Pion annihilation $\pi\pi \to \gamma\gamma$ near the chiral symmetry restoration

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Low energy photon production from the pion annihilation

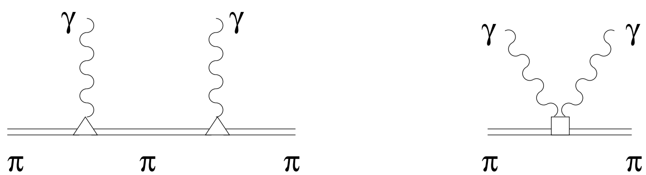
M. K. Volkov, E. A. Kuraev, D. Blaschke, G. Röpke and S. M. Schmidt, "Excess low energy photon pairs from pion annihilation at the chiral phase transition," Phys. Lett. B **424**, 235 (1998) [arXiv:hep-ph/9706350].

The amplitude for the pion annihilation to two photons

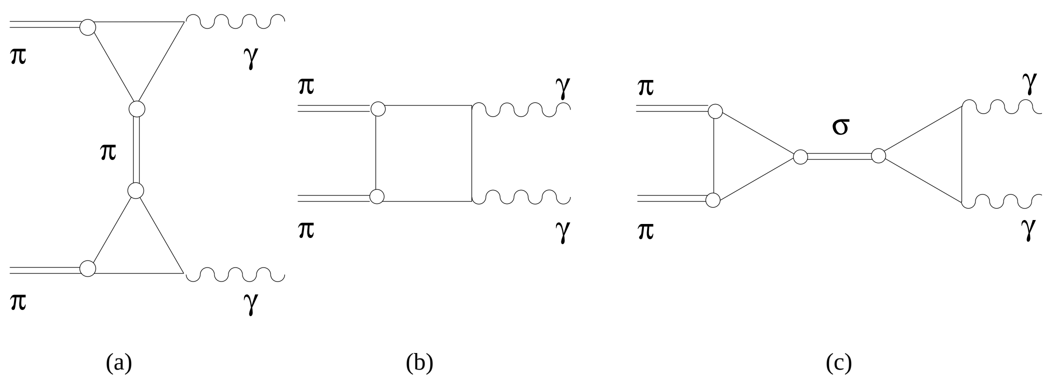
$$\pi\pi \to \gamma\gamma$$

can be divided into two parts:

i) the Born contribution T_{Born} (for charged pions)



ii) the term provided by the quark substructure of mesons T_{qs}, obtained by subtraction of T_{Born} from the following set of diagrams



Low energy photon production from the pion annihilation

$$T_{\text{Born}} = 2e^2 \left(g^{\mu\nu} - \frac{q_1^\mu q_2^\nu}{q_1 k_1} - \frac{q_2^\mu q_1^\nu}{q_2 k_1} \right) \epsilon_\mu(k_1)\epsilon_\nu(k_2)$$

where e is the elementary electric charge $e^2/4\pi = \alpha \approx 137^{-1}$,
q_1, q_2 are pion 4-momenta,
k_1, k_2 are photon 4-momenta, and
$\epsilon_\mu(k_1)$, $\epsilon_\nu(k_2)$ are corresponding photon polarization vectors.

The second part contains resonant and "box" contribution

$$T_{\text{sq}} = (T_{\text{res}} + T_{\text{box}})[(k_1 \cdot k_2)(\epsilon_1 \cdot \epsilon_2) - (k_2 \cdot \epsilon_1)(k_1 \cdot \epsilon_2)], \qquad \epsilon_i \equiv \epsilon(k_i), \quad i = 1, 2.$$

The "box"-diagram contribution T_{box} smoothly changes through a cross-over

$$T_{\text{box}}(T,\mu) = -\frac{f_2(T,\mu)}{36\pi^2 F_\pi^2(T,\mu)}.$$

Here, $F_\pi(T=0,\mu=0) = F_\pi = 93$ MeV; the factor $f_2(T=0,\mu=0) = 1$ is determined by the quark substructure of mesons.

$$f_2(T,\mu) = 3m^2(T,\mu) \int_0^\infty dp \, \frac{p^2}{E^5(p)} \left[1 - n(p|T,\mu) - \bar{n}(p|T,\mu) \right], \qquad E(p) = \sqrt{p^2 + m^2},$$

where $n(p|T,\mu) = (\exp[(E(p)-\mu)/T] + 1)^{-1}$ and $\bar{n}(p|T,\mu) = (\exp[(E(p)+\mu)/T] + 1)^{-1}$.
Both $F_\pi(T,\mu)$ and $f_2(T,\mu)$ decrease with growing T and μ.

Low energy photon production from the pion annihilation

At finite T and μ the resonant contribution is given by

$$T_{\text{res}}(T,\mu) = \frac{10m^2(T,\mu)f_1(T,\mu)}{9\pi^2 F_\pi^2(T,\mu)} \cdot \frac{1}{M_\sigma^2(T,\mu) - s - i\sqrt{s}\Gamma_\sigma(s|T,\mu)}$$

The factor $f_1(T=0,\mu=0)=1$ is determined by the quark substructure:

$$f_1(T,\mu) = 1 - \frac{3}{2}m^2(T,\mu)\int_0^\infty dp\,\frac{p^3}{E^6(p)}\ln\left[\frac{E(p)+p}{E(p)-p}\right]\left[n(p|T,\mu)+\bar{n}(p|T,\mu)\right],$$

$f_1 \to 0$ at $T\to\infty$ and/or $\mu\to\infty$.

The resonant contribution is maximal when $s=(q_1+q_2)^2=M_\sigma^2$, in this case

$$T_{\text{res}} \propto \frac{1}{M_\sigma\Gamma_\sigma}.$$

The resonant contribution may become dominant when the width of the σ-meson decreases, which is expected in hot and/or dense medium!

Mesons in hot and/or dense medium.

Properties of mesons are supposed to modify in hot and/or dense hadron (quark-gluon) medium. This modification is in part governed by the predicted from QCD restoration of chiral symmetry, which, after the phase transition, would reveal itself through the following effects:

i) the constituent quark mass and the pion weak decay constant drop;

ii) the pion becomes heavier;

iii) the chiral partner of the pion, the σ-meson becomes as heavy (or light) as the pion; ...

Apart from any model, one therefore can expect that the σ-meson can be as light as twice the pion mass amidst the cross-over from the hadron phase to QGP. As a consequence, at appropriate conditions, the main decay channel that determines the width of the σ-meson ($\sigma \to \pi\pi$) closes and the σ-meson appears as a relatively narrow resonance with $\Gamma_\sigma \approx \Gamma(\sigma \to \pi\pi) \to 0$, which may lead to amplification of some processes mediated by the σ-resonance, whose cross section becomes (at the maximum)

$$\sigma \propto \frac{1}{\Gamma_\sigma^2} + \ldots$$

NJL model

Chiral symmetry: $SU(N_F)_L \times SU(N_F)_R$,
color symmetry: $SU(N_c)$, $N_c = 3$.

