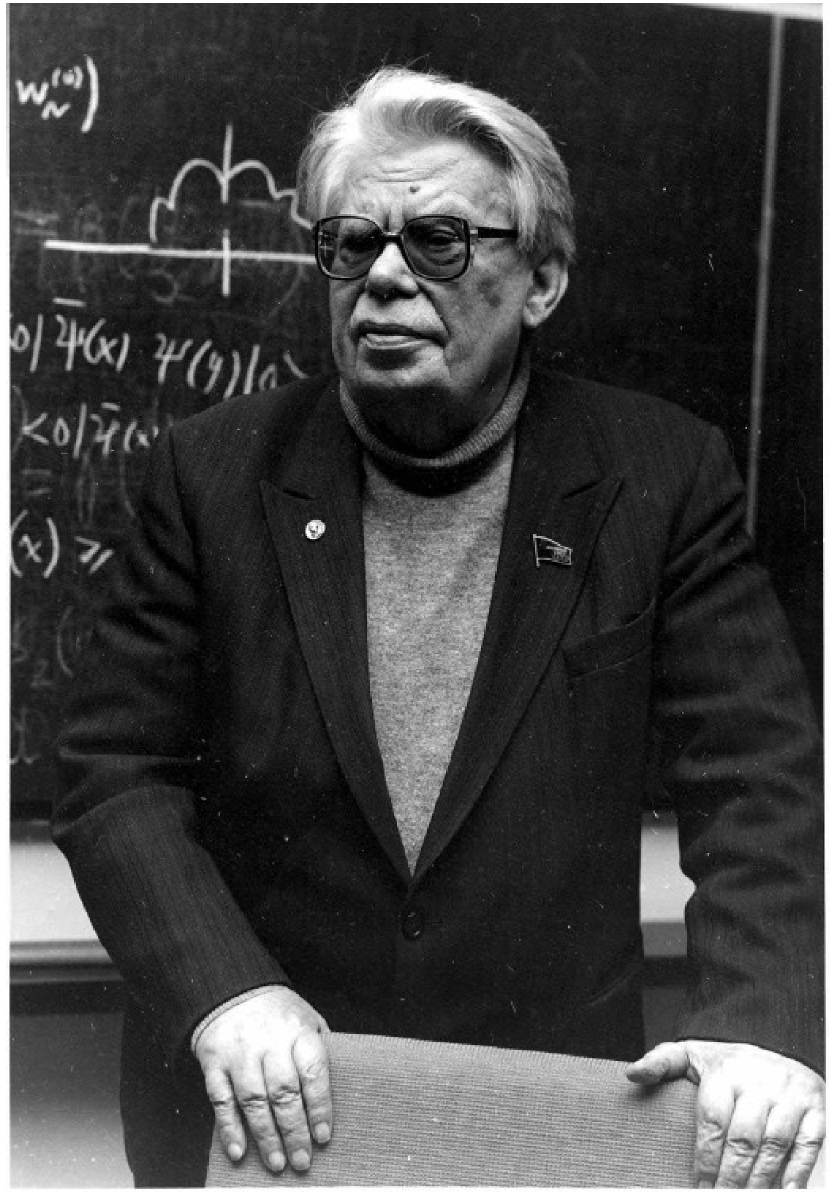
JOINT INSTITUTE FOR NUCLEAR RESEARCH

BLTP Bogoliubov Laboratory of Theoretical Physics

«Сверхтекучесть ядерной материи: от ядер к звездам»

В.В.Воронов

“ОИЯИ в 100-летие открытия ядра”, Дубна, 10-11 марта, 2011



1. Основатель Лаборатории теоретической физики Н.Н. Боголюбов

Several ideas related to the name of N. N. Bogoliubov have made a strong impact on the development of Nuclear theory. There are:

- the idea of superfluidity of nuclear matter;
- u-v transformation;
- time-dependent Hartree-Fock-Bogoliubov method;
- idea of *broken* symmetries of the selfconsistent mean field;
- conception of quasiaverages.

In 1958 (N. N. Bogoliubov, DAN SSSR v. 119, p. 52 (1958)) N. N. Bogoliubov was the first who has indicated on a possibility of superfluidity of nuclear matter.

Then A. Bohr, B. Mottelson and D. Pines (Phys. Rev. v. 110, p. 936 (1958)) formulate a problem of existence of the superfluid state of atomic nuclei. The theory of pair correlations of superfluid type in atomic nuclei has been developed independently by S. T. Belyaev and V. G. Soloviev.

From that time the theory of pair correlations not only explain many nuclear properties which have not been understood before. That was a beginning of the modern stage in the development of the nuclear theory "— microscopic approach to the nuclear structure. Due to simplicity of the u-v Bogoliubov transformation this theoretical technique was used practically by all theoreticians and experimental groups for an interpretation of the *experimental data*.

The interest to pair correlations of nucleons in atomic nuclei was recreated in connection with studies of the properties of nuclei far from stability valley.

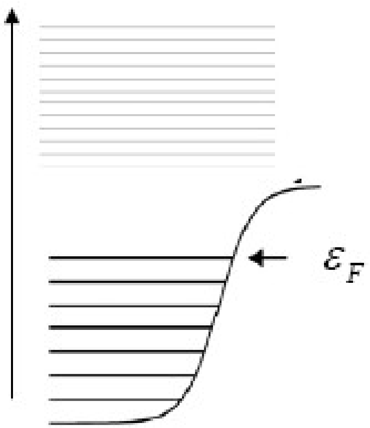
(

- **1953** – *V.G.Soloviev is introduced to N.N.Bogoliubov*
- **1954** – *the first paper in which the radiation correction to the pi-meson lifetime was calculated*
- **1956** – *PhD thesis "Constuction of Approximate Green Functions in the Pseudoscalar Meson Theory"*
- **1958** – *ZhETPh "On Nucleon Interaction Resulting in a Superfluid State of an Atomic Nucleus"*
- **1962** – *the Doctor of Science degree "Superconducting Pairing Correlations in Atomic Nuclei"*
- **1963** – *the establishment of the nuclear theory division in the Laboratory of Theoretical Physics*

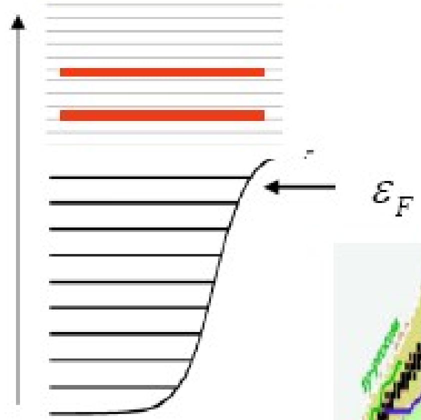

ТЕОРЕТИЧЕСКОЙ
ФИЗИКИ
·1959·

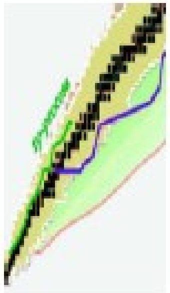
Nuclear Superfluidity

stable nuclei

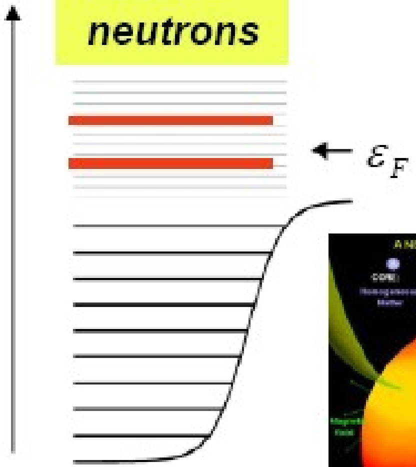


neutron-rich nuclei

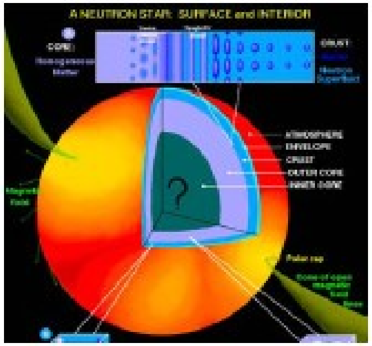




nuclei+ neutrons



neutron matter



neutrons: superfluidity of $^1S_0$ type

- ***Consequences***

  - excitations (energy gap)

  - moment of inertia

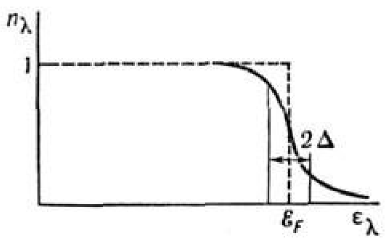
- **Crust**: - neutrons: superfluidity of $^1S_0$ type

