

Financial Institutions and Services



# Financial Markets

Recent Developments, Emerging Practices  
and Future Prospects

*Mohsen Bahmani-Oskooee, Ph.D.*

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*Editors*

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**FINANCIAL INSTITUTIONS AND SERVICES**

# **FINANCIAL MARKETS**

## **RECENT DEVELOPMENTS, EMERGING PRACTICES AND FUTURE PROSPECTS**

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**MOHSEN BAHMANI-OSKOOEE, PH.D.**

AND

**SAHAR BAHMANI, PH.D.**

**EDITORS**



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## PREFACE

Financial markets are said to be an important source of providing capital to other sectors in every economy. Their establishment, operation, development, and practices have important implications in the development process of every country. There are also challenges that every country must deal with. Institutional rigidities in some countries could curtail financial development and introduce severe challenges. Dealing with the challenges and introducing new practices to smooth out the operation of the financial markets is an important goal. This book focuses on some of the issues related to recent developments in financial markets, their rigidities, and implications for the future of these markets in several chapters.

Chapter 1 - In this chapter Mohsen Bahmani-Oskooee and Ruixin Zhang investigate whether financial development has any implication on income distribution. They argue that in countries where financial markets are well developed, opportunities are higher as compared to countries where opportunities are less. Within an individual country, not everyone has the same opportunities to accumulate wealth. Thus, it is possible for income distribution to get worse or better with the improvement in financial markets. Clearly, if more people understand the markets and invest, income distribution is expected to improve. Otherwise, if only a limited number of individuals engage in investment activities, income distribution could get worse. In this chapter, the authors test this hypothesis by using a measure of financial development and its impact on income distribution in Australia, Austria, Belgium, Bolivia, Canada, Denmark, Ecuador, Egypt, Finland, Greece, India, Israel, Italy, Japan, Kenya, Korea, Mexico, Malta, Malaysia, Netherlands, Norway, the Philippines, Senegal, Singapore, Turkey, and the U.S. They find support for both views.

Chapter 2 - In this chapter Paulo Parracho, Antonio Canelas, Rui Neves, and Nuno Horta describe a new computational finance approach which combines pattern recognition techniques with an evolutionary computation kernel applied to financial markets time series. The goal is to optimize trading strategies. Moreover, for pattern matching a template-based approach is used in order to describe the desired trading patterns. The parameters for the pattern templates, as well as for the decision making rules, are optimized using a genetic algorithm kernel. The approach is tested considering actual data series and presents a robust profitable trading strategy which clearly beats the market, S&P 500 index, reducing the investment risk significantly. In order to reduce portfolio risks, a low volatility selection stocks strategy is tested. A study on outlier removal to improve data series quality is also presented.

Chapter 3 - A modern trend in equity investing is to combine a low risk objective with fundamental portfolio weighting. In this chapter, Kris Boudt and Marjan Wauters provide examples that include the S&P's GIVI index and the RAFI low volatility index. The chapter aims at investigating the gains in combining low-risk and fundamentally weighted portfolios for the universe of S&P 500 constituents over the period 1987-2012. The authors find that, compared to the market capitalization weighted benchmark (the S&P 500), there are economically significant gains in low risk and fundamental investing for the complete sample. Combining the two investment styles reduces the market regime dependence of the portfolio performance. The choice of allocation scheme has a minor impact on the gross portfolio performance, but matters for the portfolio turnover.

Chapter 4 - In this chapter Andrew Clark discusses alternative investments in a multi-asset portfolio. He introduces the topic by touching on the similarity and differences between the retail and institutional use of alternatives. A definition of alternatives is given for retail investors, and a brief examination of how retail investors and financial advisors can use alternatives is presented. Institutional alternatives are defined, as were the initial steps typically taken to evaluate private equity, real estate, and hedge fund purchases. In the last section, the author deals with risk in its various forms. The measurement of private equity and real estate risk is examined as is liquidity risk.

Chapter 5 - In this chapter Hui Ding and Jaeboem Kim investigate whether inflation targeting (IT) macroeconomic policy matters for real interest rate parity (RIP). They employ two panel unit root tests with and without cross-sectional dependence to examine the stationarity of real interest rate differentials for panel data on eleven OECD countries from 1974Q1 to 2011Q3. Comparisons are made, together with CPI and PPI, between IT and non-IT, and with and without cross-sectional dependence. Their empirical results present favorable evidence for RIP especially in countries that adopted IT. Moreover, when it comes to RIP, IT seems to outweigh other issues, including the choice of price indices, the choice of base currencies, and cross-sectional dependence, proposed by previous studies.

Chapter 6- Almost all known econometric models applied on a long term basis on financial forex market does not work sufficiently. The reason is that transaction costs and arbitrage opportunity are not included which is not a simulation of the real financial markets. Analysis is not done on the non equidistance date but rather on the aggregate date which is also not a real financial case. Almost all known prediction models are not stable for a long time in trading on the financial forex market. In this chapter Richard Pincak shows a new way as to how to analyze and moreover forecast financial markets. He utilizes the projections of the real exchange rate dynamics onto the string-like topology. The approach is inspired by the contemporary movements in the string theory. Inter-strings information transfer is analyzed as an analogy with the dynamics of prices or currency at specified exchange rate options.

Chapter 7- This chapter by Ekaterina Svetlova and Karl-Heinz Thielmann aims to improve the insufficient communication of risks in financial markets, particularly in the retail sector. The communication of risks is usually based on only one number, usually related to volatility, or on an endless list of risks. The authors suggest a tool for communicating investment risks that combines four types of risk, is simple and easy to understand, and takes investors' time perspective into consideration. The authors believe that the simplicity of this communication tool renders it particularly effective in explaining and communicating risks to retail clients.

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Chapter 8 - This chapter is about mergers and acquisitions (M&A) in China. Relatively little is known in western countries about the legal framework under which M&A activities are conducted in China. Yet, a detailed understanding of the rules and regulations that govern this area is essential if the research agenda on Chinese M&A activities, which is now emerging in the western literature, is to be properly conducted. Xiaojing Song and Mark Tippett provide a detailed summary of the laws and regulations as they affect the conduct of M&A activities in China. They begin with a brief summary of China's main stock exchanges and their listing requirements. They then outline some of the unique characteristics of Chinese capital markets, in particular the distinction between "A" shares and "B" shares issued by Chinese firms. The Takeover Measures, which is the principal law regulating M&A activities in China, is then considered. The focus is on the mandated bid rules, the disclosure of substantial shareholdings, the tender offer rules and the defense mechanisms, which may be used in Chinese M&A activities. The authors conclude by identifying some of the important issues that are likely to affect the legal framework surrounding Chinese M&A activities in the years ahead.

Chapter 9 – This chapter is about speculative attack on government bond markets. David Carfi and Fabrizio Lanzafame analyze a kind of speculative-attack on government-bond markets, through Game Theory, applying the Complete Analysis of a Differentiable Game. They propose a way to stabilize government bond prices in European "periphery". They focus on two economic operators: an Investment Bank, the Speculator, and the European Central Bank. The Speculator, the first player, can influence the market and gain by creating arbitrage opportunities, as the crisis of Euro-bond has shown. In the model, the European Central Bank, the second player, purchases government bonds to stabilize their prices and the normative authority introduces a Tobin-tax on financial transactions, preventing only "extra-profits" of speculation.

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*Chapter 1*

## **DOES FINANCIAL DEVELOPMENT IMPROVE OR WORSEN INCOME DISTRIBUTION?**

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### **ABSTRACT**

An individual's endowment and wealth depends on many factors including the skills, education, inheritance, direct and financial investments. The degree to which one can engage in such activities that lead to wealth accumulation will depend upon the opportunities that exist in every country and globally as well. In countries where financial markets are well developed opportunities are higher as compared to countries where opportunities are less. Within an individual country, not every one has the same opportunities to accumulate wealth through different channels. Thus, it is possible for income distribution to get worse or better with improvement in financial markets. Clearly, if more people understand the markets and invest, income distribution is expected to improve. Otherwise, if only limited number of individuals engage in investment activities, income distribution could get worse. In this chapter we test this hypothesis by using a measure of financial development and its impact on income distribution in Australia, Austria, Belgium, Bolivia, Canada, Denmark, Ecuador, Egypt, Finland, Greece, India, Israel, Italy, Japan, Kenya, Korea, Mexico, Malta, Malaysia, Netherlands, Norway, the Philippines, Senegal, Singapore, Turkey, and the U.S.

### **I. INTRODUCTION**

In 1955 Kuznets (1955) hypothesized that economic growth worsens income inequality first and improves it later. This has resulted in a literature that is known as Kuznets inverted-U hypothesis. In his migration based two-sector model, Kuznets described growth as the transition from the traditional agricultural sector to modern sector. Due to economic growth

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and increased standard of living, workers tend to migrate from agriculture sector to urban and industrial sector. Since income is lower in the agriculture sector and higher in industrial sector, such migration increases income inequality. However, there is a limit to this process. After reaching a threshold level, income inequality declines resulting in an inverted-U pattern of relationship between income inequality and the level of economic development.

Despite the realization by Kuznets (1955, p. 26) that his proposal “is perhaps 5 per cent empirical information and 95 percent speculation” many researchers have tried to test his hypothesis and have provided mixed results. While Berry (1974), Ram (1991), Anand and Kanbur (1993), Fields (1994), and Deininger and Squire (1998) failed to support the hypothesis, Ahluwalia (1974), Papanek and Kyn (1986), Campano and Salvatore (1998), Bourguignon and Morrison (1990), Jha (1996), and Eusufzai (1997) supported the hypothesis. Others have tried to identify other determinants of income distribution. For example, Bourguignon and Morrison (1998) argued and found that relative labor productivity between agriculture and the rest of the economy (as a good proxy for rural-urban income differences) has an impact on overall level of income inequality. They then argued that increasing the level of productivity in the traditional agriculture sector could be the most efficient way of reducing income inequality. Williamson (1997), on the other hand found demographic forces, particularly age distribution in society as another factor effecting income inequality. Other determinants of income inequality includes corruption by Gupta, Davoodi, and Terme (1998), institutional factors by Durham (1999), asset distribution by Deininger and Squire (1998), initial factor endowment by Figueroa (1998) and Spilimbergo, Londono, and Szekely (1999), price stability and financial deepening by Law and Tan (2009), trade liberalization by Corden (1987), Woods (1997), and Savvides (1998), role of the government by Evans (1998), globalization and trade by Norman (1994), and Richardson (1995), devaluation by Bahmani-Oskooee (1997), black market exchange rate premium by Bahmani-Oskooee *et al.* (2006), and saving-investment gap by Bahmani-Oskooee *et al.* (2012).

In this chapter we consider the role and impact of financial development on income inequality. To that end in Section II we introduce our model and method as well as theoretical reasons as to why financial development could affect income distribution. The model is then tested using time-series data for each of the following countries: Australia, Austria, Belgium, Bolivia, Canada, Denmark, Ecuador, Egypt, Finland, Greece, India, Israel, Italy, Japan, Kenya, Korea, Mexico, Malta, Malaysia, Netherlands, Norway, the Philippines, Senegal, Singapore, Turkey, and the U.S. and the results are discussed in Section III. Finally a summary and conclusion is provided in Section IV.

## II. THE MODEL

Since our modeling approach is based on time-series with limited number of observations for each country, we cannot include all the determinants, especially those that are qualitative variables. Therefore, the long-run model adopted in this chapter takes the following form:

$$\text{LnGini}_t = \alpha_0 + \alpha_1 \text{LnFin}_t + \alpha_2 \text{LnY}_t + \alpha_3 \text{LnCPI}_t + \alpha_4 \text{LnGov}_t + \alpha_5 \text{LnTrade}_t + \varepsilon_t \quad (1)$$

where Gini is the measure of income inequality. The first determinant is a measure of financial development identified as *Fin*. What are the channels through which financial developments could affect income inequality? Financial development provides different opportunities for different market participants. Some may use the credit available through improved banking system to gain higher education and more skills which definitely provide better economic opportunities and higher income. Some could use the available credits from financial sectors to either initiate a new business or expand existing business. Even governments could borrow from financial institutions to engage in government-induced projects that help create jobs and again, boost income. Through multiplier effects, increased opportunities, higher skills, and newly established government projects will lead to a higher standard of living and therefore, a decline in inequality.<sup>1</sup> On the other hand, it is possible that the available opportunities and credits through financial sector find their ways into the hands of well-connected as well as well- to do individuals since they can provide collateral on the borrowed money (Clarke *et al.* 2006). In this later case, financial development could worsen income inequality. Therefore, we expect an estimate of  $\alpha_1$  to be positive or negative. This is indeed our main interest to determine in which country it is positive and in which country it is negative.

In equation (1), *Y* denotes per capita GDP and is included to account for Kuznets's inverted-U hypothesis. Of course, as it will be clear later, estimating (1) yields only the long-run estimates. To test the inverted-U hypothesis we must introduce dynamic adjustment process so that we can determine whether at early stages income inequality worsens and it improves later. The second control variable we include is price level that is denoted by *CPI*. Previous studies which used cross-sectional or panel data, relied upon rate of inflation. However, since our methodology is based on cointegration and error-correction modeling, we include the price level itself such that the first differenced variable will reflect rate of inflation. It is expected that an increase in price level worsen income inequality. It is argued that monetary instability reflected in rising prices hurt the poor relatively more than the rich, mostly because the rich have easy access to credit markets. Therefore, we expect an estimate of  $\alpha_3$  to be positive.<sup>2</sup>

The third control variable we include in model (1) is government consumption denoted by *Gov*. The effects of government consumption on income inequality is ambiguous. Clarke *et al.* (2006) have argued that if government uses tax revenues to help poor through transfer payments, then an increase in government consumption will result in less inequality. However, if rich households use their political power to exploit the poor or pay less taxes, then government consumption could boost income inequality. Therefore, an estimate of  $\alpha_4$  is expected to be negative or positive. Finally, the last control variable is a measure of trade. Here too, the effect of trade on Gini could be ambiguous, depending whether a country is labor abundant or capital abundant and exports labor intensive or capital intensive commodities. If a country is labor abundant and exports mostly labor intensive goods, then wages will rise and this is expected to reduce income inequality. On the other hand, a capital abundant country which exports capital intensive goods will enjoy an increase in return to capital or profit. This could worsen income inequality.<sup>3</sup>

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<sup>1</sup> For review of the literature see Demircuc-Kunt (2009) and for support of this view see Greenwood and Jovanovic (1990).

<sup>2</sup> For more on inflation and income distribution see Easterly and Fischer (2001) and Clarke *et al.* (2006).

<sup>3</sup> For more see Papanek and Kyn (1986).

If we estimate equation (1) by any method using time-series data for any country, we only get the long-run coefficient estimates. Furthermore, estimates do not account for Kuznets inverted-U hypothesis. In order to address inverted-U hypothesis, following Bahmani-Oskooee *et al.* (2008) we express (1) in an error-correction modeling format. However, since some of the variables such as Gini could be stationary and some like CPI could be non-stationary, we rely upon a specification and an estimation method that account for this issue and variables could be stationary, non-stationary or combination of both. Therefore, following Pesaran *et al.*'s (2001) bounds testing approach we rely upon the following error-correction specification:

$$\begin{aligned} \Delta \text{LnGini}_t = & a + \sum_{i=1}^{n1} b_i \Delta \text{LnGini}_{t-i} + \sum_{i=0}^{n2} c_i \Delta \text{LnFin}_{t-i} + \sum_{i=0}^{n3} d_i \Delta \text{LnY}_{t-i} + \sum_{i=0}^{n4} e_i \Delta \text{LnCPI}_{t-i} + \\ & \sum_{i=0}^{n5} f_i \Delta \text{LnGov}_{t-i} + \sum_{i=0}^{n6} g_i \Delta \text{LnTrade}_{t-i} + \lambda_0 \text{LnGini}_{t-1} + \lambda_1 \text{LnFin}_{t-1} + \\ & \lambda_2 \text{LnY}_{t-1} + \lambda_3 \text{LnCPI}_{t-1} + \lambda_4 \text{LnGov}_{t-1} + \lambda_5 \text{LnTrade}_{t-1} + \mu_t \end{aligned} \quad (2)$$

Without lagged level variables, equation (2) will resemble an autoregressive distributed lag model. However, the addition of lagged level variables as a substitute for lagged error term from equation (1) turns it to an error-correction model. The first task, therefore, is to justify the addition. Pesaran *et al.* (2001) propose using standard F test to justify the joint significance of lagged level variables. This will also support cointegration among the variables. However, the F test has new critical values that they tabulate and these critical values do account for integrating properties of all variables. They provide a lower bound critical values at different significance level by assuming all variables in a model to be stationary or integrated of order zero, I(0). They also provide an upper bound critical values by assuming all variables to be integrated of order one or I(1). They demonstrate that the upper bound critical value could be used to establish joint significance of lagged level variables or cointegration even if some variables are I(0) and some are I(1).

From specification (2), coefficients attached to first-differenced variables reflect short-run effects of those variables. For example, short-run effect of per capita income on Gini is judged by estimates of  $d_i$ 's. The Kuznets's hypothesis is also inferred by these estimates. Namely, initially positive  $d$ 's followed by negative ones will indicate that economic growth worsens income inequality first and improves it later. What will be the overall long-run effect of economic growth on Gini. That is judged by the estimate of  $\lambda_2$  normalized on  $\lambda_0$ . Similarly, long-run effects of all right-hand side variables on Gini are inferred by the estimates of  $\lambda_1$ -  $\lambda_5$  that are normalized on  $\lambda_0$ .

### III. THE RESULTS

In this section we estimate error-correction model outlined by equation (2) for countries for which time series data are available. The main constraint is availability of Gini coefficient.



**Table 1. Coefficient Estimates of Equation (2)**

	Variables	Australia	Austria	Belgium	Bolivia	Canada	Denmark	Ecuador	Egypt	Finland
Short-run	$\Delta \text{LnFin}_t$	-	0.07(1.92)	-0.06(1.49)	0.11(0.93)	-0.12(3.57)	-0.14(6.41)	0.27(6.42)	0.28(3.31)	-0.15(1.39)
	$\Delta \text{LnFin}_{t-1}$	-	-0.31(6.42)	-0.02(0.44)	0.23(2.11)	0.38(5.91)	0.04(1.25)	0.10(2.02)	-0.08(1.06)	-0.74(4.91)
	$\Delta \text{LnFin}_{t-2}$	-	-0.15(3.98)	0.14(3.34)		-0.11(2.36)	0.12(4.26)	-0.09(1.51)	-0.26(3.42)	-0.91(8.32)
	$\Delta \text{LnFin}_{t-3}$	0.08(2.0)		0.07(1.65)		0.43(5.32)	0.05(2.20)	-0.55(9.42)		0.50(4.97)
	$\Delta \text{LnCPI}_t$	0.19(1.8)	-0.43(5.71)	-0.57(1.94)	0.21(5.40)	-0.44(4.15)	0.40(2.83)	-0.24(2.80)	1.61(3.04)	-2.38(8.10)
	$\Delta \text{LnCPI}_{t-1}$	-		-0.89(2.51)	-0.24(4.62)	-0.78(6.86)	0.16(1.15)	-0.46(4.91)	2.10(4.40)	1.32(2.69)
	$\Delta \text{LnCPI}_{t-2}$	-		1.59(4.66)	0.28(5.73)	0.41(2.85)		0.22(2.37)	0.57(2.42)	-0.80(1.80)
	$\Delta \text{LnCPI}_{t-3}$	-		-1.85(6.85)		-0.39(3.74)		1.07(8.97)		-2.11(6.86)
	$\Delta \text{LnY}_t$	0.00(1.6)	0.00(0.25)	0.01(2.46)	0.00(0.58)	0.00(1.82)	0.00(1.37)	0.03(7.16)	0.08(5.05)	0.03(6.88)
	$\Delta \text{LnY}_{t-1}$	0.00(3.7)	0.01(6.32)	-0.01(4.28)		0.02(8.15)	0.00(2.22)	0.12(11.28)	-0.19(4.34)	-0.06(8.03)
	$\Delta \text{LnY}_{t-2}$	0.01(4.5)	0.00(3.46)	-0.01(2.75)		0.02(8.43)		0.10(10.26)	-0.14(4.65)	-0.03(4.29)
	$\Delta \text{LnY}_{t-3}$	0.00(2.4)				0.01(4.72)		0.06(8.09)		-0.01(2.16)
	$\Delta \text{LnGov}_t$	-	0.39(6.21)	0.19(1.34)	-0.33(2.27)	-0.04(0.69)	-0.07(0.87)	0.22(4.08)	-0.02(0.14)	-0.10(0.87)
	$\Delta \text{LnGov}_{t-1}$	0.14(3.1)	0.17(2.78)	-0.24(2.36)	0.01(0.10)	0.01(0.13)	-0.11(1.34)	0.38(5.32)		2.23(7.18)
	$\Delta \text{LnGov}_{t-2}$	0.26(4.6)		0.20(1.81)	1.51(5.20)	-0.44(4.92)	0.17(1.87)	-0.11(2.13)		0.84(4.87)
	$\Delta \text{LnGov}_{t-3}$	0.19(3.1)		0.45(4.08)		-0.16(3.75)	0.23(3.74)	0.19(3.53)		1.48(7.69)
$\Delta \text{LnTrade}_t$	-	0.05(1.81)	-0.23(2.74)	0.73(4.32)	-0.24(5.91)	-0.45(7.25)	0.60(8.23)	-0.24(3.18)	0.98(7.46)	
$\Delta \text{LnTrade}_{t-1}$	0.24(4.5)	0.13(4.22)		-0.19(1.39)	0.07(2.19)	0.35(6.62)	0.46(7.30)		-0.31(3.93)	
$\Delta \text{LnTrade}_{t-2}$	0.17(4.0)			0.80(4.02)	-0.21(4.36)		0.52(7.20)		-0.27(3.01)	
$\Delta \text{LnTrade}_{t-3}$	0.08(1.7)				-0.09(2.45)		-0.31(5.96)			
Long--Run	constant	4.59(3.9)	-4.40(0.57)	2.68(3.17)	0.79(0.40)	2.42(4.59)	-0.16(0.03)	3.49(4.83)	2.42(4.83)	1.23(0.15)
	LnCPI	0.19(1.5)	-0.25(0.81)	-0.04(0.54)	0.02(4.51)	0.01(0.20)	-0.13(0.91)	0.00(0.28)	0.04(1.14)	-0.74(1.17)
	LnY	0.00(0.1)	-0.02(0.86)	0.02(2.94)	0.00(0.62)	-0.01(0.31)	0.01(0.34)	0.00(0.10)	0.16(4.96)	-0.16(1.90)
	LnGov	-	0.74(0.80)	0.26(2.34)	0.17(0.85)	0.54(1.64)	0.33(0.99)	-0.14(1.67)	0.03(0.28)	5.54(2.08)
	LnFin	-	1.60(1.10)	-0.02(0.23)	-0.18(0.66)	-0.08(0.19)	0.11(0.50)	0.19(2.48)	0.26(3.34)	-1.11(1.43)
	LnTrade	-	-0.10(0.31)	0.05(0.20)	0.89(2.71)	0.01(0.02)	0.60(0.77)	-0.08(0.62)	-0.31(5.42)	-1.21(1.57)

**Table 1. (Continued)**

	Variables	Australia	Austria	Belgium	Bolivia	Canada	Denmark	Ecuador	Egypt	Finland
Diagnostic	ECM <sub>t-1</sub>	-	-0.25(8.01)	-1.05(7.81)	-1.28(7.54)	-0.87(8.34)	0.63(7.15)	-2.67(10.66)	-1.73(4.90)	0.69(10.20)
	F test	4.94	7.35	6.36	5.53	5.8	5.24	7.1	2.78	6.51
	Adj. R <sup>2</sup>	0.87	0.77	0.82	0.81	0.82	0.86	0.92	0.69	0.96
	LM	6.06	10.63	3.46	5.81	1.65	10.76	4.55	3.43	17.81
	RESET	0.52	0.28	3.85	2.44	2.44	2.38	0.21	0.48	0.01

	Variable	Greece	India	Israel	Italy	Japan	Kenya	Korea, Rep.	Luxembourg	Mexico
Short-run	$\Delta \text{LnFin}_t$	0.86(8.03)	-2.17(6.90)***	0.03(5.83)	-0.04(0.32)	-0.01(0.21)	0.02(0.19)	-0.08(3.21)	0.19(10.74)	0.24(12.99)
	$\Delta \text{LnFin}_{t-1}$	-0.07(0.92)		-0.02(3.72)	-0.02(0.16)	-0.05(1.56)		-0.03(1.35)	-0.02(1.50)	-0.28(7.41)
	$\Delta \text{LnFin}_{t-2}$	-			0.40(3.29)	0.17(6.01)		-0.38(9.24)		-0.03(1.98)
	$\Delta \text{LnFin}_{t-3}$	-						-0.37(5.63)		
	$\Delta \text{LnCPI}_t$	-	-0.49(2.45)	0.05(7.04)	-	0.06(2.21)	-0.41(3.43)	-0.05(1.16)	3.08(10.97)	0.08(1.84)
	$\Delta \text{LnCPI}_{t-1}$	1.91(6.83)	1.26(5.45)	0.01(1.17)	-0.09(0.22)	0.01(0.39)		0.30(5.25)	0.82(2.59)	0.31(3.20)
	$\Delta \text{LnCPI}_{t-2}$	-0.99(2.55)	1.66(6.14)	0.04(3.44)	-0.83(2.65)	0.12(3.98)		0.19(3.42)	-1.07(5.39)	0.08(2.15)
	$\Delta \text{LnCPI}_{t-3}$	2.20(6.55)	0.78(3.23)	0.01(1.95)				-0.32(6.16)		
	$\Delta \text{LnY}_t$	0.03(6.15)	0.02(2.25)	0.01(7.70)	0.01(3.61)	0.00(0.55)	-0.01(1.79)	-0.01(10.79)***	-0.01(4.51)	0.00(3.39)
	$\Delta \text{LnY}_{t-1}$	-	-0.17(7.45)***	-0.01(7.06)		0.00(2.41)		0.00(2.21)	-	-0.02(5.76)
	$\Delta \text{LnY}_{t-2}$	-	-0.13(8.14)***	-0.01(6.69)		0.00(2.12)			-0.03(9.05)	0.00(1.59)
	$\Delta \text{LnY}_{t-3}$		-0.06(4.31)***							
	$\Delta \text{LnGov}_t$	-0.36(3.23)	1.05(4.79)	-	-0.52(2.72)	0.07(2.58)	0.07(0.47)	0.18(5.47)	-0.06(1.07)	0.48(9.06)
	$\Delta \text{LnGov}_{t-1}$	0.12(1.06)	-0.47(1.07)	-0.15(8.02)	-0.23(1.53)	-0.08(3.40)***		-0.04(1.63)		0.22(3.57)
	$\Delta \text{LnGov}_{t-2}$	0.04(0.36)	0.63(1.52)	-0.10(5.66)	-0.36(2.04)			0.02(0.64)		0.20(3.12)
	$\Delta \text{LnGov}_{t-3}$	-0.31(2.43)	2.07(4.27)					-0.19(4.24)		
	$\Delta \text{LnTrade}_t$	0.00(0.03)	1.13(6.13)	-0.03(1.84)	0.22(2.49)	-0.01(1.80)	0.04(0.58)	-0.10(4.62)	0.12(1.67)	0.28(5.57)
	$\Delta \text{LnTrade}_{t-1}$	1.58(6.27)	0.26(1.24)	-0.01(0.74)	0.00(0.05)	0.02(2.82)	0.14(1.88)	0.31(6.86)	-0.43(7.46)	-0.72(6.61)
$\Delta \text{LnTrade}_{t-2}$	0.27(1.57)	-0.77(3.36)***	-0.08(4.48)	-	-0.02(3.99)***		0.02(1.91)	-0.12(2.40)	-0.53(7.09)	

	Variable	Greece	India	Israel	Italy	Japan	Kenya	Korea, Rep.	Luxembourg	Mexico
	$\Delta \text{LnTrade}_t$	0.48(3.11)	-0.99(4.25)***					0.05(4.93)		
Long-Run	constant	3.71(9.96)	0.09(0.04)	3.91(34.13)	3.18(2.35)	12.75(0.24)	3.88(12.11)	4.88(4.00)	1.21(2.85)	2.01(17.49)
	LnCPI	0.08(5.40)	-0.85(2.87)	0.01(4.21)	-0.06(1.72)	-0.20(0.19)	-	-0.06(1.96)	0.08(5.87)	-
	LnY	0.02(1.50)	0.20(2.74)	0.03(5.50)	0.01(1.72)	0.01(0.16)	-0.01(1.93)	0.04(1.90)	0.02(6.03)	0.00(1.76)
	LnGov	0.12(0.44)	0.32(0.61)	-0.09(3.93)	0.12(0.42)	0.54(0.25)	0.09(2.15)	-0.78(1.27)	-0.02(0.49)	0.09(3.39)
	LnFin	0.31(4.30)	0.02(0.04)	0.04(7.88)	-0.03(0.45)	-1.98(0.18)	0.01(0.32)	-0.19(1.40)	0.06(5.26)	0.10(8.54)
	LnTrade	-0.56(2.38)	1.64(5.28)	-0.08(2.16)	0.13(0.77)	-0.33(0.18)	-0.06(1.31)	0.38(2.04)	0.30(4.13)	0.30(17.80)
Diagnostic	$\text{ECM}_{t-1}$	-3.16 (7.57)	-1.21 (8.08)	-1.40 (12.4)	-1.60 (9.12)	-0.05 (9.26)	-2.02 (5.33)	0.96 (9.60)	-2.84 (13.6)	-5.74 (11.3)
	F test	3.59	5.94	18.03	8.93	7.14	3.97	6.83	16.95	9.49
	Adj. R <sup>2</sup>	0.92	0.94	0.89	0.82	0.89	0.61	0.96	0.91	0.97
	LM	5.97	15.18	0.21	7.75	20	1.25	2.71	1.2	9.15
	RESET	1.83	0.01	2.4	4.19	0.21	0.52	3.74	0.13	0.9