$N_F = 2$: u- and d-quarks: the pion and the σ-meson.

The Lagrangian density

$$L = \bar{q}(i\,\partial\!\!\!/ - m_0)q + \frac{G}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$

The chiral symmetry is spontaneously broken at normal conditions.
$\Longrightarrow$ A gap equation should be satisfied:

$$m_0 = m\left(1 - 8GI_1^\Lambda(m)\right) \equiv m = m_0 - 2G\langle\bar{q}q\rangle \gg m_0,$$

$$I_1^\Lambda(m) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{p^2}{E(p)}dp,$$

$$F_\pi = \frac{m}{g_\pi}, \qquad g_\pi^{-2} = \frac{N_c}{2\pi^2}\int_0^\Lambda \frac{p^2}{E(p)}dp.$$

The pion is almost a Goldstone boson.
The GMOR relation is valid: $M_\pi^2 \approx -2m_0\langle\bar{q}q\rangle/F_\pi^2 \approx 140$ MeV.

NJL model

The model has 3 parameters parameters to be fixed in vacuum: the current quark mass m_0, the four-quark interaction constant G, the UV-cutoff Λ. There are diffenrent ways to fix them. For example, the Type I scheme satisfies the following conditions:

1. the pion mass in vacuum is 140 MeV;
2. the pion weak decay constant is $F_\pi = 93$ MeV;
3. the quark condensate is fixed to $\langle \bar{q}q \rangle = -(245 \text{ MeV})^3$.

The Type II scheme takes into account the $\pi - a_1$ transitions and replaces the third condition by

3. the model reproduces the decay $\rho \to \pi\pi$.

	G [GeV^{-2}]	Λ [MeV]	m_0 [MeV]
Type I	11.7205	618.7	5.76
Type II	3.4105	1037.4	2.08

Quark condensate in hot and/or dense medium

The most important quantity, the constituent quark mass is given at finite T and μ by the gap equation

$$m^0 = m(T,\mu)\left(1 - 8\,G\,I_1^{\Lambda}(m(T,\mu)|T,\mu)\right),$$

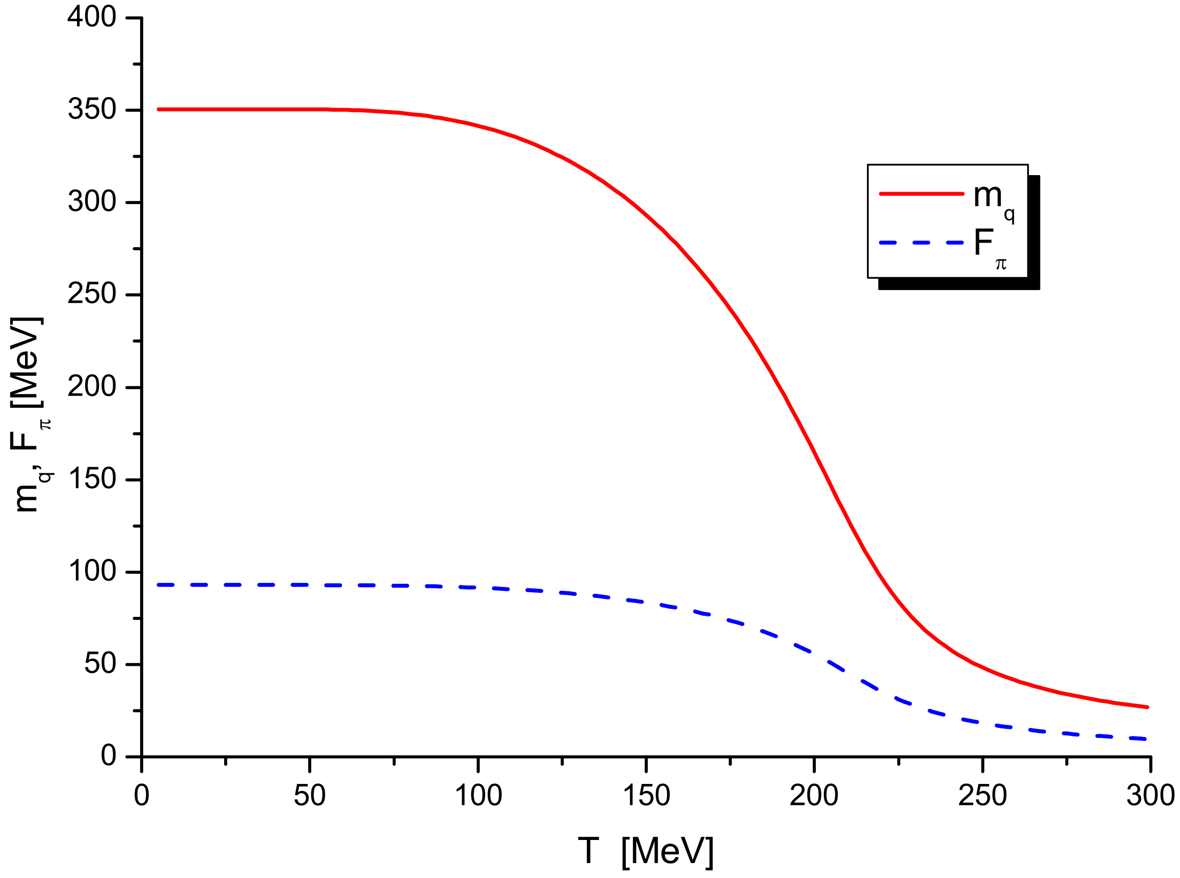
$$I_1^{\Lambda}(m|T,\mu) = \frac{N_c}{4\pi^2}\int_0^{\Lambda}\frac{k^2}{E(k)}\,(1 - n(k|T,\mu) - \bar{n}(k|T,\mu))\,dk,$$

$$F_\pi(T,\mu) = \frac{m(T,\mu)}{g_\pi(T,\mu)},$$

$$g_\pi^{-2}(T,\mu) = \frac{N_c}{2\pi^2}\int_0^{\Lambda}\frac{p^2}{E^3(p)}\,(1 - n(p|T,\mu) - \bar{n}(p|T,\mu))\,dk.$$

The current quark mass m^0, the interaction constant G, and the cutoff Λ are supposed not to change with T and μ.