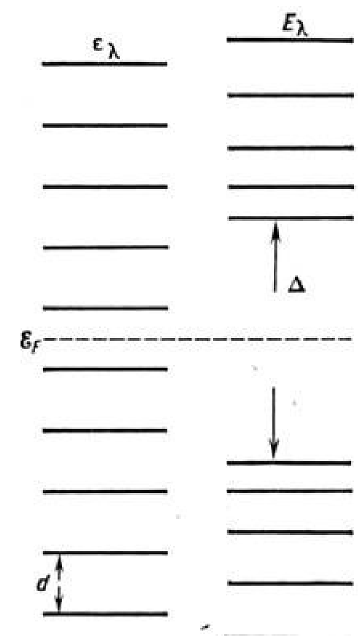
- ***Core*** : - neutrons: superfluidity of ^3PF type
  - protons: superfluidity of $^1S_0$ type

- ***Conequences*** : - geant glitches
  - cooling

$$\mathscr{E}(p) = (p - p_F)p_F/m_e$$

$$E(p) = \sqrt{\mathscr{E}^2(p) + \Delta^2}.$$





$$H = \sum_{\tau}^{(n,p)} \{ \sum_{jm} (E_j - \lambda_\tau) a^\dagger_{jm} a_{jm} - \frac{1}{4} G_\tau : (P^\dagger_0 P_0)^\tau : \}$$

$$P^\dagger_0 = \sum_{jm} (-1)^{j-m} a^\dagger_{jm} a^\dagger_{j-m},$$

$$a^+_{jm} = u_j \alpha^+_{jm} + (-1)^{j-m} v_j \alpha_{j-m},$$

$$H = h_0 + h_{pp}$$

$$h_0 + h_{pp} = \sum_{jm} \varepsilon_j \, \alpha^\dagger_{jm} \, \alpha_{jm},$$

$$\varepsilon_j = \sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}$$

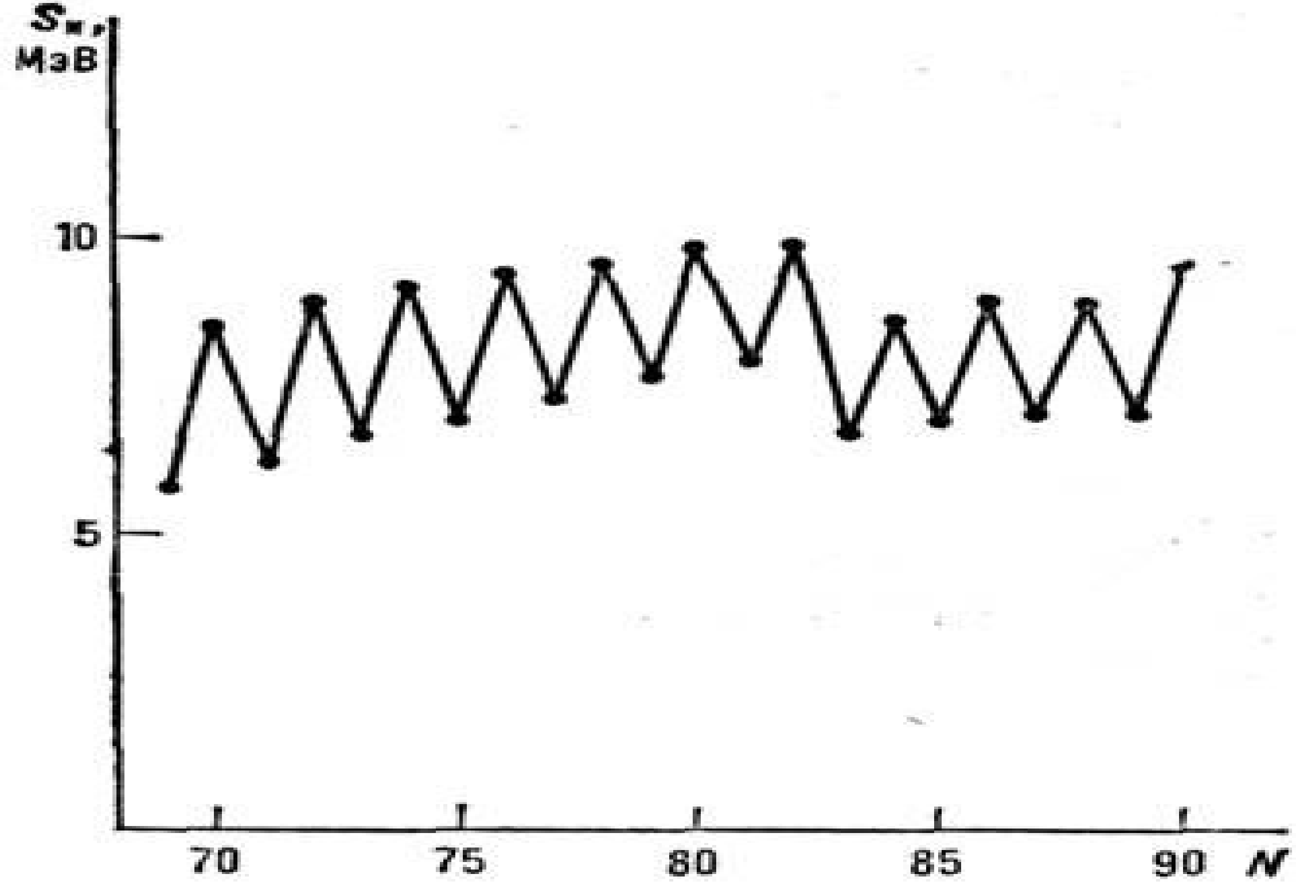
$$\frac{1}{2} G_\tau \sum_j \frac{1}{\sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}} = 1$$

$$N_\tau = \sum_j (j + 1/2) \left(1 - \frac{E_j - \lambda_\tau}{\sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}} \right)$$

$$E_{g.s.} = \sum_{\tau} E_0^{\tau}$$

$$E_0^{\tau} = \sum_{j} (2j+1) E_j v_j^2 - \frac{\Delta_{\tau}^2}{G_{\tau}}$$

$$\bar{\Delta} = \frac{12}{\sqrt{A}}\,\mathrm{MeV}.$$



S_n,
МэВ
10
5
70
75
80
85
90
N

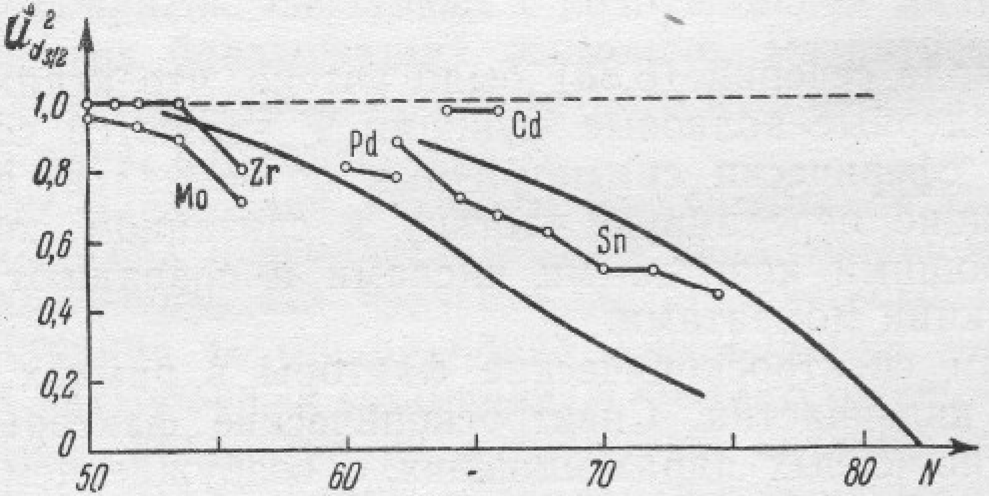


Рис. 6.12. Величины $u^2_{d_{3/2}}$ для подоболочки $d_{3/2}$ в нейтронной системе, полученные из (dp)-реакций в изотопах Zr, Mo, Pd, Cd и Sn, и рассчитанные значения $u^2_{d_{3/2}}$.

По оси абсцисс отложено число нейтронов; величины $u^2_{d_{3/2}}$, полученные из (dp)-реакций, обозначены точками, соединенными прямыми линиями; рассчитанные значения $u^2_{d_{3/2}}$ даны сплошными кривыми, левая — для ядер первой трети заполнения оболочки, правая — для ядер последней трети заполнения оболочки $N = 50-82$. Рисунок составлен на основе данных, приведенных в [214].

