Neutrino oscillations

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Idea of neutrinos was proposed by Pauli on December 4th 1930 in order to solve nuclei problems

During many years after Rutherford's discovery nuclei were considered as bound states of protons and electrons

There were two fundamental problems in the framework of this assumptions

I. β-decay of nuclei in this model is two particle decay

$$(A, Z) \to (A, Z+1) + e^-$$

Monochromatic electron must be produced. In experiment continuous β-spectrum was observed

II. Problem of spins of some nuclei.

$$^7N_{14} = (14p + 7e) \to \mathrm{half\ integer\ spin}$$

From molecular spectra followed that $^7N_{14}$ satisfird Bose-Einstein statistics; spin must be integer

Pauli came to idea that only existence of a new particle could solve these nuclear problems

In order to come to continuous β-spectrum β-decay of nuclei must be three-particle decay

Additional particle must not be visible in an experiment

It must have spin 1/2 and be constituent of nuclei (problem of spin can be solved)

Thus, Pauli assumed that exist a neutral, spin 1/2, particle with interaction which is much weaker that the interaction of photon.

Pauli called a new particle neutron

In 1932 neutron was discovered by Chadwick

Heisenberg, Majorana, Ivanenko came to correct idea of proton-neutron structure of nuclei

No problem of spin. For example, $^{7}N_{14} = (7p + 7n)$, integral spin

What about β-decay and continues β spectrum?

The problem was solved by E. Fermi in 1933-34

F. Fermi accepted Pauli hypothesis of the existence of a new light particle (much lighter than neutron) which E. Fermi proposed to call neutrino (from Italian, *neutral, small*)

Fermi assumed that (e, ν) pair is produced in the quantum transition of neutron to proton

$$n \to p + e + \nu$$

On the basis of analogy with electrodynamics Fermi proposed the first Hamiltonian which provides this transition

$$\mathcal{H}_I = G_F \bar{p} \gamma^\alpha n \ \bar{e} \gamma_\alpha \nu + \mathrm{h.c.}$$

After discovery of parity violation in β-decay and other weak processes (1957-58) Two-component neutrino theory was proposed by Landau, Lee and Yang, Salam

They connected large observed violation of parity with neutrino

Landau and others assumed that neutrino mass is equal to zero and neutrino field is left-handed (or right-handed) field

$$\nu_{L,R}(x) = \frac{1 \mp \gamma_5}{2} \nu(x)$$

For massless neutrinos

$$i\gamma^{\alpha}\partial_{\alpha}\nu_{L,R}(x) = 0$$

Two main consequences

I. Large violation of parity (in agreement with the Wu et al experiment)

$$\mathcal{H}_I = \sum_i G_i \, \bar{p} \; O_i n \; \bar{e} \; O^i \frac{1}{2}(1 \mp \gamma_5)\nu + \mathrm{h.c.}$$

$$O \to 1,\, \gamma_{\alpha},\, \sigma_{\alpha\beta},\, \gamma_{\alpha}\gamma_5,\, \gamma_5$$

II. If neutrino field is $\nu_L(x)$, neutrino is left-handed particle and antineutrino is right-handed particle (in the case of $\nu_R(x)$ neutrino is right-handed and antineutrino is left-handed)

Neutrino helicity was measured in spectacular Goldhaber et al experiment (1958)

$$
\begin{array}{lll}
e^- + {}^{152}\mathrm{Eu} \to \nu + & {}^{152}\mathrm{Sm}^* & \\
 & \downarrow & \\
 & {}^{152}\mathrm{Sm} & + \gamma
\end{array}
$$

Measurement of the circular polarization of γ allows to determine neutrino helicity

Goldhaber et al proved that neutrino has negative helicity

Two-component neutrino theory with neutrino field $\nu_L(x)$ was confirmed

After this success of the two-component theory physicists during many years believed than neutrinos are massless particles ($V - A$ theory, Standard model were build for massless two-component neutrinos)

The first physicist who started to think about a possibility of small neutrino masses was B. Pontecorvo (1957-58)

He believed in analogy between weak interaction of hadrons and leptons and looked for analogy of $K^0 \leftrightarrows \bar{K}^0$ oscillations in the lepton sector

In such a way B. Pontecorvo came to an idea of neutrino oscillations

"If the two-component neutrino theory turn out to be incorrect (which at present seems to be rather improbable) and if the conservation law of neutrino charge would not apply, then in principle neutrino $\rightleftarrows$ antineutrino transitions could take place in vacuum."

