DIFFRACTION
WITHOUT MULTIPLE DIFRACTIVE DIPS

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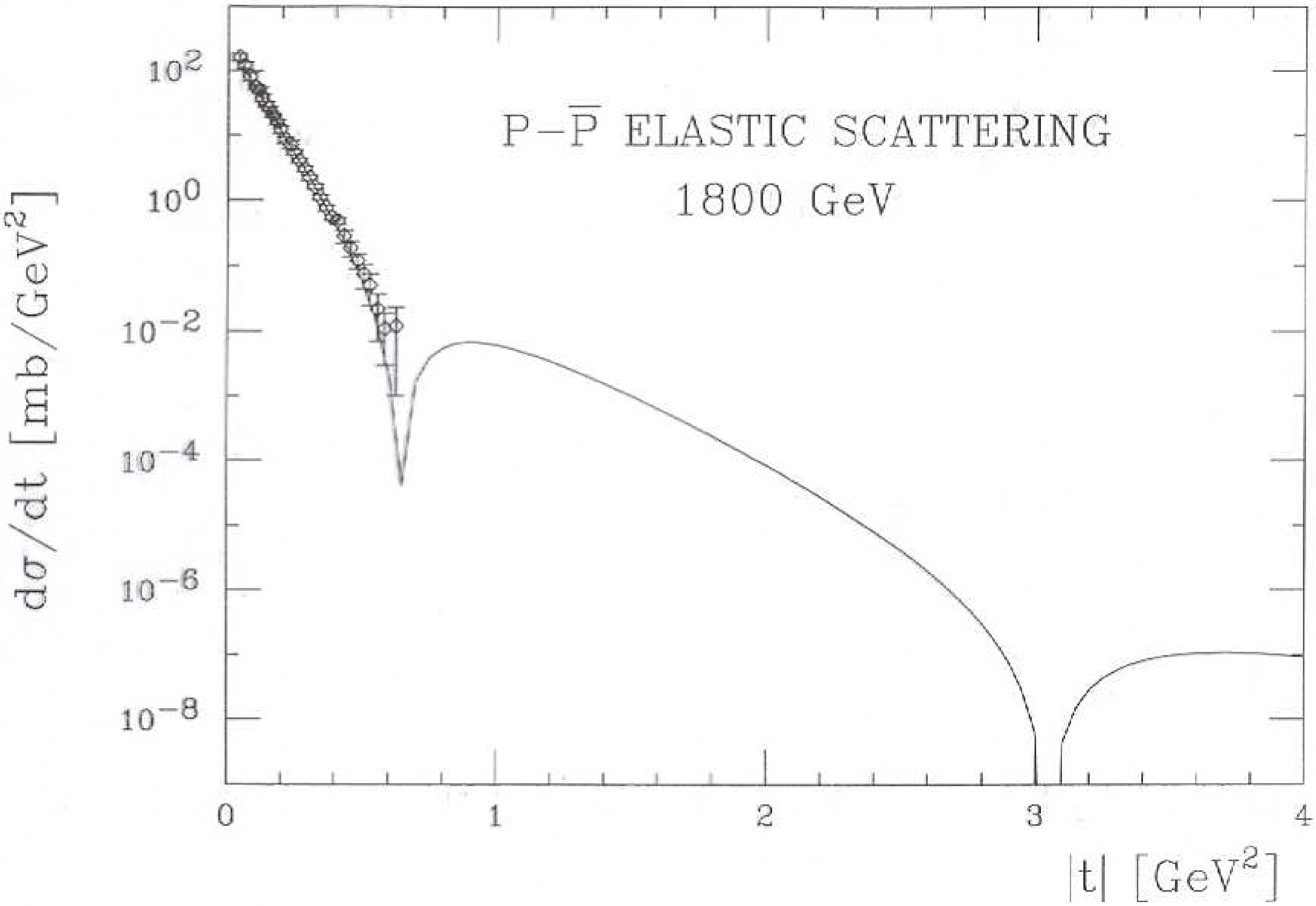
High energy hadron-hadron scattering is strongly absorptive: production of a very rich set of inelastic final states with direct two-body channels suppressed.

As a consequence of unitarity substantial elastic scattering takes place through a feed-back from the inelastic channels: shadow or diffraction scattering.