The reduced width of α-decay is proportional to

$$\gamma_{rs} \sim \sum_r u_r(Z-2)v_r(Z) \sum_s u_s(N-2)v_s(Z)$$

$$W_{\alpha I} = \sum_l \gamma_{Il}^2 P_l \nearrow 10^3$$

$$g.s. \longrightarrow 2qp\,(E_{2qp} < 2\,MeV)\; W_{\alpha I} \searrow 10^2$$

$$\Delta \sim exp(-\frac{1}{G\bar{\rho}})$$

The properties of nuclei on the way from stable to drip line can change strongly with variation of the particle number. This require a restoration, at least partial, of the symmetries broken by the mean field. In the case of pairing it means that the particle number conservation violated in the standard BCS wave function should be restored using the projection technique

$$\mid \Psi\rangle \equiv P^N \mid \Phi\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\Phi e^{i\Phi(\hat{N}-N)} \mid \Phi\rangle$$

$$E^N(\rho, k) = \frac{\langle\Phi \mid HP^N \mid \Phi\rangle}{\langle\Phi \mid P^N \mid \Phi\rangle} = \frac{\int d\Phi\langle\Phi \mid He^{i\Phi(\hat{N}-N)} \mid \Phi\rangle}{\int d\Phi\langle\Phi \mid e^{i\Phi(\hat{N}-N)} \mid \Phi\rangle},$$

where ρ is the particle-hole density; k is the pairing density.

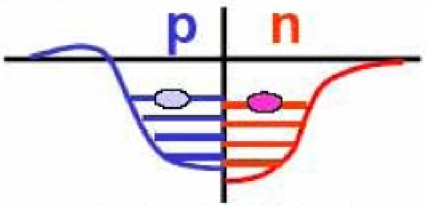
Stable nuclei

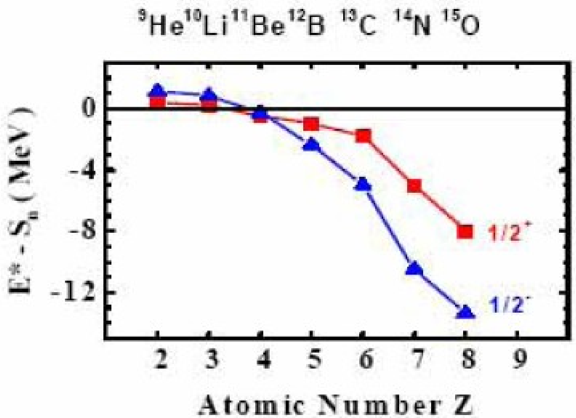
N / Z ~ 1 - 1.5

ε_S ~ 6 - 8 MeV

ρ_0 ~ 0.16 fm $^{-3}$

proton and neutrons homogeneously mixed, *no decoupling* of proton and neutron distributions





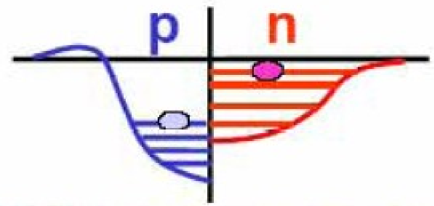
Unstable nuclei

N / Z ~ 0.6 - 4

ε_S ~ 0 - 40 MeV

decoupling of proton and neutron distributions

neutron halos and neutron skins



Prerequisite of the *halo* formation :

low angular momentum motion for halo particles and few-body dynamics

1s - intruder level

^{11}Be parity inversion of g.s.

^{10}Li g.s. : $\left[\pi 0p_{\frac{3}{2}} \otimes \nu 1s_{\frac{1}{2}}\right]_{2^-}$

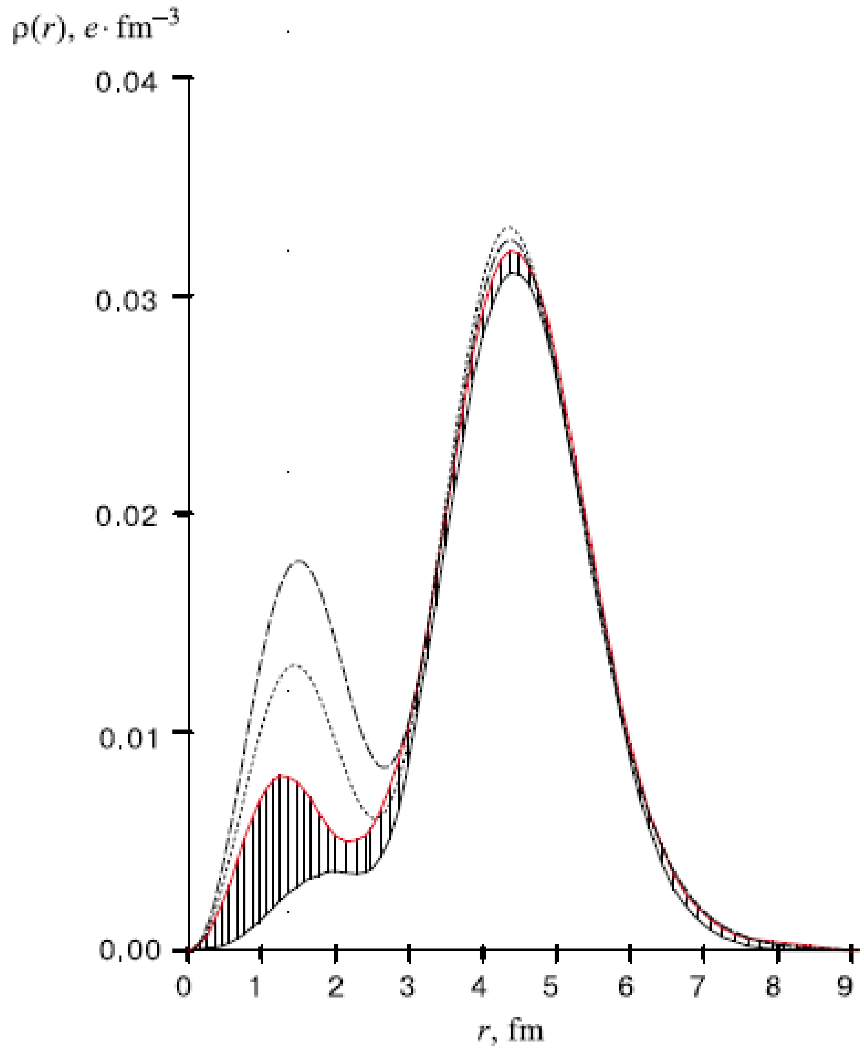
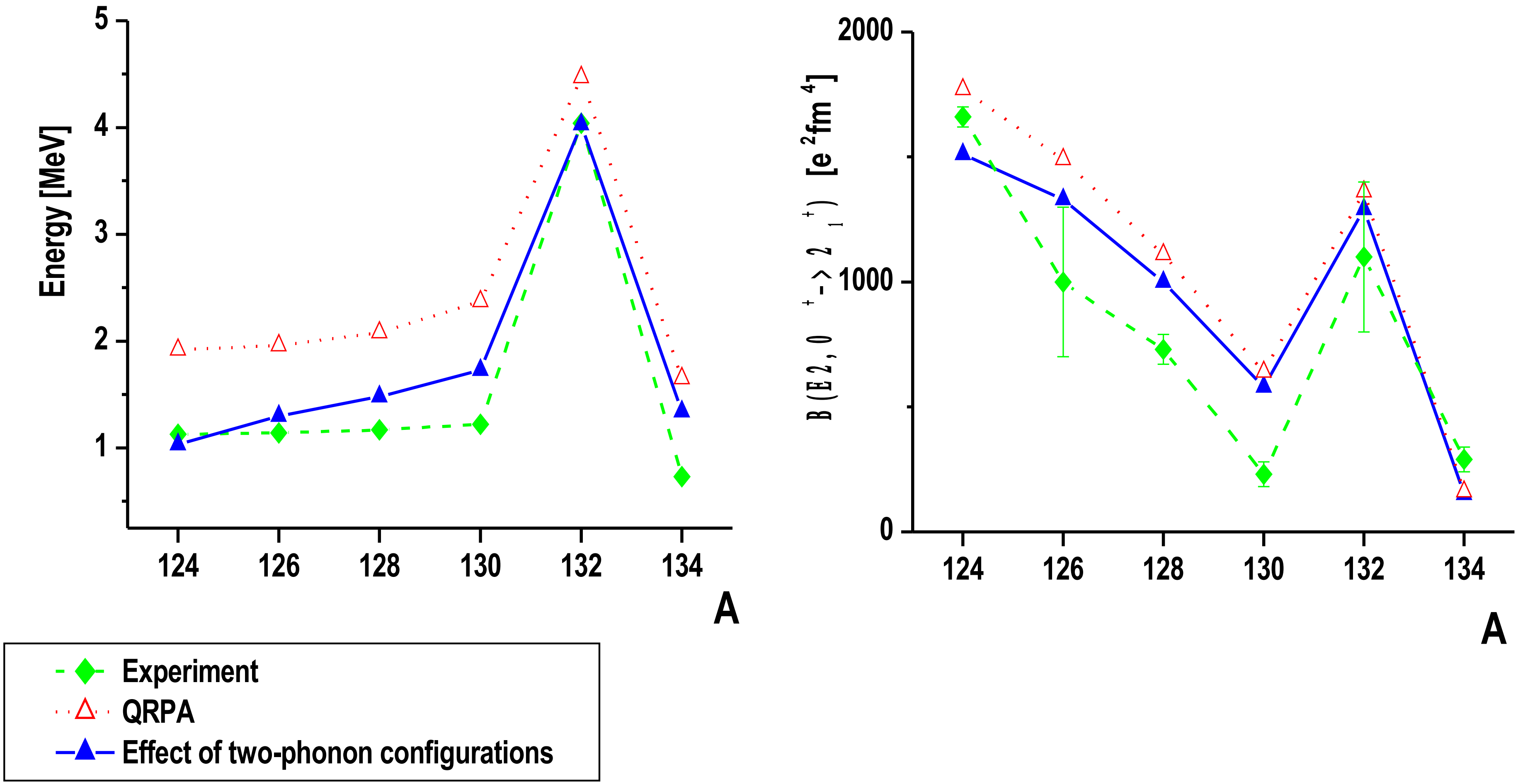