By analogy with $K^0 - \bar{K}^0$ B. Pontecorvo assumed

$$|\nu_L\rangle = \frac{1}{\sqrt{2}}(|\nu_{1L}\rangle + |\nu_{2L}\rangle), \quad |\bar{\nu}_L\rangle = \frac{1}{\sqrt{2}}(|\nu_{1L}\rangle - |\nu_{2L}\rangle)$$

where ν_1 and ν_2 are Majorana neutrino with masses m_1 and m_2

"neutrino and antineutrino are *mixed particles*, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2"

In the first paper (1958) B. Pontecorvo wrote ...the number of events $\bar{\nu} + p \to e^+ + n$ with reactor antineutrino would be smaller than the expected number. "It would be extremely interesting to perform the Reins-Cowan experiment at different distances from reactor"

In 1967 B. Pontecorvo considered all possible transitions between ν_e and ν_μ and apply idea of neutrino oscillations to solar neutrinos

"..due to neutrino oscillations the observed flux of solar neutrinos could be two times smaller than the expected flux"

Three years later R. Davis obtained first solar neutrino data

Upper bound of the solar neutrino flux was 2-3 times smaller than SSM predicted flux It was called solar neutrino puzzle

B. Pontecorvo envisaged "the puzzle"

In 1969 Gribov and Pontecorvo paper on two neutrino oscillations was published. Majorana mass term was considered and two-neutrino transition probability was obtained and applied to solar neutrinos.

In 1975-1989 many papers on neutrino oscillations by B.P. and S.B. All possible neutrino mass terms were considered including Dirac and Majorana mass term which is the basis of the seesaw mechanism. Neutrino oscillations in vacuum. All possible experiments. First review on neutrino oscillations...

IDEA OF NEUTRINO OSCILLATIONS WAS BORN AND DEVELOPED IN DUBNA

In eighties special reactor and accelerator experiments on the search for neutrino oscillations started. No indications. Model dependent evidence for oscillations from solar experiments. Atmospheric neutrino anomaly

GOLDEN YEARS OF NEUTRINO OSCILLATIONS

1998 Super-Kamiokande discovery of neutrino oscillations in atmospheric experiment (zenith angle dependence of the number of ν_μ's)

2001 SNO Model independent proof of the transition of solar ν_e into ν_μ and ν_τ (ratio of the flux of ν_e 's to the total flux of ν_e, ν_μ and ν_τ is about $1/3$)

2002-2004 KamLAND reactor experiment (significant distortion of the spectrum of reactor $\bar{\nu}_e$'s)

Basic assumptions

I. Standard Model interaction

$$\mathcal{L}_{\mathcal{I}}^{\mathcal{CC}}(x) = -\frac{g}{\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\,\gamma_{\alpha}\, l_L(x)\ W^{\alpha}(x) + \mathrm{h.c.}$$

II. Mixed flavor fields

$$\nu_{lL}(x) = \sum_i U_{li}\,\nu_{iL}(x).$$

$\nu_i(x)$ is the field of neutrinos with mass m_i, $U^{\dagger}U = 1$

III. States of the flavor neutrinos ν_e, ν_μ and ν_τ

$$|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^{*}\ |\nu_i\rangle$$

$|\nu_i\rangle$ is the state of neutrino with mass m_i

IV. Transition probability in vacuum

If at $t=0$ flavor neutrino ν_l is produced in a weak process

$$|\nu_l\rangle_t = e^{-iHt}\sum_{i=1} U^*_{li}\,|\nu_i\rangle = \sum_{i=1} e^{-iE_i t} U^*_{li}\,|\nu_i\rangle = \sum_{l'} |\nu_{l'}\rangle \sum_i U_{l'i} e^{-iE_i t} U^*_{li}$$

$$E_i \simeq p + \frac{m_i^2}{2E}$$

Probability of the transition $\nu_l \to \nu_{l'}$

$$\mathrm{P}(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i}\, e^{-i\frac{\Delta m^2_{ji} L}{2E}}\, U^*_{li}|^2 = |\sum_{i\neq j} U_{l'i}\,(e^{-i\frac{\Delta m^2_{2i} L}{2E}} - 1)\, U^*_{li} + \delta_{l'l}|^2.$$

$$\Delta m^2_{ji} = m_i^2 - m_j^2$$

All data are in agreement with the assumption that the number of massive neutrinos is equal to the number of the flavor neutrinos (three, LEP).