	Variable	Malta	Malaysia	Netherland	Norway	Philippines	Senegal	Singapore	Turkey	United States
Short-run	$\Delta \text{LnFin}_t$	-0.01(0.15)	0.16(4.37)	-0.22(1.71)	-0.35(3.83)	-0.17(12.23)	-0.19(1.50)	-0.08(3.82)	0.00(0.27)	0.05(1.58)
	$\Delta \text{LnFin}_{t-1}$		0.03(0.86)	-0.26(1.94)	0.16(1.47)	0.11(5.26)	0.51(4.15)	-0.06(2.87)	-0.04(3.65)	-0.05(1.88)
	$\Delta \text{LnFin}_{t-2}$		-0.24(5.05)		-0.17(2.28)	-0.12(7.31)	0.60(3.24)	-0.14(5.48)	-0.08(5.07)	
	$\Delta \text{LnFin}_{t-3}$							-0.02(1.35)	-0.03(2.03)	
	$\Delta \text{LnCPI}_t$	0.35(1.85)	-0.27(2.22)	0.04(0.16)	0.27(1.51)	0.08(3.13)	0.99(3.15)	-0.08(1.06)	0.07(3.65)	-0.10(1.35)
	$\Delta \text{LnCPI}_{t-1}$				0.08(0.33)		-0.89(2.73)	0.12(1.54)	0.21(7.65)	0.12(1.22)
	$\Delta \text{LnCPI}_{t-2}$				0.10(0.38)			-0.39(4.77)	0.14(4.03)	0.07(1.29)
	$\Delta \text{LnCPI}_{t-3}$				-1.17(5.42)			0.15(3.90)		
	$\Delta \text{LnY}_t$	0.00(0.06)	0.00(1.69)	0.00(0.87)	-0.01(1.11)	0.01(7.83)	-0.02(1.77)	-0.01(5.50)	0.01(4.34)	0.00(0.53)
	$\Delta \text{LnY}_{t-1}$	-0.01(3.06)	-0.01(3.04)	-0.01(2.33)	0.02(3.71)	-0.02(9.15)	0.09(4.41)	0.03(10.18)	0.00(1.85)	0.00(3.14)
	$\Delta \text{LnY}_{t-2}$			-0.01(1.71)	0.02(2.91)	0.00(1.88)	0.06(5.22)	0.02(10.08)	0.00(1.82)	
	$\Delta \text{LnY}_{t-3}$				0.01(2.58)	0.01(9.97)		0.01(8.94)		
	$\Delta \text{LnGov}_t$	0.01(0.09)	0.18(3.84)	-0.81(3.65)	-0.36(1.85)	0.22(7.58)	-0.37(2.17)	-0.01(0.51)	-0.11(4.69)	0.01(0.46)
	$\Delta \text{LnGov}_{t-1}$			0.46(2.04)	0.85(4.55)	-0.03(1.05)	-1.79(7.33)	0.11(5.26)	0.24(10.11)	-0.03(1.23)
$\Delta \text{LnGov}_{t-2}$				0.11(0.44)	-0.19(8.70)	-1.70(4.25)	0.05(1.74)	0.32(6.69)	-0.08(2.84)	

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**Table 1. (Continued)**

	Variable	Malta	Malaysia	Netherland	Norway	Philippines	Senegal	Singapore	Turkey	United States
	$\Delta \text{LnGov}_{t-3}$				0.77(2.79)	-0.52(11.85)		0.06(2.65)	0.15(4.62)	
	$\Delta \text{LnTrade}_t$	0.24(4.24)	-0.20(3.34)	0.26(2.44)	-0.30(2.66)	-0.20(8.12)	-0.64(2.75)	-0.17(7.17)	-0.04(2.82)	0.04(2.20)
	$\Delta \text{LnTrade}_{t-1}$	0.09(1.30)	0.33(4.72)		0.63(3.82)	-0.50(15.47)	-0.19(1.98)	-0.03(1.74)	-0.04(2.85)	
	$\Delta \text{LnTrade}_{t-2}$	0.22(3.15)			-0.36(1.98)	-0.79(16.76)	-0.35(4.28)	0.08(4.09)	0.01(0.41)	
	$\Delta \text{LnTrade}_{t-3}$				0.35(1.82)	-0.62(11.56)		0.11(6.43)	0.03(1.93)	
Long--Run	constant	2.66(1.90)	3.54(11.28)	5.74(3.10)	5.45(4.27)	3.08(33.46)	4.30(4.45)	3.27(8.40)	4.99(5.81)	2.52(2.48)
	LnCPI	0.47(1.08)	0.04(0.90)	0.12(0.64)	0.14(2.45)	-0.03(4.10)	0.09(1.59)	-0.34(5.88)	0.01(2.15)	0.07(1.41)
	LnY	0.02(0.33)	0.02(2.84)	0.03(1.19)	-0.02(0.99)	0.02(5.90)	-0.03(0.51)	0.04(2.85)	0.01(1.26)	-0.01(1.05)
	LnGov	0.38(1.41)	0.15(2.39)	-1.13(3.83)	-0.29(1.44)	0.18(7.80)	0.92(2.12)	0.17(2.72)	-0.21(1.29)	0.14(0.97)
	LnFin	-0.12(0.79)	0.25(5.15)	0.14(0.58)	-0.11(1.30)	-0.08(11.57)	-0.66(2.65)	-0.03(1.19)	-0.08(1.17)	0.12(0.52)
	LnTrade	-0.37(0.68)	-0.34(4.15)	0.00(0.00)	-0.21(0.73)	0.17(4.49)	0.00(0.05)	0.22(2.37)	-0.19(1.89)	0.00(0.03)
Diagnostic	$\text{ECM}_{t-1}$	-0.29(4.58)	-1.17(7.36)	-0.60(3.87)	-1.32(7.32)	-2.88(17.36)	-2.80(7.20)	1.09(9.95)	-0.88(8.75)	-0.35(5.09)
	F test	2.67	5.8	1.84	5.21	25.11	3.24	7.34	6.95	3.05
	Adj. $R^2$	0.59	0.79	0.7	0.85	0.98	0.84	0.93	0.93	0.76
	LM	1.43	5.02	5.04	20.01	22.62	5.4	16.92	12.7	4.32
	RESET	0.08	3.07	0.01	0.5	0.06	5	0.02	0.11	0.3

Note: Numbers inside parentheses are the absolute value of t-ratios.

Fortunately, the University of Texas Inequality Project provides time-series data for certain countries which we use them here.<sup>1</sup> The list includes Australia, Austria, Belgium, Bolivia, Canada, Denmark, Ecuador, Egypt, Finland, Greece, India, Israel, Italy, Japan, Kenya, Korea, Mexico, Malta, Malaysia, Netherlands, Norway, the Philippines, Senegal, Singapore, Turkey, and the U.S. We therefore, estimate model (2) for each of these countries. Since data are annual, we impose a maximum of four lags on each first differenced variable and estimate all possible lag combination. We then use Akaike's Information Criterion (AIC) and select an optimum model for each country. Full information results from each optimum model are reported in Table 1.

Due to volume of the results they are reported in three sections identified as short run, long run, and diagnostics. From the short-run coefficient estimates, there is a clear indication that the variable representing financial development (Fin) carries at least one significant coefficient in the results for all countries except India, Kenya, and Malta. Indeed, all variables seem to have short-run effects on income distribution in most countries. The Kuznets's inverted-U hypothesis receive support in the results for Belgium, Egypt, Finland, Israel, Mexico, and Malaysia, since only in these countries per capita income carries positive coefficients first followed by negative ones. Do these short-run effects last into long run?

From the long-run coefficient estimates we gather that the price level carries significant and positive coefficient in the cases of Bolivia, Greece, Israel, Luxembourg, Norway, and Turkey, implying that rising inflation in these countries worsens income inequality. It also carries significantly negative coefficients in the cases of India, Italy, Korea, the Philippines, Singapore, and Turkey. Economic growth seems to have worsening effect on income distribution in Belgium, Egypt, India, Israel, Italy, Korea, Luxembourg, Mexico, Malaysia, the Philippines, and Singapore, since per capita income carries a significantly positive coefficient. Government spending also affects income inequality in limited number of countries. It carries significant coefficient in the cases of Belgium, Canada, Ecuador, Finland, Israel, Kenya, Mexico, Malaysia, the Philippines, Senegal, and Singapore and the coefficient is positive in most cases. As for our variable of interest, i.e., the measure of financial development, eight countries are affected in the long run. They are: Ecuador, Egypt, Greece, Israel, Luxembourg, Mexico, Malaysia, and Singapore. Financial development worsens income inequality in all of these eight countries except Singapore. Finally, international trade seems to reduce income inequality in Australia, Egypt, Greece, Israel, Malaysia, and Turkey. However, it worsens inequality in Bolivia, India, Korea, Luxembourg, Mexico, the Philippines, and Singapore.

The above long-run analysis, however, will be valid only if we establish joint significance of lagged level variables as a sign of cointegration. To this end we shift to diagnostics and the results of the F test. As mentioned before, Pesaran *et al.* (2001) provide new critical values, though for large sample. The counterpart for small samples like ours is provided by Narayan (2005, p. 1988). Using 10% significance level and an upper bound critical value of 3.763, clearly cointegration among the variables is supported in the results for all countries except Egypt, Greece, Malta, Netherland, Senegal, and the United States. For these countries, however, we can use an alternative test to establish cointegration. Following other studies, we use the long-run coefficient estimates and use equation (1) to generate the

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<sup>1</sup> See University of Texas Inequality Project, available at <http://utip.gov.utexas.edu/data.html>. For more on data see the Appendix.

error term, say  $ECM$ . We then replace the lagged level variables with  $ECM_{t-1}$  and estimate each model after imposing the same optimum lags. A significantly negative coefficient will support cointegration and adjustment toward long run. From the results we gather that  $ECM_{t-1}$  carries a significantly negative coefficient in almost every country.

Three other statistics are reported for each country. First is the Lagrange Multiplier (LM) statistic which is used autocorrelation among the residuals of the error-correction model. It is distributed as  $\chi^2$  with one degree of freedom. Given its critical value of 3.84, there is evidence of serial correlation in many countries. The second statistic is Ramsey's RESET statistic which is also distributed as  $\chi^2$  with one degree of freedom. This statistic is used to determine whether each optimum model is correctly specified. As can be seen, in almost every case this statistic is less than its critical value of 3.84, implying that optimum models are correctly specified. Finally, the size of adjusted  $R^2$  seems to be fairly high in every case, supporting a good fit.

## CONCLUSION

Financial market development is said to provide different opportunities for different people. Individuals could use the available credit to enrich their education background, skills, and new or old business, improving their standard of living. In this case, financial market development will reduce income inequality. However, if developments help those who are already well-connected and well-off then financial market development could have unequalizing effect on income distribution. Although our concern is to determine impact of financial market development on income inequality, we also account for other determinants of income inequality such as economic growth, government spending, international trade, and inflation.

Using time series data, we estimate a reduced form model for every country for which time-series data on a measure of income inequality are available. The list includes Australia, Austria, Belgium, Bolivia, Canada, Denmark, Ecuador, Egypt, Finland, Greece, India, Israel, Italy, Japan, Kenya, Korea, Mexico, Malta, Malaysia, Netherlands, Norway, the Philippines, Senegal, Singapore, Turkey, and the U.S. Using bounds testing approach to cointegration and error-correction modeling, we find that all variables have short-run effects on income distribution in most countries. In the long-run however, only limited number of countries are affected.

## APPENDIX

### Data Definition and Sources

Annual data are used to estimate the model. The time period differed from one country to another due to availability of the measure of income in equality and are as follows: Australia (1963-2001), Austria (1963-1995), Belgium (1963-1997), Bolivia (1970-2000), Canada (1963-2001), Denmark (1963-1998), Ecuador (1963-1999), Egypt, Arab Rep. (1964-1998), Finland (1963-1998), Greece (1963-1998), India (1965-2001), Israel (1963-2001), Italy

(1967-1998), Japan (1963-1993), Kenya (1963-2002), Korea, Rep. (1966-2001), Luxembourg (1963-1992), Mexico (1970-2000), Malta (1970-2001), Malaysia (1968-1997), Netherlands (1963-1997), Norway (1963-2001), Philippines (1963-1997), Senegal (1974-2002), Singapore (1965-2002), Turkey (1963-2000), United States (1963-1995).

The data come from the following sources:

- a) University of Texas Inequality Project, available at <http://utip.gov.utexas.edu/data.html>.
- b) World Bank Open Data: Database on Financial Development and Structure : <http://econ.worldbank.org/WBSITE/EXTERNAL/EXTDEC/EXTRESEARCH/0,,contentMDK:20696167~pagePK:64214825~piPK:64214943~theSitePK:469382,00.html>

## Variables

*Gini* = each country's level of income inequality measured by the country's Gini Coefficient. The data come from source a.

*Y* = each country's real GDP, from source b.

*Fin* = each country's measure of financial development. It is defined as ratio of bank credit / bank deposits, source b.

*CPI* = each country's price level measured by Consumer Price Index, source b.

*Trade* = each country's level of international trade defined as (Exports + Imports)/ GDP. Nominal values for exports, imports, and GDP come from source b.

*Gov* = each country's government expenditure as a percent of GDP. Nominal figures for government consumption and GDP come from source b.

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*Chapter 2*

# OPTIMIZED UPTREND AND DOWNTREND PATTERN TEMPLATES FOR FINANCIAL MARKETS TRADING BASED ON A GA KERNEL

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## ABSTRACT

This work describes a new computational finance approach. This approach combines pattern recognition techniques with an evolutionary computation kernel applied to financial markets time series, in order to optimize trading strategies. Moreover, for pattern matching a template-based approach is used in order to describe the desired trading patterns. The parameters for the pattern templates, as well as, for the decision making rules are optimized using a genetic algorithm kernel. The approach was tested considering actual data series and presents a robust profitable trading strategy which clearly beats the market, S&P 500 index, reducing the investment risk significantly. In order to reduce portfolio risks a low volatility selection stocks strategy is tested. A study on outlier removal to improve data series quality is also presented.

**Keywords:** genetic algorithms, pattern templates, pattern recognition, optimization, financial markets, volatility, outlier removal

## INTRODUCTION

The domain of computational finance has received an increasing attention by people from both finance and computational intelligence domains. The main driving force in the field of

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computational finance, with application to financial markets, is to define highly profitable and less risky trading strategies. In order to accomplish this main objective, the defined strategies must process large amounts of data which include financial markets time series, fundamental analysis data, technical analysis data, etc. and produce appropriate buy and sell signals for the selected financial market securities. What may appear, at a first glance, as an easy problem is, in fact, a huge and highly complex optimization problem, which cannot be solved analytically. Therefore, this makes the soft computing and in general the computational intelligence domains specially appropriate for addressing the problem, recently, several works like (Atsalakis & Valavaris, 2008) and (Leigh, Frohlich, Hornik, Purvis, & Roberts, 2008), have been published in the field of computational finance, where soft computing methods are used for stock market forecasting. One of the principal support decision methods to investment, used by traders, is to observe and find graphical chart patterns in the financial time series, trying to predict the stock trend.

In this chapter a new approach is presented in order to improve the quality of decision making by optimizing both the parameters for the pattern template and for the decision making rules. Thus, in this work, templates are combined in order to generate new decision rules. The optimization of the trading strategy is here carried out by a genetic algorithm kernel (Goldberg, 1994). The proposed approach was tested considering a large period of time including different market trends and considering a sliding window for defining both training and testing periods. Additionally, a measure of risk could be associated with the volatility of the stock prices, based on this fact, a study made with two volatility measures that could assist on the picking stock decision, is presented. Finally, a study of two new methods of outlier removal, to test how the results could be improved, is also discussed.

This chapter is organized as follows: In Section II, a pattern recognition background is presented. Section III the pattern recognition approach based on a template description is analyzed outlining the parameters which will be subjected to an optimization process, also presents the studies about outlier removal and low volatility stock pick. Section IV describes the overall trading optimization methodology. Section V presents and discusses the selected case studies. Section VI provides the conclusions of the chapter.

## **PATTERN RECOGNITION BACKGROUND**

In recent years the area of chart patterns recognition has given rise to different detection methodologies applied to chart extracted from financial markets time series. Particularly, three main methodologies have been successfully applied. The first one is a rule-based approach, where the patterns are represented by a set of rules that define their main characteristics, (Lo, Mamaysky, & Wang, 2000) (S.C. Suh, D. Li, & J. Gao, 2004). The second approach is based on the application of pattern templates (Leigh, Modani, Purvis, & Roberts, 2002). The third approach uses a data dimensional reduction representation for both the time series and the patterns, e.g., works like (F.L. Chung, T.C. Fu, R. Luk, & V. Ng, 2001) (Zhou & Hu, 2009), where a representation named perceptually important points (PIP) is used to represent time series and patterns, or (Canelas, Neves, & Horta, 2012) where a symbolic representation, named Symbolic Aggregate approxImation (SAX) is used.

In the rule based approach presented by *Lo, Mamaysky, & Wang (2000)*, the patterns are defined by a set of rules or conditions, then a window of a kernel regression estimator applied to the price history is examined, to look for extreme points (maximums and minimums, local or global), after this set of points are tested against the rules, in order to evaluate the presence of patterns or not. Sometimes, this methodology is extended to more complex financial data and is applied to candlesticks charts, like in (*Ng, Liang, Chan, & Yeung, 2011*). This approach has the advantage of allowing to describe complex patterns, by defining a large set of rules. The main drawback, is building the set of rules for a large library of patterns and the matching process between the rules and time series.

The template base approach uses patterns described in a matrix format, like in Figure 1. This method was used for pattern matching by *Leigh et al.*, where a bull flag template pattern was the basis to create several market forecasting methodologies. This approach was also used by *Wang & Chan(2009)*, where the authors used a different weight strategy, than *Leigh et al.*, to represent the template patterns and also tested different patterns. This matrix representation is very easy to obtain, the visual correspondence between the matrix and the pattern is obvious. Easy is also the matching process, since it is only necessary to multiply the template matrix with a time series representation matrix, in order to obtain a correlation value, which indicates the presence of the searched pattern. This methodology has a limitation, which is the fact that it only can be used to represent simple patterns, complex patterns like head-and-shoulders implicates that the matrix dimension should be higher, and causing that the time series representation to be examined should have bigger dimensions. This will cause that if the pattern is formed in a short time window the method would not detect the pattern.

0.5		-1	-1	-1	-1	-1	-1	-1	
<b>1</b>	0.5		-0.5	-1	-1	-1	-1	-0.5	
<b>1</b>	<b>1</b>	0.5		-0.5	-0.5	-0.5	-0.5		0.5
0.5	<b>1</b>	<b>1</b>	0.5		-0.5	-0.5	-0.5		<b>1</b>
	0.5	<b>1</b>	<b>1</b>	0.5				0.5	<b>1</b>
		0.5	<b>1</b>	<b>1</b>	0.5			<b>1</b>	<b>1</b>
-0.5			0.5	<b>1</b>	<b>1</b>	0.5	0.5	<b>1</b>	<b>1</b>
-0.5	-1			0.5	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	
-1	-1	-1	-0.5		0.5	<b>1</b>	<b>1</b>		-2
-1	-1	-1	-1	-0.5		0.5	0.5	-2	-2.5

Figure 1. Bull flag pattern template.

Perceptually Important Points (PIP), is another method largely used to represent financial time series (*T.C. Fu, F.L. Chung, R. Luk, & C.M. Ng., 2008*) (*T.C. Fu, F.L. Chung, R. Luk, & C.M. Ng, 2005*). In this approach the time series are reduced to a set of points that are considered important. The process to identify PIP's in a time series is quite simple and could be seen in Figure 2.

It starts by defining that the first two important points are the first and the last of the time series, then draws a line between those points and calculates for the rest of the points in the time series which is further apart from the line, the more distant point will now become an important point. After, draws other line between the first point and this new one, and from the

new to the last point, and for each of the segments, again finds the point further apart, and in a recursive way it will detect the important points. If this process is taken to the limit, then all points from the time series will be important points. To avoid this fact, usually, a compression rate between the number PIP's and the number of points in the time series is defined. This rate will represent an acceptable level of error between time series and the PIP representation. The matching process is made by evaluating the distance, Eq. (1), between pattern and time series representation, Figure 3

$$DM(P,Q)=w AD(P,Q)+ (1-w) TD(P,Q) \quad (1)$$

where,

- P and Q     PIP time series representation;  
 AD(.)       Average amplitude distance between pattern and time series points;  
 TD(.)       Average time or horizontal distance between pattern and time series points;  
 w             Weight factor that allows to specify which distance is more important.

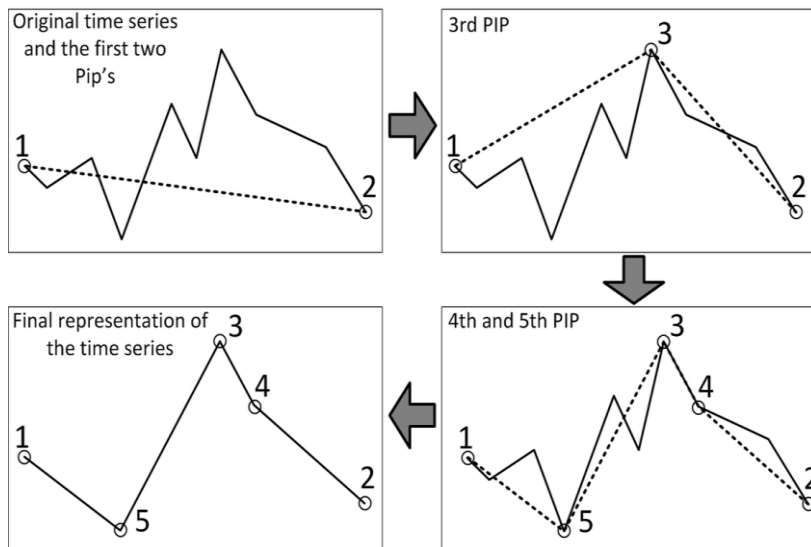


Figure 2. PIP identification process.

The time series representation using PIP's has the advantage of preserving some of the important features of financial time series. Many of the preserved points are important indicators of trend inversion. In (T.C. Fu, F.L. Chung, K.Y. Kwok, & C.M. Ng, 2008) the PIP method proved good results, in this work the authors used PIP's to create a similar string sequence representation to SAX (Lin, Keogh, Lonardi, & Chiu, 2003) (Lin, Keogh, Lonardi, & Patel, 2002). Instead of using the usual Piecewise Aggregate Approximation (PAA) (Keogh, Chakrabarti, Pazzani, & Mehrotra, 2001) the PIP method was used; and get better results due to the fact that some relevant points that were preserved. One of the problems, when using this PIP method to recognize patterns, is that it is necessary to convert the time series to the same number of PIP's present in the template, in order to evaluate the degree of similitude between them.

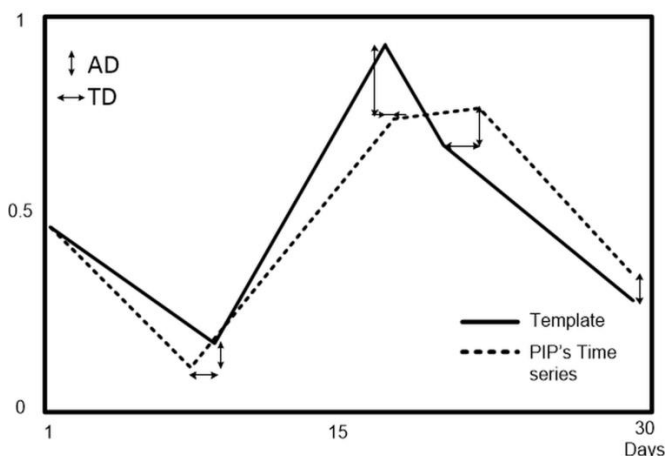


Figure 3. PIP matching process.

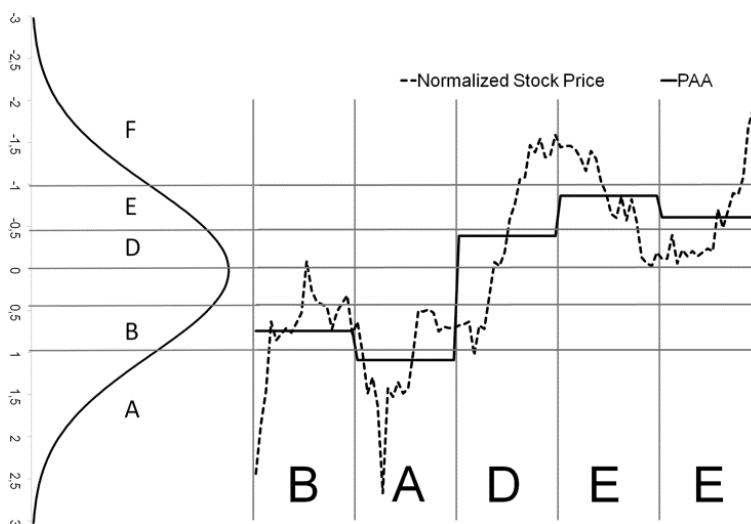


Figure 4. SAX representation process.

So, if the application has to find a head-and-shoulders pattern, it is necessary to convert the time series to a 7 important point's representation and after if template changes to a different pattern, with a different number of important points, is necessary to reprocess the time series. The SAX-GA work (Canelas, Neves, & Horta, 2012), converts time series to a symbolic representation and uses the GA to generate and discover patterns in the time series data, also the GA is used to identify rules of investment, based on the discovered pattern, to trade on the markets. The SAX methodology begins by dividing the time series in windows, if needed, and then normalizes the data in each window. After, divides the windows into segments of equal size, and for the point in each segment the arithmetic mean is calculated. Now the data is in a PAA representation format, to convert the PAA to a symbol representation, a normal distribution curve is applied to the vertical axes and then is divided in sectors of equal area, where each division is correspond to a symbol, which is assign to the correspondent level of the PAA level. The SAX process is depicted in Figure 4.

Several works that used the previous approaches are presented in Table 1.

**Table 1. Pattern recognition algorithms**

Ref.	Method	Used Data	Financial Market	Period		Algorithm Performance	
(Leigh, Frohlich, Hornik, Purvis, & Roberts, 2008)	Bull Flag Pattern w/ Matriz Template	Stock Price	NYSE Composite. Index	1967 – 2003		4.59% (Transaction average over the period)	
(Wang & Chan, 2007)	Bull Flag Pattern w/ Matriz Template	Stock Price	NASDAQ and TWI	NASDAQ	TWI	NASDAQ	TWI
				85/04/03 – 2004/03/20	71/06/01 – 2004/03/24	4.38% (Transaction average over the period)	6.48% (Transaction average over the period)
(Leigh, M. Paz, & Purvis, 2002)	Hybrid neural Network w/ Pattern detection	Stock Price and Vol.	NYSE Composite Index	1984/07/24 – 1998/06/11		66% (Days market goes up after buying order)	
(T.C. Fu, F.L. Chung, Luk, & Ng, 2007)	Template-based	Stock Price	Several	N/A		96% (Hits on pattern identification)	
(T.C. Fu, F.L. Chung, Luk, & Ng, 2007)	Rule-based	Stock Price	Several	N/A		38% (Hits on pattern identification)	
(T.C. Fu, F.L. Chung, Luk, & Ng, 2007)	PAA	Stock Price	Several	N/A		82% (Hits on pattern identification)	
(Lee, Cho, & Baek, 2003)	AANN	Stock Price	Kospi 200 Futures	2001		31% (year profit)	
(Ko, Lin, & Shih, 2008)	GACS-M	Stock Price	Stocks from TAIEX	September 21, 2000 to September 21, 2007		21,42% (Average rate return)	
(Ng, Liang, Chan, & Yeung, 2011)	MCS RBFNN	Stock Price	Hang Seng Index	10 years		167 (Average earning)	
(Canelas, Neves, & Horta, 2012)	SAX-GA	Stock Price	S&P500	Jan 1998 – April 2010		122.39% (Average return)	



## ADOPTED TEMPLATE BASED STRATEGY

In this work, the template base methodology is used, since the investment strategy is to find periods of time where an uptrend is already in place. The algorithm will search for simple ascendant pattern, which indicate the presence of a bull market. The use of GA to assist on the definition of the search parameters will provide a performance improvement over other pattern recognition methodologies. Next, the method is presented in more detail.

### TEMPLATE-BASED METHODOLOGY

The template-based methodology, illustrated in Figure 5, consists of: first, defining the desired chart patterns templates in a matrix format, then, select the data points within a temporal sliding window, next, remove noise from data points, finally, determine the fitness value by comparing the noiseless data points with the defined chart pattern template. In the next sections each methodology step will be addressed.

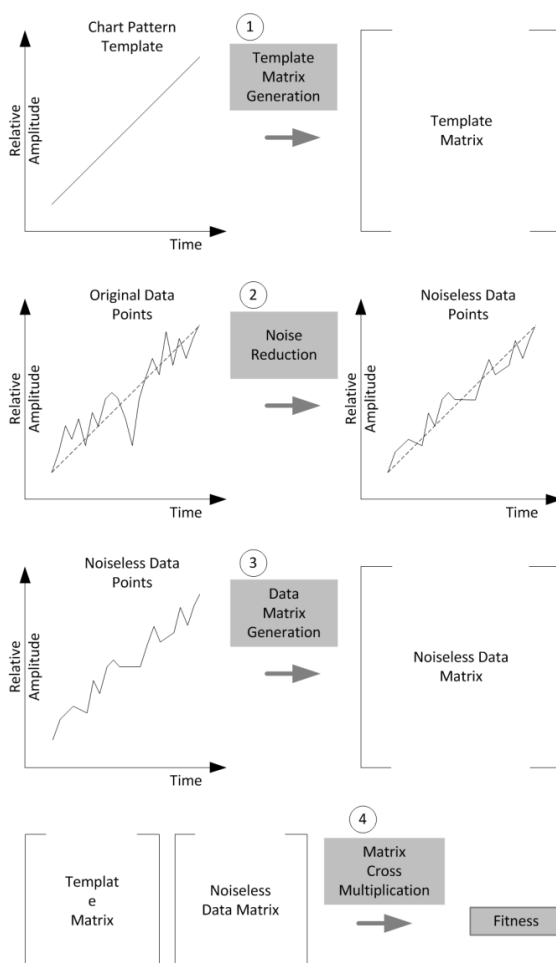


Figure 5. Pattern Recognition using Template-Based Methodology.