Fig. 7. The transition charge density from the ground to the first 2^+ state in ^{64}Zn. The dashed line corresponds to the RPA result; the dotted line — to ERPA. Experimental data are presented by a shadowed area

2_1^+

Sn isotopes

SLy4



Time-dependent HFB-method in nuclear theory

At present it is difficulty to imagine theoretical nuclear physics without such a notion as the self-consistent mean field. Numerous experimental data point out that the nucleons in the nucleus behave in a certain approximation as independent particles moving in a common potential well. However, this potential well can be time-dependent.

Owing to this fact it was reasonable to construct the nuclear theory, at least the theory of the low-lying excited states of nuclei basing on the concept of the self-consistent field + pairing.

The main equations of the method has been published by Bogoliubov in 1959.

The total Hamiltonian of the system taken in a general form is

$$H = \sum_{f,f'} T(f,f') a_f^+ a_{f'} - \frac{1}{4} \sum_{f_1,f_2,f_1',f_2'} G(f_1 f_2; f_2' f_1') a_{f_1}^+ a_{f_2}^+ a_{f_2'} a_{f_1'}$$

The basic equations have been derived for the following quantities

$$F(f_1, f_2) \equiv \langle a_{f_1}^+ a_{f_2} \rangle$$

and

$$\Phi(f_1, f_2) \equiv \langle a_{f_1} a_{f_2} \rangle$$

where the averaging is performed over the ground state of the system.

One can derived from the equations of motion the following exact equations

$$i\frac{\partial}{\partial t}F(f_1,f_2)=\langle\left[a^+_{f_1}a_{f_2},H\right]\rangle\equiv\mathcal{B}(f_1,f_2)$$
$$i\frac{\partial}{\partial t}\Phi(f_1,f_2)=\langle[a_{f_1}a_{f_2},H]\rangle\equiv\mathcal{U}(f_1,f_2)$$

In the self-consistent field method $\mathcal{B}(f_1,f_2)$ and $\mathcal{U}(f_1,f_2)$ can be expressed in terms of $F(f_1,f_2)$ and $\Phi(f_1,f_2)$.

To investigate the spectrum of the elementary excitations due to small deviations from the ground state we should consider small additions to the stationary solutions F_0 and Φ_0

$$\begin{aligned} F(f,f') &= F_0(f,f')+\delta F(f,f') \\ \Phi(f,f') &= \Phi_0(f,f')+\delta\Phi(f,f') \\ i\frac{\partial}{\partial t}\delta F(f,f') &= \delta\mathcal{B}(f,f') \\ i\frac{\partial}{\partial t}\delta\Phi(f,f') &= \delta\mathcal{U}(f,f') \end{aligned}$$

Finally, the basic equations have been derived far the amplitudes $R^{(\mp)}(f_1,f_2)$ which are linearly related to δF and $\delta\Phi$. They are

$$\begin{aligned} \omega R^{(\mp)}(f_1,f_2) &= \left(\mathcal{E}(f_1)+\mathcal{E}(f_2)\right)R^{(\pm)}(f_1,f_2) \\ &-\sum_{f_1',f_2'}G^{\xi}(f_1f_2;f_2'f_1')v^{(\pm)}_{f_1f_2}v^{(\pm)}_{f_1'f_2'}R^{(\pm)}(f_1',f_2') \\ &-2\sum_{f_1',f_2'}G^{\omega}(f_1f_2;f_2'f_1')u^{(\pm)}_{f_1f_2}u^{(\pm)}_{f_1'f_2'}R^{(\pm)}(f_1',f_2'), \end{aligned}$$

where $u^{(\pm)}_{ff'}$ and $v^{(\pm)}_{ff'}$ are related to the u-v Bogoliubov transformation.

In this general form the equations are used up to now.

1.D.2

Nuclear Physics **A224** (1974) 93–115;

AN ALGEBRAIC TREATMENT OF THE NUCLEAR QUADRUPOLE DEGREE OF FREEDOM

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Received 18 January 1974

Abstract: The collective quadrupole degree of freedom is described by a SU(6) algebra, generated by five generalized coordinates and conjugated momenta and their commutators. On this basis a collective Hamiltonian is derived where the parameters of the realistic nuclear Hamiltonian (single particle energies, matrix elements of the interaction) appear in terms of a few constants. The algebraic properties of the collective variables lead to a new quantum number N which restricts the maximal number of phonons contained in the collective states. The collective Hamiltonian is applied to the transitional nuclei ^{152}Gd and ^{150,152}Sm. The transformation into the intrinsic frame of reference yields explicit formulae for the potential energy, the mass coefficients and the moments of inertia in terms of the intrinsic deformation parameters.

$$Q^{\dagger}_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\{\psi^{\lambda i}_{jj'} A^{\dagger}(jj';\lambda\mu) - (-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'} A(jj';\lambda-\mu)\},$$

$$A^{\dagger}(jj';\lambda\mu) = \sum_{mm'}\langle jmj'm' \mid \lambda\mu\rangle \alpha^{\dagger}_{jm}\alpha^{\dagger}_{j'm'}$$

$$[Q_{\lambda\mu i}, Q^{\dagger}_{\lambda'\mu'i'}] = \delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{ii'} - \sim \alpha^{\dagger}_{jm}\alpha_{j'-m'}$$

$$[[Q_{2\mu}, Q^{\dagger}_{2\mu'}]Q^{\dagger}_{2\nu}] = const\, Q^{\dagger}_{2\nu'}$$

$$Q_{2\mu} \;\; , Q^{\dagger}_{2\mu} \;\; and \;\; [Q_{2\mu}, Q^{\dagger}_{2\mu'}] \;\Rightarrow\; U(6) \;\; algebra$$

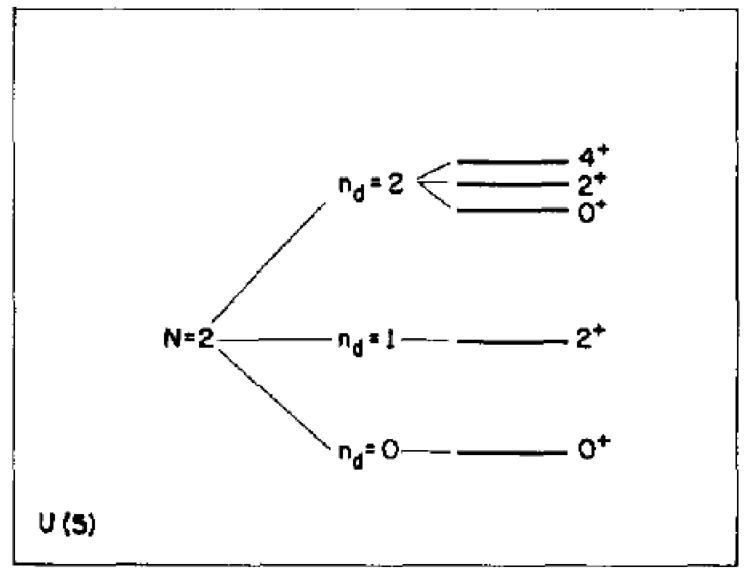
$$Q^{\dagger}_{2\mu} \;\Rightarrow\; d_{\mu}s\,; \;\; Q_{2\mu} \;\Rightarrow s^{+}(-1)^{\mu}d_{-\mu}\,; \;\; [Q_{2\mu}, Q^{\dagger}_{2\mu'}] \;\Rightarrow\; d^{\dagger}_{\mu}d_{\mu'}$$

$$
U(6) \begin{cases} \nearrow U(5) \supset O(5) \supset O(3) \supset O(2)\,, & \text{(I)} \\ \rightarrow SU(3) \supset O(3) \supset O(2)\,, & \text{(II)} \\ \searrow O(6) \supset O(5) \supset O(3) \supset O(2)\,. & \text{(III)} \end{cases}
$$