No sterile neutrinos

However, the data of LSND and MiniBooNE ($\bar{\nu}_\mu$) experiments require additional massive neutrinos and, correspondingly, sterile neutrinos

In the case of three neutrinos six parameters: $\Delta m^2_{12}, \Delta m^2_{23}, \theta_{12}, \theta_{23}, \theta_{13}, \delta$

From experimental data

$\Delta m^2_{12} \simeq \frac{1}{30}\Delta m^2_{23}, \quad \sin^2\theta_{13} \leq 4 \cdot 10^{-2}$

In the leading approximation in atmospheric, accelerator region of $\frac{L}{E}$ $(\frac{\Delta m^2_{23} L}{2E} \gtrsim 1)$ dominant transition is $\nu_\mu \to \nu_\tau$

Survival probability

$$\mathrm{P}(\nu_\mu \to \nu_\mu) \simeq 1 - \frac{1}{2}\ \sin^2 2\theta_{23}\ (1 - \cos\Delta m^2_{23}\frac{L}{2E})$$

In the reactor KamLAND region of $\frac{L}{E}$ $(\frac{\Delta m^2_{12} L}{2E} \gtrsim 1)$ dominant transitions are $\bar\nu_e \to \bar\nu_{\mu,\tau}$

Survival probability

$$\mathrm{P}(\bar\nu_e \to \bar\nu_e) \simeq 1 - \frac{1}{2}\ \sin^2 2\theta_{12}\ (1 - \cos\Delta m^2_{12}\frac{L}{2E}).$$

Good description of the data

From the analysis of the LBL accelerator neutrino MINOS data

$$\Delta m_{23}^2 = (2.43 \pm 0.13) \cdot 10^{-3}\ \mathrm{eV}^2, \quad \sin^2 2\theta_{23} > 0.90$$

Good agreement with atmospheric neutrino data

From the global analysis of the reactor KamLAND and solar data

$$\Delta m_{12}^2 = (7.50^{+0.19}_{-0.20}) \cdot 10^{-5}\ \mathrm{eV}^2, \quad \tan^2\theta_{12} = 0.452^{+0.035}_{-0.032}$$

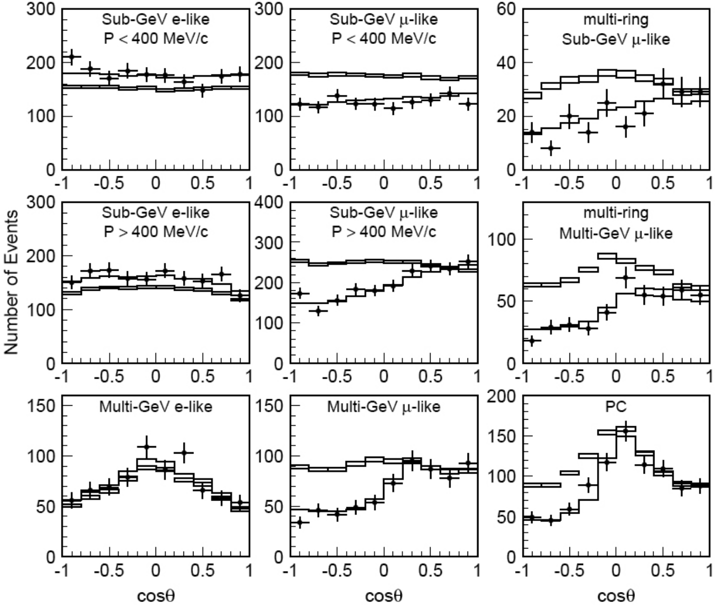
From reactor CHOOZ data $\sin^2\theta_{13} \leq 4 \cdot 10^{-2}$

Four neutrino oscillation parameters are known with accuracies (3-10)%. Upper bound on $\sin^2\theta_{13}$. No information about *CP* phase δ