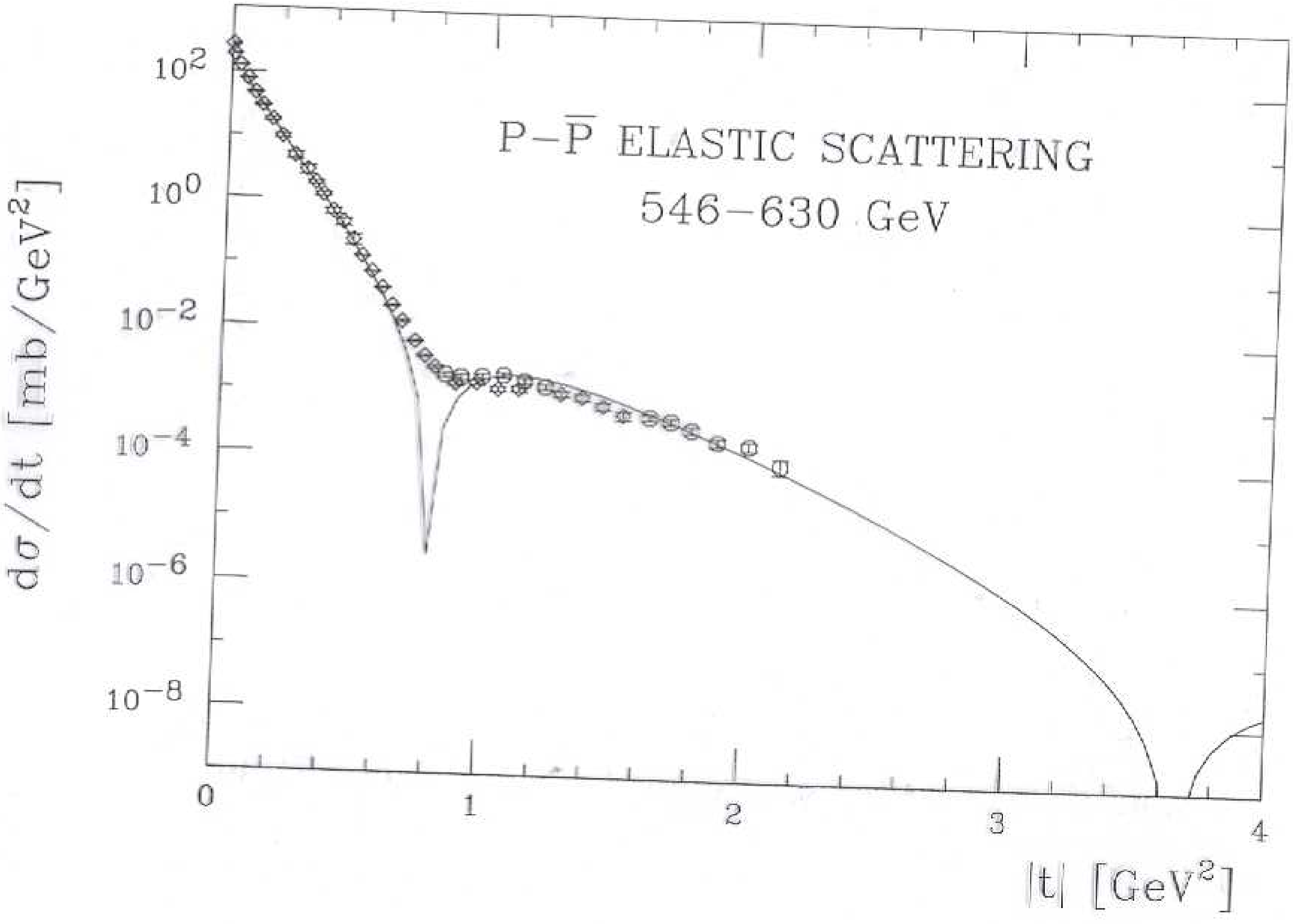
According to the optical analogy (suppression of the intensity of light by opaque/semitransparent obstacles) diffraction should be associated with multiple dips in elastic differential cross-section, but the behaviour of experimental cross-sections does not agree with standard ideas about diffraction. In particular, it does not agree with the geometrical models of scattering which were supposedly patented to work at high energies. In fact, experiments