<u>Group Chain I</u>

The classification for this chain is

$$
\left| \begin{array}{ccccccccc} U(6) & \supset & U(5) & \supset & O(5) & \supset & O(3) & \supset & O(2) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N] & & n_d & & v(n_\Delta) & & L & & M_L \end{array} \right\rangle
$$



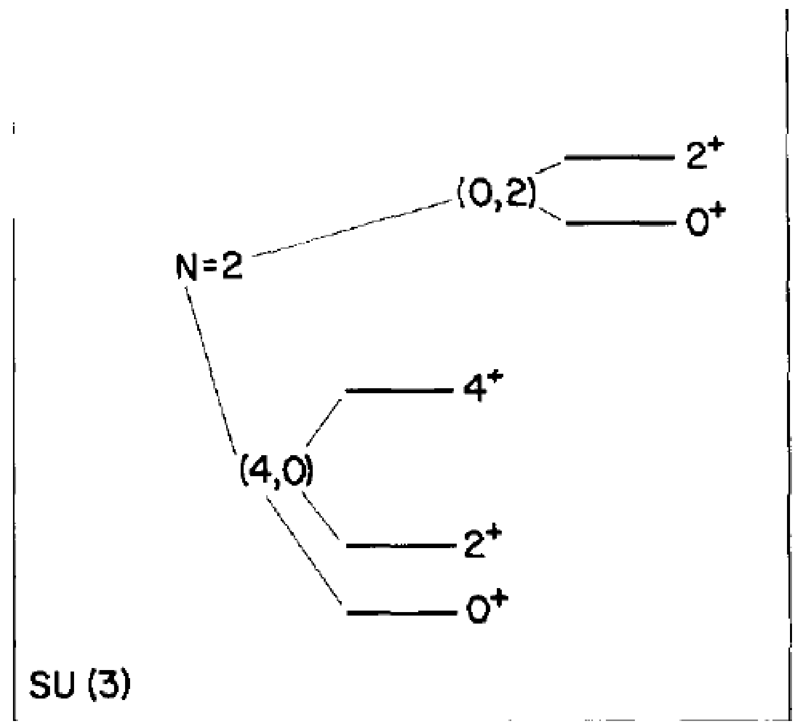
Group Chain II

The classification for this chain is

$$\left| \begin{array}{cccc} U(6) \supset & SU(3) \supset & O(3) \supset & O(2) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ [N] & (\lambda,\mu)(K) & L & M_L \end{array} \right\rangle ,$$

The most general Hamiltonian with this dynamic symmetry can

$$H_B^{(II)} = \alpha' \ \mathcal{C}_2(O3) + \beta' \ \mathcal{C}_2(SU3) ,$$



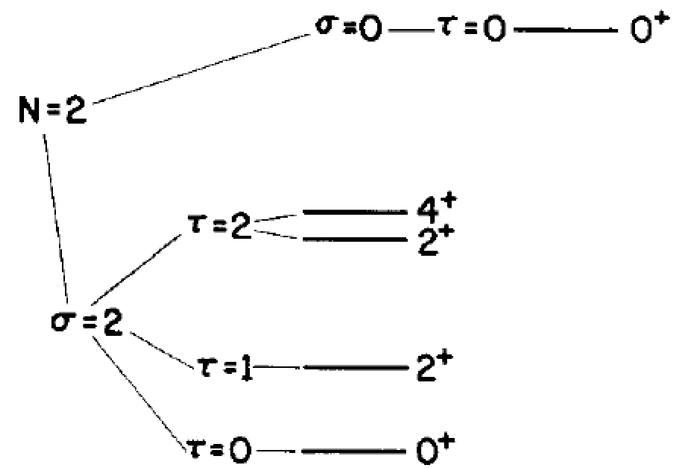
Group Chain III

The classification for this chain is

$$\left| \begin{array}{ccccccccc} U(6) & \supset & O(6) & \supset & O(5) & \supset & O(3) & \supset & O(2) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N] & & \sigma & & \tau(\nu_\Delta) & & L & & M_L \end{array} \right\rangle .$$

The corresponding Hamiltonian is

$$H_B^{(III)} = A\ \mathcal{C}_2(O6) + B\ \mathcal{C}_2(O5) + C\ \mathcal{C}_2(O3) ,$$

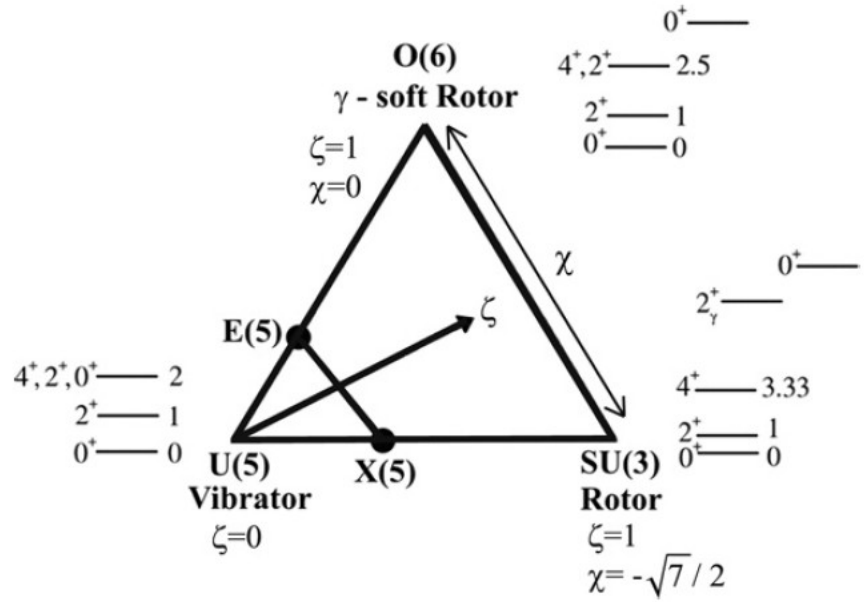


O (6)

$$H = \epsilon n_d - \kappa Q \cdot Q = c\left[(1-\zeta)n_d - \frac{\zeta}{4N_B} Q \cdot Q\right]$$

where n_d is the number operator $(d^\dagger d)^0$, giving the number of d bosons, ϵ is the d boson energy, and the boson quadru operator Q is defined as

$$Q = s^\dagger \tilde{d} + d^\dagger s + \chi [d^\dagger \tilde{d}]^2 \quad \text{and} \quad \zeta = \frac{4N_B}{4N_B + \epsilon/\kappa}.$$



e symmetry triangle of the IBA with the addition of the geometric critical point descriptions $X(5)$ and $E(5)$.

The interacting boson model (IBM) is suitable for describing intermediate and heavy atomic nuclei. Adjusting a small number of parameters, it reproduces the majority of the low-lying states of such nuclei. Figure 0.1 gives a survey of nuclei which have been handled with the model variant IBM2. Figures 10.7 and 14.3 show the nuclei for which IBM1-calculations have been performed.