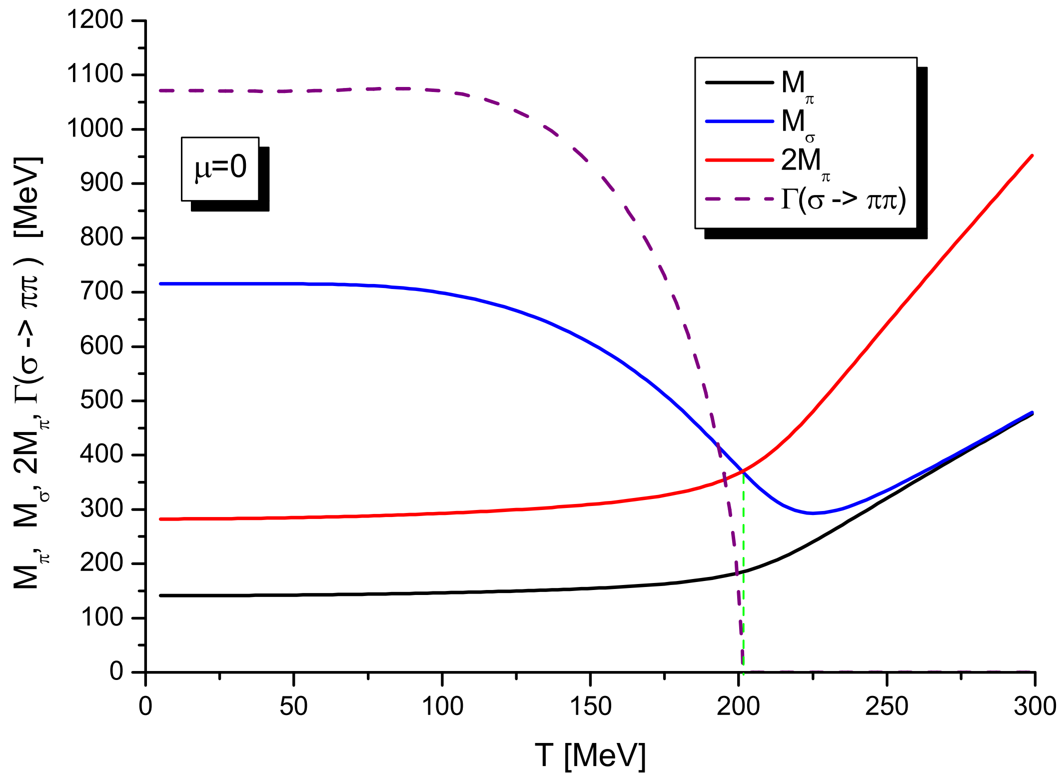
Quark condensate in hot and/or dense medium



Mesons in hot and/or dense medium

The masses and widths of mesons

$$
\begin{aligned}
M_\pi^2(T,\mu) &= g_\pi^2(T,\mu)\left(\frac{1}{G} - 8I_1^\Lambda(m|T,\mu)\right) = \frac{m^0 m(T,\mu)}{F_\pi^2(T,\mu)G} \\
M_\sigma^2(T,\mu) &= M_\pi^2(T,\mu) + 4m^2(T,\mu) \\
\Gamma_\sigma(M_\sigma|T,\mu) &= \frac{3m^2(T,\mu)\sqrt{M_\sigma^2(T,\mu) - 4M_\pi^2(T,\mu)}}{2\pi\, M_\sigma^2(T,\mu)\, F_\pi^2(T,\mu)} \coth\left(\frac{M_\sigma(T,\mu)}{4T}\right)\theta(M_\sigma(T,\mu) - 2M_\pi(T,\mu))
\end{aligned}
$$



Cross section

$$\sigma^{\pi^+\pi^-\to\gamma\gamma}(s) = \frac{1}{16\pi^2}\frac{1}{s\kappa}\int\frac{d^3k_1}{2\omega_1}\frac{d^3k_2}{2\omega_2}\delta^4(q_1+q_2-k_1-k_2)|T_{\mathsf{Born}}(s)+T_{\mathsf{qs}}(s)|^2$$

$$= \sigma_1(s)+\sigma_2(s)+\sigma_3(s)\ ,$$

$$\sigma_1(s) = 16\sigma_0\left(2-\kappa^2-\frac{1-\kappa^4}{2\kappa}\ln\left[\frac{1+\kappa}{1-\kappa}\right]\right),$$

$$\sigma_2(s) = 4\sigma_0\ s\ \mathrm{Re}\left[\mathcal{A}_{\pi^+\pi^-\to\gamma\gamma}(s)\right]\frac{1-\kappa^2}{\kappa}\ln\left[\frac{1+\kappa}{1-\kappa}\right],$$