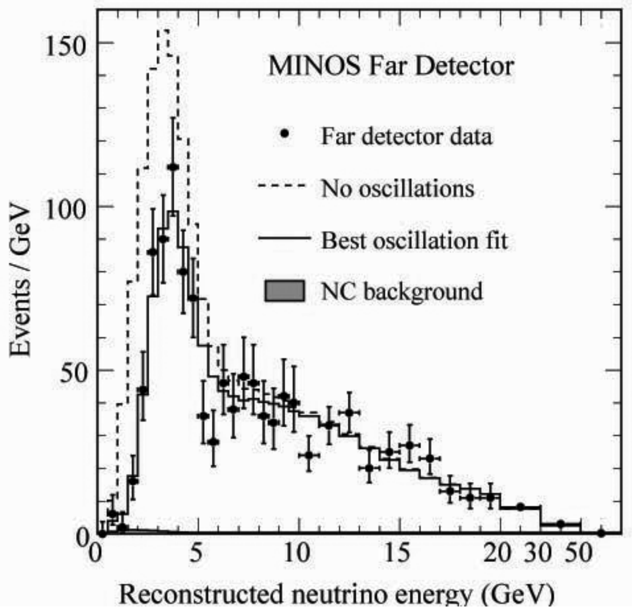
From TROITSK and MAINZ tritium experiments on the measurement of the absolute value of the neutrino mass

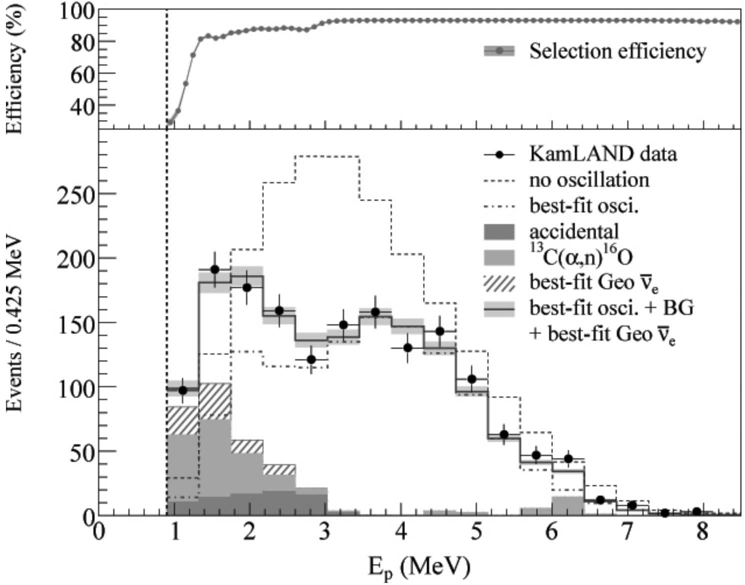
$m_\beta < 2.3$ eV

From cosmology $\sum_i m_i < (0.6 - 1.0)$ eV

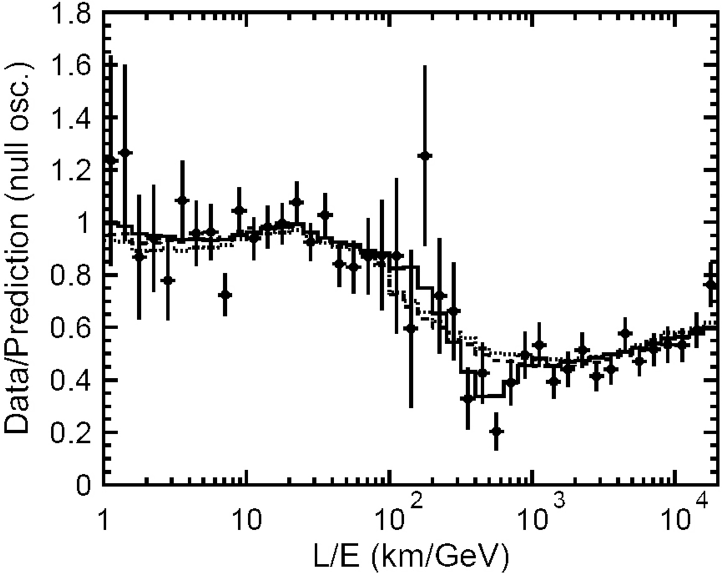


Sub-GeV e-like
P < 400 MeV/c
Sub-GeV μ-like
P < 400 MeV/c
multi-ring
Sub-GeV μ-like
Sub-GeV e-like
P > 400 MeV/c
Sub-GeV μ-like
P > 400 MeV/c
multi-ring
Multi-GeV μ-like
Multi-GeV e-like
Multi-GeV μ-like
PC
Number of Events
cosθ
cosθ
cosθ