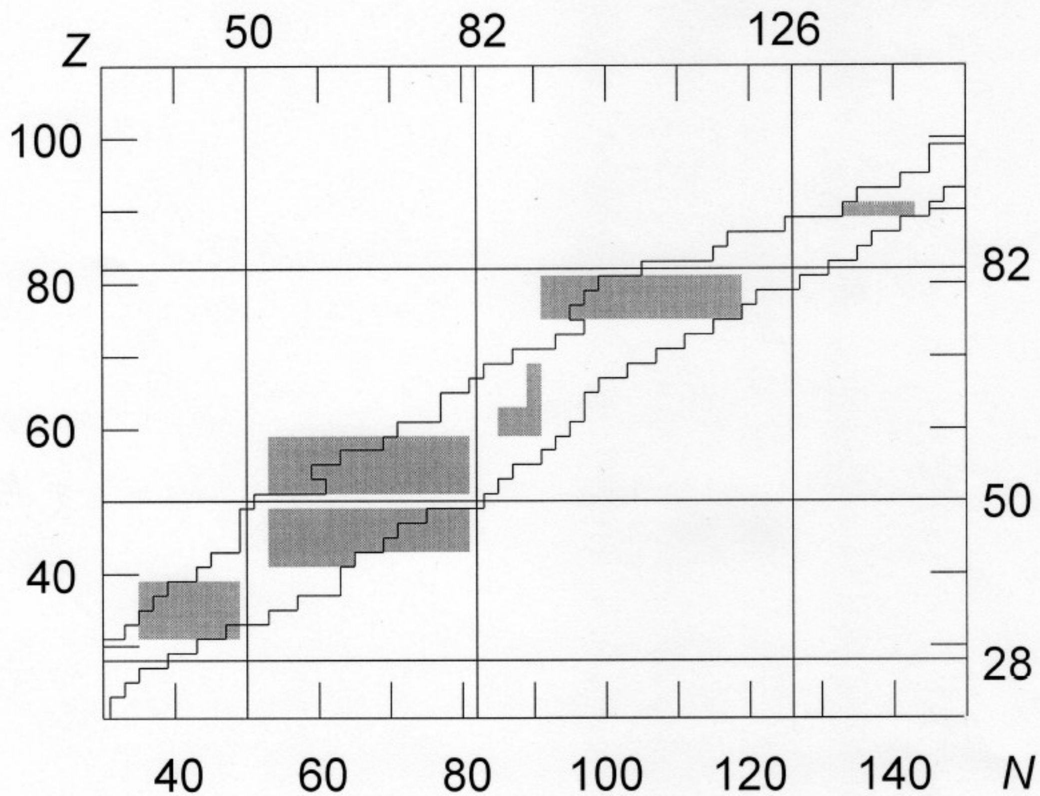


Figure 0.1. Card of even-even nuclei. Z = number of protons, N = number of neutrons. The dark areas denote nuclei which have been calculated using the IBM2 approximation (Iachello, 1988, p. 110).

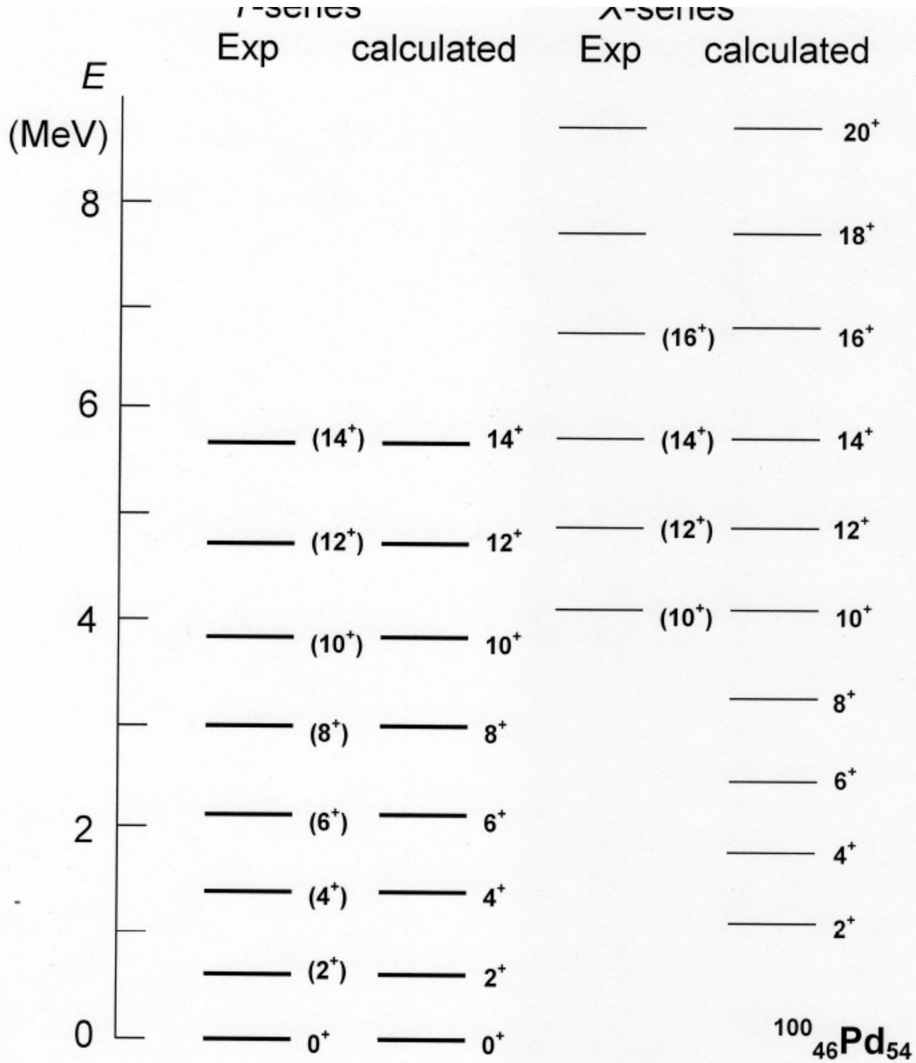


Figure 10.6. Comparison between the measured and the calculated energies of the *Y*-series and the *X*-series in ^{100}Pd. The parameters employed are ε = 680 keV, c_2 = -160 keV und c_4 = 45 keV (Arima and Iachello, 1976a, p. 275).

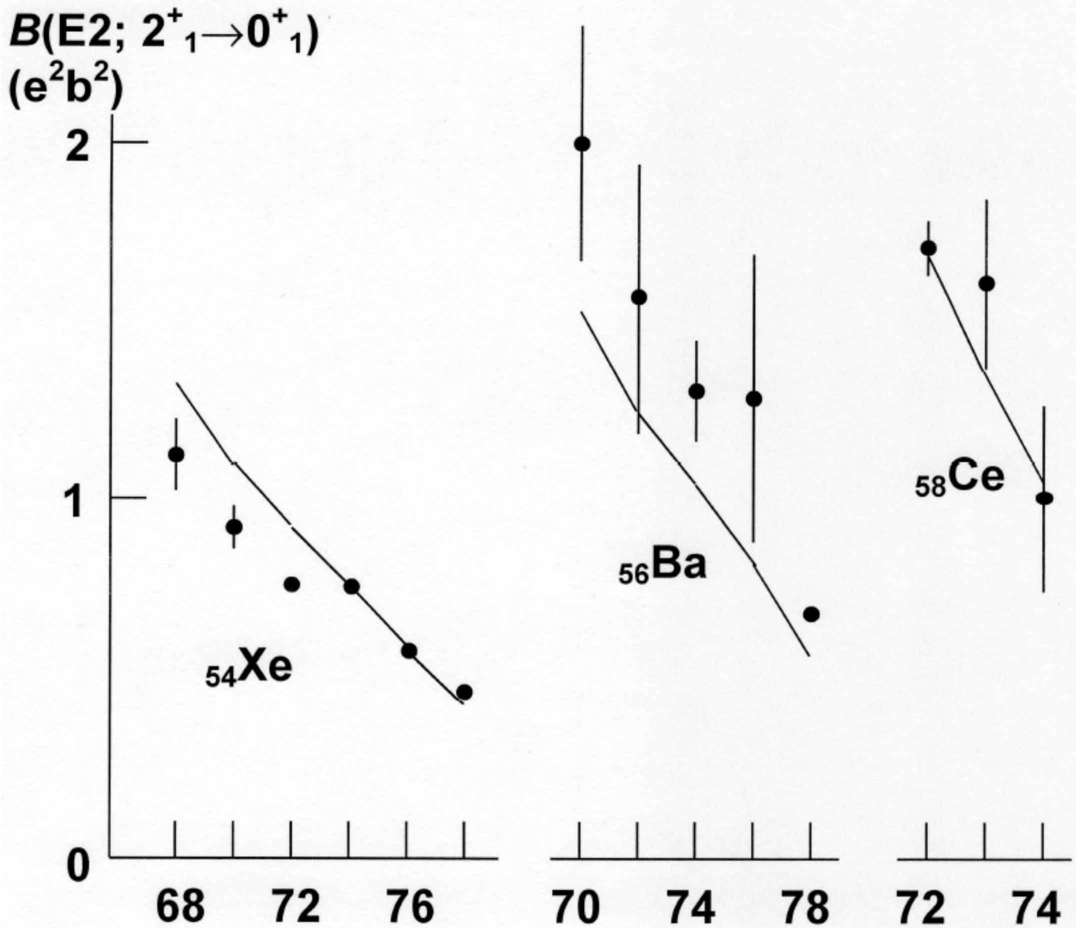


Figure 15.6. Comparison between measured and calculated reduced transition probabilities for quadrupole radiation of even-even nuclei with neutron numbers 68 up to 78. The parameters of figures 15.4 and 15.5 have been employed (Scholten, 1980, p. 58).