### Template Matrix

Although the presented approach can be generalized, an uptrend pattern is here considered for its relevance on practical cases such as detecting the right market trend.

The chart pattern template must be described in a matrix format, as illustrated in Figure 7. This matrix will be referred as matrix “T”. The matrix size is selected based on the desired template resolution, for practical purposes, in this case a 10x10 matrix was chosen. The cells values are selected within a range where the maximum corresponds to the maximum fit, the desired pattern, and the minimum corresponds to the minimum fit. The values for each cell were chosen according to the rules defined in (Wang & Chan, 2009), where the values of each column are calculated at a linear decrement from the pattern maximum value 1, and the sum of the values for in each column should be equal to zero. In Figure 6 is shown how the cell weights are calculated, for the first column of the pattern template in Figure 7.

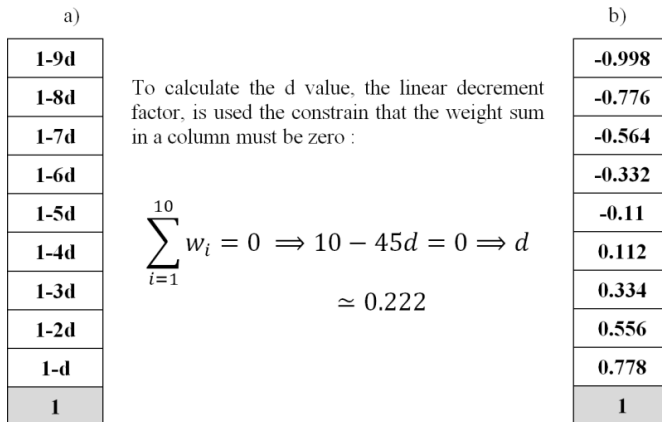


Figure 6. a) Weight calculation with linear decrement b) Final weights values.

-0.998	-1.224	-1.415	-1.496	-1.38	-1	-0.428	0.162	0.655	1
-0.776	-0.946	-1.07	-1.08	-0.908	-0.5	0.048	0.584	1	1
-0.564	-0.668	-0.72	-0.674	-0.428	0	0.524	1	1	0.723
-0.332	-0.39	-0.38	-0.248	0.048	0.5	1	1	0.655	0.446
-0.11	-0.112	-0.035	0.162	0.524	1	1	0.584	0.31	0.161
0.112	0.166	0.31	0.584	1	1	0.524	0.168	-0.035	-0.108
0.334	0.444	0.655	1	1	0.5	0.048	-0.248	-0.38	-0.385
0.556	0.73	1	1	0.524	0	-0.428	-0.674	-0.72	-0.682
0.778	1	1	0.584	0.048	-0.5	-0.908	-1.08	-1.07	-0.939
1	1	0.655	0.168	-0.428	-1	-1.38	-1.496	-1.415	-1.216

Figure 7. Matrix Template (T) for the Uptrend Pattern.

Some additional chart patterns templates will be consider and technical indicators that were made to support the decision of buying or selling some financial asset, namely, the bull flag and the breakout pattern. The bull flag pattern and the breakout patterns, illustrated in

Figure 8 and Figure 9, were used simultaneously with the already existing technique for the detection of the uptrend pattern. Also, it was added an additional sliding window responsible for the detection of secondary graphical patterns, like the downtrend, presented in Figure 10. Results obtained from these new additions to the algorithm are depicted by several examples in section 5.

0.655	0,162	-0,428	-1	-1,38	-1,496	-1,415	-1,224	-1,5	1
1	0,584	0,048	-0,5	-0,908	-1,08	-1,07	-0,946	-0,875	1
1	1	0,524	0	-0,428	-0,674	-0,72	-0,668	-0,25	1
0.655	1	1	0,5	0,048	-0,248	-0,38	-0,39	0,375	1
0.31	0,584	1	1	0,524	0,162	-0,035	-0,112	1	0,524
-0.035	0,168	0,524	1	1	0,584	0,31	0,166	1	0,048
-0.38	-0,248	0,048	0,5	1	1	0,655	0,444	1	-0,428
-0.72	-0,674	-0,428	0	0,524	1	1	0,73	0,375	-0,904
-1.07	-1,08	-0,908	-0,5	0,048	0,584	1	1	-0,25	-1,38
-1.415	-1,496	-1,38	-1	-0,428	0,168	0,655	1	-0,875	-1,856

Figure 8. Bull Flag Pattern.

-1.224	-1,224	-1,224	-1,224	-1,224	-1,224	-1,224	-1,224	-1,496	-0,875	1
-0.946	-0,946	-0,946	-0,946	-0,946	-0,946	-0,946	-0,946	-1,08	-0,25	1
-0.668	-0,668	-0,668	-0,668	-0,668	-0,668	-0,668	-0,668	-0,674	0,375	1
-0.39	-0,39	-0,39	-0,39	-0,39	-0,39	-0,39	-0,39	-0,248	1	1
-0.112	-0,112	-0,112	-0,112	-0,112	-0,112	-0,112	-0,112	0,162	1	0,524
0.166	0,166	0,166	0,166	0,166	0,166	0,166	0,166	0,584	1	0,048
0.444	0,444	0,444	0,444	0,444	0,444	0,444	0,444	1	0,375	-0,428
0.73	0,73	0,73	0,73	0,73	0,73	0,73	0,73	1	-0,25	-0,904
1	1	1	1	1	1	1	1	0,584	-0,875	-1,38
1	1	1	1	1	1	1	1	0,168	-1,5	-1,856

Figure 9. Breakout Pattern.

1	0,655	0,162	-0,428	-1	-1,38	-1,496	-1,415	-1,224	-0,998
1	1	0,584	0,048	-0,5	-0,908	-1,08	-1,07	-0,946	-0,776
0.723	1	1	0,524	0	-0,428	-0,674	-0,72	-0,668	-0,564
0.446	0,655	1	1	0,5	0,048	-0,248	-0,38	-0,39	-0,332
0.161	0,31	0,584	1	1	0,524	0,162	-0,035	-0,112	-0,11
-0.108	-0,035	0,168	0,524	1	1	0,584	0,31	0,166	0,112
-0.385	-0,38	-0,248	0,048	0,5	1	1	0,655	0,444	0,334
-0.682	-0,72	-0,674	-0,428	0	0,524	1	1	0,73	0,556
-0.939	-1,07	-1,08	-0,908	-0,5	0,048	0,584	1	1	0,778
-1.216	-1,415	-1,496	-1,38	-1	-0,428	0,168	0,655	1	1

Figure 10. Downtrend Pattern.

## Data Acquisition

After defining the desired template, the data must be selected from the whole data set, which is done by defining a sliding window with a specific size and search for the desired window moving the window  $n$  days on each iteration, as illustrated in Figure 11.

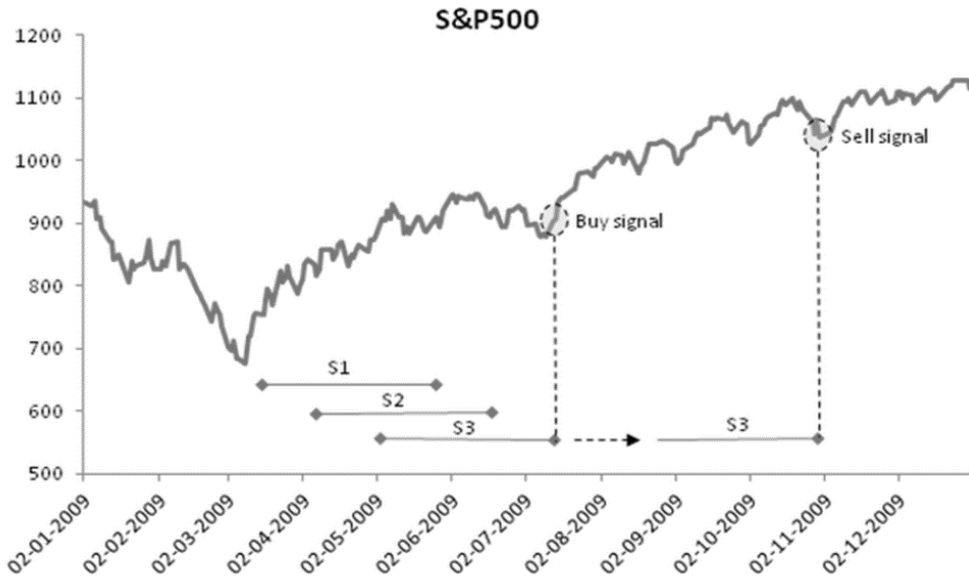


Figure 11. Sliding Window for Data Acquisition and Pattern Evaluation.

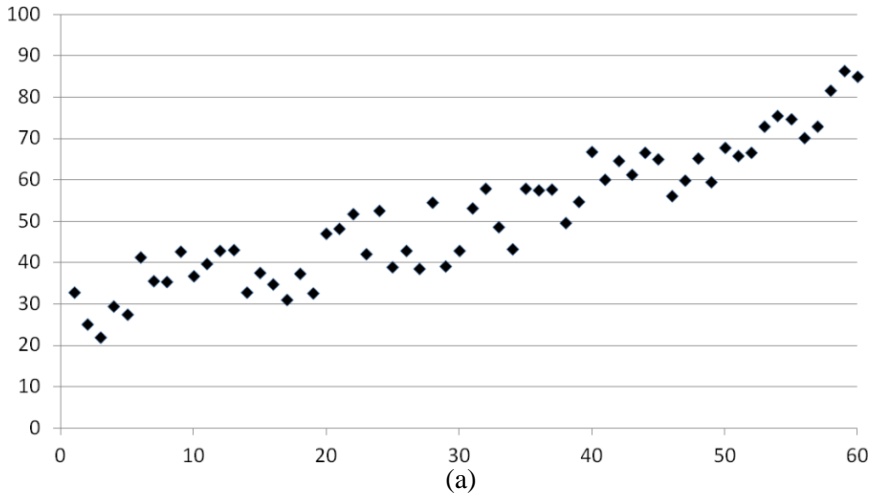
In order to avoid the spikes that usually appear in data values a noise reduction procedure should be applied where a defined percentage of points will be removed leading to a smooth data curve, this methods are described in the next section 3.2.

## Data Matrix

The noiseless data obtained in the previous step must now be described in a matrix format with a similar size to the matrix "T". Therefore, the new matrix will have the size 10x10 and will be referred as matrix "I". Thus, for each sliding window a new matrix "I" will be produced.

The algorithm then subdivides the sliding window in 10 equal groups, mapping each of these groups in one of the columns of matrix "I". This process will compress the time frame considered and maintains the essential features of the original pattern.

Each cell in the new matrix is subjected to a factorization function, so that the field values range between 0 and +1. Figure 12 illustrates the generation of a matrix "I" for a sliding window of 60 days.



									0.50
								0.17	0.17
								0.33	0.33
						0.33	0.67	0.33	
				0.17	0.50	0.50	0.33	0.17	
			0.50		0.33	0.17			
0.17	0.33	0.17	0.33	0.33	0.17				
	0.67	0.50		0.50					
0.33		0.33	0.17						
0.50									

(b)

Figure 12. (a) Temporal window of 60 days; (b) Matrix “I”.

**Fitness**

Once matrix “I” is obtained, a function of Fit is computed based on a Cross-Multiplication of both matrixes.

$$Fit_k = \sum_{i=1}^{10} \sum_{j=1}^{10} (T(i, j) \cdot I_k(i, j)) \tag{2}$$

Thus, the highest values for the Fit function will occur when the image in matrix “I” is in the highest conformance with matrix “T”. For example, in Figure 12, the Fit with the uptrend template, Figure 7, is 8.17 out of 10.

## Outlier Removal

An outlier can be defined, according to *Gumbel* (1960) and quoted in (Kaya, 2010), as:

‘The outliers are values which seem either too large or too small as compared to the rest of the observations.’

Applying this definition to financial time series, outliers are points far away from the trend. So, in this section several methods are used to find and reduce the effect of those points. The methods of noise reduction and outlier removal are compared with raw data performance, i.e. without any noise removal.

The need of this filtering operations, comes from the fact that when the data window is transformed in the matrix “P”, to match with the template pattern “T”, this kind of points lay in regions of negative coefficients, affecting negatively the fit function Eq. (2). In the next example it is presented this situation, the data window used is a 30 days period from the stock Alcoa, Figure 13.

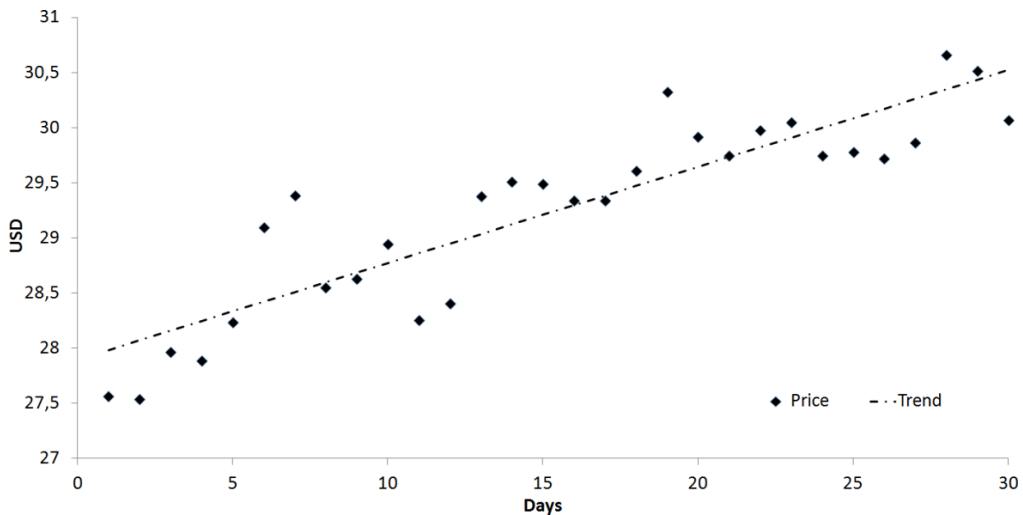


Figure 13. Financial data from Alcoa, Inc.

The template tested is the 45° ascendant pattern, the next matrixes (Figure 14, Figure 15) are the normalized products of the converted data window with the template. The fitness function will be the sum of all the terms of the matrix.

In Figure 14 is presented the product of the data, without any noise removal, with the template pattern coefficients, the next figure is the same data with the outlier removal, presented in section 3.2.2, where is clear the removal of several outliers.

The previous results show the fitness is quite different, so is expected that the filtered data will achieve more accurate results and that the decisions supported by this algorithm will bring better results.

Next, three methods of outlier detection and removal are presented; the first one is a noise reduction process, based on the extreme values of the data. The other two methods are based on data trend. In all methods, is applied the strategy of search and replace and is considered

that the outliers are described by the additive model. The search is based on the noise parameter optimized by genetic algorithm (GA) and the replace is visible in the substitution of the outlier by a value on the boundary of classification which identifies the point as an outlier or not outlier.

									0.67
						0.02	0.19		0.33
						0.35	0.67	0.67	
				0.03	0.17				0.21
	-0.04	-0.01		0.17	0.67				
			0.19						
		0.44							
	0.24		0.67						
	0.33								
1									

Figure 14. Match with raw data and template pattern - Fit = 6.97.

									0.67
							0.19		0.33
						0.53	0.67	0.67	
				0.03	0.17				0.21
				0.17	0.67				
		0.10	0.19						
	0.15	0.44							
	0.24		0.67						
	0.33								
1									

Figure 15. Match with filtered data and template pattern - Fit = 7.43.

The few number of outliers per window allows to implement algorithms that are light from the computational point of view.

### Method I: Noise Reduction

This method first begins to sort the data by value and then, based on the noise level parameter from the GA are defined two thresholds, one to limit the maximum admissible value of the points and other limit the minimum.

So in this method, data points are processed according to:

$$x_k^* = \begin{cases} T^M & \text{if } x_k > T^M \\ T^m & \text{if } x_k < T^m \\ x_k & \text{otherwise} \end{cases}$$

where,

$x_k$  data points;

$T^M$  value of the data point at the transition between data and the top noise level;

$T^m$  value at the lower noise level.

In the next example, Figure 16, sixteen data points are present. The first step is to sort the data, and then get the noise parameter, that in this case is 25%, resulting that the top and bottom three points are outliers, so the fourth and 13<sup>th</sup> points are the thresholds that limit the level of noise in our sample. The last step is to replace all points that lie outside the limits, by the thresholds to remove the outliers.

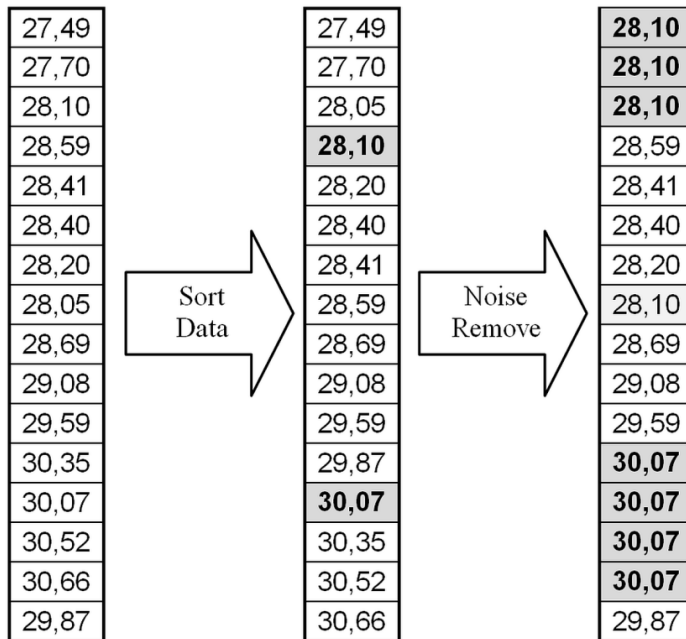


Figure 16. Noise reduction example.

The next figure shows this method applied to the previous data from Alcoa.

As can be seen by Figure 17 this method reduces the dynamic range of the sample, and eliminates points in the limit regions of the window and does not remove some outliers, this noise reduction provides better results with graphics with more horizontal trends, where it just eliminates the “spikes” caused by the outliers. This is the method applied in the tests of pattern template methodology.



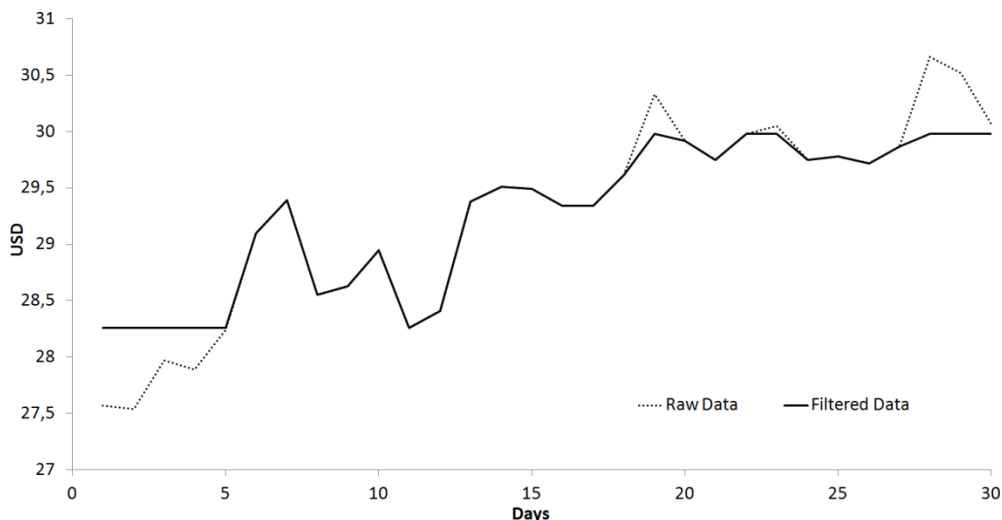


Figure 17. Noise reduction of 20%.

### Method II: Outlier Removal with Linear Trend

This outlier detection begins with the creation of a linear trend, based on linear regression, and then defines two boundary lines, above and below the original trend, the distance of these lines to the trend is given by the noise parameter returned by the GA. The outliers are points outside these lines. So, bigger values of the noise parameter move boundaries farther away from the trend leaving more points intact, while in the first method bigger values replaces more points, applying a more aggressive filtering.

Using a noise parameter of 1.3% to the test data from Alcoa, defines the boundaries shown in Figure 18.

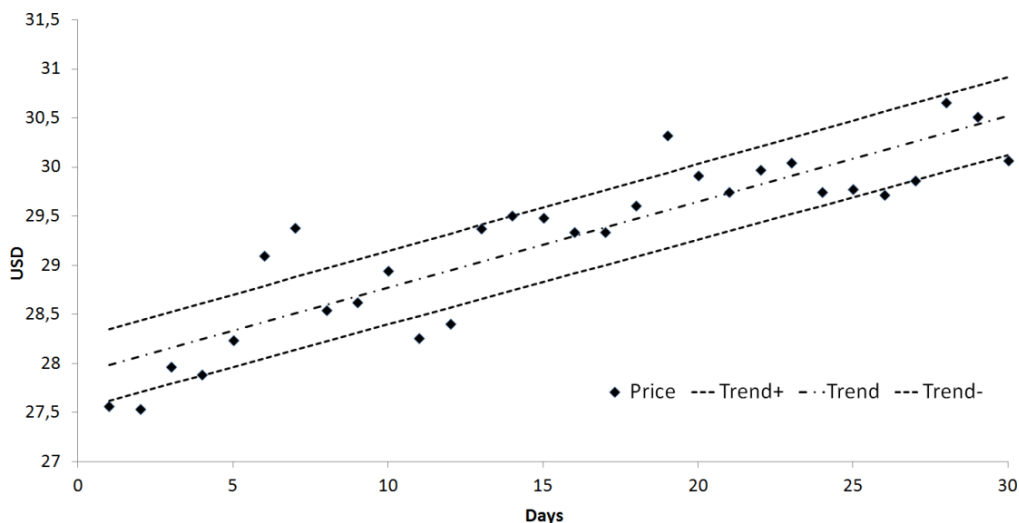


Figure 18. Outlier detection with 1.3% noise level.

The noise parameter gives the vertical distance of the boundary lines to the trend, in the present example the points of these lines are 1.3% above and below the points that define the trend line. So, the points outside those lines are outliers and must be replaced by the corresponding point in the nearest threshold line. In Figure 19, the data after filtering is shown.

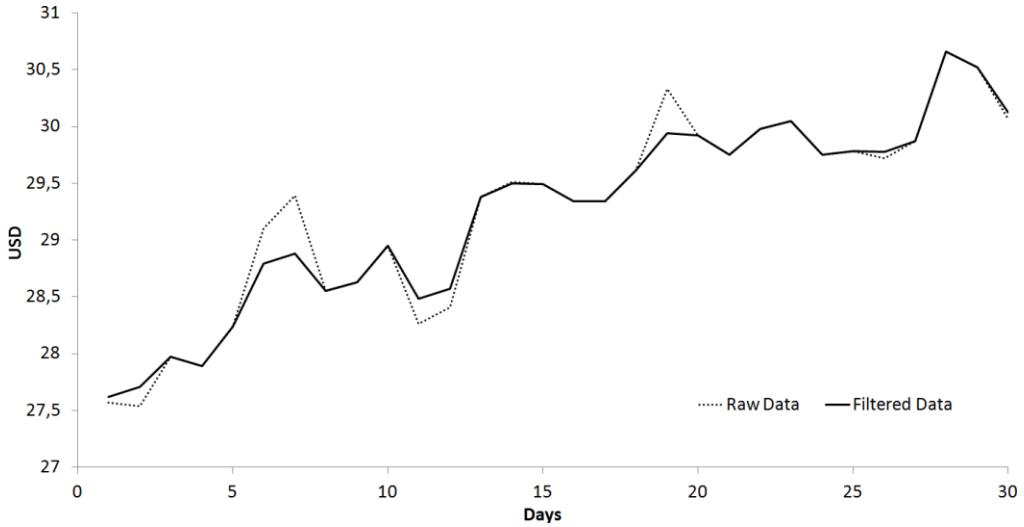


Figure 19. Outlier removal with 1.3% noise level.

Comparing Figure 17 and Figure 19, it is clear how the two methods are different in their results, this last method does not reduce the dynamic range of data, and is more adjusted to the trend of the market. The trend is a very important indicator of the market behavior, but data cannot just be replaced by the trend, because the information is in the ups and downs of the graph and an aggressive filtering will dismiss all capability of “reading” possible changes in trend.

### Method III Outlier Removal with Polynomial Trend

In order to test a more intense filtering and keep the ability of predicting changes in market trend, this new method defines a trend line using a 2<sup>nd</sup> order polynomial regression. The use of this regression allows filtered data to keep the information of possible inverting market trend as shown in Figure 20.

This last algorithm is pretty much like the second, in section 3.2.2, the only change is the definition of the trend line. Like in the previous method after calculating the trend line, two boundary lines must be found based on the noise parameter:

$$Trend_k^- = Trend_k(1 - N_{GA})$$

$$Trend_k^+ = Trend_k(1 + N_{GA})$$

where,

$N_{GA}$  noise parameter from the GA;

$Trend_k$   $k$  point in the trend where the correspondent point of the boundary is calculated.

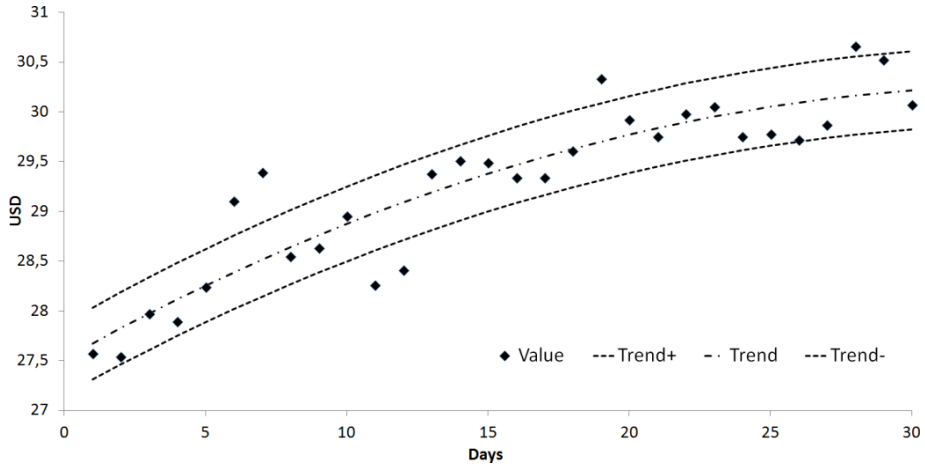


Figure 20. Alcoa data with 2nd order polynomial regression.

For comparing purposes, the influence of the 2<sup>nd</sup> order regression in the outlier detection, in Figure 20 is presented the boundary lines using a noise level of 1.3%.

As in the previous method, next is presented a Figure 21 with an example of data filtered using this algorithm. The filtered points are calculated according to:

$$x_k^* = \begin{cases} Trend_k^+ & \text{if } x_k > Trend_k^+ \\ Trend_k^- & \text{if } x_k < Trend_k^- \\ x_k & \text{otherwise} \end{cases}$$

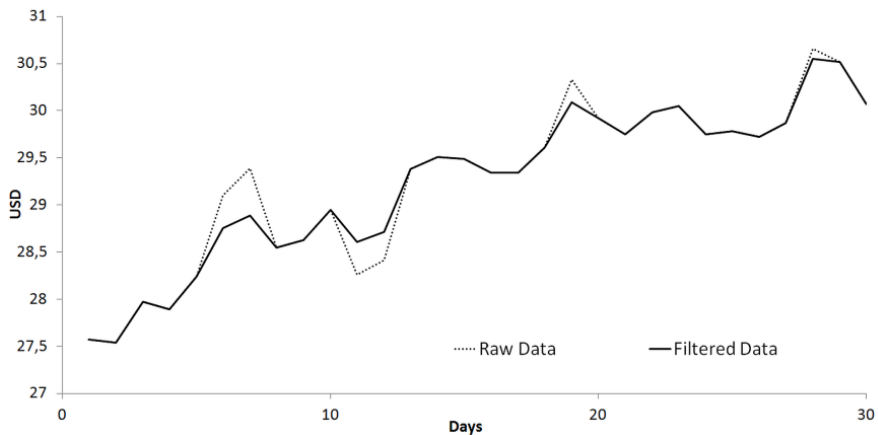


Figure 21. 2nd order polynomial regression with noise level 1.3%.

Although this method allows a more intense filtering, the data must not be replaced by the trend for the reasons previously presented. The tests made when data was replaced by its trend, showed poor results of earnings, implying bad prediction capability of market movement, in fact they were worst than analyzing raw data.

## Volatility

Creating a profitable portfolio highly depends on how the stocks are chosen, here is tested the effect of picking low volatility stocks to invest. The training and testing data are collected from a set of 567 stocks from the S&P 500 index, from the beginning of 2001 to September 3, 2010. The set is of 567 and not just 500 stocks, because in this period of time some companies were removed from the index and replaced by others.