have seen no multiple dips. There is hardly any dip in p-$\bar{p}$ elastic cross-section at Tevatron ($\sqrt{s}$=1800 GeV) and SPS collider (546-630 GeV) while p-$\bar{p}$ and p-p cross-sections at ISR (20-60 GeV) have only one dip.

Elastic differential cross-sections of lightest nuclei from Saclay (several GeV) reveal one unambiguous dip.

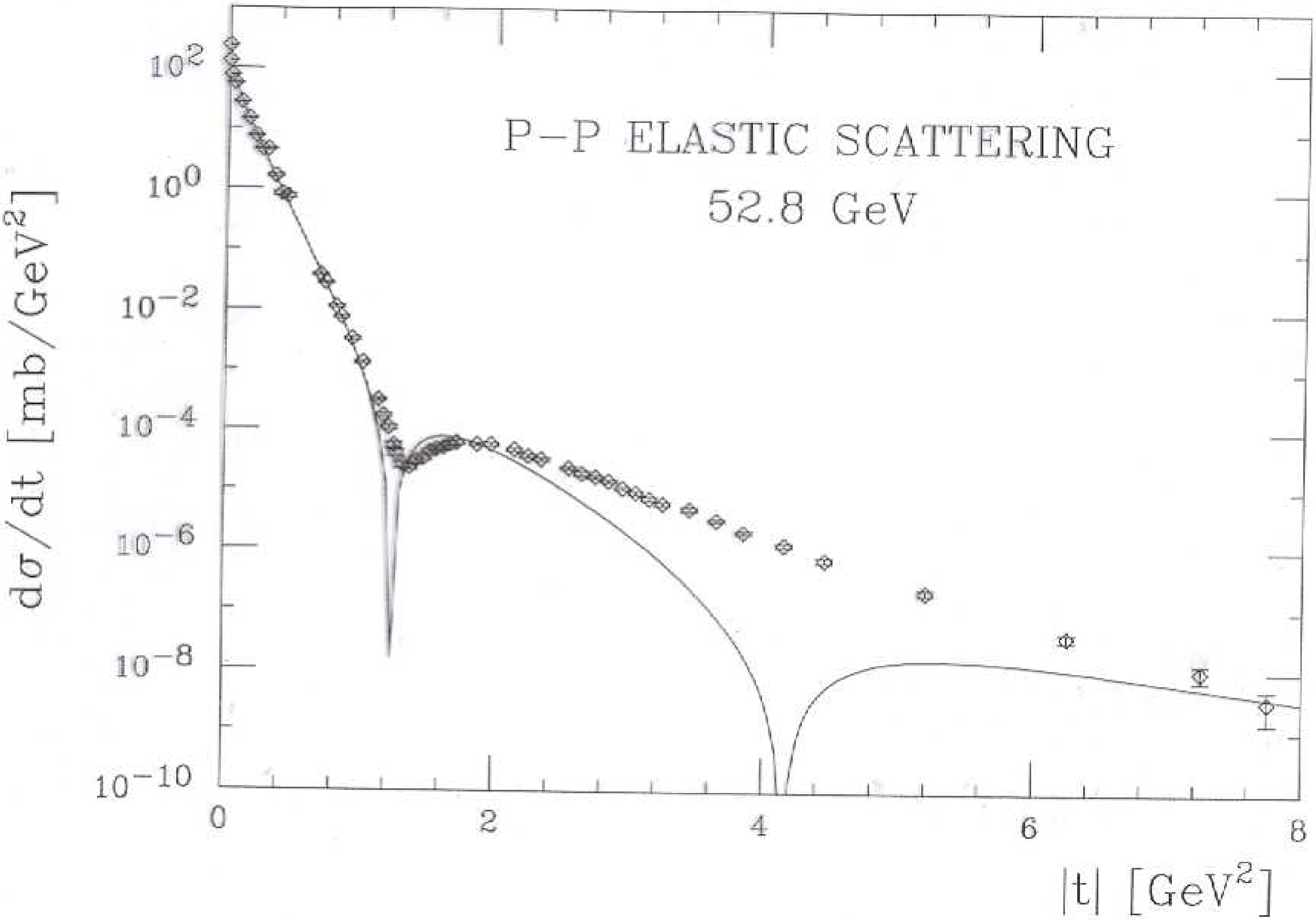
Experimental data constitute a puzzling problem: a diffractive phenomenon without characteristics of diffraction.