SHELL CORRECTIONS TO THE DEFORMATION ENERGIES OF VERY HIGH SPIN NUCLEI ($I \lesssim 100$)

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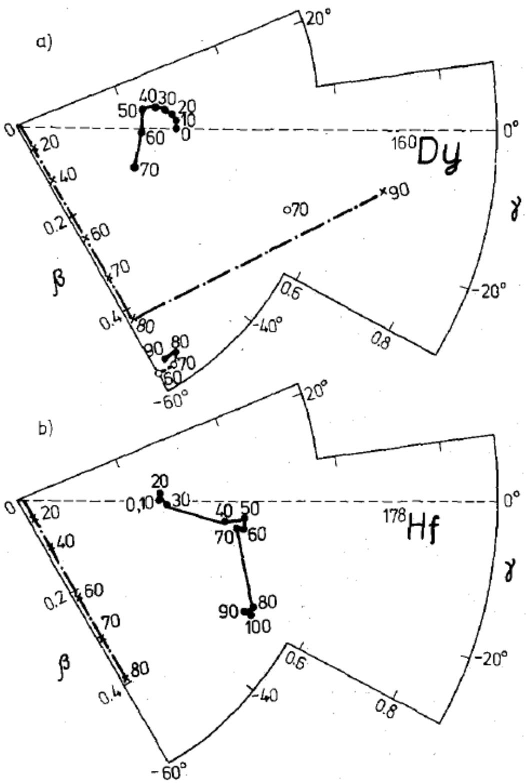


Fig. 2. Various minima in the energy functions of the nuclei ^{160}Dy and ^{178}Hf. A filled circle denotes an absolute minimum, an open circle a second minimum, and a cross a minimum in the classical energy function. Points which belong to a continuous family of shapes are connected by straight lines.

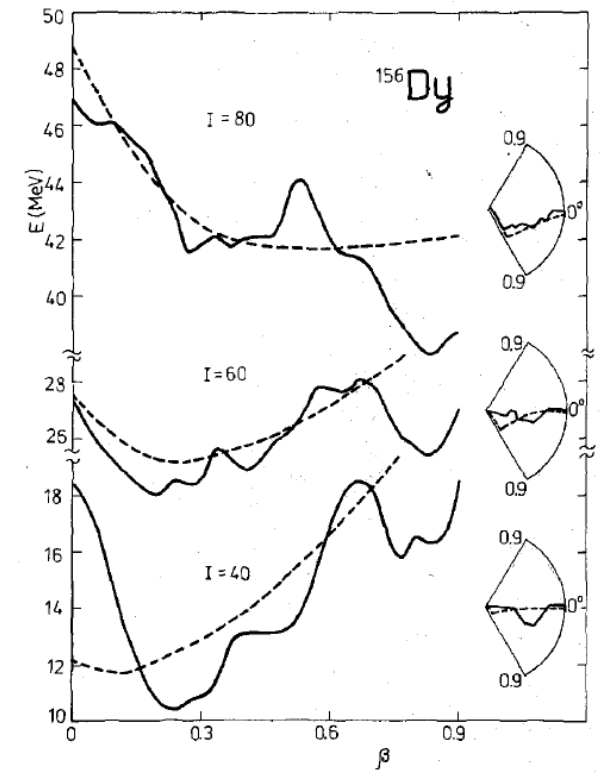
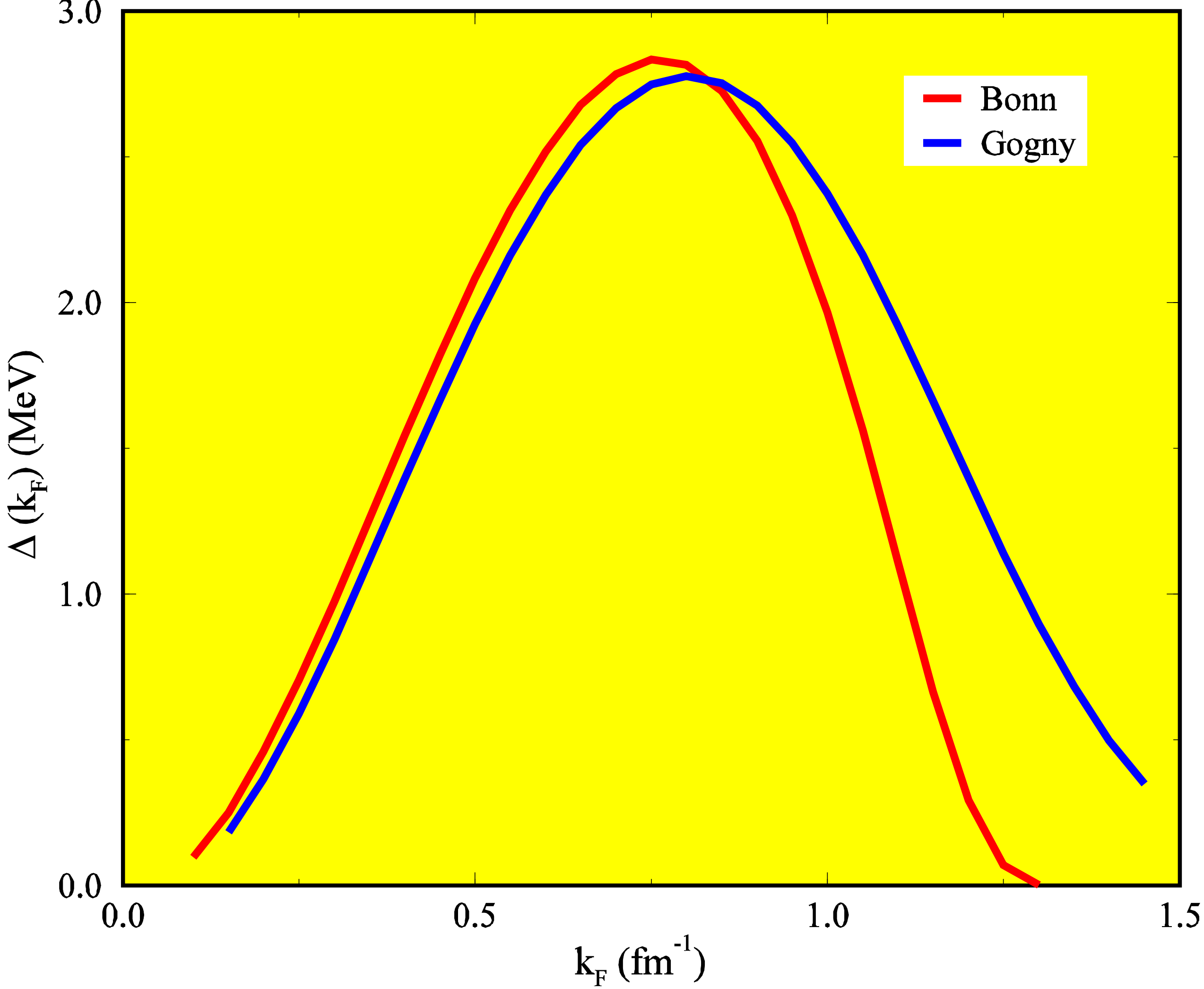


Fig. 3. Cuts through the energy function of ^{156}Dy along an approximate "fission valley" for three different spin values. In the inserts is indicated how the cuts are taken. Corresponding classical curves are shown as dashed lines.

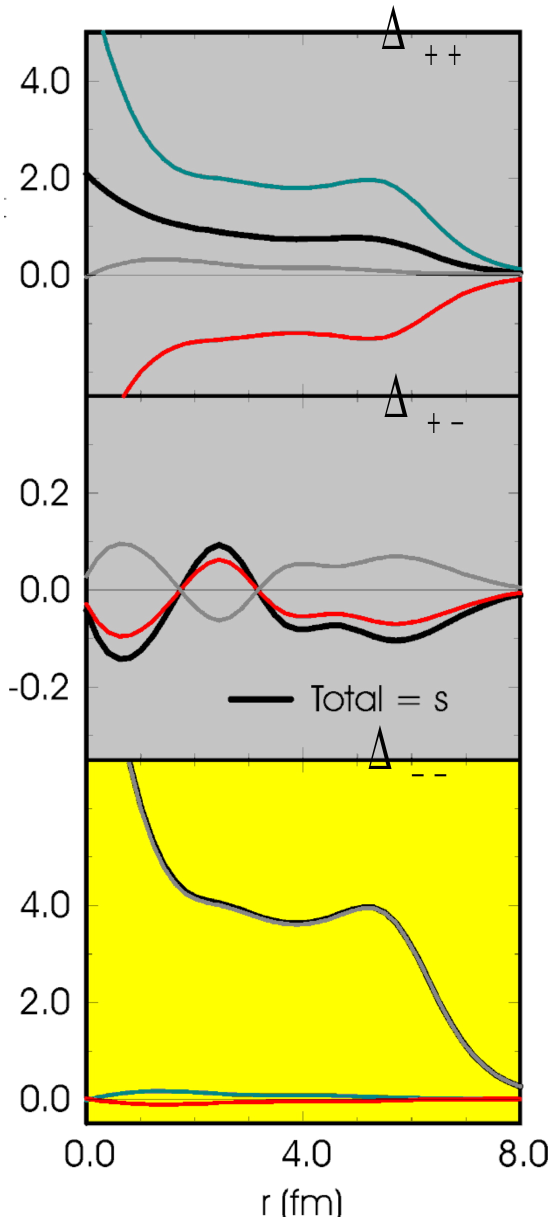


free NN-forces, which reproduce the phase shift in the 1S0 channel, give pairing similar to the Gogny force

It is seen that the gap vanishes at very small and at very large densities. The maximum is located at $k_F = 0.8\,\mathrm{fm}^{-1}$. This value of k_F corresponds to relatively low density which in finite nuclei is found only in the surface region.