It was used two daily measures of volatility, the first one is the “Historical Volatility”, described below, and the second is the standard deviation of the stock prices. Both measures were calculated over 10 days.

The “Historical Volatility” (HV) value is calculated by the following set of Eq. (3) and Eq. (4):

$$R_n = \text{Ln} \left( \frac{P_n}{P_{n-1}} \right) \quad (3)$$

where

$P_n$  is the stock price at day  $n$ ;

$\text{Ln}(\ )$  is the natural logarithm.

$$H_v = \sqrt{\frac{\sum R_n^2}{d}} \quad (4)$$

where

$d$  is the number of days where the volatility is being calculated. The sum is calculated over  $d$  days;

$R_n$  is defined by Eq. (3).

This volatility is known as “Non-centered Historical Volatility”, the calculation is the standard deviation of the earnings,  $R_n$ , where the mean is not subtracted, that justify the non-centered name.

The second method of volatility, as refer before, is the standard deviation of the close price stock, calculated by the following Eq. (5):

$$V_\sigma = \sqrt{\frac{\sum (P_n - \mu)^2}{d}} \quad (5)$$

where

- $P_n$  is the stock price at day  $n$ ;
- $\mu$  is the mean of the prices over  $d$  days;
- $d$  is the number of days.

## OPTIMIZATION METHODOLOGY

Determining the optimal parameters for a trading strategy based on chart pattern templates may be extremely profitable but is clearly a huge task, particularly, when considering a realistic case of using several templates and technical indicators together. In this case, the search space rapidly rises to unacceptable numbers and there is no analytical model to solve the problem, therefore, evolutionary computation appears as a natural solution.

### Optimization Methodology Overview

The proposed optimization methodology, illustrated in Figure 22, is implemented with a three tier architecture (Data, Application and Interface levels). The data level includes the description of several chart templates, the market data which includes updated historical data from most relevant financial markets, and, finally, the technical indicators for the market data. The application level is mainly composed by the optimization kernel and the data acquisition system. The interface level includes the selection of the templates, market data and technical indicators to be considered during the optimization process, the fitness function, and desired output parameters.

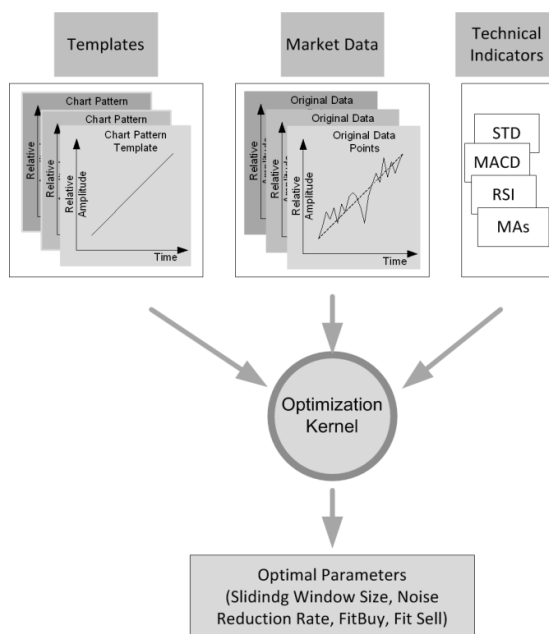


Figure 22. Optimization Methodology Overview.

## Optimization Parameters

The selected optimization parameters, in this study, will be the sliding window size, the noise removal percentage and two additional parameters based on the fitness, *FitBuy* and *FitSell*. The *FitBuy* is the limit that should be crossed up to produce a buy signal, while the *FitSell* is the limit that should be crossed down to originate a sell signal.

## Optimization Kernel

The optimization kernel is based on a genetic algorithm implementation where the previously referred optimization parameters are the genes that composed the chromosome structure, Figure 23.

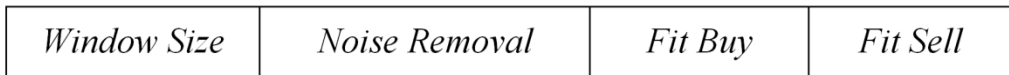


Figure 23. Chromosome Structure for the uptrend approach.

The algorithm uses a single point crossover and the selection method adopted is a uniform ranking method (Zhang & Kim, 2000). The optimization kernel evaluates each population element during a predefined training period and considering a sliding window which advances with a 1 day step. The population is rated, for the examples presented in the next section based on the return achieved on the evaluation/training period.

## RESULTS

In this section several case studies using the proposed methodologies are presented. The application was tested in real market conditions. Data was retrieved from 1998 to 2010 for three main stock indexes: S&P500, Dow Jones Industrial Average and NYSE Composite Index. These indexes are representative of the United States stock markets, the chosen time span is large enough to embark a bull and a bear market. Particularly, the training and testing period were, respectively, from 1998 to 2004 and from 2005 to 2010.

### Case Study I. Uptrend Detection

In this case study the application is going to find the uptrend pattern. It gives the indication of an upward trend, which develops because the demand for shares is larger than the supply of shares. So the algorithm is based on the assumption that once this pattern is detected, the upward trend will continue to follow until at some point a reversal is detected. Once this pattern is detected a buying signal is generated, and when the uptrend eventually breaks the software will sell this position.

In Figure 24 the evolution of the S&P500 from 2005 until 2010, testing period, is presented. Additionally two more curves are depicted with the best and average GA run

obtained during the training period. For the first three years when the market is in an uptrend the GA just follows the market. Then the best GA sells its position around January 2008 and only reenters the market in July 2009. As for the average GA, it leaves the market on September 2007, only to reenter temporarily from April to July 2008 and then finally from June 2009 onward. It is clearly shown that the algorithm also has the capability of detecting bear markets and staying out of them.

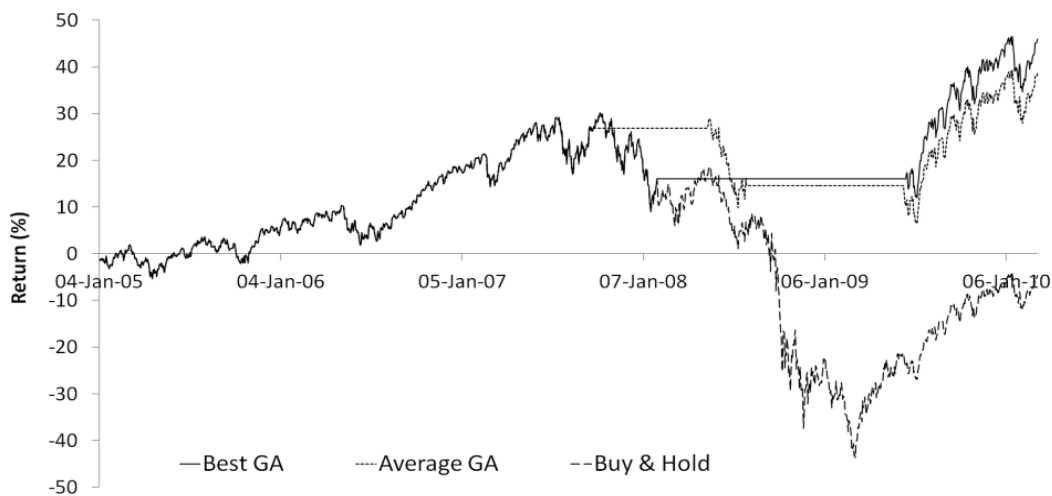


Figure 24. Profitability Growth for three Strategies on S&P500.

In Figure 25 a histogram is presented, which demonstrates the profitability distribution for 50 different executions of the GA. The results show the algorithm is capable of producing consistent and far better results than the B&H strategy, with the results centered around 35% return and the B&H at almost -5%.

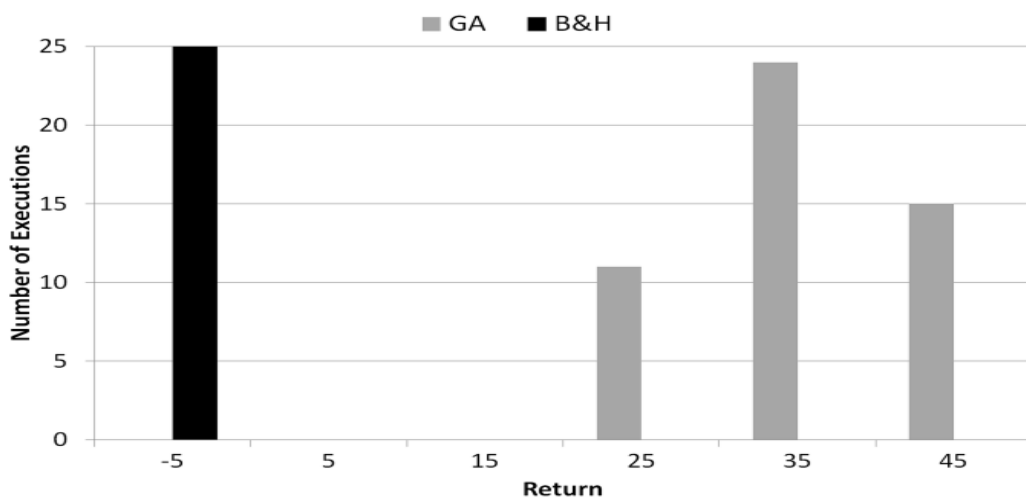


Figure 25. GA approach vs. B&H on S&P500.

Finally, Table 2 shows the results obtained for three stock indexes. The robustness and superiority of the genetic algorithm is evident by the returns presented.

**Table 2. Profitability for Different Stock Indexes**

Curve\Market	S&P500	Dow Jones	NYSE
Average GA	36.92%	16.33%	10.02%
Best GA run	46.02%	30.54%	14.57%
Buy and Hold	-4.69%	0.12%	-1.41%

## Case Study II. Uptrend and Downtrend

In this section the use of the uptrend and downtrend patterns with independent parameters is going to be evaluated.

The data used for this case study are 100 stocks from the S&P500, for the periods mention before. Also, the system was changed, by adding more graphical patterns that were responsible for the identification of the downtrend formation and by joining two new additional genes to the chromosome:

- FitDescent - This new gene will be associated to the new pattern templates characterizing the downtrend formation. This way the algorithm will be more accurate in the search of this particular graphic and will have a better precision when selling an acquired market position.
- Downtrend Sliding Window - Previously the algorithm had only one sliding window available. Technical analysis dictates that different pattern formations may have different time periods associated with them.

The Downtrend pattern is a graphical formation used to determine the moment where ceases the uptrend/horizontal tendency. Once this new pattern arises the algorithm will sell its position and stay out of the market during its presence.

Table 3 depicts the average results obtained for all of the 100 stocks that were retrieved from S&P500.

**Table 3. Uptrend Pattern + Downtrend Pattern**

Methodology	Fit Buy	Fit Descent	Noise Removal (%)	Sliding Window (days)	10 Runs Average Testing Period
Uptrend Pat. + Downtrend Pattern	5,45	6,96	27,70%	39,7/22,8	35,08%

In this new methodology the algorithm will acquire a buying position once the uptrend pattern is detected by one of the three matrixes that are used for the uptrend detection. On the



other hand the system will sell its position once it detects the inversion/downtrend pattern. Sliding window sizes for both patterns are different.

Next, in Figure 14 the return for the testing period is shown. In this figure it is apparent the time period in which the genetic algorithm detects the downward trend, leaving the market in late 2008. Afterwards the algorithm reenters the market when it detects an upward trend.

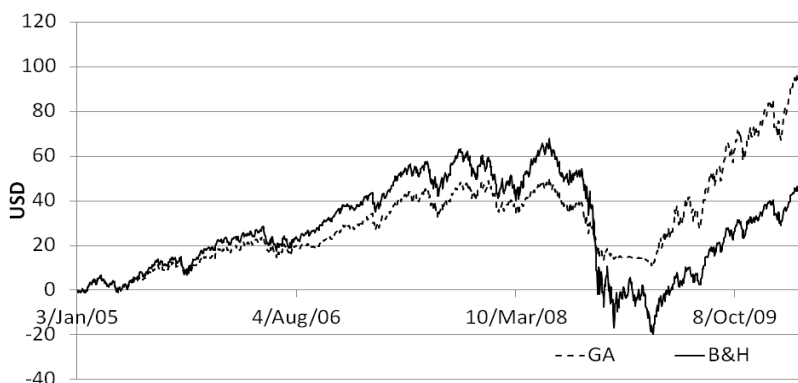


Figure 26. Return value of the GA and B&H methodologies for all of the 100 stocks.

### Case Study III: Outlier Removal

To test the several outlier removal methods, was used data from 100 stocks of the S&P 500 index, where the training set is from 1998 to the end of 2004, and the test period is January 2005 to April 21, 2010. Also for comparison purposes the same data, in raw format, was feed to the genetic algorithm.

The application and the genetic structure is the same for all methods and is the program used in the previous section, and the transaction costs are considered for the results.

In this tests, the algorithm is trying to find uptrend periods to invest and when to go out the market, for that the methods described in the previous section, '*Detection of the Uptrend*' (*DoU*) and '*Uptrend and Downtrend*' (*UaD*), are used.

Beginning with the *DoU* pattern detection method, the earnings results when applying the several outlier removal algorithms are presented in Table 4, this results show the average return of each methodology.

**Table 4. Average returns of each methodology for pattern detection *DoU***

Methodology	Average return of 10 runs of all stocks	Improvement to raw data	Average return of best run of each stock	Impr. to raw data
Raw data	41.30%	-	59.80%	-
Noise Rem.	41.41%	0.2%	61.28%	2.4%
Linear Regr.	45.55%	10.2%	84.22%	40.8%
Poly. Regr.	42.08%	1.8%	82.05%	37.2%

By the results presented in Table 4, the linear regression based method is the one that gives better returns, the improvement in the average return in relation to the unfiltered raw data system is around 10%, which clearly justifies the application of this methodology, this effect is more clear when the best runs are analyzed.

Next are presented the results for *UaD* method, the average earnings are shown in Table 5.

**Table 5. Average returns of each methodology for pattern detection *UaD***

Methodology	Average return of 10 runs of all stocks	Improvement to raw data	Average return of best run of each stock	Impr. to raw data
Raw data	50.54%	-	81.78%	-
Noise Rem.	50.17%	-0.73%	82.49%	0.87%
Linear Regr.	49.78%	-1.50%	89.94%	9.98%
Poly. Regr.	50.73%	0.38%	90.48%	10.6%

The results from Table 5, show that all the outlier removal methods have earnings very close to each other, in fact just one, 2<sup>nd</sup> order polynomial regression, gets better result than no data filtering. This *UaD* method of pattern detection is more effective than *DoU*, because in this method the algorithm is detecting the uptrend to enter the market and the down trend to exit and the genetic algorithm can optimize all the other parameters. So, the noise reduction is not a very important factor in decision process.

In conclusion, the method that benefits more from noise removal is the *DoU*, and in this method the linear regression is the one that takes more advantage of it, this is caused by the effect of linearization of the regression and the noise removal, that causes the data window to become very close to the linear template pattern.

The low values of improvement in the *UaD* method could be explained by the fact that the selling decision is taken in the presence of a downtrend pattern, and usually the down movements are fast and with no spikes, or with low noise, so the noise removal has less effect in this method.

## Case Study IV: Volatility

For testing this method of composing a portfolio, the data used is from January 2008 to September 3 of 2010, and 100 stocks were chosen by the criteria of low volatility, also an experiment with 75 stocks was made.

The fact that the list changes every day, poses the problem that only for a small number of stocks is possible to get a complete window of data and the portfolio list has less than 100 stable stocks.

So, the solution found was once a stock enters the list is kept there until goes over some threshold, in which case a stock with lower volatility will enter the portfolio. Based on this, 3 threshold values were tested, these limit values were chosen based on several preliminary experiments made in a small window of data outside the test period. The values obtained from these tests were rounded to 150, 200 and 250. All this process, of stock picking, is

represented in Figure 27, where the stocks were order based on their volatility value and only exit the portfolio when the threshold value is reached.

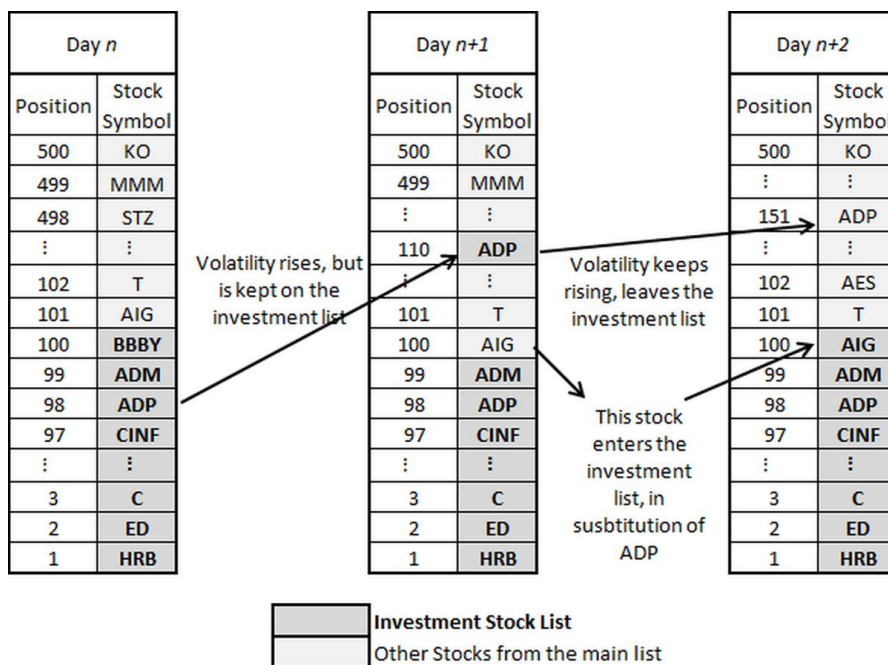


Figure 27. Stock picking based on volatility value, with 150 limit position.

After choosing the portfolio composition, the 100/75 stock were tested with the algorithms of section 5.1 and section 5.2, the 'Detection of the Uptrend' (DoU) and 'Uptrend and Downtrend' (UaD) algorithm, respectively. The portfolio was evaluated on a daily basis and the stocks were tested by the algorithms in order find patterns and to identify investment opportunities. In case a stock leaves the investment stock list, all open market positions must be closed. The results of the investment tests, using the historical volatility (HV) as a measure of volatility are presented in Table 6.

**Table 6. Earnings obtained by choosing low historical volatility stocks**

Method	Choice Parameter	Earnings (%)	Std.dev. Earnings
B&H	-	-14,60	40,53
DuO	75/250	-0,04	18,48
	100/150	0,95	15,70
	100/200	1,35	16,46
	100/250	0,84	18,58
UaD	75/250	-0,91	19,82
	100/150	-1,48	16,83
	100/200	-2,71	20,74
	100/250	-1,60	19,64

As can be seen by the previous Table 6, in all combinations of the test parameters, the earnings are better than the “Buy & Hold” (B&H) strategy. The values are not very interesting, but comparing with the loss of 14% by the B&H it is a sign that taking some criteria of choosing the stocks it can lead to better results, proving the advantage of picking low volatility stocks and supporting the idea that volatility is a measure of risk. Also the standard deviation of the earnings is lower than the B&H, which indicates the less spread of the earnings, this is presented in Figure 28, where is shown the frequency distribution of the earnings, comparing the B&H and the best historical volatility method (*DuO* 100/200). In this Figure 28, it is clear the spread and the deviation of B&H method to the more negative part of the graphic, contrasting with the tested methodology where the stocks are chosen by its low volatility.

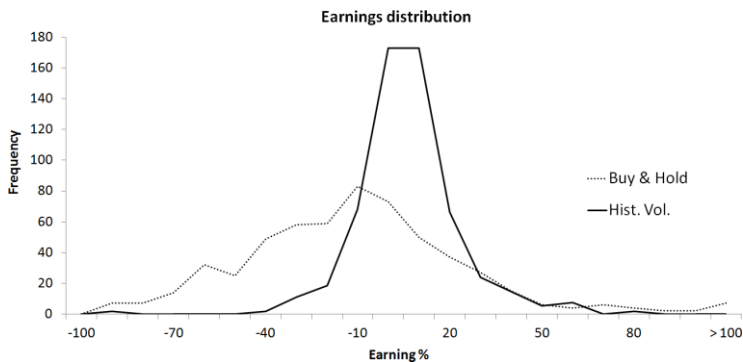


Figure 28. Freq. distribution of earnings comparing Buy&Hold with best method of Hist. Volatility

In Table 7 the tests results of using the standard deviation as volatility measure are presented. These results shows, when compared to the previous results Table 6, that the standard deviation is a more efficient measure of volatility for choosing the stocks to the investment portfolio. From the results, is also clear, that a more stable portfolio composition, threshold of 250, allows to get better earnings. The better performance may be explained by the fact that in a more unstable portfolio composition, each time a stock leaves the portfolio, all open positions must be closed and most of the times the transactions were closed even before making enough earnings to pay the transaction costs.

**Table 7. Earnings obtained by choosing low volatility stocks, using standard deviation as volatility measure**

Method	Choice parameter	Earnings (%)	Std.dev. Earnings
B&H	-	-14,60	40,53
DuO	75/250	12,29	70,36
	100/150	8,28	60,44
	100/200	15,55	98,40
	100/250	18,86	112,4
UaD	75/250	17,93	78,16
	100/150	10,73	68,59
	100/200	14,62	59,43
	100/250	16,52	72,90

In Figure 29 is presented the best of the tests for choosing stocks, based on low standard deviation of price,  $DuO$  100/250. From the graphic was a surprise the value of standard deviation of earnings, 112, which apparently from the curve should be smaller than the value for the B&H. The larger value is explained by the earning values that exist above the 500%, in fact they are away above this number, the highest is around 1500%, causing a bigger spread of data and an increase in standard deviation of earnings.

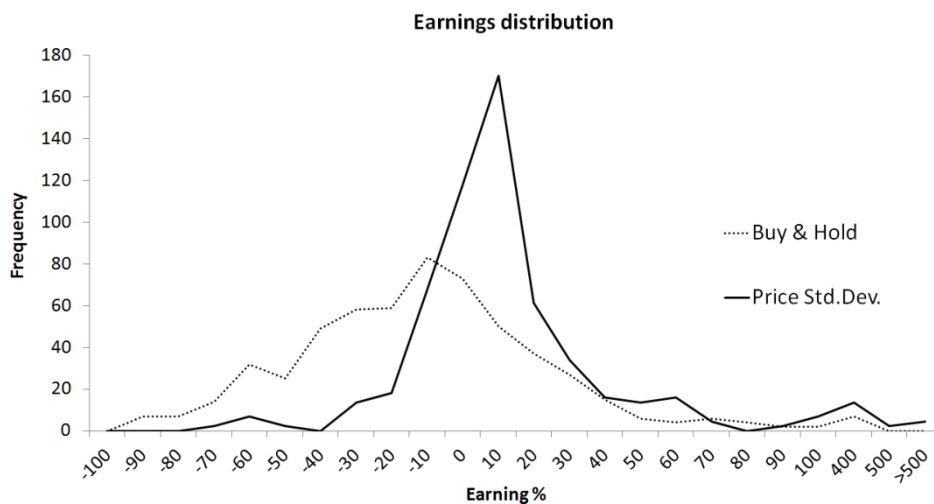


Figure 29. Freq. distribution of earnings comparing Buy&Hold with best method of Price Std.Dev.

From the tests made for the two volatility measures is possible to conclude that the one that gives better results is the standard deviation (SD) of the stock price, this could be explained by the fact that as soon the price begins to have oscillations the SD will increase and causing the exclusion of the stock from the portfolio. In the HV measure, the fact that is calculated based on the earnings of day to day close price, results in stock volatility that are very close to each other, only when a stock has big drawdown's the value of volatility will increase compared to others and will be eliminated from the portfolio. So, if the day to day earning is kept stable the stock will be maintain in the portfolio, even if risk investment increases.

## CONCLUSION

This chapter presents a new methodology where pattern recognition techniques, based on a template approach, together with an evolutionary computation technique are used to obtain efficient trading strategies. The proposed approach as tested with real data from world relevant stock markets. The achieved results proof both the robustness and effectiveness of the solution once it always perform better than B&H and assures a high return rate. Even, in economic crisis scenarios like the 2008 year.

Combined with this new technique, the studies made about outlier removal and portfolio composition based with low volatility measures, prove that is still possible to improve the results of this investment strategy.

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*Chapter 3*

# IS THERE A GAIN IN COMBINING LOW RISK AND FUNDAMENTAL INVESTMENT OBJECTIVES IN PORTFOLIO ALLOCATION?

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## ABSTRACT

A modern trend in equity investing is to combine a low risk objective with fundamental portfolio weighting. Examples include the S&P's GIVI index and the RAFI low volatility index. This paper aims at investigating the gains in combining low-risk and fundamentally weighted portfolios for the universe of S&P 500 constituents over the period 1987-2012. We find that, compared to the market capitalization-weighted benchmark (the S&P 500), there are economically significant gains in low risk and fundamental investing for the complete sample. Combining the two investment styles reduces the market regime dependence of the portfolio performance. The choice of allocation scheme has a minor impact on the gross portfolio performance, but matters for the portfolio turnover.

## 1. INTRODUCTION

For many investors, the market capitalization-weighted portfolio has long been the standard choice of risky asset to invest in. The market portfolio thanks its popularity to the well-known Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964) and Lintner (1965), which, under heroic assumptions, predicts that the market portfolio is mean-variance efficient in the sense that it provides the highest possible expected return above the risk free rate per unit of volatility. Many of the CAPM assumptions are in practice violated, explaining also why several empirical studies have rejected the mean-variance efficiency of the market

capitalization-weighted portfolio. E.g. Clarke et al. (2006) find that minimum variance portfolios based on the 1000 largest U.S. stocks over the 1968-2005 period achieve a volatility reduction of about 25% while delivering comparable or even higher average returns than the market portfolio.

For this reason, an increasing number of institutional investors have adopted an alternative weighting scheme searching for diversification based on fundamental accounting data that weight stocks by firm characteristics such as earnings, dividends, or book value (Arnott et al., 2005) or using risk forecasts. Over the past few years such alternative investment strategies attracted an increasing number of investors. In 2011, more than 40 % of North American professional investors already adopted an alternative weighting scheme to their portfolio (North American Index Survey, 2011). Since the beginning of 2010, advanced beta funds generated a total inflow of \$81.6bn, the largest part attracted by exchange traded funds (\$ 66.2bn). The total value of assets held in smart beta portfolios grew from \$58bn at the end of 2010, up to \$142bn at the beginning of 2013 (Flood, 2013). The increasing value is reflected in the growing number of smart beta funds traded. The number of smart beta indexes increased radically from 55 at the beginning of 2006 to 457 at the end of the first quarter of 2013. One example is the commercialization of the fundamental weighting scheme. Over the past decade, several FTSE RAFI indexes, inspired by the idea of fundamental indexing, were launched. Other advanced beta strategies exploit different investment styles. The low-volatility, value and momentum indexes represent only a limited part of possibilities.

Several studies have shown that both low volatility and fundamental investing outperform the market on a risk-adjusted basis (E.g., Baker and Haugen, 2012, Arnott et al., 2005). In this context, portfolios combining low risk and fundamental value based investing have been proposed (e.g., GIVI and RAFI low-volatility). The idea is to harvest jointly the low risk and fundamental value premiums and enjoy diversification benefits due to the specificity of the life cycle of each investment style.

An important question is the design of such multi-style portfolios. We are the first to compare different allocation methods. We apply an equally weighted (EW), minimum portfolio variance (MinVar) and an equal risk contribution (ERC) approach to allocate over a low-risk and fundamental portfolio. We also discuss a sequential method, where the most risky stocks (resp. stocks with lowest fundamental value) are first removed and a fundamental weighting (resp. low risk weighting) is applied on the remaining stocks.

We show the existence of diversification opportunities over the life cycle of investment styles. We show fundamental portfolios underperform market cap-weighted indexes in the run-up of speculative bubbles. Though after the bubble burst, the fundamental portfolio again outperforms. Our results show low-risk investments outperform market cap-weighted indexes in bear markets and underperform in bull markets. By combining both investment styles, the influence of the market regime on the portfolio return is reduced.

We study the exposure of the portfolios to the market, size, value and momentum factor. The results indicate the existence of highly significant positive alphas over the total investment period from 1987 to 2012.

Section 2 includes a literature review and hypotheses development. In Section 3, we give an overview of the data and the methodology. Section 4 discusses the results for single-style portfolios. In Section 5, we present our findings for the multi-style portfolios. We conclude in Section 6.

## 2. LITERATURE REVIEW AND HYPOTHESES DEVELOPMENT

Markowitz (1952) derives an optimal asset allocation framework. In the mean-variance efficient setting, investors only care about average expected returns and the corresponding variance. Mean-variance efficient portfolio theory led to the two-fund separation theorem, stating every investor holds a combination of a risk-free asset and a risky portfolio. The CAPM states mean-variance optimizing investors should hold the market portfolio. The implementation of the mean-variance efficient portfolio is a complex process. However, many researchers have criticized the cap weighted index for being inefficient. Siegel (2006) develops the noisy market hypothesis under which prices are noisy estimates when stock prices diverge temporarily from their true value. According to the noisy market hypothesis, market cap-weighted indexes overweight overvalued stocks and underweight underpriced stocks leading to a drag on returns (see e.g., De Moor et al., 2012). Furthermore, cap-weighted indexes are often highly concentrated in a limited number of stocks. A wide variety of alternatively weighted indexes has been developed, in order to avoid the weaknesses of market capitalization-weighted portfolios.

We start with an overview of fundamental investing literature. Next, we present the literature on low-risk investing. Section 2.3 concludes with a discussion on combining both investment styles.