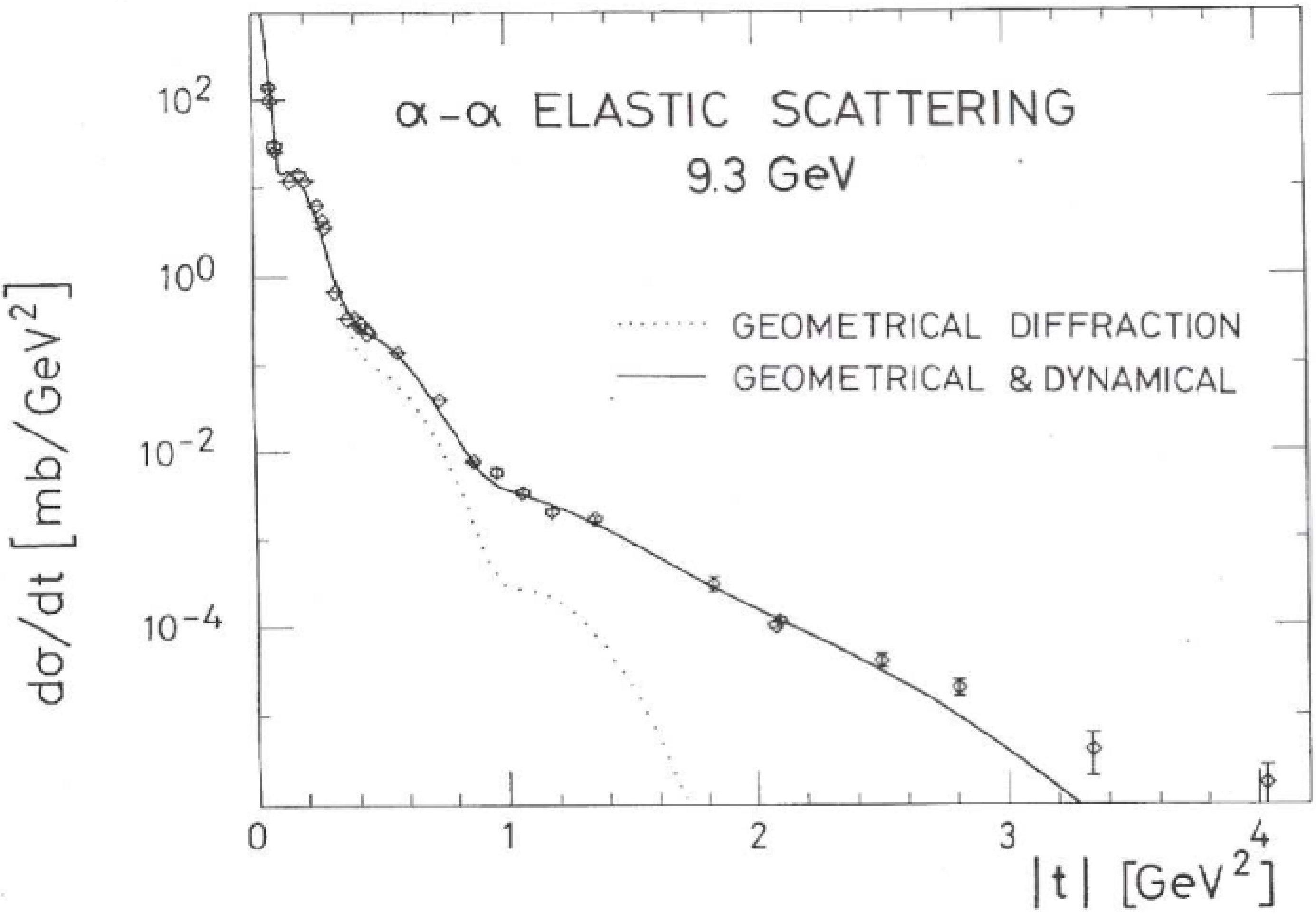


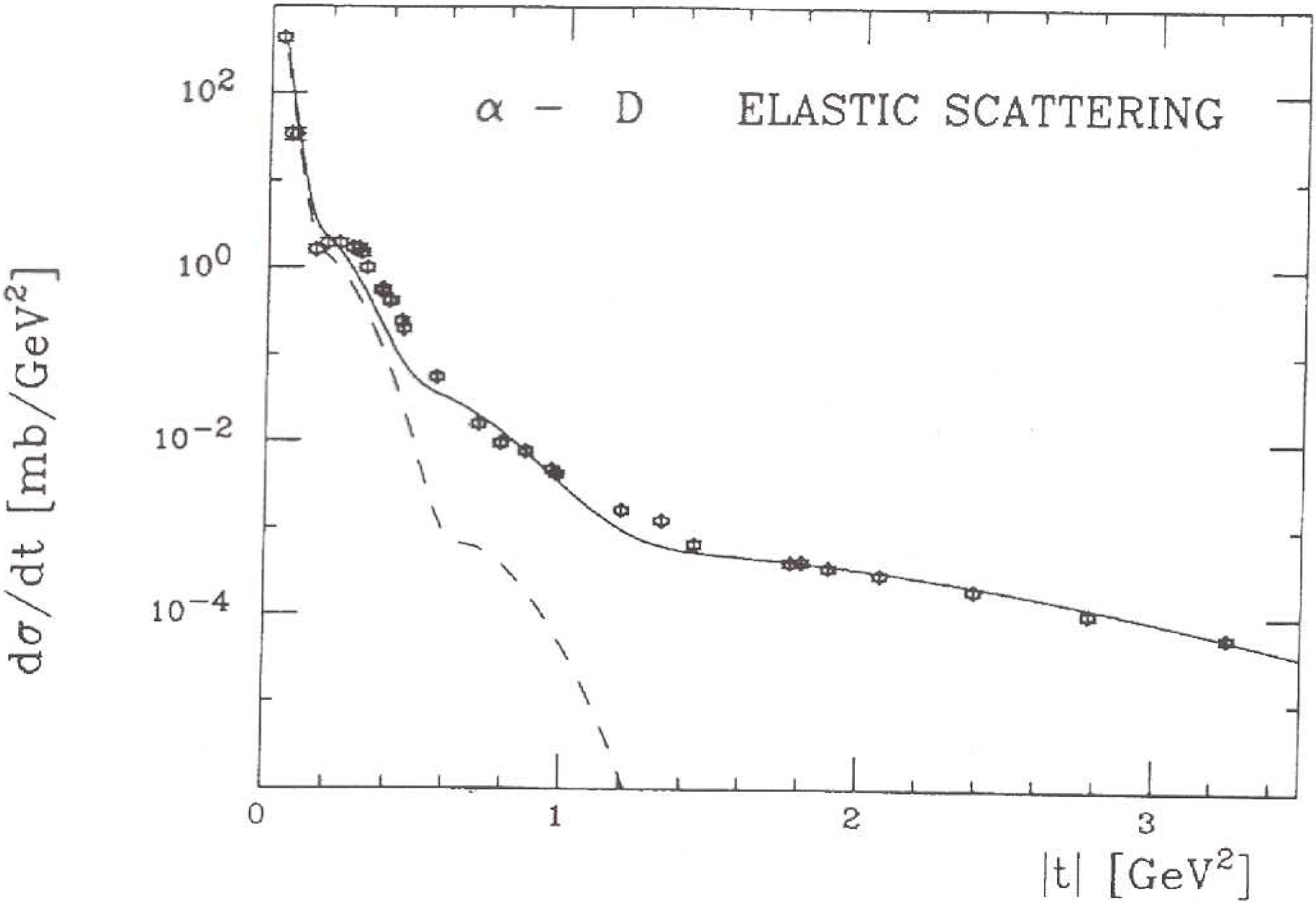
P-$\overline{P}$ ELASTIC SCATTERING
1800 GeV
$d\sigma/dt$ [mb/GeV^2]
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
1
2
3
4
|t| [GeV^2]