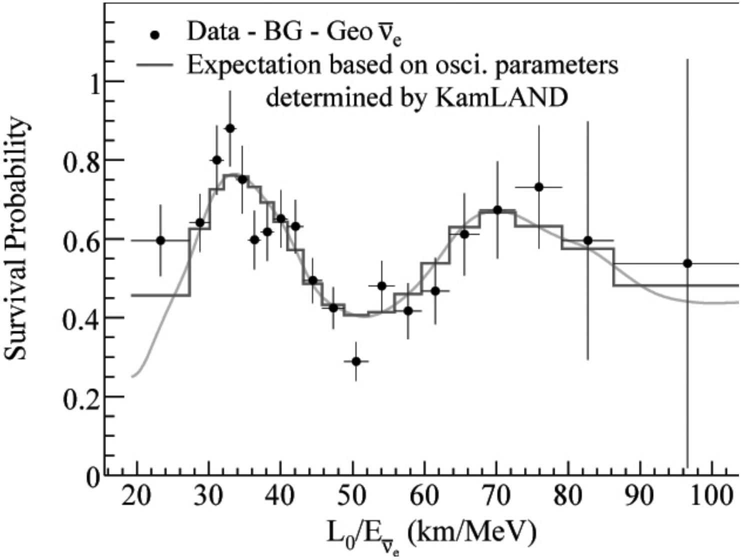

MINOS Far Detector
Far detector data
No oscillations
Best oscillation fit
NC background
Events / GeV
150
100
50
0
0
5
10
15
20
30
50
Reconstructed neutrino energy (GeV)



Efficiency (%)
100
80
60
40
Selection efficiency
KamLAND data
no oscillation
best-fit osci.
accidental
$^{13}C(\alpha,n)^{16}O$
best-fit Geo $\overline{\nu}_e$
best-fit osci. + BG
+ best-fit Geo $\overline{\nu}_e$
Events / 0.425 MeV
250
200
150
100
50
0
0
1
2
3
4
5
6
7
8
E_p (MeV)



1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Data/Prediction (null osc.)
1
10
10^2
10^3
10^4
L/E (km/GeV)


Data - BG - Geo $\overline{\nu}_e$
Expectation based on osci. parameters determined by KamLAND
Survival Probability
0
0.2
0.4
0.6
0.8
1
20
30
40
50
60
70
80
90
100
$L_0/E_{\overline{\nu}_e}$ (km/MeV)

In future accelerator T2K, reactor Double CHOOZ, RENO and Daya Bay experiments sensitivities to $\sin^2\theta_{13}$ will be at least one order of magnitude better than in the CHOOZ experiment

If the value of the parameter $\sin^2\theta_{13}$ will be measured, in the experiments of the next generation CP violation in the lepton sector and neutrino mass spectrum will be studied

New Facilities (Super beam, β-beam, Neutrino factory) under R&D

NEUTRINO MASSES ARE DIFFERENT FROM ZERO BUT VERY SMALL

Much smaller than masses of quarks and leptons

What does this mean?

The Standard Model and neutrino masses

Quark and lepton masses of the third family

$$m_t \simeq 1.7 \cdot 10^2 \ \mathrm{GeV}, \quad m_b \simeq 4.7 \ \mathrm{GeV}$$

$$m_3 \leq 2.3 \ 10^{-9} \ \mathrm{GeV}, \quad m_\tau \simeq 1.8 \ \mathrm{GeV}$$

We believe that quark and lepton masses are generated in the framework of the Standard Model after spontaneous breaking of the elecroweak symmetry

It is very unlikely that neutrino masses are of the same Higgs origin

Additional (or new) beyond the SM mechanism of neutrino mass generation is necessary

The most plausible and popular is SEESAW MECHANISM

A beyond the SM physics generate non-renormalizable effective Lagrangian of the form $\sum_n \frac{1}{\Lambda^n}\mathcal{L}^{\mathrm{eff}}_{4+n}$