$$\sigma_3(s) = \sigma_0\ s^2\ |\mathcal{A}_{\pi^+\pi^-\to\gamma\gamma}(s)|^2$$

where $\sigma_0=\pi\alpha^2/4s\kappa,\ \kappa^2=1-4M_\pi^2/s$, and

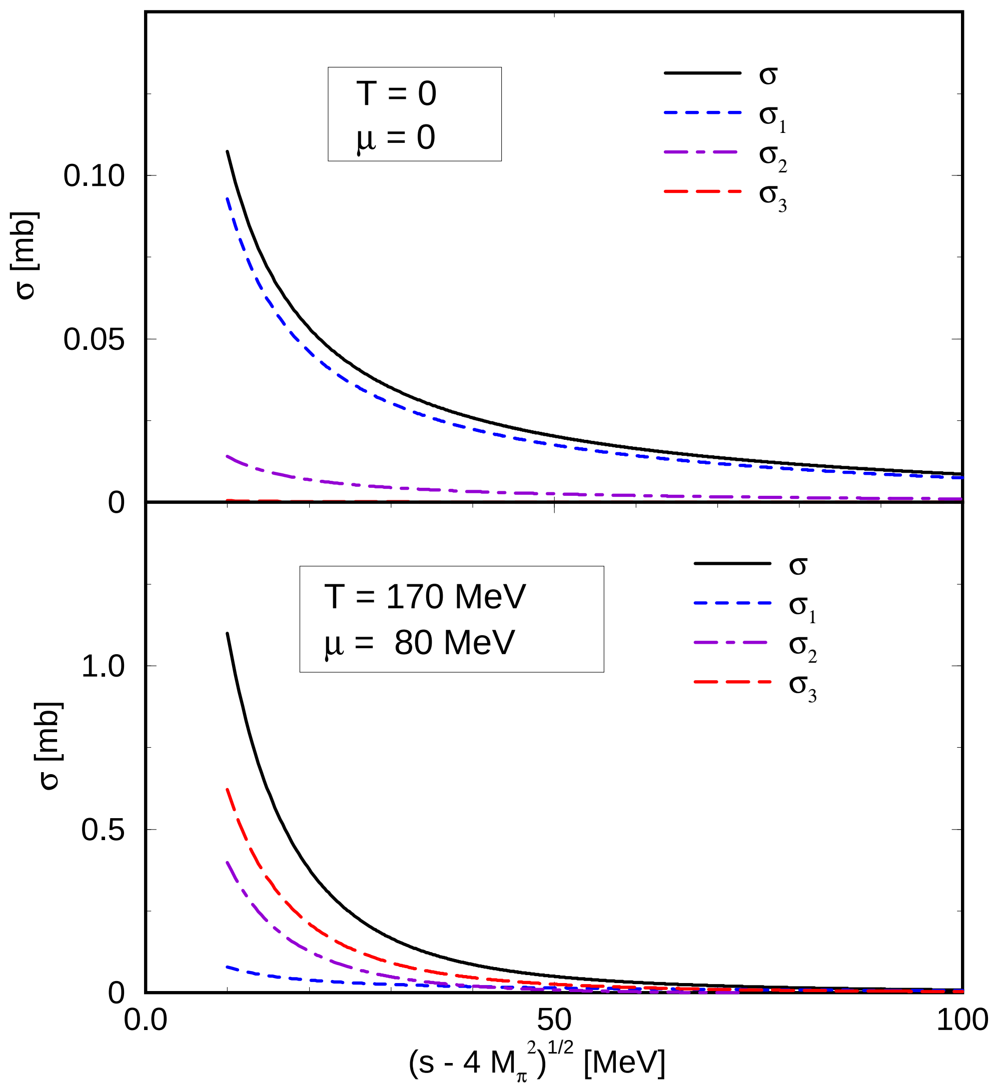
$$\mathcal{A}_{\pi^+\pi^-\to\gamma\gamma}(s)=\frac{1}{(6\pi f_\pi(T,\mu))^2}\left[\frac{40m^2(T,\mu)}{M_\sigma^2(T,\mu)-s-i\sqrt{s}\,\Gamma_\sigma(s|T,\mu)}f_1(\mu,T)-f_2(\mu,T)\right].$$

The contribution from neutral pions is

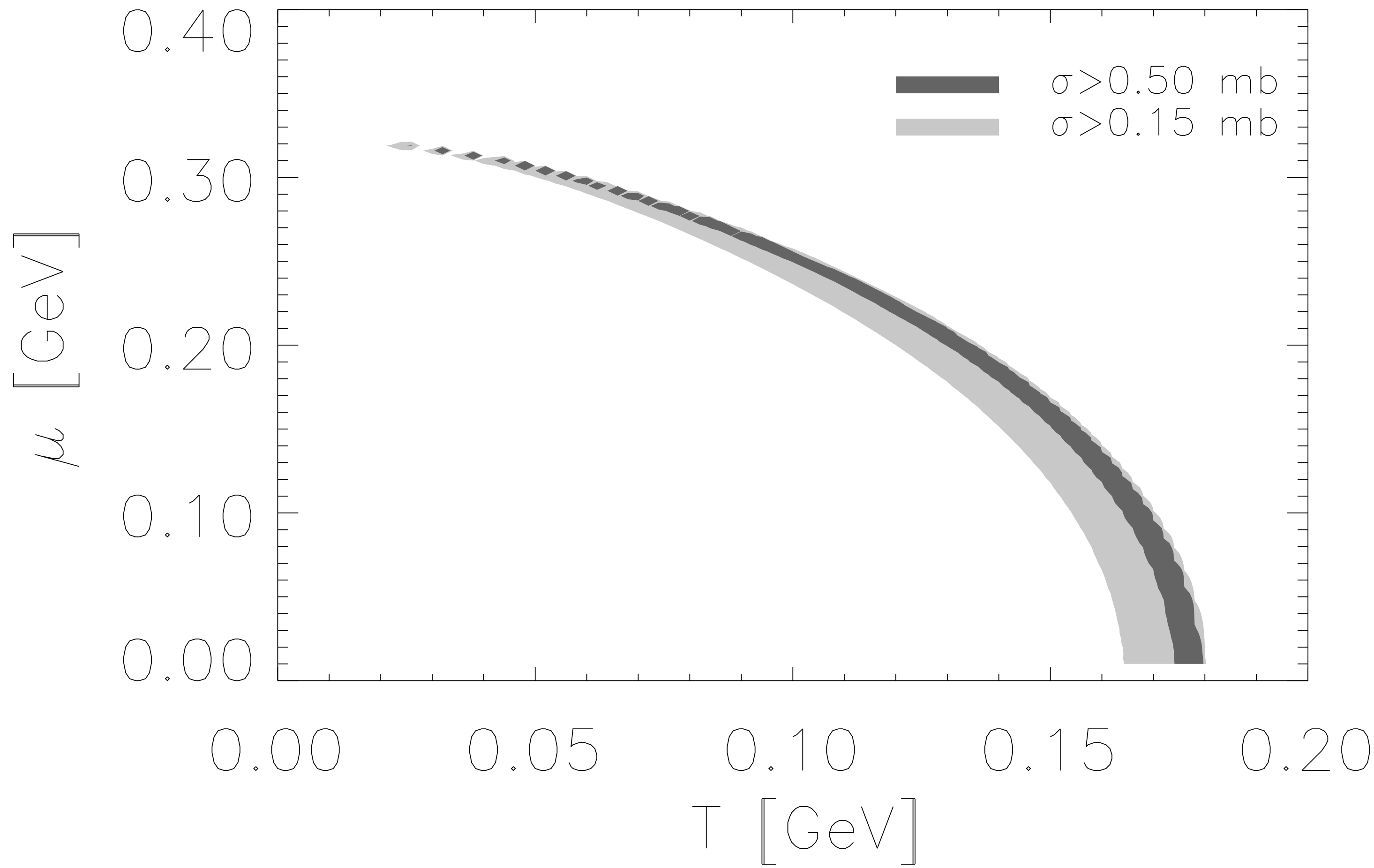
$$\sigma^{\pi^0\pi^0\to\gamma\gamma}(s)=\sigma_0\ s^2\ |\mathcal{A}_{\pi^0\pi^0\to\gamma\gamma}(s)|^2,$$

$$\mathcal{A}_{\pi^0\pi^0\to\gamma\gamma}(s)=\frac{1}{(6\pi f_\pi(T,\mu))^2}\left[\frac{40m^2(T,\mu)}{M_\sigma^2(T,\mu)-s-i\sqrt{s}\,\Gamma_\sigma(s|T,\mu)}f_1(\mu,T)-10f_2(\mu,T)\right].$$