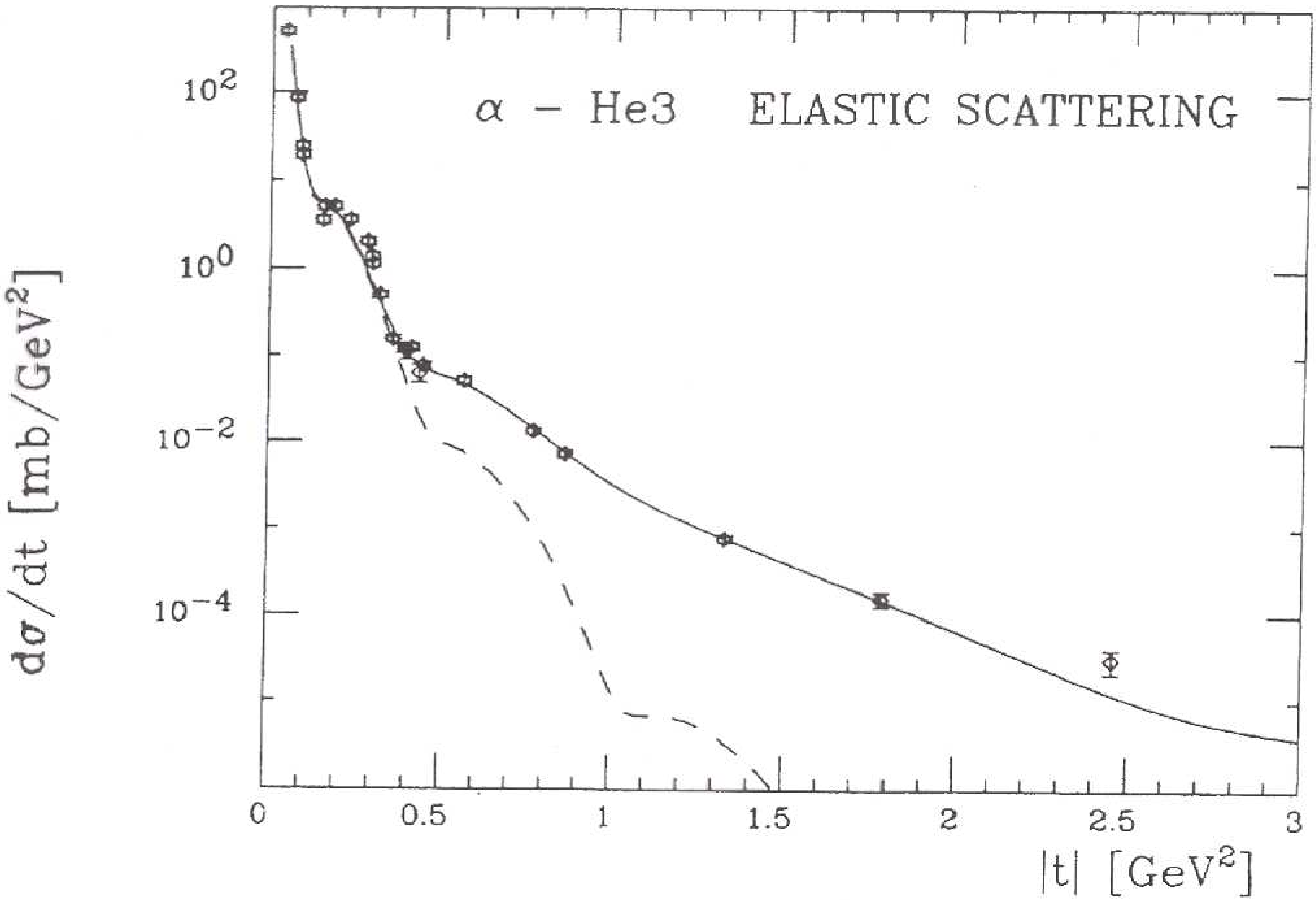
P−$\overline{\text{P}}$ ELASTIC SCATTERING
546−630 GeV
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$d\sigma/dt$ [mb/GeV^2]
0
1
2
3
4
$|t|$ [GeV^2]


P–P ELASTIC SCATTERING
52.8 GeV
$d\sigma/dt$ [mb/GeV^2]
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
2
4
6
8
$|t|$ [GeV^2]


α - α ELASTIC SCATTERING
9.3 GeV
GEOMETRICAL DIFFRACTION
GEOMETRICAL & DYNAMICAL
dσ/dt [mb/GeV²]
10²
10⁰
10⁻²
10⁻⁴
0
1
2
3
4
|t| [GeV²]



α − D ELASTIC SCATTERING
dσ/dt [mb/GeV²]
10²
10⁰
10⁻²
10⁻⁴
0
1
2
3
|t| [GeV²]



α – He3 ELASTIC SCATTERING
10^2
10^0
10^{-2}
10^{-4}
dσ/dt [mb/GeV^2]
0
0.5
1
1.5
2
2.5
3
|t| [GeV^2]

The puzzle of elastic diffraction would be resolved if there could be diffraction in the sense of unitarity driven shadow scattering which is not necessarily accompanied by multiple dips. This is what the experimental data seem unambiguously to be suggesting.

We claim that this is indeed the case also theoretically. It will be shown that structures in differential cross-sections depend crucially on the effective strength of the interaction governing the scattering process and hence on the characteristic length scales involved. Optical-like diffraction with its characteristic forward peak and multiple dip structure is governed by long distance dynamics and large values of the effective coupling strength. On the other hand, shadow scattering in hadron-hadron collisions at medium and large momentum transfer is governed by short distance dynamics and small values of the coupling strength.

We reach these conclusions by re-examining geometrical models and studying them as exact mathematical formulations. With this purpose in mind the model parameters are to be free to vary in all possible ways and not only within the bounds allowed by a reasonable fitting to the experimental data.

The results of our analysis are consistent with a two-component model of high energy elastic hadron scattering based on two sources of elastic diffraction: geometrical diffraction on an absorbing bulk and dynamic short range diffraction as the effect of intermediate dif-fractive [illegible]

In geometrical models of diffraction the eikonalized scattering amplitude is:

$$T(q)=\frac{i}{(2\pi)^2}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}\,T(b);\quad T(b)=1-\exp[-\Omega(b)]$$

$q=\sqrt{|t|}$ being the momentum transfer (c.m.) and $\Omega=\Omega(b)$ is the real, dimensionless opacity function depending on a relative impact parameter b.

Under an assumption of the rotational symmetry:

$$T(q)=\frac{i}{2\pi}\int_0^\infty db\, b\, J_0(qb)\,T(b)=-\frac{i}{2\pi}\int_0^\infty db\, b^2\,\frac{J_1(qb)}{qb}\,\frac{dT(b)}{db}$$

where the second form makes clear that in geometrical models elastic diffraction takes place mainly at the edge of the absorbing profile $T(b)$.

The optimistic conjecture was that opacity $\Omega(b)$ can be expressed in terms of hadronic shapes known from other experiments, e.g. in the Chou-Yang model was assumed that the Fourier transform of $\Omega(b)\sim F_{H_1}(t)F_{H_2}(t)\leftarrow$ product of the electromagnetic form factors of the colliding hadrons.

