RESIDUE Package for REDUCE

Wolfram Koepf
email: Koepf@zib.de

April 1995 : ZIB Berlin

This package supports the calculation of residues. The residue $\operatorname{Res}_{z=a} f(z)$ of a function $f(z)$ at the point $a \in \mathbb{C}$ is defined as

$$\operatorname*{Res}_{z=a} f(z) = \frac{1}{2\pi i} \oint f(z)\, dz \,,$$

with integration along a closed curve around $z = a$ with winding number 1.
If $f(z)$ is given by a Laurent series development at $z = a$

$$f(z) = \sum_{k=-\infty}^{\infty} a_k \,(z-a)^k \,,$$

then

$$\operatorname*{Res}_{z=a} f(z) = a_{-1} \,. \tag{1}$$

If $a = \infty$, one defines on the other hand

$$\operatorname*{Res}_{z=\infty} f(z) = -a_{-1} \tag{2}$$

for given Laurent representation

$$f(z) = \sum_{k=-\infty}^{\infty} a_k \,\frac{1}{z^k} \,.$$

The package is loaded by the statement

```
1: load residue;
```

It contains two REDUCE operators:

- `residue(f,z,a)` determines the residue of f at the point $z = a$ if f is meromorphic at $z = a$. The calculation of residues at essential singularities of f is not supported.
- `poleorder(f,z,a)` determines the pole order of f at the point $z = a$ if f is meromorphic at $z = a$.

Note that both functions use the `taylor` package in connection with representations (1)–(2).

Here are some examples:

```
2: residue(x/(x^2-2),x,sqrt(2));

 1
---
 2

3: poleorder(x/(x^2-2),x,sqrt(2));

1

4: residue(sin(x)/(x^2-2),x,sqrt(2));

 sqrt(2)*sin(sqrt(2))
----------------------
          4

5: poleorder(sin(x)/(x^2-2),x,sqrt(2));

1

6: residue(1/(x-1)^m/(x-2)^2,x,2);

 - m

7: poleorder(1/(x-1)/(x-2)^2,x,2);

2

8: residue(sin(x)/x^2,x,0);
```

```
1

9: poleorder(sin(x)/x^2,x,0);

1

10: residue((1+x^2)/(1-x^2),x,1);

-1

11: poleorder((1+x^2)/(1-x^2),x,1);

1

12: residue((1+x^2)/(1-x^2),x,-1);

1

13: poleorder((1+x^2)/(1-x^2),x,-1);

1

14: residue(tan(x),x,pi/2);

-1

15: poleorder(tan(x),x,pi/2);

1

16: residue((x^n-y^n)/(x-y),x,y);

0

17: poleorder((x^n-y^n)/(x-y),x,y);

0
```

```
18: residue((x^n-y^n)/(x-y)^2,x,y);

  n
 y *n
------
  y

19: poleorder((x^n-y^n)/(x-y)^2,x,y);

1

20: residue(tan(x)/sec(x-pi/2)+1/cos(x),x,pi/2);

-2

21: poleorder(tan(x)/sec(x-pi/2)+1/cos(x),x,pi/2);

1

22: for k:=1:2 sum residue((a+b*x+c*x^2)/(d+e*x+f*x^2),x,
    part(part(solve(d+e*x+f*x^2,x),k),2));

 b*f - c*e
-----------
     2
    f

23: residue(x^3/sin(1/x)^2,x,infinity);

  - 1
------
  15

24: residue(x^3*sin(1/x)^2,x,infinity);

-1
```

Note that the residues of factorial and Γ function terms are not yet supported.