Cross section



Cross section

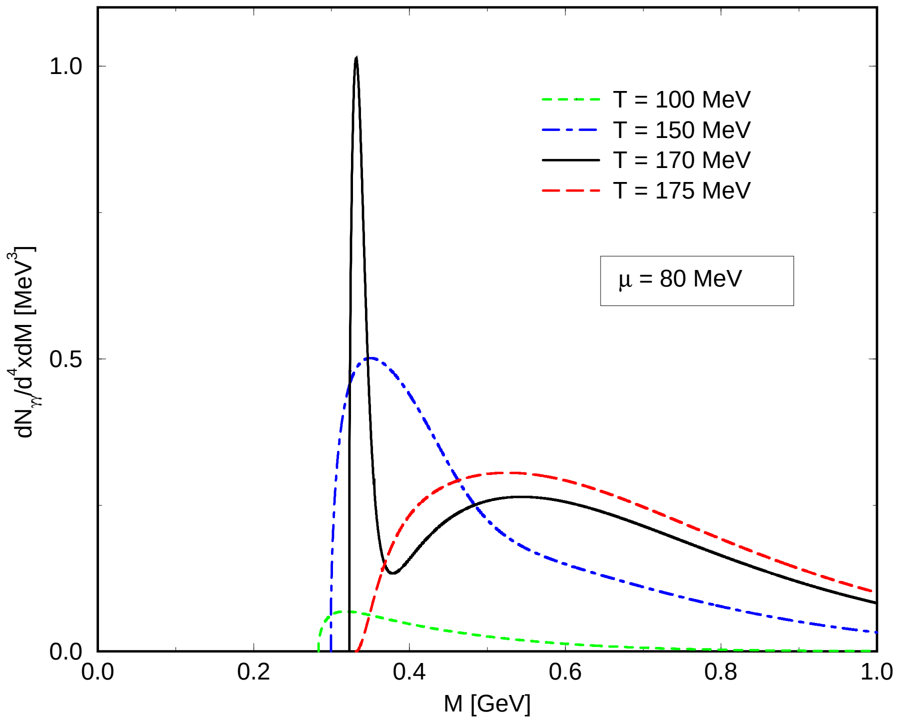


Photon production rate

The invariant-mass spectrum of produced photon pairs is given by

$$\frac{dN_{\gamma\gamma}}{d^4x dM} = 4M \int \frac{d^3q_1}{(2\pi)^3} \int \frac{d^3q_2}{(2\pi)^3} v_{\mathrm{rel}} \sigma^{\pi^+\pi^-\to\gamma\gamma} n_\pi(q_1) n_\pi(q_2) \delta\big(M^2 - (q_1+q_2)^2\big)$$

Here, $v_{\mathrm{rel}}(q_1, q_2) = \sqrt{1 - M_\pi^4/(q_1 \cdot q_2)^2}$ is the relative velocity, and $n_\pi(q) = \{\exp(\sqrt{q^2 + M_\pi^2}/T) - 1\}^{-1}$ is the pion distribution function.



Conclusion

According to the investigation of the photon pairs production from the pion annihilation, the results of which were published in paper M. K. Volkov, E. A. Kuraev, D. Blaschke, G. Röpke and S. M. Schmidt, Phys. Lett. B **424**, 235 (1998) , the in-medium modification of the σ-meson width and mass can lead to noticeable amplification of the low-energy photon production rate at certain conditions.

This excess of low-energy photons, if observable, may serve as an indicator of approaching the conditions at which the chiral symmetry is restored.

Similar effect can take place in other processes, e. g., in $\pi\pi \to \pi\pi$ as discussed in Ref. M. K. Volkov, A. E. Radzhabov and N. L. Russakovich, Phys. Atom. Nucl. **66**, 997 (2003) [Yad. Fiz. **66**, 1030 (2003)], arXiv:hep-ph/0203170.

Outlook

Further investigation

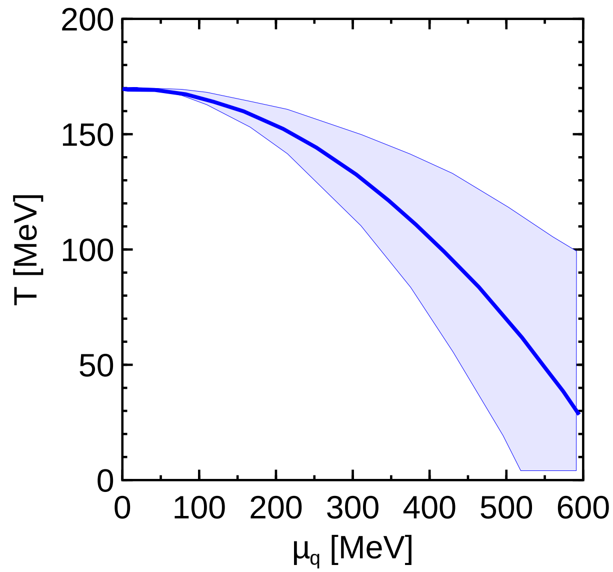
- T- and μ- dependent model parameters G and Λ
- Meson broadening from in-medium collisions
- Direct decay $\sigma \to \gamma\gamma$
- ...

T- and μ- dependent model parameters

The model parameters G and Λ can depend on the temperature and chemical potential. This dependence is not given by the NJL model and should taken from other sources (QCD, Lattice, phenomenology, ...).

Lattice QCD: $T_c \approx 170$ MeV, $\mu_c \sim 700$ MeV

C. R. Allton, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann and C. Schmidt, Nucl. Phys. Proc. Suppl. **141**, 186 (2005) [arXiv:hep-lat/0504011].