The large parameter Λ has dimension M and characterizes a scale of a new physics

The only dimension five effective Lagrangian has the form

$$\mathcal{L}^{\mathrm{eff}}_5 = -\frac{1}{\Lambda}\sum_{l',l,i} \overline{L}_{l'L}\tilde{H} Y_{l'l} C \tilde{H}^T (\overline{L}_{lL})^T + \mathrm{h.c.}.$$

$$L_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix} \qquad H = \begin{pmatrix} H^{(+)} \\ H^{(0)} \end{pmatrix} \qquad \tilde{H} = i\tau_2 H^*$$

The Lagrangian $\mathcal{L}^{\mathrm{eff}}_5$ does not conserve the total lepton number L.

After electroweak symmetry breaking

$$\tilde{H} = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \quad v = (\sqrt{2} G_F)^{-1/2} \simeq 246 \text{ GeV}$$

(parameter v (Higgs vacuum expectation value) characterizes scale of the electroweak breaking)

the left-handed Majorana mass term is generated

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2} \sum_{l'l} \bar{\nu}_{l'L} M^L_{l'l} \, C \bar{\nu}^T_{lL} + \mathrm{h.c.}$$

$$M^L_{l'l} = \frac{v^2}{\Lambda} Y_{l'l}$$

Performing the standard diagonalization of the mass term ($Y = UyU^T$) we have

$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2} \sum_i m_i \bar{\nu}_i \nu_i$

$\nu_i = \nu_i^c$ is the field of Majorana neutrino with mass m_i Neutrino and antineutrino are identical (no conserved lepton number L)

Neutrino masses and mixing

$$m_i = \frac{v^2}{\Lambda} y_i \quad \nu_{lL} = \sum_i U_{li} \nu_{iL}$$

Neutrino masses are determined by the seesaw factor

$$\frac{v^2}{\Lambda} = \frac{(\mathrm{EW\ scale})^2}{\mathrm{scale\ of\ new\ physics}}$$

We can estimate $\Lambda \simeq (10^{14} - 10^{15})$ GeV

Small Majorana neutrino masses are the only signature of a beyond the SM physics at a very large GUT scale where the total lepton number L is violated

How can we test this idea?

First of all we need to prove that neutrinos with definite mass ν_i are Majorana particles?

This can not be done in neutrino oscillation experiments. We need to observe processes in which the total lepton number is violated

If neutrinos with definite masses are Majorana particles, some processes with virtual neutrinos in which lepton number is violated are allowed

The most sensitive to small neutrino masses process is $0\nu\beta\beta$-decay

$$(A, Z) \to (A, Z+2) + e^- + e^-$$

Neutrino propagator enters into the matrix element of $0\nu\beta\beta$-decay the form

$$\sum_i U_{ei}^2 \frac{1-\gamma_5}{2} \frac{\gamma \cdot + m_i}{p_2 - m_i^2} \frac{1-\gamma_5}{2} \simeq m_{\beta\beta} \frac{1}{p^2} \frac{1-\gamma_5}{2}$$

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

is the effective Majorana mass

The probability of the $0\nu\beta\beta$-decay is extremely small

I. It is second order in the Fermi constant process

II. Additional suppression factor $m_{\beta\beta}\frac{1}{p^2}$ due to $V-A$ structure of currents ($|m_{\beta\beta}| \leq 1$ eV and $\bar{p}^2 \simeq 10^2\mathrm{MeV}^2$)

Half-life is given by the expression

$$\frac{1}{T^{0\,\nu}_{1/2}(A,Z)} = |m_{\beta\beta}|^2\,|M(A,Z)|^2\,G^{0\,\nu}(E_0,Z)$$

$M(A,Z)$ is the nuclear matrix elements and $G^{0\,\nu}(E_0,Z)$ is known phase-space factor

Theoretical problems of calculation of NME (five models, all give different results)

From existing data

$$|m_{\beta\beta}| < (0.20-0.32)\,\mathrm{eV}(^{76}\mathrm{Ge}), \quad |\mathrm{m}_{\beta\beta}| < (0.19-0.68)\,\mathrm{eV}(^{130}\mathrm{Te})$$

Future experiments will be sensitive to $|m_{\beta\beta}| = \mathrm{a\ few}10^{-2}\ \mathrm{eV}$