag{7}$$

In the expression (6) the variables are chosen in the center-of-mass system (c.m.s.) and the axis z is directed along the momenta of the incident particles.

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x/ We note, that in this expression only the renormalization of the mass has been made.

As is shown in the article [1], the second term of the scattering amplitude, obtained with the aid of the replacement $p_1 \leftrightarrow p_2$ or $T = (p_1 - q_1) \leftrightarrow U = (p_2 - q_1)$ has in the region $\frac{t}{s} \to 0$ a higher order of smallness with respect to s.

III. Thus, we see, that in the approximation used at $s \to \infty$ and t-fixed the account of radiation corrections leads to the multiplication of the eikonal form of the elastic amplitude by a certain factor, depending only on the momentum transfer t.

Consequently, in the supposition made in the sum of all the ladder and crossladder graphs radiation corrections do not affect the asymptotic in s at high energies.

However, we note that the problem of a correct account of radiation corrections is rather complicated and would be interesting to perform further studies of different aspects of this problem.

Apparently our result corresponds to the correct account of contributions of the so-called "soft mesons", for which the approximation $k_i k_j = 0$ is valid [4].

In conclusion the authors express their sincere gratitude to N.N. Bogolubov, A.V. Efremov, A.N. Tavkhelidze for valuable discussions and critical remarks.

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Received by Publishing Department
on March, 13, 1970.