### 2.1. Fundamental Investing

Fundamental indexing weights stocks in proportion to accounting measures of company size. Proponents of fundamental investing argue that market cap-weighted indexes overweight overvalued stocks and underweight underpriced stocks (e.g., Siegel, 2006). Fundamentally weighted indexes tackle this hurdle by imposing a price-insensitive weighting scheme. Wood and Evans (2003) suggest a profit-based index. Arnott et al. (2005) recommend the use of several fundamental measures such as book value of equity, revenues and number of employees. They show an outperformance of the S&P 500 US benchmark by on average 1.97 pps a year over the 43 year sample period (1962-2004). According to Arnott et al. (2005), the superior performance is the result of portfolio construction, inefficient prices, an additional exposure to distress risk and a combination of the former reasons.

Fundamental indexing is inherently biased towards value and small stocks (Jun and Malkiel, 2007). Among others, Asness (2006), Perold (2007) and Blitz and Swinkels (2008) criticize fundamental investing for being a simple repackaging of long-existing and well-documented investment strategies. Using European data, Stotz et al. (2010) show fundamental indexes produce positive, often significant alphas. Walkhausl and Lobe (2010) expand the geographical scope of fundamental indexing towards a global perspective. They conclude that fundamentally weighted global portfolios exhibit genuine alpha after correcting for additional exposure to the Fama and French (1993) and Carhart (1997) risk-factors.

Opponents also criticize the sensitivity of fundamental indexes to methodological assumptions. Fundamental indexes need to be rebalanced at a regular interval. Investors therefore need to decide on the rebalancing frequency and moment. Blitz et al. (2010) have provided evidence for the dispersion in performance between different rebalancing moments.

In 2009, an annual March rebalanced portfolio outperformed the capitalization-weighted benchmark by 10 %, whereby rebalancing in September resulted in an underperformance. Besides the rebalancing frequency, index performance is also influenced by the choice of the fundamental metric. For the 1982-2010 sample period, fundamental portfolio returns can differ up to 10.8 % (Amenc et al., 2013).

Several authors have shown an outperformance of fundamentally weighted indexes over the market cap-weighted indexes, but the relative performance is time-dependent. Chen et al. (2007) propose a framework in which smoothed cap weights represent estimates of a company's fundamental weight. Their results show fundamental indexes tend to outperform in bear markets. Using European data, Hemminki and Puttonen (2008) construct fundamental portfolios outperforming the cap-weighted benchmark by an average of 1.76 percentage points per year over the 1996-2006 period.

Hsu et al. (2006) find an underperformance over the dot-com bubble (late 90s). The fundamental portfolios underperform because of their anti-bubble nature. Compared to market cap-weighted indexes, fundamental portfolios underweight companies for which stock price growth is not reflected in the growth of their fundamentals. Contrary to value indexes, fundamental indexes outperform market cap-weighted indexes during bull markets (Hsu et al., 2006).

Based on this discussion, we state the following hypotheses:

Hypothesis 1a: Fundamentally weighted portfolios outperform the market cap-weighted benchmark on a risk-adjusted basis in the long-run.

Hypothesis 1b: Fundamental indexes tend to underperform compared to market cap-weighted indexes during speculative bubbles and outperform them in bear markets.

## **2.2. Low-Risk Investing**

Following the CAPM, the relationship between risk and returns should be positive. Many authors have shown the outperformance over the long-term of low-volatility stocks over highly volatile stocks. Haugen and Heins (1972) show US stocks with smaller volatility exhibit larger returns over a period ranging from 1926 to 1971. Clarke et al. (2006) document a decrease in the volatility of 25 % while having comparable returns as the market over the period 1968-2005 in the US. Furthermore, over the 1968-2008 period, low-volatility and low-beta US stocks both possess higher returns (Baker et al. 2011). Baker and Haugen (2012) also find a flat relationship between risk and returns.

Several behavioral drivers of the volatility anomaly exist, namely the lottery effect, representativeness and overconfidence (Baker et al., 2011). Also, institutional investors' performance is likely to be benchmarked with the tracking error of market cap-weighted indices. As low-risk investing increases the tracking error, these securities are anticipated as unattractive. Institutional investors are often constrained with respect to leveraged investments. Consequently, instead of using leverage, they overweight risky stocks (Frazzini and Pedersen, 2013). All these biases cause a gap between market prices and the true, fundamental prices causing non-compensated volatility (Shiller, 2000).

A further explanation for the outperformance of low risk strategies on the long run is because of the asymmetric effect of positive and negative returns of the same magnitude on

compounded returns. If a portfolio loses 20% in one month, and gains 20% the next month, the portfolio will still have an overall loss of 4%. To recover fully from the loss of 20%, the portfolio actually needs to gain 25% the next month rather than just 20%. The asymmetric impact of returns increases with volatility (Boudt and Peeters, 2011). Assume that a portfolio loses 50% in one month: to recover this loss the portfolio needs to gain 100% the next month.

Soe (2012) illustrates the time-varying performance of low-risk strategies in the short-run. They show an underperformance of low-risk investments in bull markets. As a result of its less-than unity market beta, the low risk investment style is expected to underperform the market in market rallies. However, low-volatility strategies tend to outperform market cap-weighted benchmarks during down-markets. The outperformance in bear markets is more consistent and larger in magnitude. As a result, low-volatility strategies outperform in the long-run. Earlier, Ang et al. (2006) documented superior performance of high idiosyncratic volatility stocks during up periods in the US and global markets. Contrarily, high-risk stocks exhibit an underperformance during bear markets.

Different low-risk weighting schemes are available. Low-risk indices can be constructed by minimum variance or maximum diversification optimizing techniques or by heuristic approaches (Kang, 2012). To apply the joint optimization approaches, investors need to estimate the covariance matrix. Alternatively, low-risk indexes can be created by applying a less complex heuristic approach. Stocks are selected or weighted based on a risk criterion. Examples of such criteria are the volatility, variance, beta, semi-deviation etc. One of the advantages over optimization approaches is its simplicity. Kang (2012) shows for both optimized and non-optimized strategies an overall portfolio volatility reduction of 20 to 30% and a reduction of the portfolio downside risk.

Following earlier research, we can state a second set of hypotheses:

Hypothesis 2a: Low-risk investment strategies outperform the market cap-weighted indexes over a long-term investment period.

Hypothesis 2b: Low-risk investments exhibit asymmetric payoffs. They underperform compared to capitalization-weighted indexes in bull markets and outperform in bear markets.

### **2.3. Combining Fundamental and Low-Risk Investing**

Recently, portfolios that combine low risk and fundamental value based investing have been proposed to harvest jointly the low risk and fundamental value premiums and enjoy diversification benefits due to the specificity of the life cycle of each investment style (Shleifer and Barberis, 2001). As stated above, each style appears to perform differently depending on the equity market regime. As a result, diversification opportunities exist across the life cycles of these equity investment styles.

The Global Intrinsic Value Index (GIVI) and RAFI low-volatility index are only two examples of recently launched indexes combining low-risk and fundamental investing. Both are designed to capture the low-risk and value premium. Luk et al. (2012) illustrate an outperformance of the GIVI index over the S&P500 in nine different regions over an investment period from 1993 to 2011. Results point at an underperformance in the run up of accelerating markets. Combining low-risk and intrinsic value reduces exposure to thematic bubbles.

A wide variety of portfolio allocation methods are discussed in earlier research, each with its specific objectives. We distinguish between sequential and unified methods.

Sequential allocation methods disentangle the stock selection and weighting process. First, stocks are selected based on a specific characteristic. In a second step, a weighting scheme is applied. One example is the S&P GIVI. The construction starts with restricting the investment universe by removing the high-risk stocks. Afterwards the selected stocks are reweighted in proportion to their intrinsic value.

Amenc et al. (2012) discuss the performance of a portfolio selected on fundamentals and afterwards proportionally weighted to a specific characteristic. Over a 1984-2010 investment period, a fundamentally selected and weighted portfolio results in a minor volatility reduction compared to the market cap benchmark. Applying a fundamental selection and minimum volatility weighting approach sequentially results in a portfolio with an average annualized volatility of 13.20%, a reduction of over 3 % compared to the S&P 500. The volatility reduction is achieved while maintaining superior returns.

Boudt and Peeters (2013) criticize the sequential approach for being suboptimal, as these methods do not account for dependence between different investment strategies. Also, the percentage of excluded stocks should be time-dependent, as betas are dynamic.

A unified allocation approach combines both investment strategies simultaneously. A low-risk and fundamental portfolio is selected from one investment universe. In a next step, these are reweighted applying a portfolio allocation method. Possible portfolio allocation methods are: equal weighting (EW), minimum-variance portfolio (MinVar), equal risk contribution (ERC) and 60/40 allocation based on momentum (Mom).

The EW approach aims to construct a diversified portfolio. This so-called “naive” approach is intrinsically a contrarian style investment strategy that buys the losing investment style and sells the winning style. This is in contrast with the more widely observed behavior of style momentum investing (Barberis and Schleifer, 2003). In order to mimic this, we consider a 60/40 investment strategy in which 60% of the portfolio is invested in the style that has gained most value over the past year.

An alternative can be found in the minimum-variance portfolio. The MinVar approach optimizes allocation, targeting the lowest portfolio volatility. A standard approach for combining different investment styles is to buy the style as the strategy falls and sell as it rises. As investment style returns possibly are positively autocorrelated in the short run, this creates an additional risk (Barberis and Schleifer, 2003). Both equally weighting and minimum variance portfolio composition have drawbacks. The former does not include any risk measure. While the latter exhibits a high concentration level in a limited number of assets. ERC portfolios aim to ensure diversification, while controlling for risk contributions of the included assets (Maillard et al., 2010). As a result, ERC portfolios are less concentrated both in terms of risk and weights (Amenc, 2012).

Therefore, we define the third set of hypotheses:

*Hypothesis 3a:* Portfolios combining low-risk and fundamental investing outperform the market cap-weighted index in the long-run.

*Hypothesis 3b:* A unified portfolio allocation methodology has superior risk-return characteristics compared to a sequential approach.

### 3. DATA AND METHODOLOGY

#### 3.1. Data

We restrict our investment universe to the S&P 500 universe. This is a different approach compared to Arnott et al. (2005) who considered the 1000 largest capitalization stocks. The reason for this is twofold. By investing in S&P 500 stocks only, we make the investment strategy easy applicable for investors. Also, we avoid a bias towards small stocks and secure the liquidity of the portfolios.

All fundamental measures are retrieved from the WRDS COMPUSTAT database on an annual basis. Besides financial statement information, also daily and monthly price data are retrieved from the COMPUSTAT database. The sample period is taken from 1980 up to 2012.

The S&P 500 US index is taken as a benchmark. Daily and monthly closing prices are downloaded from the Thomson Reuters Datastream database.

The market, size, value and momentum factors and the risk free rate are retrieved from Kenneth French's data library.

#### 3.2. Portfolio Construction

##### 3.2.1. Construction of Fundamental Portfolios

For the construction of the fundamental portfolios, we follow Arnott et al. (2005). We construct indexes based on six different fundamentals: book value of common equity (Book Value), total dividends (Dividends), number of employees (Employment), total revenues (Revenues) and net operating cash flow (Cash Flow). Arnott et al. (2005) show similar performance for a sales and revenues weighted portfolio. Therefore, we prefer to construct a portfolio based on total earnings (Earnings). Net operating cash flow is taken as the difference between the operating income before depreciation and total accruals (Kothari et al., 2005). The accrued liabilities at time  $t$  is the change in current assets minus the change in cash and short term investments, minus the change in current liabilities excluding long term debt minus the amount of depreciation and amortization scaled at the lagged value of total assets.

Portfolio performance depends on the rebalancing assumptions (Blitz et al., 2010). In our setting, the portfolio is quarterly rebalanced. The reweighting of the portfolio takes place on 31<sup>st</sup> of December, the 31<sup>st</sup> of March, the 30<sup>th</sup> of June and the 30<sup>th</sup> of September. The calculation of the weights is based on the data available on the last trading day of the month prior to the rebalancing date. Meaning, for rebalancing on the 31<sup>st</sup> of March, we use the data available on the last trading day of February. Introducing a gap ensures that all companies have reported their fundamental data and that we do not calculate weights and returns based on non-available information.

At rebalancing date, we calculate the weight of each stock in proportion to the fundamentals. For the fundamentals Cash Flow, Dividends, Sales and Revenues, we use averages over the past 5 years. By doing so, we limit the influence of business cycles and reduce rebalancing turnover. When less than 5-year data are available, we average over the years available. Negative values are set to zero. In this way, short selling is ruled out. The stocks' weight in the fundamental portfolio is:

$$w_{k,i,t}^{fund} = \frac{\max[0, F_{k,i,t}^-]}{\sum_{k=1}^N \max[0, F_{k,i,t}^-]}, \quad (1)$$

where  $F_{k,i,t}^-$  is the value of metric  $i$  for company  $k$  available at the month end previous to the rebalancing date and  $N$  is the number of stocks in the universe.

The weight of a stock at rebalancing date is proportional to the value of the fundamental. Portfolio weights change proportional to relative price changes between two rebalancing dates. The advantage of fundamental weighting possibly vanishes when the gap between actual and true prices becomes too large. Stotz et al. (2010) show low rebalancing frequencies have only small disadvantage compared to daily rebalancing.

Besides the single-metric portfolio, we also construct a composite index (Fundamental Composite). The fundamental composite portfolio is based on the Book Value, Cash Flow, Dividends and Revenues portfolios. These four portfolios are aggregated to the composite by an equal weighting scheme. If a company does not pay dividends, the composite weight is an equally weighted combination of the remaining metrics. The non-payment of dividends is possibly the consequence of a well-informed choice rather than a sign of weakness.

### 3.2.2. Construction of Low Risk Portfolios

The low-risk approach uses volatility, beta and semideviation as weighting tools. The risk-metrics are calculated over a 250-day rolling window. The advantage of beta over standard deviations is the incorporation of the covariance. Semideviation is a downside risk measure and therefore better reflects investors' preferences as only negative returns are incorporated.

The low-risk portfolio uses an inverse approach. The weight of stock  $k$  at time  $t$  for risk measure  $i$  is:

$$w_{k,i,t}^{lr} = \frac{\max[0, 1/risk_{k,t,t}]}{\sum_{k=1}^N \max[0, \frac{1}{risk_{k,t,t}}]}, \quad (2)$$

where  $risk_{k,i,t}$  is the value of risk measure  $i$  for company  $k$  at time  $t$ . The risk metrics are estimated over a 250 day rolling window.

A low-risk composite is created by aggregating the weights of the three different metrics in equal proportions. The low-risk portfolios are constructed following the same rebalancing assumptions as the fundamental indexes.

### 3.2.3. Construction of Multi-Style Portfolios

Next, we allocate our portfolio over the low-risk and fundamental value weighted composites. Three different approaches are applied.

The equal weighting strategy implies holding each portfolio in equal proportions. For a portfolio consisting of  $N$  assets, the weight of asset  $i$  is:

$$w_{lr}^{EW} = w_{fund}^{EW} = 1/N. \quad (3)$$

In our setting, it involves holding 50% in the low-risk and 50% in the fundamental portfolio.



The EW strategy is by definition a contrarian strategy. In practice, investors tend to buy winning strategies and sell the losing strategies (Barberis and Schleifer, 2003). Therefore, we construct a 60/40 momentum portfolio. Denote  $R_{lr}$  ( $R_{fund}$ ) as the return of the low-risk portfolio over the previous 250 days. Then:

$$w_{lr}^{mom} = \begin{cases} 0.6 & \text{if } R_{lr} > R_{fund} \\ 0.4 & \text{otherwise} \end{cases} \text{ and } w_{fund}^{mom} = 1 - w_{lr}^{mom} \quad (4)$$

A minimum variance composite portfolio is constructed. The minimum variance portfolio minimizes risk without considering expected return. Consider a portfolio consisting of two assets: a low-risk and fundamental asset. Let  $\sigma_p^2$  be the portfolio variance,  $\sigma_{lr}^2$  the variance of the low-risk composite portfolio,  $\sigma_{fund}^2$  the variance of the fundamental composite portfolio,  $\sigma_{lr,fund}$  the covariance between the low-risk and fundamental composite,  $w_{lr}$  the weight of the low-risk composite in the portfolio and  $w_{fund}$  the weight of the fundamental composite in the portfolio. In a two-asset case the minimization problem is:

$$\min_{w_{lr}, w_{fund}} \sigma_p^2 = w_{lr}^2 \sigma_{lr}^2 + w_{fund}^2 \sigma_{fund}^2 + 2 w_{lr} w_{fund} \sigma_{lr,fund} \quad (5)$$

$$\text{st } w_{lr} + w_{fund} = 1$$

$$w_{lr} \geq 0, w_{fund} \geq 0,$$

with the low-risk and fundamental variances equal to the single-style composite return variances over the previous 250 days.

The first constraint ensures weights sum up to unity. The other constraints reflect the long-only character of the portfolio.

Solving this minimization problem leads to:

$$w_{lr}^{min} = \begin{cases} 0 & \text{if } \theta < 1 \\ \theta & \text{if } 0 \leq \theta \leq 1 \\ 1 & \text{if } \theta > 1, \end{cases} \quad (6)$$

$$\text{with } \theta = \frac{\sigma_{fund}^2 - \sigma_{lr,fund}}{\sigma_{lr}^2 + \sigma_{fund}^2 - 2 \sigma_{lr,fund}} \text{ and } w_{fund}^{min} = 1 - w_{lr}^{min}. \quad (7)$$

Also, we apply an equal risk contribution (ERC) weighting scheme. Weights are set such that both portfolios contribute equally to overall portfolio risk. The risk contribution of an asset is defined as the product of its portfolio weight and its marginal contribution to risk. Define  $\Sigma$  as the covariance matrix for the returns over the reference period and  $w$  the weights of the portfolio. The total risk contribution of asset  $i$  is:

$$RC_i = \frac{w_i \Sigma w}{\sqrt{w' \Sigma w}}. \quad (8)$$

The standard deviation is a homogeneous function of degree one. Consequently, Euler's theorem is satisfied implying that the sum of the asset's risk contribution is the portfolio volatility. For a  $N$ -asset portfolio:

$$\sum_{i=1}^N RC_i = \sqrt{W' \Sigma W}. \quad (9)$$

In our two-asset framework (low-risk and fundamental), the correlation cancels out in the ERC constraint. The weight of the low-risk portfolio in the ERC composite is:

$$w_{lr}^{ERC} = \frac{\sigma_{fund}}{\sigma_{lr} + \sigma_{fund}}, \quad (10)$$

whit  $\sigma_{fund}$  ( $\sigma_{lr}$ ) is the volatility of the fundamental (low-risk) composite portfolio returns over the past year (250 trading days). The allocation to the low-risk portfolio is equal to the ratio of the fundamental portfolio volatility to the sum of low-risk and fundamental volatility. As such, the weight is directly related to volatility and independent of correlation. The higher the volatility of the low-risk portfolio is, the lower its weight in the composite portfolio.

We also apply a sequential approach. We construct two sequential-portfolios. The first is based on a low-risk stock selection and a fundamental weighting approach (Sequential LR-Fund). The second portfolio applies a fundamental stock selection and a low-risk weighting approach (Sequential Fund-LR). In a first step, we exclude the 30% highest risk, respectively lowest fundamental stocks. Next, we reweight the companies in the restricted universe according to their fundamental values, respectively risk characteristics.

The multi-style portfolios are rebalanced at the beginning of each month.

### 3.3. Turnover Analysis

Despite the advantages of alternative beta-investments, the strategies have a clear drawback compared to market cap-weighted indexes. Both low-risk weighted and fundamentally weighted portfolios need to be rebalanced on a regular basis. Rebalancing is an issue as it increases turnover and transaction costs. We follow DeMiguel et al. (2007) to compute the portfolio turnover as the average absolute change in portfolio weight across the rebalancing dates:

$$\text{turnover} = \frac{1}{T-1} \sum_{t=1}^{T-1} \sum_{j=1}^N (|w_{j,t+1} - w_{j,t+1}^*|) \quad (11)$$

with  $w_{j,t+1}$  the weight of asset  $j$  in the portfolio at time  $t+1$  and  $w_{j,t+1}^*$  is the portfolio weight of asset  $j$  before rebalancing at time  $t+1$ .

### 3.4. Multi-Factor Analysis

The Carhart four factor model (1997) is applied for the multifactor-analysis. The model is used to decompose the performance of the portfolios and to study whether these exhibit

exposure to the size, value and momentum factor. The analysis is of interest as critics of the fundamental weighting technique argue that the outperformance can be attributed to the exposure to the value factor. The following regression is estimated:

$$R_{it} - R_{ft} = \alpha_{it} + \beta_{it}(R_{mt} - R_{ft}) + s_{it}SMB_{it} + h_{it}HML_{it} + m_{it}MOM_{it} + \varepsilon_{it}, \quad (12)$$

where  $R_{it}$  is the daily return of portfolio  $i$ ,  $R_{ft}$  the daily risk-free return and  $R_{mt}$  the daily market return. Small minus big (SMB) is constructed as the difference in a small-cap and a large-cap portfolio. High minus low (HML) captures the value premium and is the difference in return between a portfolio of high book-to-market stocks and a portfolio of low book-to-market stocks. The fourth factor, momentum (MOM) is the difference in return between a portfolio of past winners and a portfolio of past losers.  $\beta_{it}$ ,  $s_{it}$ ,  $h_{it}$  and  $m_{it}$  are the factor loadings and  $\alpha_{it}$  is the four-factor model alpha.

#### 4. RESULTS SINGLE-STYLE PORTFOLIOS

Annualized risk and return characteristics for the quarterly rebalanced single-style portfolios are reported in Table 1. The portfolios cover a 27 year investment horizon from 1987 to 2012. We observe a clear outperformance for both the fundamentally weighted and low-risk weighted portfolios.

**Table 1. Risk and return characteristics single-style portfolios**

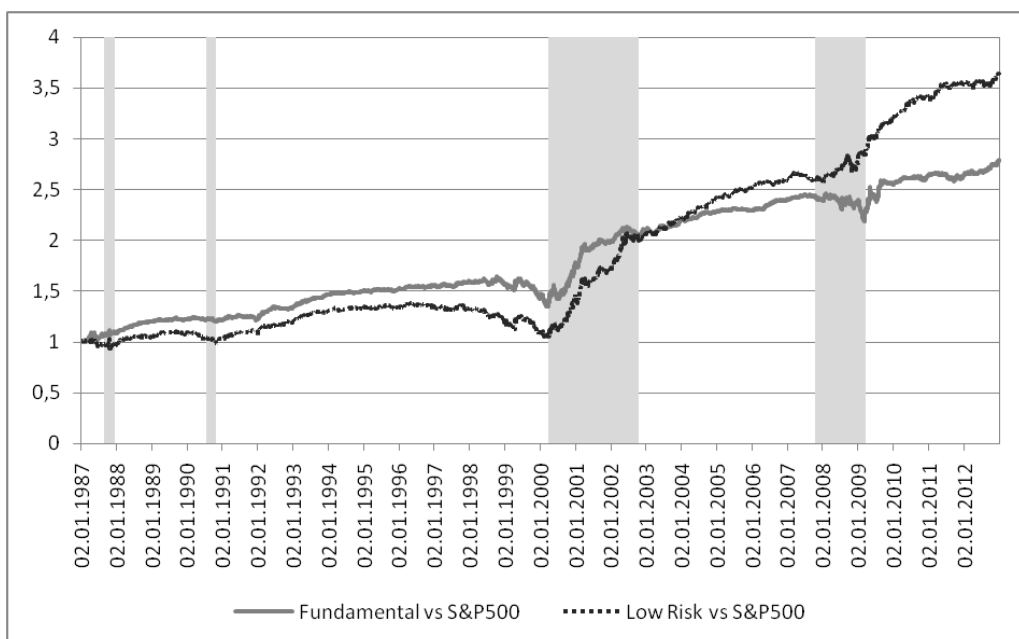
Portfolio/Index	Ending Value of \$1	Arithmetic Return	Volatility	Sharpe Ratio	Maximum Drawdown	Tracking Error
S&P 500	5.89	8.58 %	17.43 %	0.28	-56.78 %	-
Book	12.92	11.97%	17.96%	0.46	-62.92%	4.42%
Employment	16.17	12.65%	16.91%	0.53	-54.35%	4.91%
Cash Flow	21.27	14.26%	18.88%	0.57	-61.32%	6.50%
Dividends	15.27	12.38%	16.48%	0.53	-59.62%	5.71%
Earnings	15.02	12.36%	16.76%	0.52	-59.21%	3.68%
Revenues	16.41	12.95%	17.80%	0.52	-60.50%	4.54%
Fundamental Composite	16.41	12.89%	17.48%	0.53	-60.98%	4.48%
Beta	17.85	13.07%	16.81%	0.55	-52.64%	7.73%
Volatility	22.21	14.00%	16.82%	0.61	-53.23%	4.76%
Semi-deviation	22.62	14.09%	16.86%	0.61	-53.16%	4.77%
Low-Risk Composite	21.39	13.83%	16.77%	0.60	-52.94%	5.43%

The table reports the annualized risk and return characteristics for quarterly rebalanced portfolios over a 1987-2012 investment period. Fundamental Composite is an equally weighted composite of the book, cash flow, dividend and earnings single-metric portfolios. Low-Risk Composite is an equally weighted composite of the beta, volatility and semi-deviation portfolios.

The fundamental portfolio returns are on average 4.18 pps higher compared to the market cap-weighted benchmark. The portfolios exhibit similar volatilities to the S&P 500. As a result, the Sharpe Ratios of the fundamental indexes are almost twice as high. The risk and return characteristics report differences across the different portfolios. On a risk-adjusted basis, the net operating cash flow portfolio is the best performing.

The low-risk indexes exhibit similar return characteristics as the fundamental indexes. The low-risk composite portfolio reports superior returns compared to both market cap and fundamentally weighted portfolios, while volatilities are in general lower. The Sharpe Ratios are well-above those of market capitalization-weighted index and the different fundamental portfolios.

Based on our findings, we do not reject Hypothesis 1a and 2a. Both fundamental and low-risk indexes outperform the market cap-weighted benchmark in the long run.



Relative cumulative performance of fundamental and low-risk quarterly rebalanced composites versus S&P 500 over a 1987 – 2012 investment period; the shaded periods indicate the periods reporting the largest drawdowns for the S&P 500, respectively 26<sup>th</sup> Aug 1987- 4<sup>th</sup> Dec 1987, 17<sup>th</sup> July 1990 – 11<sup>th</sup> Oct 1990, 27<sup>th</sup> March 2000-9<sup>th</sup> Oct 2002 and 27<sup>th</sup> July 2007 – 9<sup>th</sup> March 2009.

Figure 1. Relative cumulative performance of the composite fundamental (black line) and low risk (grey line) versus the S&P 500.

Figure 1 charts the relative performance of the alternative portfolios to the market capitalization-weighted portfolio. In the late nineties, the cap-weighted index outperformed both the fundamental and low-risk composites. This period is also known as the dot-com bubble. In the run-up of this market rally, the low-risk portfolio underperforms the market cap-weighted benchmark. Meanwhile, underpricing tends to be corrected. As a result, the fundamental portfolio reports superior returns to the low-risk index, as it is biased towards undervalued stocks (Figure 2). The fundamental composite underperforms compared to the market as the speculative bubble grows. By construction, the weights of stocks in a

fundamentally weighted portfolio only increase if a stock's fundamental metrics grow disproportionately compared to the other constituents. Over the 2000 to 2002 period, both the fundamental and low-risk composites outperform the market again. The positive compounded returns for the low-risk index indicate an outperformance in bear markets.

In general, the graph indicates an outperformance of the low-risk investment over the market capitalization-index in bear markets. Fundamental index tend to exhibit anti-bubble behavior compared to the market capitalization-weighted index. In the run-up of a speculative bubble the fundamental portfolio underperforms, after the burst of the bubble the portfolio outperforms the market cap-weighted index.

Therefore we do not reject Hypotheses 1b and 2b.

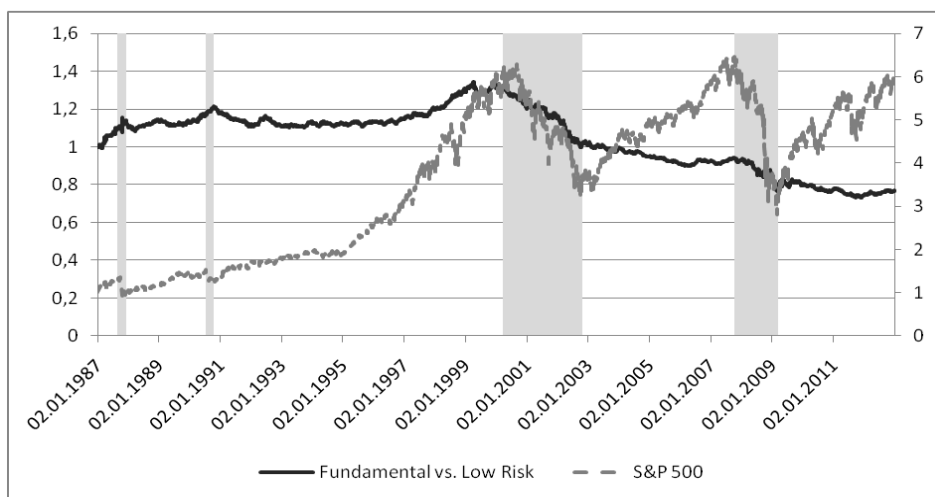


Figure 2. Relative cumulative performance fundamental versus low-risk composite.