For simplicity we take these form factors as the extrapolations to all values of $|t|$ of the asymptotic quark model behaviour:

$$F_H(t)=\left(1+\frac{|t|}{m^2}\right)^{1-\nu_H}$$

where ν_H is the number of valence quarks in H, m being a mass scale parameter.

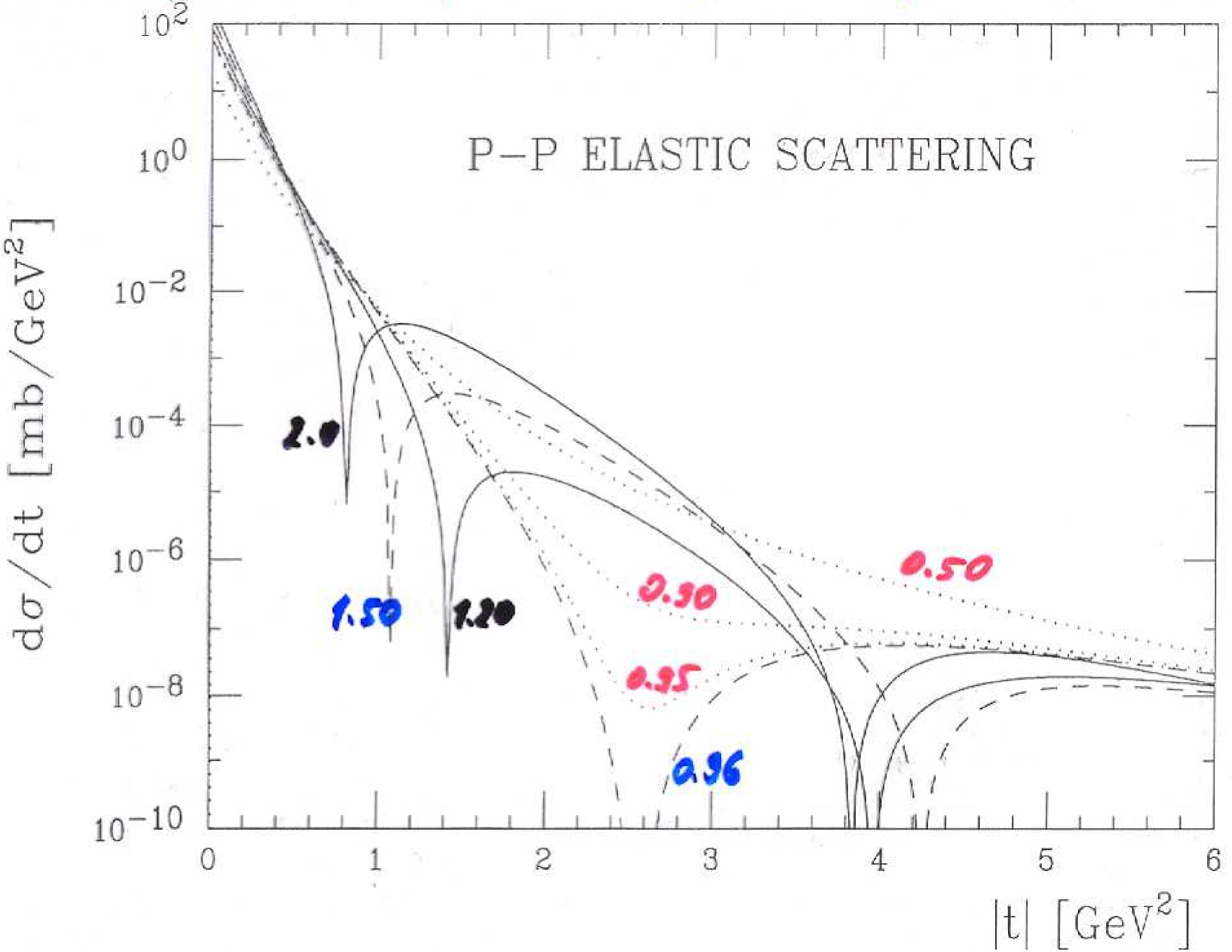
Then $\Omega(b)=g\,h_\nu(mb)$, $h_\nu=\frac{2}{\Gamma(\nu-3)}\left(\frac{mb}{2}\right)^{\nu-3}K_{\nu-3}(mb)$

where $\nu=\nu_{H_1}+\nu_{H_2}$, $K_\nu(mb)$ - the modified Bessel function and $g=\Omega(0)$ is a dimensionless coupling parameter.

