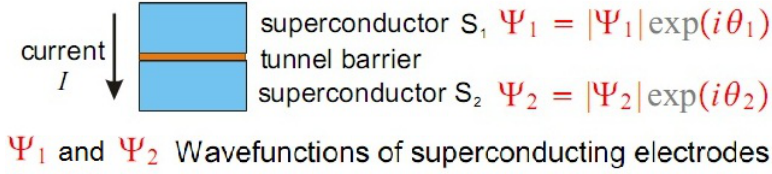


# Modeling of phase dynamics of Josephson Junction

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Obtained by B. Josephson in 1962



*dc* Josephson effect:  $I_s(\varphi) = I_c \sin \varphi \quad (1)$

*ac* Josephson effect:  $\frac{d\varphi}{dt} = \frac{2e}{\hbar} V \quad (2)$

Phase difference:  $\varphi = \theta_2 - \theta_1$

From (2) it follows that  $\varphi = \frac{2e}{\hbar} V t + \varphi_0$ . Thus,  $I_s$  oscillates

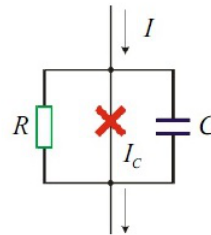
with frequency  $f = \frac{2e}{2\pi\hbar} V = \frac{1}{\Phi_0} V, \quad (3)$

where  $\Phi_0 = 2.068 \times 10^{-15} \text{ Wb}$  is the magnetic flux quantum.

Josephson junction is a *quantum dc voltage - to - frequency converter*

$$1 \mu\text{V} \leftrightarrow 483.59767 \text{ MHz}$$

*RCSJ*  $\equiv$  "resistive-capacitive shunted junction"



Trough system follows  $I_{qp} = \frac{V}{R}$ -quasiparticle,  $I_{disp} = C \frac{dV}{dt}$ -displacement and  $I_s = I_c \sin \varphi$ -superconducting currents.

$$I = I_{disp} + I_{qp} + I_s$$

$$I = C \frac{dV}{dt} + I_c \sin \varphi + \frac{V}{R}$$

$$\begin{cases} \frac{\hbar}{2e} \frac{\partial \varphi}{\partial t} = V \\ I = C \frac{dV}{dt} + I_c \sin \varphi + \frac{V}{R} \end{cases}$$

Now we should normalize system of Equations, using following parameters:

$$V_0 = \frac{\hbar\omega_p}{2e}; \tau = \omega_p t; \omega_p = \sqrt{\frac{2eI_c}{C\hbar}}; \beta = \frac{1}{R}\sqrt{\frac{\hbar}{2eI_c C}}; \frac{I}{I_c} \rightarrow I; \frac{V}{V_0} \rightarrow V$$

If we consider JJ under external radiation, then trough system follows additional curent  $I_{Rad} = A \sin(\omega t)$ .  
Normalized system of equations

$$\left\{ \begin{array}{l} \frac{\partial \varphi}{\partial t} = V \\ I = \frac{dV}{dt} + \sin \varphi + \beta \frac{\partial \varphi}{\partial t} \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} \frac{\partial \varphi}{\partial t} = V \\ \frac{dV}{dt} = I - \sin \varphi - \beta \frac{\partial \varphi}{\partial t} \end{array} \right.$$

We solve above system of equations using fourth order Runge–Kutta method

$$\left\{ \begin{array}{l} \varphi^{j+1} = \varphi^j + \Delta \varphi^j \\ V^{j+1} = V^j + \Delta V^j \end{array} \right. \quad \left\{ \begin{array}{l} \Delta \varphi^j = \frac{1}{6}(P_1^j + 2P_2^j + 2P_3^j + P_4^j) \\ \Delta V^j = \frac{1}{6}(K_1^j + 2K_2^j + 2K_3^j + K_4^j) \end{array} \right.$$

First Runge–Kutta coefficients

$$\left\{ \begin{array}{l} P_1 = V h_t \\ K_1 = \{I - \sin \varphi\} h_t - \beta P_1 \end{array} \right.$$

Second Runge–Kutta coefficients

$$\left\{ \begin{array}{l} P_2 = \left\{ V + \frac{K_1}{2} \right\} h_t \\ K_2 = \left\{ I - \sin \left( \varphi + \frac{P_1}{2} \right) \right\} h_t - \beta P_2 \end{array} \right.$$

Third Runge–Kutta coefficients

$$\left\{ \begin{array}{l} P_3 = \left\{ V + \frac{K_2}{2} \right\} h_t \\ K_3 = \left\{ I - \sin \left( \varphi + \frac{P_2}{2} \right) \right\} h_t - \beta P_3 \end{array} \right.$$

Fourth Runge–Kutta coefficients

$$\left\{ \begin{array}{l} P_4 = \{V + K_3\} h_t \\ K_4 = \{I - \sin(\varphi + P_3)\} h_t - \beta P_4 \end{array} \right.$$