Relative cumulative performance of fundamental versus low-risk quarterly rebalanced composites over a 1987 – 2012 investment period; the shaded periods indicate the periods reporting the largest drawdowns for the S&P 500, respectively 26<sup>th</sup> Aug 1987- 4<sup>th</sup> Dec 1987, 17<sup>th</sup> July 1990 – 11<sup>th</sup> Oct 1990, 27<sup>th</sup> March 2000-9<sup>th</sup> Oct 2002 and 27<sup>th</sup> July 2007 – 9<sup>th</sup> March 2009. The dashed line indicates the cumulative growth of a \$1 investment in the S&P 500 at the beginning of 1987.

Figure 2 compares the cumulative performance of the fundamental strategy to the low-risk strategy. The dashed line is the cumulative growth of a \$1 investment in the S&P 500 at the beginning of 1987. The marked periods indicate the four periods in which the S&P 500 reports the highest drawdowns.

In general, the period can be divided in two parts. Over the first half of the sample period, the fundamental portfolio tends to outperform the low-risk composite. After the burst of the technology bubble in the late nineties, the opposite is true.

Looking closer at the graph, short term differences in performance can be noticed. First, in market rallies, the fundamental strategy tends to outperform the low-risk strategy. The opposite holds in down-markets. After a crash or in bear markets, the low-risk portfolio often outperforms the fundamental index. These differences in behavior over different equity markets, indicates the existence of a diversification opportunity over both investment styles.

## 5. RESULTS ON COMBINING LOW-RISK AND FUNDAMENTAL INVESTING STYLES

### 5.1. Portfolio Characteristics

The difference in life cycles of low-risk and fundamental investments leads to a diversification opportunity. We combine both styles using six allocation methods.

Figure 3 illustrates the annualized volatility of the low-risk and fundamental composite portfolios. The dashed line is the 250-day rolling correlation between both portfolios. The volatilities are most often very similar, except for some periods where the fundamental portfolio reports higher variance. In addition, the correlations range from 0.92 to values over 0.98.

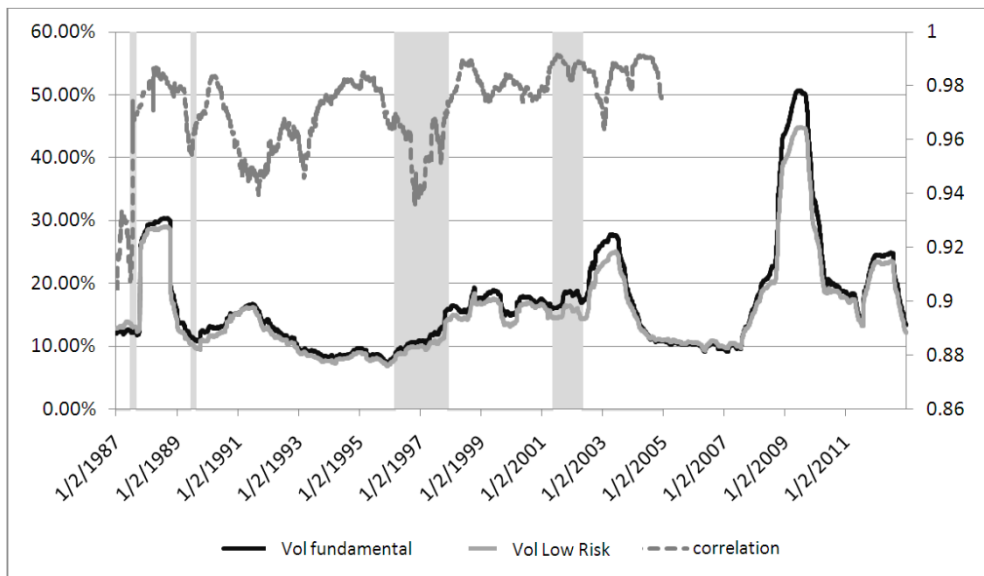


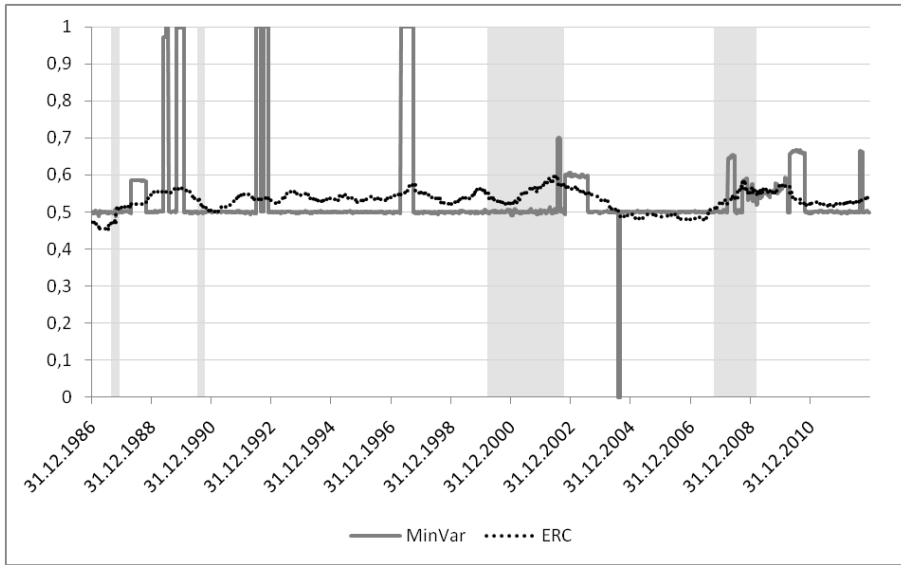
Figure 3. Annualized volatility and correlation of fundamental and low-risk portfolios.

The graph shows the annualized return volatilities for the low-risk and fundamental quarterly rebalanced composites, as well as the correlations (dashed line) over the 1987 to 2012 period. The marked periods indicate the periods reporting the largest drawdowns for the S&P 500, respectively 26<sup>th</sup> Aug 1987- 4<sup>th</sup> Dec 1987, 17<sup>th</sup> July 1990 – 11<sup>th</sup> Oct 1990, 27<sup>th</sup> March 2000-9<sup>th</sup> Oct 2002 and 27<sup>th</sup> July 2007 – 9<sup>th</sup> March 2009.

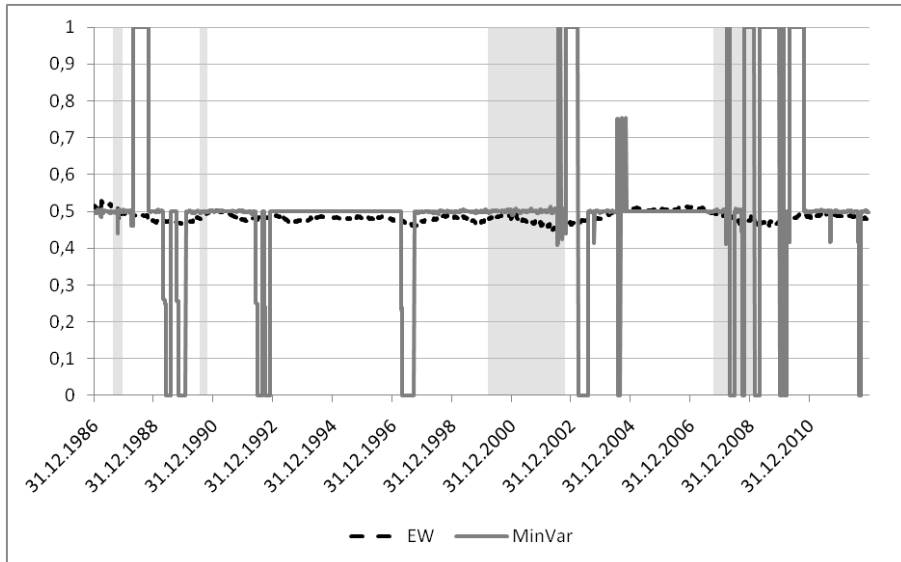
It is important to notice, that if the low-risk and fundamental variances are equal, the two-asset minimum variance portfolio corresponds to the EW portfolio.

Figure 4 (Panel A) shows the weight (%) allocation of the low-risk portfolio to the minimum variance and equal risk weighted portfolio. The MinVar portfolio is mostly an equally weighted investment. When different, the MinVar strategy tends to allocate the largest part in the low-risk portfolio.

The equal risk-contribution portfolio invests 50 to 60 % in the low-risk index. When the volatility is similar, the ERC portfolio weights are very close to the EW allocation.



PANEL A: Weights (%) invested in low-risk Portfolio under the Minimum Variance (full line) and ERC allocation (dotted line).



PANEL B: Risk contribution (%) of the low-risk portfolio component in the multi-style portfolio under the minimum variance (full line) and equal-weight (dotted line) allocation.

Figure 4. Weight (%) invested in low-risk portfolio (Panel A) Risk contribution (%) of low-risk portfolio (Panel B).

The smaller variance of the low-risk portfolio is reflected in its contribution to the overall portfolio risk (Figure 4, Panel B). For the EW portfolio, the risk contribution is usually smaller than 50%. The risk contribution (%) of the low-risk portfolio to the minimum variance portfolio risk has a similar pattern as for the weight contribution (Figure 4, Panel A).

Table 3 reports the annualized risk and return characteristics over the 1987 to 2012 investment period. The results presented for the EW, MinVar, ERC and Momentum (60/40) portfolios are for quarterly rebalanced underlying portfolios.

**Table 3. Annualized risk and return characteristics for the multi-style portfolios**

Portfolio/Index	Ending Value of \$1	Arithmetic Return	Volatility	Sharpe Ratio	Maximum Drawdown	Tracking Error
S&P 500	5.89	8.58 %	17.43 %	0.28	-56.78 %	-
Equal Weight	18.73	13.32 %	16.90 %	0.57	-56.93 %	4.54 %
Minimum Variance	19.11	13.41 %	16.92 %	0.57	-56.50 %	4.52 %
Equal Risk Contribution	19.10	13.39 %	16.84 %	0.58	-56.56 %	5.22 %
Momentum (60/40)	19.91	13.58 %	16.89 %	0.59	-56.64 %	4.51 %
Sequential (Low-Risk – Fund)	14.97	12.03 %	15.01 %	0.57	-49.28 %	6.13 %
Sequential (Fund – Low-Risk)	20.62	13.66 %	16.53 %	0.60	-55.18 %	6.61 %

The table reports the annualized risk and return characteristics for monthly rebalanced portfolios over a 1987-2012 investment period. The underlying single-style portfolios are quarterly rebalanced. Equal Weight is the equally weighted combination of the low-risk and fundamental composite portfolio. Minimum variance is the combination of the low-risk and fundamental composite achieving the minimum portfolio variance. Equal risk contribution allocates the portfolio weights such that both the low-risk and fundamental composite equally contribute to portfolio risk. Momentum (60/40) invests 60% in the portfolio producing the highest returns and 40% in the other portfolio. Sequential (Low-Risk – Fund) is the portfolio constructed by low-risk stock selection and fundamental weighting. Sequential (Fund-Low-Risk) is the portfolio constructed by the fundamental stock selection and low-risk weighting.

The realized returns are still well-above the 8.58 % average return of the marketcapitalization-weighted portfolio. Likewise, the Sharpe Ratios are twice as high. But, when comparing the single- and multi-style portfolios, only minor differences can be noticed. The results do not report a clear gain in combining both investment styles in the long-run.

Furthermore, the results across the allocation methods are also similar. The only Sequential Fund-LR portfolio underperforms in terms of realized returns compared to the other multi-style portfolios. When adjusted for the realized risk, all portfolios report similar Sharpe Ratios.

The drawdown is maximal for all portfolios in the period around the 2008 financial crisis. The multi-style maximum drawdowns closely resemble those of the low-risk and fundamental composites. In addition, there is no improvement of the tracking error.

Table 4 reports the compounded returns over subperiods, which are determined by the maximum drawdown period of the S&P 500.

The results for the single-style returns are somewhat ambiguous. In general, we can divide the sample period in two parts. In the first part, the fundamental index tends to outperform the low-risk. In the second part, the opposite is true. When looking more in detail, the fundamental portfolio tends to outperform the low risk investment in market rallies. One example is the high tech bubble in the late nineties. The S&P 500 reports the largest



drawdowns after the high-tech bubble burst and during the financial crisis. In these sharply declining markets, the low-risk portfolio outperforms the fundamental portfolio.

For the multi-style portfolios, the compounded returns are in all but one period superior compared to the market capitalization-weighted index. The different portfolio allocation methods report comparable returns. There is no single method consistently outperforming.

Based on our findings, we do not reject Hypothesis 3a. The multi-style portfolios outperform the market cap-weighted index.

**Table 4. Compounded returns over subperiods**

	S&P 500	Fund Comp	LR comp	EW	MinVar	ERC	mom	Sequential (LR- Fund)	Sequential (Fund- LR)
2nd Jan 1987	39.06	47.49	34.90	41.17	41.10	41.64	42.04	39.72	40.02
26th Aug 1987	-33.51	-31.04	-33.55	-32.25	-32.29	-32.18	-32.04	-31.34	-33.39
5th Dec 1987	64.77	82.42	77.14	79.93	77.63	79.59	79.54	80.92	73.46
17th July 1990	-19.92	-20.43	-23.08	-21.76	-21.76	-21.83	-21.49	-20.90	-23.67
12th Oct 1990	416.98	477.47	431.71	455.91	471.68	452.31	453.67	342.95	402.76
27th March 2000	-49.15	-24.53	-0.80	-13.21	-13.01	-11.78	-11.10	-9.08	7.54
10th Oct 2002	101.50	144.54	160.21	152.58	154.58	152.63	154.08	129.44	158.91
10th Oct 2007	-56.78	-60.98	-52.54	-56.85	-55.75	-56.48	-56.55	-49.12	-54.90
9th March 2009	36.79	50.63	60.01	52.96	60.34	53.40	56.79	54.44	57.11

The table reports the compounded returns for monthly rebalanced portfolios over sub-periods. The periods are based on the maximum drawdown periods of the S&P 500 index. The dates indicate the start of the subsample. Figures in bold indicate an outperformance over S&P 500.

## 5.2. Multi-Factor Analysis

The Carhart four-factor model is used to decompose the portfolio returns. The multi-factor analysis shows all four unified allocation methods (EW, MinVar, ERC and Momentum) generate genuine alpha. After correcting for style biases, the portfolios exhibit highly significant, positive alphas. The minimum variance portfolio returns the highest alpha and has, as expected, the lowest market beta compared to the EW, ERC and Momentum portfolios. The alphas are significant on a higher level and larger in magnitude compared to the sequential approaches, indicating a superior performance of the EW, MinVar, ERC and Momentum portfolio.

All portfolios tilt towards value stocks. The HML-factor loading is often almost three times as high in magnitude as the size factor. Part of the value tilt is cancelled out by the negative size factor. The negative loading of the size factor indicates that there is no additional exposure to small stocks. Alternative beta investments are often criticized for being biased towards small-cap stocks. In our setting, the investment universe is limited to S&P 500 constituents. This restriction to large-cap stocks solves for the often criticized small-cap bias. Even after correcting for the size bias, the alternative approaches still exhibit genuine alphas.

Loadings on the size factor range from -24.41% to -10.75%. The size factor loading for the LR-Fund is the largest in magnitude. This reflects a concentration in large-cap stocks for the LR-Fund portfolio.

The momentum factor is negative for all non-sequential portfolios, indicating a contrarian character of the strategies. The factor loading for the Momentum portfolio is also negative. In fact, we apply a momentum approach on the portfolios and not on the stocks. The sequential methods are biased to momentum-stocks.

**Table 5. Multi-Factor Analysis of the multi-style portfolio returns**

	alpha	market	SMB	HML	Mom	R <sup>2</sup>
Equally Weighted	7.4 10 <sup>-5</sup> *** [2.59 10 <sup>-5</sup> ]	0.9518*** [0.0024]	-0.1130*** [0.0044]	0.3195*** [0.0049]	-0.0522*** [0.0033]	0.9658
Minimum Variance	7.97 10 <sup>-5</sup> *** [ 2.61 10 <sup>-5</sup> ]	0.9491*** [0.0024]	-0.1075*** [0.0044]	0.3165*** [0.0049]	-0.0508*** [0.0033]	0.9652
Equal Risk Contribution	7.66 10 <sup>-5</sup> *** [2.63 10 <sup>-5</sup> ]	0.9485*** [0.0024]	-0.1081*** [0.0045]	0.3166*** [0.0050]	-0.0482*** [0.0033]	0.9646
Momentum (60/40)	8.13 10 <sup>-5</sup> *** [2.63 10 <sup>-5</sup> ]	0.9495*** [0.0024]	-0.1154*** [0.0045]	0.3143*** [0.0050]	-0.0418*** [0.0033]	0.9645
Sequential (LR – Fund)	4.22 10 <sup>-5</sup> [3.83 10 <sup>-5</sup> ]	0.8488*** [0.0035]	-0.2441*** [0.0065]	0.3024*** [0.0073]	0.0227*** [0.0048]	0.9122
Sequential (Fund – LR)	6.99 10 <sup>-5</sup> * [3.99 10 <sup>-5</sup> ]	0.8908*** [0.0036]	-0.1266*** [0.0068]	0.3981*** [0.0076]	0.0209*** [0.0050]	0.9104

The table presents the results from estimating the Carhart four-factor model. The data are taken on a daily basis. The time period ranges from 1987 to 2012. SMB, HML and Mom represent respectively the size, value and momentum premium. The factors are retrieved from the K. French Data Library. The estimated coefficients are presented together with the corresponding standard errors (between square brackets). \*\*\*, \*\* and \* indicate a significance level of 1%, 5% and 10 %.

Based on our observations of the risk and return characteristics, we conclude that non-sequential allocation approaches outperform sequential approaches, especially when taking into account turnover. Furthermore, we find evidence that non-sequential approaches produce positive, significant alphas. Therefore, we do not reject Hypothesis 3b: non-sequential approaches are superior compared to sequential approaches.

### 5.3. Turnover Analysis

Turnover is one of the main issues of advanced beta strategies. Where market cap-weighted indexes are self-rebalancing, advanced weighting strategies need to be updated at a regular frequency. This causes additional transaction costs and taxes for the investor. Table 6 reports the monthly average turnover for the different allocation methods. The turnover is calculated over the total underlying portfolio.

The Momentum portfolio has the lowest turnover of 5.43%, followed by the EW portfolio (5.67%). The EW portfolio – turnover differs from zero as we apply an equal weighting approach on the composite portfolio and not on the underlying stocks. The minimum variance strategy exhibits a somewhat higher turnover. The turnover of the sequential approaches is significantly higher. On an annual basis, the Fund-LR portfolio has a 165.36%. As risk-measures are highly-dynamic measures, the time-varying behavior is reflected in the turnover.

**Table 6. Monthly Turnover Characteristics**

Portfolio	Turnover
Equal Weight	5.67 %
Minimum Variance	7.34 %
Equal Risk Contribution	5.79 %
Momentum (60/40)	5.43 %
Sequential (LR-Fund)	11.97 %
Sequential (Fund-LR)	13.78 %

The table presents the average monthly turnover over a 1987-2012 investment period, applying the allocation to the underlying portfolios. Equal Weight is the equally weighted combination of the low-risk and fundamental composite portfolio. Minimum variance is the combination of the low-risk and fundamental composite achieving the minimum portfolio variance. Equal risk contribution allocates the portfolio weights such that both the low-risk and fundamental composite equally contribute to portfolio risk. Mometum (60/40) invests 60% in the portfolio producing the highest returns and 40% in the other portfolio. Sequential (Low-Risk – Fund) is the portfolio constructed by low-risk stock selection and fundamental weighting. Sequential (Fund-Low-Risk) is the portfolio constructed by the fundamental stock selection and low-risk weighting.

The risk-return characteristics of the sequential Fund-LR portfolio are superior compared to the EW and ERC allocation (Table 3) and similar for the LR-Fund portfolio. Though, when taking into account the very high turnover imposed by the sequential construction methods, the results indicate a superior performance for the non-sequential methodology. Additionally, the unified approaches actually combine portfolios, not individual stocks. Therefore, the turnover for these portfolios is probably even lower.

Overall, we cannot conclude combining both low-risk and fundamental investments is superior to a single-style investment in the long-run. There are some short run differences which can be cancelled out by combining low-risk and fundamental investing. Furthermore, the results for the different unified approaches are also comparable.

## CONCLUSION

Alternatively weighted indexes have known an increasing popularity over the past years. Recently, indexes combining the low-risk and fundamental investment strategies have been launched (e.g., S&P GIVI and RAFI low-volatility index). The aim of these indexes is to capture both the fundamental value and low-risk premium. The traded indexes most often use

a sequential approach treating each objective separately. We look at the gains of combining both strategies simultaneously. Using a US sample from 1987 to 2012, we construct single and multi-style portfolios applying different allocation methods.

First, we find an outperformance of fundamental and low-risk portfolios over the market cap-weighted portfolio in the long-run. Short-term returns, however, depend on the equity market regime. Low-risk portfolios tend to outperform market cap-weighted in down-markets. Fundamental indexes underperform cap-weighted indexes in the run-up of speculative bubbles, but outperform after the burst of the bubble. Comparing the low-risk and fundamental portfolio performance, we find an outperformance of the low-risk portfolio in bear markets, while fundamental portfolios produces superior returns in up-markets.

Next, we compare a set of portfolio allocation approaches to combine both investment styles. We distinguish between non-sequential and sequential methods. The first includes an equally weighted, minimum portfolio variance, equal risk contribution and a 60/40 allocation based on momentum approach. These methods distribute the weights directly over the low-risk and fundamental portfolios. The sequential approach deals with both objectives sequentially. In a first step, stocks are selected based on one objective. Next, the restricted universe is reweighted according to the other objective.

Our analysis shows similar performances for the multi-style and single style portfolios in the long run. Combining both strategies reduces the short term dependence on equity market regimes. Furthermore, we find only minor differences between the considered allocation approaches. The only difference is found in the turnover, which is significantly lower for the non-sequential strategies.

The four-factor model style analysis of the portfolio returns shows that the multi-style portfolios are tilted towards value stocks. After controlling for the additional exposure to the risk factors, we find statistically significant, positive alphas for the non-sequential investment strategies over the total sample period.

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*Chapter 4*

## USING ALTERNATIVE INVESTMENTS IN A MULTI-ASSET PORTFOLIO

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### ABSTRACT

In this chapter we discuss alternative investments in a multi-asset portfolio. We introduce the topic by touching on the similarity and differences between retail and institutional use of alternatives. A definition of alternatives is given for retail investors, and a brief examination of how retail investors and financial advisors can use alternatives is presented. Institutional alternatives are defined, as are the initial steps typically taken to evaluate private equity, real estate, and hedge fund purchases. The last section deals with risk in its various forms. The measurement of private equity and real estate risk is examined as is liquidity risk.

### INTRODUCTION

Institutional investors often identify seven major asset classes plus cash as the parts of a multi-asset portfolio: private equity, real estate, absolute return, foreign equity, natural resources, domestic equity, and fixed income. Endowments such as those for Harvard and Yale use this kind of a panoramic and risk-taking model (the terms panoramic and risk-taking are taken from Rice [1]). Given that those endowments have averaged 20%-or-better returns over the past 20 years, it is not surprising that other endowments, pensions, and institutional investment managers have followed Harvard's and Yale's lead—albeit more often than not with less-stellar results.

Seeing the 20%-or-more returns for both the 20-year period and from June 2000 through June 2003—when the S&P 500 dropped 30%—wealth management advisors started using the panoramic and risk-taking model (hereafter referred to as the P&R model) for their individual investors. Initially, the alternative investment strategies wealth managers used were primarily hedge funds, usually in the form of limited partnerships. This meant investors had to have a

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lot of money to qualify. Alternative investment mutual funds—vehicles designed to replicate alternative investment strategies and developed to meet this demand—have started to make alternative investment approaches broadly available. But during the global financial crisis the most prominent endowments lost 20%, 25%, or more (the S&P 500 lost 26% over that period), and some investors started to question the validity of the P&R model.

Although one difficult patch is hardly sufficient evidence to discredit the work of endowments and others that have earned so much over the years, the idea that a broadly diversified portfolio is a low-risk portfolio has proven to be wrong. A well-diversified portfolio has higher average expected returns for a lower average risk level, but that is not to say it is somehow sheltered from significant drawdowns. There are crucial lessons to be learned from the performance of the P&R model during the credit crisis. The biggest is that advisors must consider more than just asset allocation across markets. Liquidity risk cannot be ignored, especially when the portfolio could or is being used to fund current spending.

It is a good rule of thumb that individual investors generally need more liquidity than they think they do. During the crisis, when liquidity was difficult to obtain, alternative investment assets were frequently liquidated at less than 50% of the previous year-end net asset value. Simply put, institutions with insufficient liquidity in 2008 got crushed.

Despite the difficulties the credit crisis brought, individual investors are continuing to move into alternative investments. Indeed, when judiciously used, alternative investment strategies can provide outstanding opportunities to enhance portfolio strength, especially during secular bear markets. But doing so effectively is much easier said than done. When Mohamed El-Erian, the former head of the Harvard endowment, was asked if individual investors could mimic what the top endowments do, he replied, “It would be like advising my son or daughter to drop out of school to play basketball with the goal of becoming the next Michael Jordan.”

The use of the mutual fund structure itself for alternatives suggests a potential dissonance, however. A crucial virtue of the P&R model is its ability to exploit illiquidity. A growing body of research supports the idea that illiquid assets provide the potential for higher return. But strategies requiring less liquid investments (for example, distressed debt) will be difficult to implement via mutual fund structures, which require daily liquidity.

That said, the other primary virtue of the P&R model is its better diversification. Commodities exposure can be efficiently obtained via low-cost exchange-traded funds (ETFs). Real estate investment trusts (REITs) make real estate investment readily accessible. Moreover, some alternative investment strategies such as long/short equity lend themselves to replication in a mutual fund format, but not all alternative investment strategies do. In addition to strategies that exploit illiquidity, strategies requiring a significant use of leverage are difficult to employ in a mutual fund format, since there is a cap on leverage in funds subject to the Investment Company Act of 1940.

These are not the only difficulties. The high fees commanded by alternative investment mutual funds compared with those of more “vanilla” mutual fund offerings present hurdles for such funds. Perhaps most significantly, the endowment/pension fund model’s advantages stem largely from its ability to uniquely align incentives with managers and the extensive resources available to select and monitor the best managers. Those advantages are difficult to replicate in the retail space.

These issues make the general underperformance of alternative investment mutual funds understandable. A Lipper analysis in 2013 found that fewer than 10% of long/short equity



funds outperformed the S&P 500 on a risk-adjusted basis during the previous three years. Lipper notes that absolute return funds declined 1.3% for the 12 months ended March 2013, and none of those funds had the necessary risk-adjusted return with which to minimize S&P 500 index volatility. And, alternative investment mutual funds such as absolute return funds have at times struggled to achieve noncorrelation.

Prominent and highly successful hedge fund managers are increasingly entering the mutual fund space. While many of their strategies have been used successfully by hedge funds in the past, there is no guarantee that the transition of these strategies into the mutual fund world will prove successful. As a class, alternative investment mutual funds cannot be said to have been successful up to this point. Still, there is hope that the strategies endowment and pension funds use so successfully in their respective contexts can be effectively adopted by the mutual fund world. Only time will tell.

## **ALTERNATIVES—A DEFINITION FOR THE RETAIL INVESTOR**

A definition of alternative investments needs to be acknowledged as being porous or fungible. Low correlation is often included as an aspect of alternatives; however, it is important to note that:

1. Emerging-market stocks had low correlation to developed-nation stocks until sometime in the 2000s. Now, that low correlation has almost disappeared.
2. Japanese stocks had low correlation to European and U.S. stocks until sometime in the 1980s.
3. European and U.S. stocks began to be correlated in the 1970s after several decades of low correlation.

First to consider in a definition are the financial risks an investor faces. A very long list of all the risks could be made, but we can summarize the risks into three types: idiosyncratic risk, systematic risk, and systemic risk. Idiosyncratic risk is also called diversifiable risk. In other words, when you have a diversified portfolio you have reduced idiosyncratic risk. Systematic risk (also known as market risk) includes interest rate risk, inflation risk, liquidity risk, volatility risk, and sociopolitical risk. Finally, systemic risk is defined as an event at the firm level (think of Bear Stearns and Lehmann Brothers) that is severe enough to cause instability in the financial system.

It is well known that combining stocks into portfolios creates portfolios with a better risk-return tradeoff (a higher Sharpe ratio). This reduction in risk occurs because of diversification. By combining different securities in a portfolio we diversify risk, reduce the overall volatility of the portfolio, and increase the Sharpe ratio. And, a fully diversified portfolio—the market ideal—has every available type of asset (stocks, bonds, commodities, etc.) in it, with weightings determined by the market value of the asset class. But, as we learned in 2008-2009, there are limits to diversification.

Diversification is also considered to be a property of alternatives. In financial theory diversification is meant to minimize or eliminate idiosyncratic/unsystematic risk and/or systematic risk. The standard way to decrease equity unsystematic risk is to construct a

broadly diversified global portfolio, where the securities cover different economic sectors/industries. This can be accomplished using broad-based mutual funds or index funds, for example.

There is an idiosyncratic volatility component that needs to be “sold away.” As with company-specific risk, investors are not compensated for idiosyncratic volatility risk, i.e., the market does not pay a premium to investors to hold stocks that are more volatile. So, investors must sell away company-specific risk as best they can (alternatives such as market neutral, absolute return, and equity hedge long/short can help investors sell off idiosyncratic volatility).

For assets other than equities financial theory says that a fully diversified portfolio needs to include all investment types such as bonds, real estate, private equity, infrastructure, and commodities as well as a diversified equity portfolio. Alternative investment mutual funds and hedge funds can also be included in this group of diversifiers.

How can alternatives help diversify a portfolio? Since alternatives are commonly defined as having low correlation to most long-only investments (bonds, stocks, commodities, or other asset classes), their use can help investors diversify a portfolio.