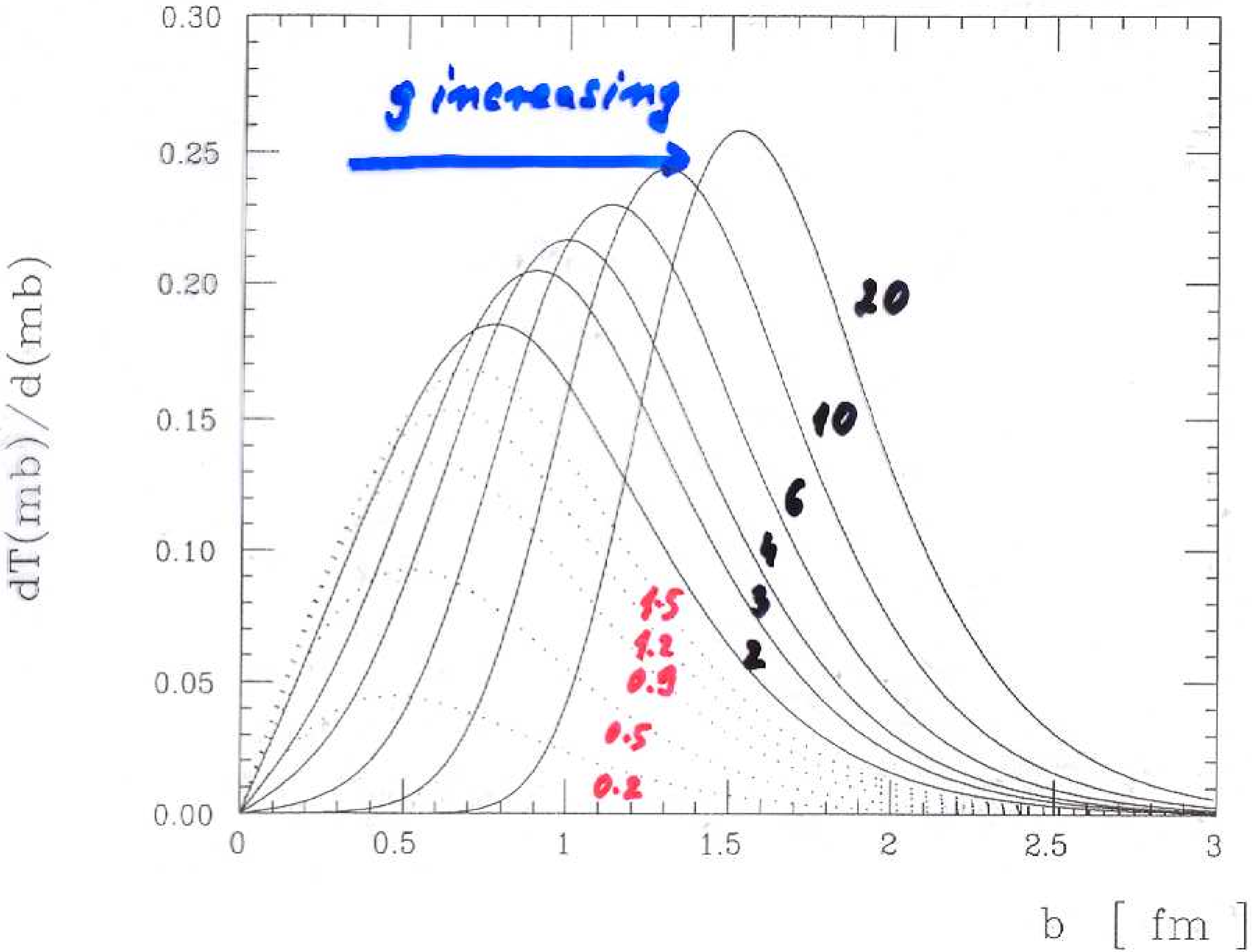
In standard phenomenological applications the proportionality constant $g(s)$ between opacity $\Omega(b)$ and the Fourier transform of the square of its form factor $F(-t)$ was adjusted to the experimental value of the total cross-section $\sigma_{tot}(s) = 8\pi^2 \operatorname{Im} T(s, t=0)$.

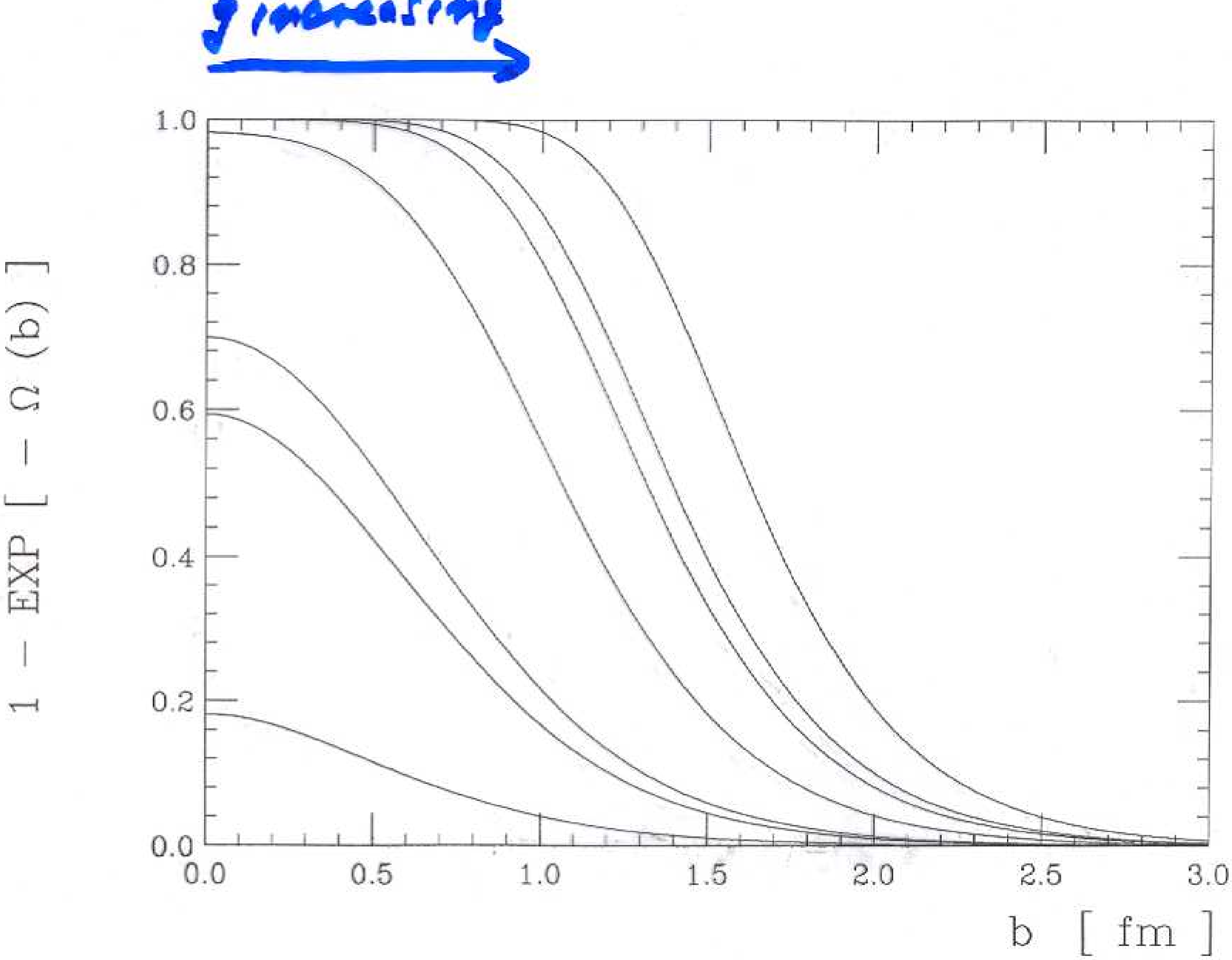
But the predictions of the Chou-Yang model are far richer than it had been usually recognized. When the experimental constraint is relaxed one finds that the model can accomodate a surprisingly wide range of behaviour of $d\sigma/dt$. Depending on the value of coupling constant g, the differential cross-section may manifest multiple dips, just one dip (a single minimum) or no dips (minima) at all.