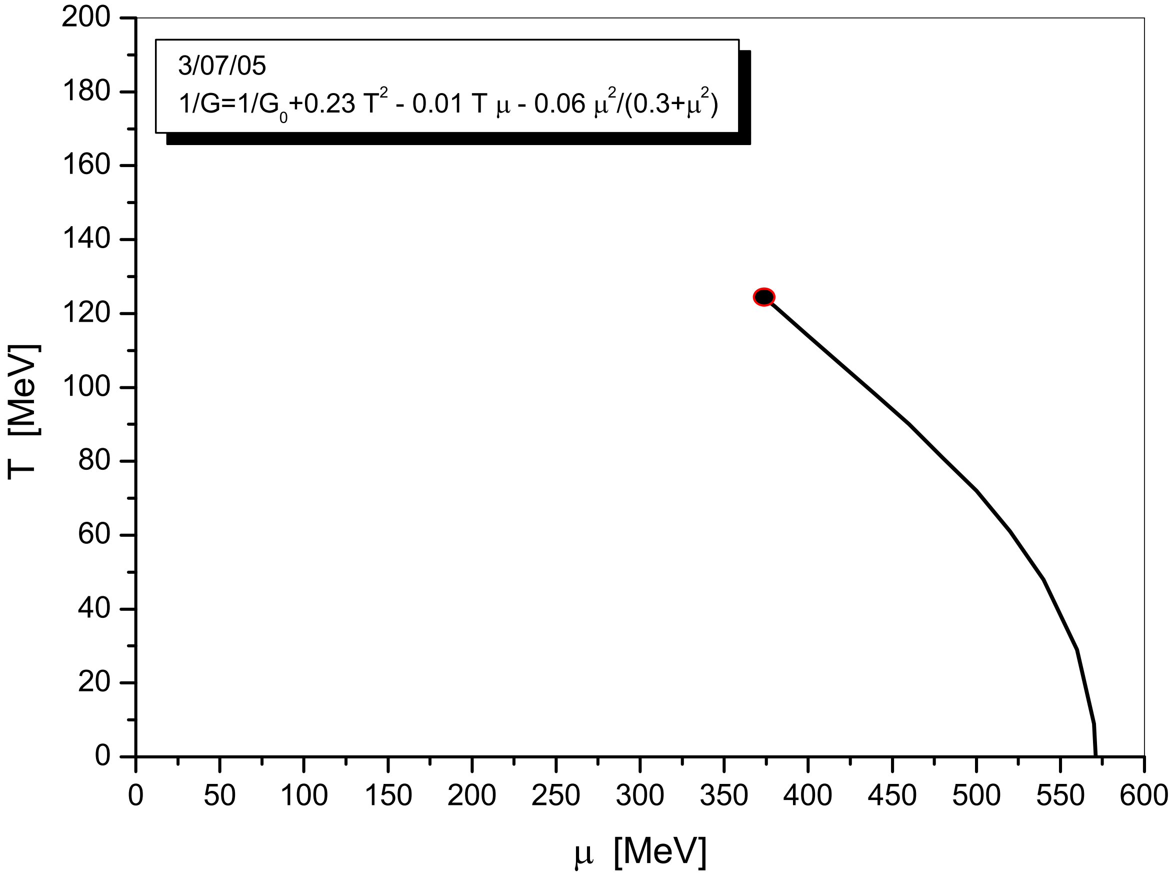


The critical temperature and chemical potentials can be shifted if the four-quark interaction constant G is replaced by an appropriate function of T and μ:

$$\frac{1}{G} \to \frac{1}{G(T,\mu)} = \frac{1}{G} + f_G(T,\mu),$$

where $f_G(T,\mu)$ is some function of dimension 2 such that $f_G(0,0) = 0$.

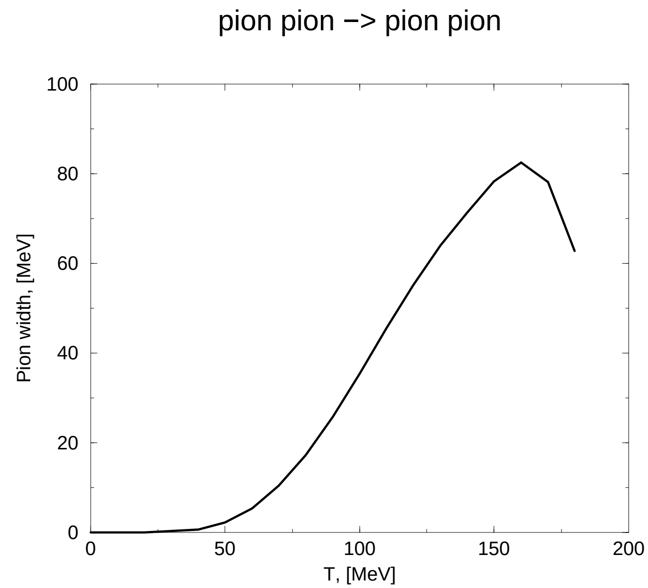
T- and μ- dependent model parameters



Meson broadening

In hot and/or dense medium, collisions may increase meson widths.

Within the approach described in [L. P. Kadanoff, G. Baym, Quantum Statistical Mechanics (W. A. Benjamin, Inc., N.Y., 1962)], the behavior of the pion width at finite temperature was investigated in D. Blaschke, M. K. Volkov and V. L. Yudichev, Phys. Atom. Nucl. **66**, 2233 (2003) [Yad. Fiz. **66**, 2285 (2003)], [arXiv:nucl-th/0303034].



An alternative approach was developed in Ref. H. van Hees and J. Knoll, Nucl. Phys. A **683**, 369 (2000) [arXiv:hep-ph/0007070]. The authors of Ref. T. Nishikawa, Y. Hidaka, M. Ohtani and O. Morimatsu, [arXiv:hep-ph/0310296] report on a pion width Γ_π of the order 100 MeV.

In hot and/or dense medium, the pion (σ-meson) can become a broad resonance due to collisions with other mesons. However, in non-equilibrium systems with short life-time, the existence of narrow resonances is not excluded.

Meson broadening

The cross section of $\pi\pi \to \gamma\gamma$ for broad pions

$$\sigma_{\pi\pi\to\gamma\gamma}(s) = \int ds_1 \int ds_2 \rho_\pi(s_1)\rho_\pi(s_2)\sigma^*_{\pi\pi\to\gamma\gamma}(s|s_1, s_2),$$

where $\sigma^*_{\pi\pi\to\gamma\gamma}(s|s_1, s_2)$ is the cross-section of the process $\pi\pi \to \gamma\gamma$ generalized for the case of off-shell pions with $s_i = q_i^2 \neq M_\pi^2$, $(i = 1, 2)$, and $\rho_\pi(s)$ is the spectral function of the pion, approximated with the Ansatz:

$$\rho_\pi(s) = N\frac{M_\pi \Gamma_\pi}{(s - M_\pi^2)^2 + M_\pi^2 \Gamma_\pi^2}, \quad \int \rho(s)\, ds = 1.$$

Direct decay $\sigma \to \gamma\gamma$

The decay width of the on-shell σ-meson

$$\Gamma(M_\sigma|T,\mu) = \frac{M_\sigma^3}{64\pi}\left(\frac{5\alpha f_1(T,\mu)}{3\pi F_\pi(T,\mu)}\right)^2 .$$

If the σ-meson has a noticeable width, it can decay off-shell. The invariant-mass spectrum of produced photon pairs is determined as

$$\frac{dN_{\sigma\to\gamma\gamma}}{d^4x\,dM} = 2M\rho_\sigma(M^2)\Gamma(M|T,\mu)\langle n_\sigma(T)\rangle,$$

where $\rho_\sigma(s)$ is the spectral function of the σ-meson

$$\rho_\sigma(s) = N\frac{M_\sigma\Gamma_\sigma}{(s-M_\sigma^2)^2 + M_\sigma^2\Gamma_\sigma^2}, \qquad \int \rho_\sigma(s)\,ds = 1,$$

and $\langle n_\sigma(T)\rangle$ is the average number of σ-mesons per unit volume.