M. Serra, A. Rummel, P. Ring, PRC 65 (2002) 014304

Relativistic structure of pairing



$$\mathcal{H} = \begin{pmatrix} m+V-S & \boldsymbol{\sigma}\mathbf{p} & \Delta_{++} & \Delta_{+-} \\ \boldsymbol{\sigma}\mathbf{p} & -m-V-S & \Delta_{-+} & \Delta_{--} \\ \Delta_{++} & \Delta_{+-} & -m-V+S & -\boldsymbol{\sigma}\mathbf{p} \\ \Delta_{-+} & \Delta_{--} & -\boldsymbol{\sigma}\mathbf{p} & m+V+S \end{pmatrix}$$

$$\Delta_{+-} = \Delta_{-+} << \Delta_{++} << \boldsymbol{\sigma}\mathbf{p}$$

therefore we neglect Δ_{+-}

total

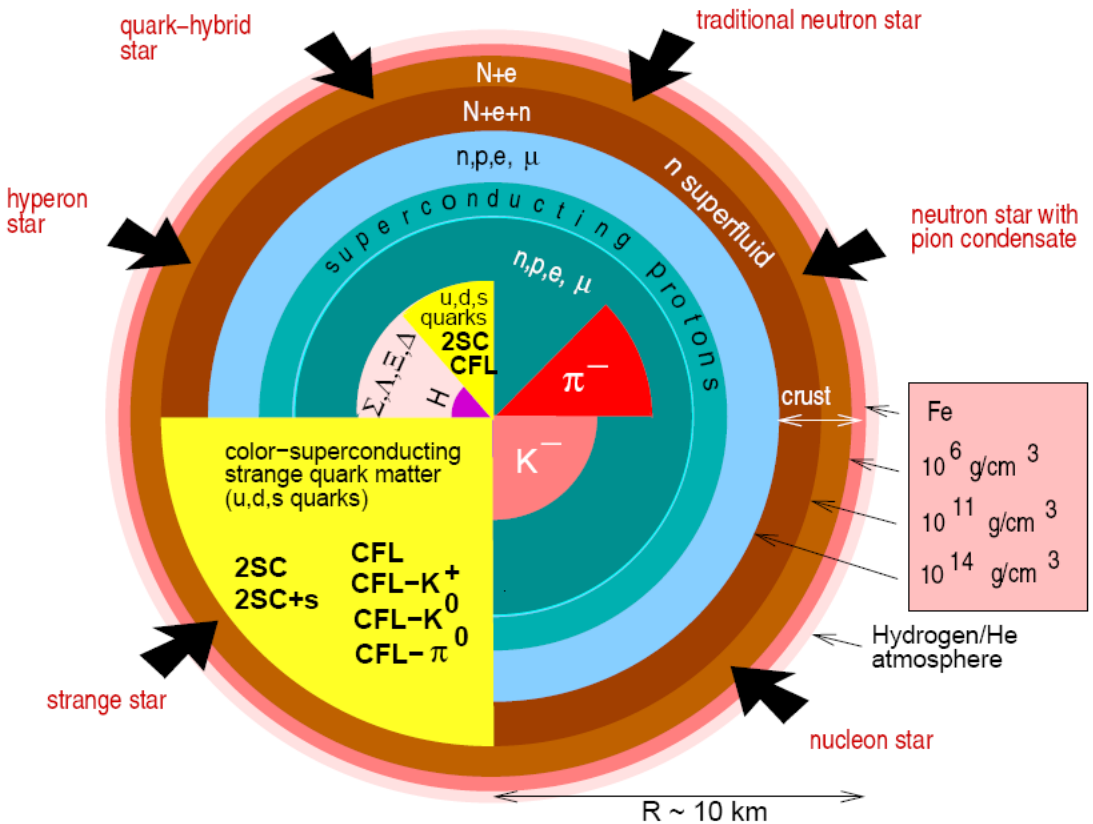
scalar

vector time-like

vector spacelike

M. Serra, P. Ring, PRC 65 (2002) 064324

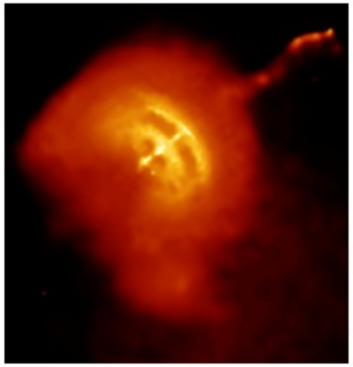
Neutron stars



Neutron star crust $\sim$ 1% mass, 10% radius

Why studying neutron star crust ?

Crust=interface between outer layers (observations) and core



⇒ Important role in the dynamics of the star :

★ electrical resistivity ⇒ evolution of magnetic field (pulsar emission, magnetars)

★ thermal conductivity ⇒ X ray emission, cooling

★ elastic properties ⇒ pulsar glitches, oscillation modes, gravitational waves

Neutron star crust as a probe for exotic nuclei

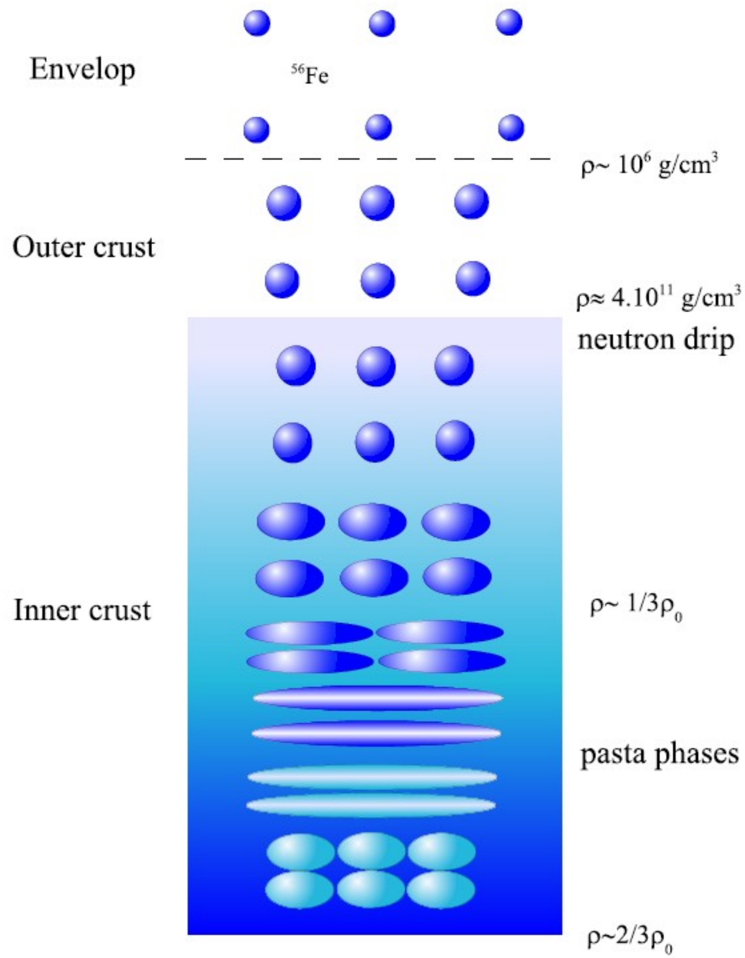
Exotic phases inaccessible on Earth !

★ Very neutron rich nuclei

★ Strongly deformed nuclei (“pasta” phases)

★ Nuclei immersed in a neutron superfluid

$\Rightarrow$ nuclear astrophysical laboratory !



Envelop
56Fe
ρ~ 10^6 g/cm^3
Outer crust
ρ≈ 4.10^11 g/cm^3
neutron drip
Inner crust
ρ~ 1/3ρ0
pasta phases
ρ~2/3ρ0

Thank you for your attention!