The coupling constant g acts effectively as a control (or order) parameter. There exists a critical value $g_c = 0.96$ (independent of m) for which one has just one dip. For small values of the coupling the differential cross-section has neither dips nor minima. As g increases towards g_c the scattering amplitude develops a minimum which gets deeper and deeper becoming a zero of the amplitude. This zero is a double one: as g increases away from g_c at once two zeroes appear. The left zero, with increasing g, always moves towards lower $|t|$. The right zero, instead, first goes in the opposite direction but when $g > 1.50$ it turns back and farther the two zeroes follow the same way. At another critical value $g = 3.20$ the third zero (also double) appears in the differential cross-section dotted lines: g=0.50, 0.90, 0.95; dashed lines: g=0.96, 1.50; solid lines: g=1.20, 2.0



dotted lines: g=0.2, 0.5, 0.9, 1.2, 1.5, solid lines: g=2.0, 3.0, 4.0, 6.0, 10.0, 20.0




g increasing
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1 − EXP [− Ω (b)]
b [fm]

The number of dips appearing in the differential cross-section depends crucially on the strength of the coupling g.

Qualitatively it follows from the fact that since $\exp[-\Omega(b)]$ falls off exponentially with g, the large value of g allow for more oscillations of the Bessel function $J_0(qb)$. The many oscillations (growing with momentum transfer q) in the integrand determine the multiple dip structure of the differential cross-section. When g decreases, the effective number of oscillations is reduced and the dip structure slowly disappears.

A more quantitative insight can be gained from studying the derivative of the scattering profile

$$\frac{dT(mb)}{d(mb)} = \exp[-\Omega(mb)]\,\Omega(mb)\,\frac{K_{\nu-4}(mb)}{K_{\nu-3}(mb)}.$$

The stronger coupling g the closer is the profile $T(mb)$ to a sharp-edged absorbing disc. The disc radius and the sharpness of its edge are determined by the coupling g. The exponential factor above cuts off the low b part of $\Omega(b)$, doing this more effectively the greater is the value of g. Since the large value of coupling increases the height of $\Omega(b)$ in the limit $g \to \infty$ the profile $T(b) \to \Theta(R-b)$, which corresponds to scattering by black sharp disc of radius R.

The number of dips depends thus crucially on the strength of the coupling g. Another way of understanding this follows from the observation that it is the coupling parameter g that governs the decomposition of the scattering amplitude into a series of multiple collisions. Expanding the exponential $\exp[-\Omega(b)]$ in the scattering amplitude one obtains upon integration over the impact plane the S-matrix ($S = 1 + iT$) of elastic scattering:

$$S(q) = e^{\langle n \rangle} \sum_{n=0}^{\infty} (-1)^n P_n \, \delta^{(2)}(\vec{q} - \vec{q}_n)$$

This formula describes the distribution of all possible partitions of the momentum transfer $\vec{q}$. It is governed by the Poissonian $P_n = e^{-\langle n \rangle} \frac{\langle n \rangle^n}{n!}$ with the mean value $\langle n \rangle = g$. Since the terms of the series alternate in sign, only for very small values of the coupling constant $g \ll 1$ the scattering amplitude is a positive function of q. But for values of g close to unity and larger the amplitude has zeroes which give rise to the dips.

The coupling g turns out to be a kind of geometrical size parameter determining the hadronic size radius and the sharpness of its edge. If the appearance of dips is interpreted as a classical-like phenomenon, g operates in a way similar to the inverse of Planck's constant $\hbar$: $g \to \infty$ corresponds to large distances in the same way as $\hbar \to 0$ corresponds to a classical macroscopic region.

On the other hand, the limit $g \to 0$ corresponds to $\hbar \to \infty$ which means a regime where quantum effects are not at all negligible. In this "microscopic" limit the absorbing disc scattering, operative at large distances, should be replaced by point-like scattering at short distances, i.e. inside the geometrical obstacle.