Another diversification attribute desirable in an alternative investment is an equivalent or higher risk-adjusted return (a higher Sharpe ratio) than the portfolio currently has. This higher Sharpe is evidence of the “substitution effect” alternatives bring to the table. The substitution effect or substitution methodology combines a long-only traditional asset class with an active component in the form of an alternative investment. Studies have shown that for most asset classes—bonds, credit, equities, commodities, and real estate—the alternative investment’s higher risk-adjusted return is passed along at least in part to the holding(s) with which it is paired.

We define alternatives then as securities that increase portfolio diversification: (1) they have low correlations to the existing portfolio or to long-only asset classes such as stocks, bonds, and commodities; and (2) they contain an active component that is often (but not exclusively) a hedge fund or a hedge fund-like strategy. We think it is the combination of these two features—low correlation and dynamic strategies—that makes alternatives an attractive way to increase portfolio diversification.

We suggest that at a minimum the following hedge fund strategies be included in any alternative investment classification schema:

1. Convertible arbitrage
2. Absolute return
3. Distressed securities
4. Emerging markets
5. Market-neutral
6. Multi-strategies
7. Market timing
8. Managed futures
9. Long/short
10. Event-driven
11. Global macro

We now give a brief description of our suggested alternative investment classification schema:

- Long/short strategies serve as the active equity, bond, or mixed-equity-and-bond benchmarks for alternative investments. Long/short managers maintain positions both long and short in equities and/or bonds and also in derivative securities. They dynamically manage their exposure to long-only indices in order to be less dependent on the direction of the underlying market.
- Global macro managers invest around the world using economic theory to justify the decisionmaking process. The strategy is typically based on forecasts and analysis about interest-rate trends, the general flow of funds, political changes, government policies, intergovernmental relations, and other broad systemic factors.
- Multi-strategies managers employ a strategy that is predicated on realization of a spread between related yield instruments in which one or more components of the spread contain a fixed income, derivative, equity, real estate, or combination of these or other instruments. The strategies are typically quantitatively driven to measure the existing relationship between instruments and in some cases to identify positions in which the risk-adjusted spread between the instruments represents an opportunity for the investment manager.
- Market-neutral managers seek to profit from both increasing and decreasing prices in single or multiple markets. An example of a market-neutral strategy is matching long and short positions in different stocks to increase the return from making good stock selections and to decrease the return from broad market movements. Market-neutral managers may also use other tools such as merger arbitrage and shorting sectors. There is no single accepted method employed in a market-neutral strategy.
- Arbitrage is the simultaneous purchase and sale of an asset in order to profit from a difference in the price. It is a trade that profits by exploiting price differences of identical or similar instruments in different markets or in different forms.
- Managed futures: Professional portfolio managers use a managed-futures strategy as part of their overall investment strategy. A managed-futures strategy provides portfolio diversification among various types of investment styles and asset classes to help mitigate portfolio risk in a way that is not possible with direct investments such as equities and bonds.
- Market timing: Commodity managers invest in a wide variety of trades: directional trades based on their opportunistic view about prices, equity trades based on commodity-linked equities, and relative-value trades that are based on spreads observed in commodity markets.
- Quantitative investment strategies are often the most difficult to define. Some managers use mathematical models they tweak, or they let the “signal” proceed on its own. In other cases the quant manager uses current economic and financial theory to define the sources of risk and to point out where opportunities may lie. Possibly the shortest, but not necessarily the best definition of quant investing, is that it is a strategy that employs sophisticated quantitative techniques of analyzing price data to gain information about future price movements. Quantitative strategies typically

maintain varying levels of net-long or -short equity market exposure over various market cycles.

- While there is more than one alternative investment currency strategy, we define here the most commonly occurring one—discretionary. A discretionary currency strategy relies on the fundamental evaluation of market data, relationships, and influences as they pertain to currency markets, including positions in global foreign exchange markets—both listed and unlisted—as interpreted by an individual or group of individuals who make decisions on portfolio positions. The strategy typically employs an investment process that is heavily influenced by top-down analysis of macroeconomic variables.
- Event-driven strategies seek to exploit pricing inefficiencies that may occur before or after a corporate event such as a bankruptcy, merger, acquisition, or spinoff.

We recognize that mutual funds for all of these strategies do not exist as yet, but we think it best to start by discussing the broad groupings of alternatives.

We do not consider long-only products—such as commodities or real estate—alternatives because we think (in the vast majority of cases) alternatives should be defined by the strategy they follow rather than by what they hold. This view is re-enforced by the following statement: Alternatives should not be treated as a separate group of assets in an investor's portfolio. The key (diversification) benefit alternatives bring to the table (as several articles have shown) is the aforementioned substitution effect. In the substitution method one or more alternatives are decked against an asset class such as stocks. A judicious choice of alternatives for each long-only asset class in most cases reduces volatility and improves risk-adjusted return on an asset-class-by-asset-class basis.

Finally in our attempt to define alternatives, we discuss the difference between diversification and hedging and show why alternatives are a diversification vehicle.

To appreciate the primary contrast between hedging and diversification, a single point needs to be kept in mind: diversification is very often based on low correlation, and hedging is based on negative correlation. In hedging the protecting asset makes money when the vulnerable asset is losing it, and the protecting asset does not lose an equal amount of money when the vulnerable asset is making money. Thus, the ideal hedging asset is an option or option-like asset that can deliver asymmetric returns. Alternatives are clearly not hedging vehicles, since they do not deliver asymmetric returns.

## **THE USE OF ALTERNATIVES BY RETAIL INVESTORS AND WEALTH MANAGEMENT CLIENTS**

The academic literature has covered the characteristics of alternatives in a variety of ways. Several optimization models such as those based on modified value at risk, conditional value at risk, omega, extreme value theory, and optimization of higher moments to create a fund-of-funds portfolio have been proposed. French [2] examines how alternatives interact with a portfolio of traditional assets (equities and bonds). He finds that the efficient frontier constructed from portfolios that include alternatives dominates portfolios restricted to equities and bonds. Amin and Kat [3] corroborate French's evidence and show that, while alternatives

improve the mean-variance characteristics of traditional portfolios, they can also lead to lower skewness and higher kurtosis—thereby mitigating their added value.

In the two studies above as well as in most other studies, alternatives are considered as a portfolio, i.e., a standalone asset class. But, there is another way of using alternatives, which arises from a lesson from the universe of traditional equities. In 2005 and 2006 an innovative approach emerged that combined buy-and-hold investment (long-only) with an active component (long/short). This approach became known as the 130/30 strategy, where a manager can short-sell 30% of identified underperformers to leverage up the weights on the long side of the portfolio, either by identifying outperformers or by considering all stocks that are not underperformers. This approach typically has appeared as a long/short equity hedge fund, but in reality the 130/30 equity manager is constrained in terms of beta, tracking error, liquidity requirements, and so on. Nevertheless, 130/30 managers outperformed prior to the financial crisis and after 2010. Three of the studies finding evidence of outperformance were Sorensen et al. for the United States [4], Johnson et al. for the EAFE countries [5], and Jarecic et al. for Australia [6].

The interesting point of the 130/30 approach is that investors can add value to a portfolio if they combine buy-and-hold investments with an actively managed component. The work of Sorensen et al. [4] and others shows that such a strategy tends to work for equities. Why should it not work for other traditional asset classes? And why should we not use alternatives as the active component?

## RETAIL INVESTOR TEST—DATA AND METHODOLOGY

We include here a study to explore these questions. We start with a buy-and-hold strategy of long-only asset classes and combine it with an actively managed component in order to improve investor diversification. In some ways we are following Jaggi et al. [7], but we do not construct funds-of-alternative-investment funds or funds-of-hedge funds for each asset class. Our approach is simpler: we look for the best alternative investment or investments to add to an asset class. We also consider different levels for the alternative investments: 5%, 15%, and 30%. We use common metrics such as Sharpe ratios and downside deviation to judge the “best” strategy. We are not arguing that our approach is better than the optimization approach; ours is an alternative solution for investors searching for a well-diversified portfolio. We think our approach could be easier to understand because it does not depend on sophisticated optimization frameworks that work with the non-normality of alternative investment returns. And, as stated in the prior section, we define alternatives as those investments that generate correlation benefits to traditional, long-only-constructed investments as well as those investments that implement a hedge fund-like strategy, often incorporating one or a combination of the following: leverage, derivatives, short positions, and/or multiple asset classes. Since our focus is on how retail investors and financial advisors can use alternatives, we limit our search for alternatives to those that have daily liquidity.

We base our diversification work on six asset classes: bonds (both U.S. and international), global equities, commodities, and real estate (both U.S. and international). We aim to activate all six asset classes with one or possibly more alternative investments. The traditional long-only asset classes and the alternatives we use in this study are listed below:

## Traditional Long-Only Funds

iShares Dow Jones U.S. Real Estate ETF (IYR), Fidelity International Real Estate (FIREX), Barclay's Aggregate Bond ETF (AGG), GSCI Commodity ETF (GSG), Vanguard Global Equity (VHGEX), and T. Rowe Price International Bond (RPIBX).

## Alternative Investment Categories

Global macro, multi-strategies, market-neutral, long-short bond, long-short, long-short equity, arbitrage, managed futures, commodity, currency, quant, and event-driven.

The alternative investment prices and returns we use are the averages of the above investment classifications as computed by Lipper. Lipper also is the source for the long-only asset class prices and returns.

The methodology we use to determine if an alternative investment is likely to add diversification to an asset class is beta co-moments. Many investors and advisors are familiar with the term beta—the relationship between a security's return and the market (or benchmark) return—along with skewness and kurtosis. Martellini and Ziemann [8] developed a framework that assesses the potential diversification benefit of an asset relative to a portfolio. They used higher-moment betas to estimate how much portfolio risk (symmetric risk [volatility], asymmetric risk [skewness], and extreme risk [kurtosis]) is impacted by adding an asset.

Their work allows us to see if adding an alternative investment to an asset class reduces the asset class's variance when the second-order (volatility) beta of the alternative investment is less than one. The technique also can assess the impact on the third and fourth moments, i.e., skewness and kurtosis. With skewness and kurtosis we can see if the diversification extends to asymmetric and extreme risks as well.

The evaluation metrics we use here are: return, volatility, downside deviation, both the Sharpe and Sortino ratios, skewness, and excess kurtosis. To ensure diversification is being improved each existing long-only asset class has its metrics computed both before and after alternatives are added. In this way the addition of the alternatives can be judged fairly.

We form the "buy-and-hold portfolios" with actively managed components and compute our metrics over two periods: March 2, 2007, through September 24, 2012, and September 28, 2007, through September 30, 2009. We choose 2007 as a starting point since enough alternatives were trading in the early part of that year that good classification averages could be formed. The second period is meant to proxy the time of the credit crisis of 2008-2009—a good test of the possible diversification benefit alternatives can bring.

## Results: March 2007–September 2012

Table 1 shows the evaluation metrics for buy-and-hold investments, while Table 2s shows the same metrics for alternatives.

**Table 1. Risk and return values—buy-and-hold investments**

Metric	U.S. Real Estate	Int'l Real Estate	U.S. Bonds	Int'l Bonds	Commodities	World Equity
Return	0.6%	-2.1%	6.0%	4.5%	0.7%	2.7%
Volatility	29.3%	28.6%	6.4%	9.1%	28.6%	26.2%
Downside Deviation vs. Tbill	2.0%	1.9%	0.4%	0.6%	1.9%	1.8%
Sharpe Ratio	0.01	-0.08	0.91	0.48	0.02	0.10
Sortino Ratio	0.02	-0.01	0.86	0.45	0.02	0.10
Skewness	-0.4	0.1	-2.8	-1.0	-0.3	-0.2
Kurtosis	4.2	6.9	68.7	14.2	2.0	5.9

NB: In Tables 1 and 2 return values have been annualized as have volatility (standard deviation) and the Sharpe and Sortino ratios.

**Table 2. Risk and return values—alternatives**

Metric	Global Macro	Multi-Strategies	Market-Neutral	Long/Short Bond	Long/Short	Arbitrage
Return	1.8%	0.0%	2.3%	4.7%	10.5%	3.9%
Volatility	9.7%	5.7%	6.5%	11.1%	16.1%	8.7%
Downside Deviation vs. Tbill	0.7%	0.4%	0.4%	0.4%	1.1%	0.5%
Sharpe Ratio	0.16	-0.03	0.32	4.00	0.64	0.48
Sortino Ratio	0.16	0.00	0.32	2.70	0.64	0.48
Skewness	-0.3	-0.7	-0.5	-0.3	-0.1	2.2
Kurtosis	11.3	3.5	9.6	0.0	3.2	54.1

Metric	Long/Short Equity	Managed Futures	Commodities	Currency	Quant	Event-Driven
Return	1.0%	-0.1%	0.1%	-4.6%	-0.6%	7.2%
Volatility	15.9%	5.6%	17.5%	5.7%	4.5%	7.5%
Downside Deviation vs. Tbill	1.0%	0.4%	1.1%	0.4%	0.3%	0.5%
Sharpe Ratio	0.05	0.00	0.00	-0.80	-0.16	0.95
Sortino Ratio	0.06	0.00	0.00	-0.80	-0.11	0.95
Skewness	-0.1	-0.7	-0.4	-0.4	-0.6	-0.3
Kurtosis	8.5	3.7	2.5	2.0	12.3	3.8

Comparing our metrics, we have a number of the alternatives showing greater returns, lower volatility, less extreme risk, and often but not always better risk-adjusted returns. It is only for asymmetric risk (skewness) that the two groups of investments match up.

Leaving the relatively good performance of the two bond benchmarks out for the moment, most of the alternatives—based upon their return and volatility—could be good diversifying candidates for the remaining benchmarks. The alternatives that would not be are: multi-strategies, managed futures, currency, and quant. We check these statements by first using the beta co-moments test and by forming portfolios.

Based on our beta tests, arbitrage performs well (except in the case of U.S. bonds) for reducing volatility and for skewness. Its kurtosis or extreme risk, however, is the highest of all the assets. We still construct arbitrage + passive portfolios despite the very high kurtosis. Market-neutral also performs well except for international bonds. Long/short equity works, as does global macro. Given our beta co-moments results, we form the following portfolios: arbitrage, market-neutral, long/short equity, and global macro.

Our discussion of portfolio results in the next section is in two parts: one covering the period March 2007–September 2012 and a second covering October 2007–September 2009. For each period three portfolios were formed: one with 5% of the portfolio in alternatives, another with 15%, and a third with 30%, but we will discuss only the 30% allocation in any detail.

### **Portfolio Results—Adding One Alternative Investment at the 30% Level, March 2007-September 2012**

Adding alternatives to a portfolio (in our case by asset class) reduces volatility, increases risk-adjusted return, and can increase returns. On the negative side each alternative investment can increase asymmetric risk (skewness) and extreme risk (kurtosis). Our results to a certain extent confirm Amin and Kat [3]: alternatives can improve the mean-variance characteristics of traditional portfolios, but they can also lead to increased skewness and higher kurtosis, thereby mitigating (or possibly mitigating) their added value. We have more of a mixed picture than Amin and Kat generated. It could be that because we are adding alternatives by asset class (they did so as a separate asset class assembled via mean-variance optimization) that we are not consistently seeing asymmetric and extreme risk detract from the alternatives' benefits.

We show the results of the 30% investment level, since it is here that investors with low to moderate tolerance for risk see the most pickup. At the 15% level the modification of risk just begins to kick in, while at the 5% level there are only minor modifications to the asset profiles.

At the 30% level return, volatility, downside deviation, and risk-adjusted return are better. Skewness tends to be better, while kurtosis is typically worse, especially for commodities and world equity. This “better” performance at the 30% level is typical of all the alternatives tested and seems to indicate that a higher allocation to alternatives benefits a portfolio more than a lower level of investment such as 5%.



**Table 3. Risk and return values—buy-and-hold investments**

Metric	U.S. Real Estate	Int'l Real Estate	U.S. Bonds	Int'l Bonds	Commodities	World Equity
Return	0.6%	-2.1%	6.0%	4.5%	0.7%	2.7%
Volatility	29.3%	28.6%	6.4%	9.1%	28.6%	26.2%
Downside Deviation vs. Tbill	2.0%	1.9%	0.4%	0.6%	1.9%	1.8%
Sharpe Ratio	0.01	-0.08	0.91	0.48	0.02	0.10
Sortino Ratio	0.02	-0.01	0.86	0.45	0.02	0.10
Skewness	-0.4	0.1	-2.8	-1.0	-0.3	-0.2
Kurtosis	4.2	6.9	68.7	14.2	2.0	5.9

**Table 4. Risk and return values—buy-and-hold investments + one alternative investment at the 30% level**

Metric	U.S. Real Estate	Int'l Real Estate	U.S. Bonds	Int'l Bonds	Commodities	World Equity
Return	1.5%	-0.3%	5.4%	4.3%	1.6%	3.0%
Volatility	21.9%	21.9%	5.2%	6.7%	20.8%	20.1%
Downside Deviation vs. Tbill	1.5%	1.4%	0.4%	0.4%	1.4%	1.4%
Sharpe Ratio	0.06	-0.02	1.00	0.60	0.07	0.14
Sortino Ratio	0.06	-0.02	1.00	0.60	0.06	0.13
Skewness	-0.4	0.2	0.5	-0.6	-0.4	0.0
Kurtosis	5.9	9.7	68.3	13.3	9.0	13.3

### **Portfolio Results—Adding One Alternative Investment at the 30% Level, Credit Crisis—October 2007-September 2009**

**Table 5. Risk and return values—buy-and-hold investments**

Metric	U.S. Real Estate	Int'l Real Estate	U.S. Bonds	Int'l Bonds	Commodities	World Equity
Return	-18.4%	-18.2%	7.3%	4.1%	-17.8%	-14.5%
Volatility	38.3%	40.1%	9.6%	11.7%	37.6%	35.2%
Downside Deviation vs. Tbill	2.6%	2.6%	0.6%	0.7%	2.4%	2.3%
Sharpe Ratio	-0.49	-0.46	0.74	0.33	-0.48	-0.42
Sortino Ratio	-0.43	-0.43	0.71	0.32	-0.46	-0.39
Skewness	-0.4	0.2	-2.2	-1.3	-0.2	0.0
Kurtosis	2.3	3.9	36.4	13.6	0.6	3.7

**Table 6. Risk and return values—buy-and-hold investments  
+ arbitrage alternative at the 30% level**

Metric	U.S. Real Estate	Int'l Real Estate	U.S. Bonds	Int'l Bonds	Commodities	World Equity
Return	-11.2%	-11.1%	6.8%	4.6%	-10.8%	-10.3%
Volatility	29.3%	30.8%	8.1%	8.8%	27.9%	27.7%
Downside Deviation vs. Tbill	2.0%	2.0%	0.5%	0.6%	1.8%	1.8%
Sharpe Ratio	-0.39	-0.38	0.81	0.49	-0.40	-0.31
Sortino Ratio	-0.35	-0.35	0.84	0.51	-0.37	-0.29
Skewness	-0.4	0.3	0.3	-0.7	-0.3	0.1
Kurtosis	3.6	5.5	31.6	11.6	1.3	5.6

As before, the 30% investment in an alternative gives the best protection and the best return performance. It may very well be that the 30% investment level is for those investors who have a low to moderate risk tolerance and would like protection in the majority of environments.

## **Final Comments on the Retail Investor Test**

We have shown in this section the benefit of using alternatives at various investment levels as a way to diversify a retail investor's long-only asset holdings. We take as our model 130/30 equity funds where there is a long-only component and an active component. Our look at the 30% investment level shows a significant improvement in returns and risk-adjusted measures, especially during the credit crisis. The only downside to the 30% investment level is the occasional uptick in extreme risk, i.e., kurtosis.

## **INSTITUTIONAL ALTERNATIVES—PRIVATE EQUITY, REAL ESTATE, AND HEDGE FUNDS—A BRIEF DESCRIPTION**

### **Private Equity Funds**

Private equity funds act as an intermediary between institutional investors and high-net-worth individuals on one side and privately held companies that are seeking capital to finance crucial stages of their corporate activity on the other. Private equity funds are collective investment schemes that gather the capital of a number of investors to invest in from ten to thirty companies. A management team (typically called the general partner) deploys the capital provided by the investors (called the limited partners) according to a predefined investment strategy. Investors do not typically take part in the day-to-day management of the fund, and their liability to the financial exposure of the fund is limited to their capital commitment made to the fund.

The risk profile of investments in private equity varies greatly; individual direct investments in privately held companies are the riskiest. Structured products such as hedge funds or securitization notes allow risk to be tailored to individual investors' needs.

Funds are a good way to diversify risks associated with investments in privately held companies, and diversification benefits increase with the number of fund investments held by an investor. Fund-of-fund investments have a risk profile comparable to holding a diversified portfolio of fund investments individually.

Private equity is an asset class that promises high returns for taking high risks. Private equity might even outperform other asset classes on a purely risk-adjusted basis, since the inefficiencies of private equity markets allow the investors to benefit from information asymmetry, legally obtained insider information, and the superior investment skills of individual managers. These elements are typically not relevant for public market investments, where information is generally shared and reflected in market prices and where residual minor market inefficiencies are difficult to exploit through arbitrage because of the transaction costs.

However, the higher return opportunities in private equity come at a price. In addition to relatively high management cost, investors in this asset class need to accept low liquidity for their investment and long holding periods before returns may materialize. Investments in individual companies typically have a holding period of between three to five years, fund investments have a lifetime of eight to twelve years, and fund-of-fund investments can last up to fifteen years before being divested. Also, investors in private equity—no matter at what level—need superior selection skills in their investment managers; the quality of fund managers varies greatly and the natural selection of successful teams through investment performance is slow.

Private equity funds can target a wide range of segments of private equity. Typical segments include:

- Seed financing in the creation stage of companies
- Start-up capital for companies developing prototypes of a marketable product
- Expansion capital for market penetration
- Buy-out capital and replacement capital for dealing with changing ownership structures in companies and succession issues
- Pre-IPO financing to prepare companies for public listing
- Turnaround capital to support distressed companies in the restructuring of their business activities
- Mezzanine capital as an instrument to leverage equity investments in buy-out or development financing

Risk profiles of these segments differ and so does the skill set required of managers investing in the various segments. For example, investment assessment in early-stage technology companies requires a deep understanding of the technology risk and the market potential of the product. Market risks and product risks are typically less pronounced in late-stage companies or in buy-out situations. In these stages it is often the superior financial structuring and organizational development skills of the fund manager that make the difference in a successful investment.

## Direct Investment in Real Estate

In this section we draw on an institutional investor survey conducted in 2005 by Dahr and Goetzman [9]. In the literature the top reasons for investing in real estate are: diversification, cash generation, potential for capital gains, and inflation hedging. Diversification and inflation hedging are the leading reasons for having real estate in a portfolio as opposed to investment in real estate for long-term return or income generation.

The main risk factors associated with real estate investing are liquidity risk, lack of reliable data, and the risk of making a poor investment. Contrary to what might be expected from a classical asset allocation model, asset volatility is often not regarded as a major risk factor for real estate.

The main factors institutional investors noted in the survey [9] in terms of measuring risk are: statistical estimates of risk and return, advice from internal staff, advice from an external consultant, advice from other investors, economic forecasts, current market values of the asset, recent trends in the market, long-term historical performance, expected changes in the economic outlook, actions taken by industry peers, and the relative skill of the external manager with this asset class.

The investors' responses to the question on major factors driving the investment decision are largely consistent with modern portfolio theory, which stresses long-term historical performance and relies upon statistical estimates of risk and return. This framework is not exactly congruent with the replies to the question about major factors. The risk factor in the standard portfolio optimization model is quantified as asset volatility. Uncertainty about the inputs, i.e., lack of reliable data, was more important to the respondents than was volatility, suggesting that uncertainty as opposed to risk looms large as a determinant of how much to allocate to real estate investments.

## Hedge Funds

While hedge funds are familiar to many readers, we think a few brief statements about their birth and historical use are of value.

Hedge funds were originally products created by unregulated stock pickers and held exclusively by high-net-worth individuals. Over time they became better known as evidenced by financial publications, analyst reports, and boardrooms. Their success was largely fuelled by the wealth created during the long bull equity market of the 1990s and is now supported by the difficult and highly volatile environment that has prevailed since the early 2000s. Indeed, by focusing on absolute performance and abstracting from benchmarks, hedge funds are able to generate superior returns in virtually all types of market environments. They offer much-needed diversification to portfolios that are invested in traditional asset classes such as equities and bonds. This diversification provides a strong argument for using hedge funds in wealth management and contributes to making this asset class an increasingly popular investment choice.

However, there is no such thing as a “free lunch” in finance. Private and institutional investors are willing to include hedge funds in their portfolios because of hedge funds' ability to deliver favorable return/risk characteristics. Hedge funds do, however, carry additional risks—risks that are not common to traditional stock and bond investments. These risks are

inherent to the strategies pursued, the instruments and markets used, the amount of leverage employed, and last but not least the specific skills of the hedge fund manager. Since choosing a bad manager can easily wipe out all the benefits of a hedge fund investment, investing in only one hedge fund is typically suboptimal. The reasons are threefold: First, performance differentials between competing funds raise the issue of whether a single investment instrument can deliver consistent returns close to those of the broad hedge fund indices that are used at the asset-allocation level. Second, a number of individual hedge funds have collapsed under the weight of spectacular fraud or investment debacles. This has raised concerns among investors, who often lack sufficient information to evaluate comparative hedge fund performance, as to whether they can perform the necessary onsite due-diligence checks. Finally, investing only with managers who have a good reputation and an established track record does not provide a complete hedge, as illustrated by the debacle that was Long Term Capital Management LP.

Risk-conscious investors are coming back, then, to the central tenet of modern portfolio theory: diversification, but not the diversification of traditional portfolio theory. By combining several hedge funds with differing return distributions and risk profiles in a portfolio, investors can diversify away specific risk and ensure a more disciplined exposure to the overall hedge fund asset class. This is likely to result in better long-term risk-adjusted returns. Those seeking to avoid the logistical problems and record-keeping headaches of tracking several hedge funds may even delegate the portfolio construction and monitoring activities to a fund of hedge funds. This is the preferred investment structure for most institutional investors, since it gives them instant diversification and frees them from the responsibility of monitoring managers. It also gives the institutional manager the freedom to align incentives with managers and the extensive resources available to select and monitor the best managers.

## **WORKING WITH RISK: AN INSTITUTIONAL PERSPECTIVE**

We focus in this section on the hows and whys of risk assessment for private equity and real estate investors. Hedge fund risk has received extensive treatment in the literature, and the following sources are recommended reading: Martin [10], Rachev's several papers and the books listed on his website [11], and Rahl [12].

### **Private Equity**

Interest in private equity funds involves investing in an illiquid asset class; private equity funds have specific characteristics that make measuring their value at risk in a way similar to the risk of tradable assets very difficult. Nevertheless, and despite conceptual challenges, investors increasingly view a proper measurement of private equity risk as a necessity.

Historically, most investors have taken relatively simplistic approaches to measuring and reporting the risks of investing in private equity. However, with investors' growing exposure to private equity, it has become important for investors to fully understand and correctly quantify the risks of investing in this asset class to strengthen their risk management

capabilities. It is also important that risk measurements used for investment decisions and internal capital allocation are of high quality to reinforce the sound corporate governance from which investment policies are implemented.

The most widely used methodologies to assess the risk of private equity investments are those based on discounted cash flow (DCF) and on net asset value (NAV). For both methodologies the period over which the risk is measured depends on the particular environment in which the investor operates, e.g., duration of the portfolio, ability to hold the investment, and liquidity constraints.

NAV-based risk measurement approaches generally look at the volatility of a fund's NAV in order to estimate the value at risk of the investment. This method has the advantages of being simple and giving a good approximation of the risks of mature funds and diversified funds of funds spread over several years.

Volatility is estimated by measuring the returns reported at different time intervals, based on quoted indices or other private equity benchmarks available in the market. The measured volatility over the relevant period is then applied to the fund's or portfolio's NAV.

It is important to note that NAV calculation can give an incomplete or even distorted picture of risk because:

- A fund's NAV fluctuates naturally with the development of the fund's underlying assets and liabilities from investment and divestment, not just because of risk exposure.
- NAV-based models need to reflect the limited partner's exposure to the fund's undrawn commitments and the future use of distributions. Both aspects might present a different risk profile than the one applied to the NAV.
- Publicly quoted private equity indices are rarely representative of the portfolio held by the investor, and the illiquid contractual nature of investments in private equity makes them fundamentally different from the shares of listed private equity funds.
- NAV-based methods may fail for young funds that are still in the early phases of their J-Curve<sup>1</sup> and for portfolios with limited diversification in their vintage years, which undermines the representativeness of the indicator used to estimate the volatility of the NAV.

Limited partners also face a number of other risks that can impact the value of their investments:

### 1. Funding risk

The unpredictable timing of cash flow over the life of an equity investment poses funding risk for the limited partner. Fund managers call most or all of the committed capital over the investment period of the fund, and limited partners have to meet their commitments within a

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<sup>1</sup> Because of private equity's unique characteristics in terms of return and cash-flow profile, the pattern is called the J-Curve. The J-Curve illustrates the tendency of private equity funds to deliver negative returns and cash flows in the early years and to have investment gains and positive cash flows later in life as the portfolio companies mature and gradually exit. Portfolios of funds have a similar J-Curve pattern, but usually the J-Curve effect is more pronounced in the sense that it takes longer to report a positive internal rate of return (IRR), since capital calls of funds are drawn over a longer period.

fixed short-notice period. Because commitments are contractually binding, a limited partner who cannot meet the obligations is forced to default on payments and loses a substantial portion of the share in the partnership. In practice negotiations can occur between the limited partner and the fund manager to adapt the size of the fund and/or the capital-call requirement.