Optimistic scenario for the σ-meson in $\pi\pi \to \gamma\gamma$

The life-time of the fireball essentially increases (~ 20 fm/c) if the it undergoes the first order phase transition.

Because of uncertainty in predictions for meson properties from quark models, it is not excluded that $M_\sigma \approx 2M_\pi$ in the conditions corresponding to the mixed phase.

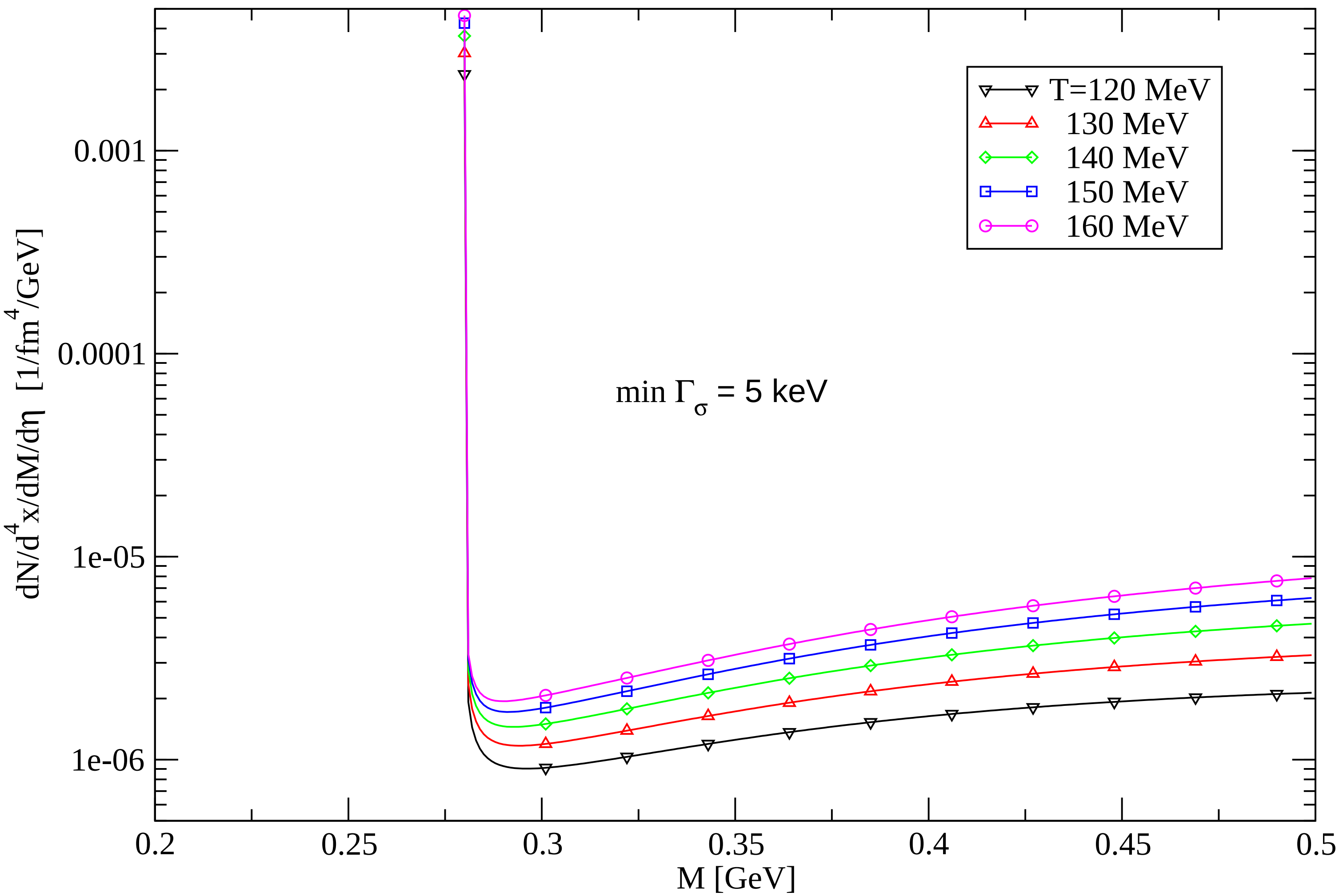
How much can be the contribution from a narrow σ-meson in this case, e. g., from Bjorken scenario?

Suppose:

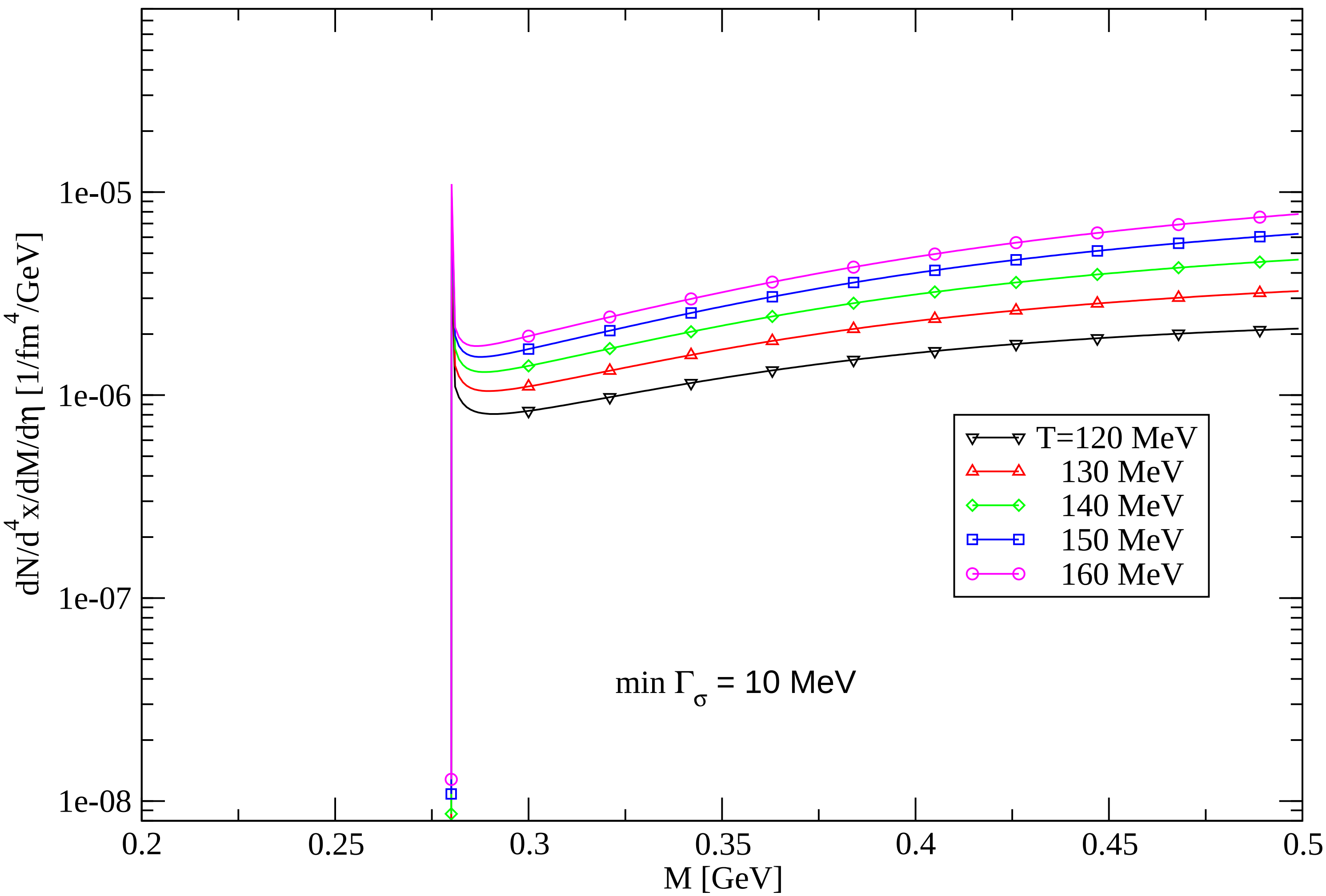
1. The pion mass does not change while in the hadron phase: $M_\pi(T,\mu) = M_\pi = 140$ MeV
2. The ratio $m/F_\pi \approx 4$ and does not change with T and μ.
3. The σ-meson width is undetermined and treated as an external parameter.
4. All particles are on their mass-shells.

$$\frac{dN_{\pi\pi\to\gamma\gamma}}{dM\,d\eta} \approx \frac{dN_{\pi\pi\to\gamma\gamma}}{d^4x dM\,d\eta} \cdot f \cdot V_4$$

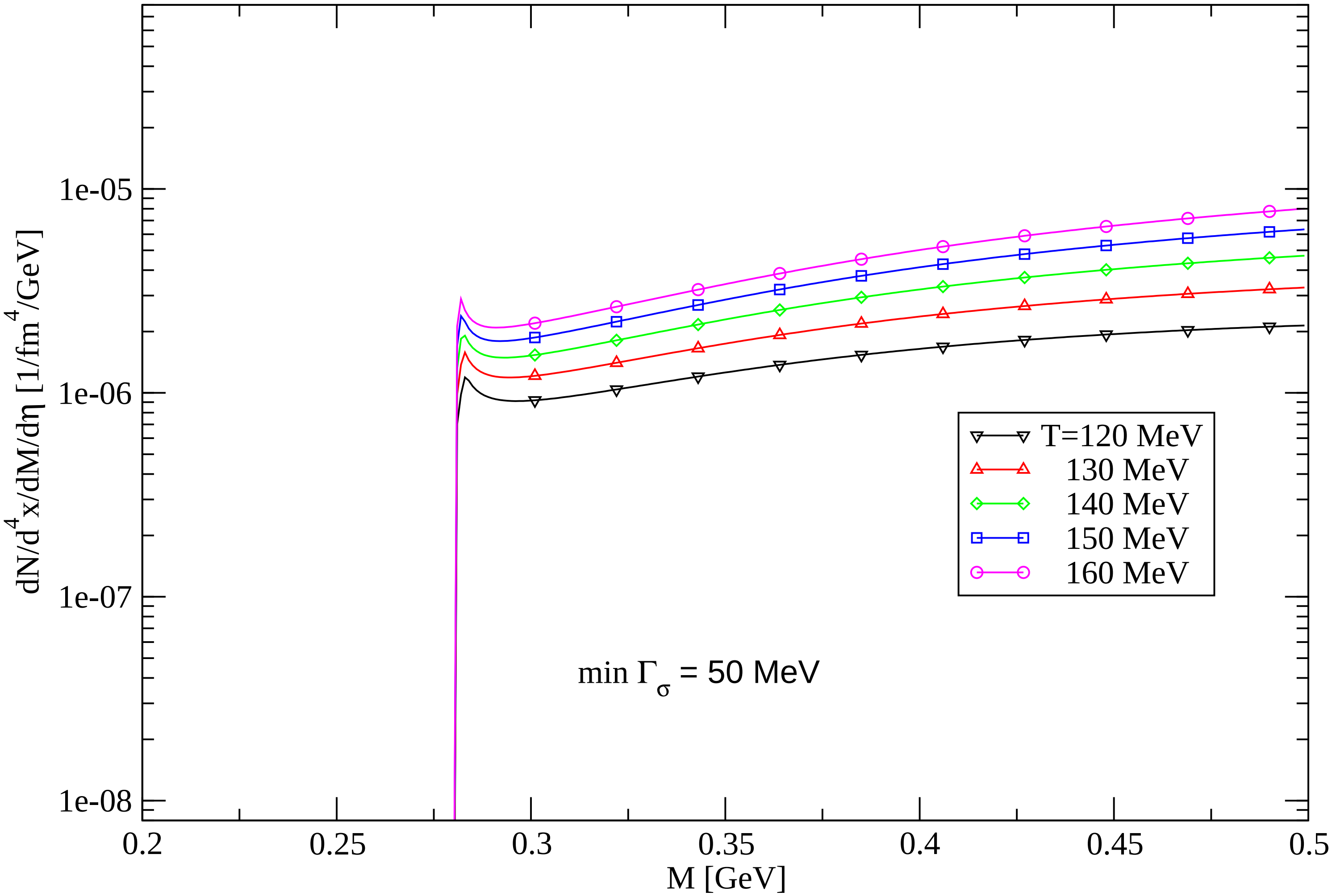
Optimistic scenario for the σ-meson in $\pi\pi \to \gamma\gamma$



Optimistic scenario for the σ-meson in $\pi\pi \to \gamma\gamma$



Optimistic scenario for the σ-meson in $\pi\pi \to \gamma\gamma$



Optimistic scenario for the σ-meson in $\pi\pi \to \gamma\gamma$