## 2. Liquidity risk

Limited partners can sell their stakes in private equity partnerships to fund their open commitments. However, the secondary market for private equity investments is relatively small and highly inefficient. The characteristics of the secondary market expose investors to liquidity risk. Moreover, secondary market prices are often influenced by factors beyond the fair value of the partnership, which can translate into a discounted price. For instance, investors selling from a distressed position often have to accept discounts to reported NAV.

## 3. Market risk

Since private equity is an illiquid asset, the treatment of market risk poses important challenges. There are two principal methods for valuing an asset. The first is its current market valuation, or an estimate of what that might be. The second is the present value of the estimated future cash flows from that asset. Normally, liquidity and arbitrage in the market force these two alternative methods of valuation into close alignment. Lack of liquidity and other market dysfunctions can cause these two alternative approaches to diverge—occasionally sharply; this is most clearly observed in secondary private equity transactions.

## Real Estate

Parts of this section use the work of DiBartolomeo et al. [13] to discuss the risk characteristics and risk management techniques for real estate investments.

Real estate is a particularly difficult risk challenge because of its lack of liquidity. Even published indices in real estate are based on annual appraisals of large properties, not actual transactions. The lack of transaction-based pricing and the long periods between transactions have the statistical effect of smoothing the ups and downs of the real estate market, compared to the actual but unobservable real market conditions. Traditional real estate indices such as the National Council of Real Estate Investment Fiduciaries (NCREIF) Index typically underestimate the true volatility of real estate and have a high degree of serial correlation that arises from the smoothing. The problems with real estate measurement are well documented in Young and Graff [14]. Partial solutions to these problems are proposed in Geltner and Fisher [15], where available transaction prices on repeated sales of the same property and concurrent appraisal data are used to adjust the published time series of index returns.

In a similar vein time series of returns can be modeled as a first-order autoregressive process to neutralize the serial correlation. Sample statistics regarding real estate index returns are then based on the adjusted time series. Unfortunately, even if we were able to fully resolve all the statistical problems of broad real estate indices, this method is of no help with respect to measuring the marginal impact of individual properties on portfolio returns.

Another approach to measuring the unobservable true returns in the real estate market has been to create a “synthetic” real estate returns series by thinking of unobservable real estate returns as being the returns of REITs with the influence of the general stock market hedged away. The idea of measuring real estate returns through a hedged REIT index began with Giliberto [16] and was put into the context of asset allocation decisions by Webb and Liang [17]. MacKinnon and Clayton [18] extended this line of research of REIT returns to those of other financial assets, providing an approach that may be helpful when considering the inclusion of real estate in an asset allocation exercise. But, the approach is of limited value when considering decisions at the individual property level.

In a linear model for financial assets two types of specifications are popular. In economic models the factors are defined to be exogenous variables such as interest rates or oil prices, such that the statistically significant factor returns (their  $F$  values) can be observed in the real world. A separate time-series regression generally is used to estimate the factor exposures (the  $B$  values) of each asset. In such regressions the dependent variable is the periodic returns of the particular asset, and the independent variables are the observable returns of the factors.

Alternatively, a fundamental model could be used where endogenous characteristics of the assets (e.g., market cap of a stock) are used to specify observable values of the factor exposures (the  $B$  values), and the factor returns (the  $F$  values) are estimated for each period in a separate cross-sectional regression for each period. In these cross-sectional regressions the dependent variable is the vector of asset returns for the period, and the independent variables are the factor exposures for all the assets at the beginning of the period. Normally, we distinguish between the two types of models via notation. In fundamental models the factor exposures can be time varying, so the factor exposures (the  $B$  values) also carry a time subscript.

One particular model of the economic type that is widely used by institutional investors to evaluate the risk of their marketable securities portfolios is the “everything, everywhere” (EE) model. This model links global public securities performance to over 50 factors, including stock and bond market performance across five global geographic regions and six broad economic sectors. Factors meant to measure investor confidence (e.g., the spreads in yields for different qualities of bonds) and macroeconomic conditions (interest rates, energy costs, exchange rates) are also included. The EE model breaks discount-rate risk into two components: the risk of Treasury curve movements and the risk of changes in credit-related yield spreads. Bond risk is estimated by measuring a bond’s price sensitivity to both the credit factors and the Treasury factors, using a binomial model that incorporates prepayment options.

Since the EE model is of the economic type, the estimation of factor exposures is normally carried out by time-series regressions. However, since real estate investments do not typically have observable periodic returns, we have no information to use as the independent variable in these regressions. Instead, we take advantage of various available techniques to estimate in closed form the exposure of a financial asset’s returns to the factors. For example, one might compute the sensitivity of a bond’s return to changes in the level of interest rates by the sort of time-series regression discussed above. However, there are other well-known closed-form methods for calculating the duration of a bond—the sensitivity measure that would have arisen as the result of our time-series regression.

Traditional real estate appraisals use one of three basic methods to value a property: (1) replacement cost, (2) comparable sales, and (3) capitalizing the expected income. One way



real estate investors have tried to evaluate the risk of individual properties is to do Monte Carlo simulations of their valuation models. By varying the valuation inputs across their expected range, one can obtain an expectation of the range of property values at any future moment in time. This allows an estimate of the uncertainty of return on that property over a known time horizon. However, such procedures are not tractable over large portfolios, nor do the “bottom up” estimates of real estate-specific variables such as rents or operating expenses allow for any insight into the interrelationships between real estate properties and other asset classes.

Following DiBartolomeo et al. [13], another way of assessing real estate risk is to estimate factor exposures using methods closely related to the EE method. From the perspective of typical real estate analysis DiBartolomeo et al. use financial market data external to real estate to forecast the possible range of inputs to the valuation process across time. By doing so they derive a direct assessment of risk. For example, they use observed volatility of bond interest rates to frame the range of potential capitalization rates for a property. They assess the potential demand for office space in a location such as Lower Manhattan based on the recent strength of the stock market performance of the financial services sector of the economy. The more volatile the expected stock market performance of the financial services sector, the greater the uncertainty of demand for such office space. Directly using information from the stock and bond markets, the model (DM hereafter) can automatically provide consistent assumptions across all asset classes to the institutional investor.

DM takes a complicated problem and breaks it into its parts by disaggregating a portfolio into buildings and buildings into their constituent sources of risk, including the cash flow from tenants, tenant credit risk, rent volatility, and the property’s financing structure. Each of these sources of risk is represented by a hypothetical proxy portfolio of marketable securities that the model believes will have the same economic payoff properties as the aspect of the real estate of concern. Once the model has taken these factors into account, it applies the EE method to value and estimates the risk of each piece. The components are then reassembled and the risk can be examined at any level the investor chooses.

Given the model’s framework, it can examine the sensitivity of each contributor to change in property value to the common set of underlying factors. With estimates of the potential range for factors, e.g., how volatile oil prices are expected to be, the mathematics of a risk assessment for a single property or the entire portfolio can be computed. The risk estimate contains the future range of interest rates, existing cash flows streams, rent volatility, and financing structure risk.

DM, then, decomposes real estate risk into four components: operating cash flow valuation, financing structure, credit, and rent/occupancy volatility risk. Each of these risks is expressed as a function of observable factors in financial markets or in the general economy. The model’s approach is congruent with methodologies used for risk management in securities market portfolios, thereby allowing seamless integration of risk assessment into multi-asset portfolios. Most importantly, the model provides a framework for determining how much of the risk of investing in a property arises from characteristics of the specific property and how much of the risk arises from common influences across all properties—such as interest rates and levels of economic activity. We believe this new level of transparency with respect to real estate risk can encourage investors to be more confident in their understanding of real estate and consequently more willing to allocate resources to

property investments. Readers are strongly encouraged to read the DiBartolomeo et al. paper [13] for its insights into real estate risk assessment.

## Liquidity Risk

Given the commonality of liquidity risk across institutional investments such as private equity and real estate, we now address liquidity risk in some detail in this section.

Investors often associate illiquidity with the difficulty of buying and selling assets. But quantifying the degree of illiquidity in formal investment analysis is not easy. In the securities market illiquidity is typically measured by the price discount or the bid-ask spread necessary for an immediate sale. This concept is implicit in a number of papers such as those of Laroque and Grossmand [19] and Luciano and Dumas [20]. Succeeding authors have attempted to refine the measure by using additional trading information such as trading volume, order size, and trading costs. However, applying these measures of illiquidity to the private asset market poses both practical and conceptual difficulties.

The practical challenge is that computation of these measures requires extensive data on trades and quotes that are simply unavailable in thinly traded private markets where there is no centralized quotation system to track and report bid/ask prices.

The conceptual problem is that sellers typically do not take a price discount in order to sell private assets immediately. Real estate trading, for example, is typically conducted through a lengthy search and random matching process that can take weeks, months, even years. Immediate sale of real estate is virtually impossible even for owners under extreme liquidity constraints, and a quick sale typically results in a significant price discount from the property's fair value. While the often deep discounts may reflect the degree of distress of the seller, they do not properly reflect the trading strategy of typical sellers who would take the time necessary under the given market conditions and wait until a desirable offer arrives.

Private assets need an alternative method to measure illiquidity. A natural choice for gauging the difficulty of trading is the time it takes to sell a private asset. For example, a house that takes longer to sell than others can be said to be less liquid. However, since the selling time is also a function of the price at which it is sold, a quicker sale may not necessarily suggest the asset was more liquid but that the price was set too low. Observing the trade-off between price and selling time, McCall and Lippman [21] propose a formal definition of illiquidity: "the expected time until the asset is sold when following the optimal policy." One way to define the optimal selling policy is to take the necessary time to wait for higher bids, as long as the expected benefit of waiting exceeds the cost of doing so. In the past two decades the McCall-Lippman proposition (often dubbed "time-on-market," TOM) has become a widely accepted definition of illiquidity for thinly traded private assets; it is certainly one of the most extensively studied concepts in housing economics.

TOM, however, suggests to some that it is a matter of individual preference. For typical sellers the optimal policy is to take whatever time is necessary to sell the private asset at or near its fair market value. For sellers who are under various financial and personal constraints, the optimal policy is to expedite the sale with a lower price in order to limit the search costs. On the other hand, the average TOM is also a function of overall market conditions. For example, in hot markets all properties are sold rather quickly, while in cold markets the average selling time is substantially longer.

To recognize the effect of market conditions on illiquidity, Cheng et al. [22] propose a concept called the normal selling time (NST) to indicate the expected (average) TOM necessary for private assets—real estate in particular—to be sold by normal unconstrained sellers under a given market condition. The NST varies under different market conditions; shorter NST indicates a more liquid market in which properties are sold more quickly, whereas longer NST indicates a less liquid market. From an *ex ante* perspective the NST at the end of a typically lengthy holding period is a function of future market conditions, which are uncertain and pose the risk that the seller may not be able to trade out the position when needed. In other words, illiquidity of private assets is a market risk. Individual sellers can determine their individual selling time based on their financial constraints, but they cannot control the NST under a given market condition.

Distressed sellers may have to sell more quickly than normal sellers, but they cannot do so without suffering negative consequences on their sales price. Just as the bid-ask spread is uninformative for measuring illiquidity of thinly traded private assets, the NST or TOM in general is uninformative to security assets that trade almost instantaneously. According to the National Association of Realtors, the national average selling time for the U.S. residential market was about six months during the period from 1989 to 2006. For the more recent 2007-2010 period when the housing market was bad, the average selling time was about nine and a half months. In certain areas such as Miami and Palm Beach, Florida, the average selling time was as high as 36 months. Therefore, by the standard of the private market, nearly all securities are extremely liquid, and the time it takes to trade even the least liquid securities, e.g., some seasoned bonds, is negligible.

At a more fundamental level, how illiquidity is measured in public markets versus private markets lies in the difference in the microstructures of the two markets, which dictate different optimal trading policies and strategies. In the relatively efficient public market immediate sale may be optimal because there is little to be gained from waiting, assuming all market participants are well informed. In an inefficient private market selling immediately is usually less optimal because it takes time for information to disperse and for high bidders to arrive in a search environment. The notion that all assets are illiquid to different degrees and that private assets such as real estate are just less liquid than stocks fails to recognize the different nature of the two markets. Private assets are not just less liquid than securities; they are less liquid in a different way.

Given the above, the next logical question is: how should illiquidity be incorporated into formal investment analysis of private assets? The expected TOM (or NST), although properly indicating the difficulty of trading in terms of time, does not quantify the risk that is due to illiquidity. For example, assume that while trying to decide whether to invest in common stock or in real estate, an investor finds the following: the real estate is expected to have returns comparable to the stock but with only half the return volatility. (This “find” is generally consistent with many empirical observations in both the literature and in practice. In fact, the high return and low volatility of real estate performance has been repeatedly documented for over half a century and is widely known as the “real estate risk premium puzzle” [Lusht [23]]).

In the past a popular explanation of the low volatility of real estate was the “appraisal smoothing theory.” Recent research, however, has shown that the appraisal-smoothing theory is based on biased empirical facts and flawed logic. It has been demonstrated that the formal model of the smoothing theory (the so-called partial adjustment model) suggests appraisal-

based real estate indices (such as the NCREIF Index) may exaggerate rather than understate the true volatility of real estate returns. In other words, the alleged appraisal smoothing is more myth than fact. But, if low volatility of real estate is a valid observation, an important question facing an investor would be: in which asset should he invest? Real estate appears to have lower volatility for comparable returns, but it is subject to significant liquidity risk because of the lengthy and uncertain TOM. How can an investor make a fair performance comparison between the two assets? To put it differently, if a 10% expected return reflects a fair price for a 24% stock volatility, is it a fair price for a 12% real estate volatility plus the additional liquidity risk?

Such questions highlight the necessity for quantifying the collective impact of price risk and liquidity risk. As the example shows, compared to real estate, stocks are highly liquid and can be considered to have trivial liquidity risk; but real estate poses two risks—price risk and liquidity risk. In order to answer the question of which to buy, an investor needs to somehow combine the two risks of real estate into a single integrated risk metric so he can compare the two assets on a risk-per-unit-return basis. The difficulty, obviously, is that the two risks are measured in dissimilar units (percentage and time). However, this difficulty must be overcome in order to make a fair comparison between the performance of the stock and real estate.

For a discussion of comparing price and liquidity risk across asset classes, the reader is referred to Cheng et al. [22]. Their paper develops a concept called liquidity risk factor (LRF), which is broadly defined in such a way as to capture the difference between the overall risk of a private and illiquid asset and the risk of an otherwise identical but completely liquid asset. Using commercial real estate as a model asset, they find that under various scenarios or market conditions and investment horizons, the magnitude of the liquidity risk of real estate is too large to be ignored by any rational investor, especially in down markets when liquidity risk is a greater concern.

The closed-form formula of LRF provides a practical tool for investors to obtain liquidity-adjusted risk estimates and enables a fair comparison between the performance of thinly traded private assets and publicly traded financial assets. The closed-form formula can be used to construct optimal mixed-asset portfolios that include both financial and private assets. McCall and Lippman [21] define the liquidity of private assets but do not quantify the liquidity risk; the Cheng et al. [22] paper extends McCall and Lippman's work by developing the LRF to capture such risk, thus making their liquidity measure something that can be incorporated into formal investment analysis and mixed-asset portfolios.

## CONCLUSION

In this chapter we have discussed alternative investments in a multi-asset portfolio. We introduced the topic by touching on the similarity and differences between retail and institutional use of alternatives. A definition of alternatives was given for retail investors, and a brief examination of how retail investors and financial advisors can use alternatives was presented. Institutional alternatives were then defined, as were the initial steps typically taken to evaluate private equity, real estate, and hedge fund purchases. The last section dealt with

risk in its various forms. The measurement of private equity and real estate risk was examined as was liquidity risk.

We close with the following final comments. As many analysts have noted since the credit crisis of 2008-2009, diversification as typically practiced by investors (and some institutional managers) is not enough. In this chapter we have presented a number of important questions to ask when one is analyzing a new alternative investment diversifier from either a standalone, asset-only, or asset-liability point of view. The frameworks can be simple or complex, but they are also highly effective. Apart from the diversifier's statistical properties, our discussion emphasizes the importance of properly accounting for parameter uncertainty and illiquidity—two elements often ignored by investors. We also show the importance of taking the correct perspective when evaluating a diversifier. What looks good from a standalone perspective might not look good in a portfolio context and vice versa. Application of these methodologies to alternatives, whether in a retail or institutional portfolio, underlines the advantages and disadvantages of alternatives as diversifiers and as ways to improve returns.

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*Chapter 5*

# RECONSIDERING RIP UNDER INFLATION TARGETING: AN EMPIRICAL INVESTIGATION\*

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## Abstract

This paper investigates whether inflation targeting (IT) macroeconomic policy matters for real interest rate parity (RIP). We employ two panel unit root tests with and without cross-sectional dependence to examine the stationarity of real interest rate differentials for panel data on eleven OECD countries from 1974Q1 to 2011Q3. Comparisons are made, together with CPI and PPI, between IT and non-IT, and with and without cross-sectional dependence. Our empirical results present favorable evidence for RIP especially in countries that adopted IT. Moreover, when it comes to RIP, IT seems to outweigh other issues, including the choice of price indices, the choice of base currencies, and cross-sectional dependence, proposed by previous studies.

**Keywords:** Real interest parity, real interest rate, inflation targeting, cross-sectional dependence, panel unit root

**JEL Classification:** F30, F31

## 1. Introduction

The main purpose of this paper is to study whether inflation targeting (IT) macroeconomic policy matters for real interest rate parity (RIP). The equality of real interest rates across countries in a world characterized by high capital mobility has played an important role in international macroeconomics. In foreign exchange markets, the uncovered interest parity (UIP) condition, together with purchasing power parity (PPP), implies the equalization of real interest rates. As numerous studies point out, the cross-country equality of real

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interest rates has been an important assumption in the early monetary approach to exchange rate determination.<sup>1</sup> Furthermore, the validity of RIP is important for the analysis of issues related to macroeconomic policy, including fiscal and monetary policies. If real rates are equal across countries, the ability of monetary authorities to influence their real interest rates is limited by the degree to which such policy is able to influence the world real interest rate.<sup>2</sup> Furthermore, as pointed out by Feldstein (1991), unless real rates can differ across countries, policies directed at increasing domestic savings cannot increase the rate of capital formation and, hence, productivity. Since the proposition that real rates are equal across countries has theoretical importance, as well as implications for policy analysis, a number of empirical studies have undertaken cross-country comparisons of real interest rates. Although the theoretical importance of RIP is obvious, empirical support for this parity is elusive.

The important question in this study is if IT adopted by the central banks in a sample helps provide favorable evidence for RIP. A number of panel data studies of OECD countries provide favorable evidence for long-run PPP.<sup>3</sup> One common explanation for the finding is that by increasing the amount of information in the tests across real exchange rates, the power of the tests is usually increased and therefore the issue of low power of early univariate unit root studies is resolved.<sup>4</sup> As Svensson (2000) and Mishkin and Schmidt-Hebbel (2007) point out, however, another important factor we need to consider for the finding is that, under IT, a high degree of transparency and accountability of monetary policy by the central banks does not limit only the variability of inflation but also that of the real exchange rate at a long horizon, and thereby eventually stabilizes real exchange rates to a significant amount relative to the cases under alternative monetary policy. Moreover, Ding and Kim (2012) find empirical support for PPP in countries under IT. If this is the case and PPP does hold with countries under IT, then it is likely to result in favor of RIP.

International capital markets have become more integrated since the 1970s, particularly among the industrialized countries. However, the RIP condition is not likely to receive full support from previous studies due to some issues under the current system of highly integrated capital markets. Issues such as the choice of price indices, the choice of base currencies, and cross-sectional dependence, as shown by previous literature, are also important for understanding long-run RIP.<sup>5</sup> It would be interesting to see whether these issues still matter for RIP under IT.

To investigate the hypothesis, we employ eleven OECD countries from 1974Q1 to 2011Q3 and they are divided into three groups. One group consists of countries under IT policy (IT group). Another one is of countries under non-IT or other monetary policy (NIT group), and the other is all countries, which includes both IT and NIT countries (ALL group). For each group, real interest rate differentials are constructed separately with consumer price index (CPI) and producer price index (PPI) to see if the choice of price indices matter for RIP under IT. We employ two panel unit root tests with and without cross-

<sup>1</sup>See Dornbusch (1976), Frenkel (1976), and Mussa (1982).

<sup>2</sup>As long as the exchange rate is a variable and exchange rate expectations are not static, there is room for the central bank to manage the nominal interest rate at least in the short run.

<sup>3</sup>See Lothian (1998), Papell (2002) and Taylor and Taylor (2004) for details.

<sup>4</sup>However, Taylor and Sarno (1998) issue an important warning for spurious interpretations of the finding with panel data. See Taylor and Sarno (1998) for details.

<sup>5</sup>Refer to literature review section for detail explanations.



sectional dependence, proposed by Im, Pesaran, and Shin (IPS, 2003) and Pesaran (2007), to examine if the cross-sectional dependence is an important factor for the stationarity of real interest rate differentials for each group. The empirical results present favorable evidence for RIP in especially countries that adopted IT macroeconomic policy. Furthermore, when it comes to RIP, IT seems to outweigh other issues such as the choice of price indices, the choice of base currencies, and cross-sectional dependence.

## 2. Empirical Evidence on RIP

RIP can be investigated by testing the stationarity of real interest rate differentials through panel unit root tests, which are believed to be more powerful and efficient than the traditional augmented Dickey Fuller test. If the null hypothesis of non-stationarity can be rejected, the parity condition is to hold, implying that the real interest rate differentials have mean reverting properties. Otherwise, the parity condition is not likely to hold.

Empirical evidence for RIP using panel unit root tests is mixed.<sup>6</sup> As previous studies pointed out, one of the important issues in this type of empirical studies is about appropriate measure of the real interest rate.<sup>7</sup> If all of the goods in the consumption basket are perfectly arbitrated, then an overall price deflator like CPI is a good measure of the real interest rate. However, the results of tests with the CPI based real interest rates have not been favorable to RIP. As can be seen in Mishkin (1984a, 1984b), and Cumby and Obstfeld (1984), they found little support for RIP with CPI. Mark (1985) employs CPI based real interest rates for six OECD countries for the equality of the pre-tax real rates, but he fails to find support for RIP. Chinn and Frankel (1995) use CPIs for Pacific-Basin countries, and they find support for RIP for only some countries. Wu and Chen (1998) employ panel unit root tests, and Fountas and Wu (1999) use cointegration tests with structural shifts and find support for RIP. Fujii and Chinn (2001) use WPI and CPI and focus on long-term yields. They find support for RIP.

If PPP does not hold, then this is likely to result in evidence against RIP. When PPP was previously tested using general price indices, non-traded goods became a major problem in the deviation of the real exchange rate from its PPP value, because general price indices contain the prices of both traded and non-traded goods. Under the assumption of perfect arbitrage in the goods market, the results of tests using the overall consumption deflator become difficult to interpret in light of existing theories.

Another issue is the choice of a base currency. In empirical studies of RIP, one country is chosen as a base country to construct the differentials with other countries. Therefore, the fluctuations of the value of a base currency may influence the differentials presenting a non-stationary process. Several studies find supportive evidence of international parities when Germany is used as a base country instead of the U.S.<sup>8</sup> One explanation is that the U.S. dollar had been substantially appreciating from 1980 to early 1985 and then depreciating

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<sup>6</sup>Mishkin (1984a,b), Cumby and Mishkin (1986), Meese and Rogoff (1988), Edison and Pauls (1993) are unable to find supportive evidence of real interest rate parity. Lothian (2000) finds mixed evidence for real interest rate parity. Fujii and Chinn (2001) find the evidence for weak version of real interest rate parity. Moosa and Bhatti (1996) and Wu and Chen (1998) find supportive evidence of real interest rate parity.

<sup>7</sup>Refer to Mishkin (1984a,b), and Dutton (1993) for details.

<sup>8</sup>Refer to Papell (1997), Jorion and Sweeney (1996), and Wei and Parsley (1995) for details.

until 1987. Consequently, the huge volatility of the U.S. dollar may result in failure of international parities when the U.S. dollar is used as a base currency.

The other is cross-sectional dependence. Conventional panel unit root tests such as the IPS test assume an independent relationship across cross-sectional units. However, since real interest rate differentials of all countries share the same base country, they are correlated at least whenever the value of the base currency changes. According to O'Connell (1998), the standard panel unit root tests have some flaws in terms of lack of power and size distortion in the presence of correlation among contemporaneous cross-sectional error terms. Failure to consider cross-sectional dependence might result in a rise of the significance level which in turn overturns the original conclusion. Consequently, the supportive evidence based on any unit root tests without cross-sectional dependence might not be reliable.

One way to handle the issue is to consider cross-sectional dependence implicitly and to remove the cross-sectional mean across cross-sectional units. Maddala and Wu (1999) propose a non-parametric bootstrap procedure to capture contemporaneous correlation of error terms and use seemingly unrelated regression (SUR) method instead of OLS regression in a non-parametric bootstrap procedure. As suggested by Phillips and Sul (2003), Moon and Perron (2004), Bai and Ng, (2004), and Pesaran (2007), the other technique is to consider cross-sectional dependence explicitly. The underlying ideas regarding the tests are very similar to each other in that the deviations from the parity are driven by a group of common factors and it is possible to distinguish the idiosyncratic and the common components. The major difference among the tests is how to represent the common factors. A number of empirical studies fail to find evidence of RIP through panel unit root tests with cross-sectional dependence, which implies that cross-sectional dependence accounts for RIP.<sup>9</sup>

As shown by Ding and Kim (2012), however, empirical supports for PPP are stronger as more IT countries are involved and issues such as the choice of price indices, the choice of base currencies, and cross-sectional dependence under IT may not be important for long run PPP. If this is the case and PPP does hold under IT, then it would be interesting to see whether the above issues still matter for RIP, especially under IT monetary policy.

### 3. Econometric Model and Estimation

Real interest parity involves both UIP and PPP. To see this, UIP between two countries can be written as

$$i_t - i_t^* = s_{t+1} - s_t + \epsilon_t \quad (1)$$

where  $i_t$  ( $i_t^*$ ) is the domestic (foreign) nominal interest rate,  $s_t$  is the natural logarithm of exchange rate between domestic country and foreign country (domestic price of foreign currency),  $\epsilon_t = E(s_{t+1}|I_t) - s_{t+1}$  is a composite error term arising from expectational errors, which is assumed to be white noise, and  $E(\cdot|I_t)$  is the conditional expectations operator based on the information at time  $t$ . PPP is given by:

$$s_t = p_t - p_t^* \quad (2)$$

<sup>9</sup>Refer to Singh and Banerjee (2006) and Camarero and Tamarit (2009) for details.

where  $p_t$  and  $p_t^*$  are the log of the domestic price level and the log of the foreign price level at time  $t$ , respectively.

Combining the two parities, Equations (1) and (2), yields:

$$r_t - r_t^* = \epsilon_t \quad (3)$$

where  $r_t = i_t - (p_{t+1} - p_t)$  and  $r_t^* = i_t^* - (p_{t+1}^* - p_t^*)$ . Under the condition of perfect arbitrage in the goods and capital markets, Equation (3) is relevant for tests of international parity. Given the fact that the composite error arising from expectational errors in Equation (1), conditional on the current information set, is stationary, Equation (3) indicates that *ex post* RIP, defined in terms of the price levels between domestic and foreign countries, holds.<sup>10</sup>

Univariate unit root tests are imprecise. As the power of the tests is very poor, non-rejection of the null hypothesis of a unit root provides little reliable information. As a result, in this paper, we employ two different panel unit root tests to overcome this problem. The IPS (2003) and the CIPS by Pesaran (2007) tests are employed to examine the stationarity of real interest rate differentials. To test the long-run relationship (3), we consider the following regression:

$$\Delta\epsilon_{it} = \alpha_i + (\rho_i - 1)\epsilon_{it} + \sum_{j=1}^{k_i} \delta_{ij} \Delta\epsilon_{it-j} + \eta_{it} \quad (4)$$

where  $\Delta\epsilon_{it}$  is the first difference of the real interest differential between domestic country  $i$  ( $i = 1, \dots, N$ ) and foreign country at time  $t$ ,  $\alpha_i$  is an individual specific effect or heterogeneous intercept,  $k_i$  is the lag length determined by Hall's general-to-specific method recommended by Campbell and Perron (1991)<sup>11</sup> and  $\eta_{it}$  is the idiosyncratic disturbance which is assumed to be cross-sectionally independent.

The IPS's T-BAR statistic is the cross-sectional average of the studentized coefficients from the univariate ADF test using Equation (4). It is simply  $\bar{t} = (1/N) \sum_{i=1}^N t_i$ , and is used to test the null hypothesis of a unit root for  $i = 1, 2, \dots, N$ . The IPS test shows that  $\bar{t}$  is asymptotically normal with mean  $E(\bar{t}) \neq 0$  and  $Var(\bar{t}) \neq 1$ , under the assumption that  $\eta_{it}$  is cross-sectionally independent. When  $\eta_{it}$  exhibits cross-sectional dependence, however, the distribution of  $\bar{t}$  is not known. As a result, we conduct inference based on a non-parametric bootstrap.

To handle the issue of a single factor common time effect, a simple method is employed to remove the common specific-time effect of a group, cross-sectional mean. That is, we let  $\epsilon'_{it} = \epsilon_{it} - \bar{\epsilon}_t$ , where  $\bar{\epsilon}_t = \sum_{i=1}^N \epsilon_{it}/N$  and  $\epsilon'_{it}$  is the real interest rate differential between country  $i$  and a base country without a single factor common time effects. Then  $\epsilon'_{it}$  instead of  $\epsilon_{it}$  is employed to handle the issue for RIP.<sup>12</sup>

<sup>10</sup>Differential tax treatment and transactions costs may result in the existence of a neutral band for financial market speculation within which profitable trading opportunities are impossible. Thus, international financial integration will result in the stationarity of real interest rate differentials. For details, see Wu and Chen (1998), and Fountas and Wu (1999).

<sup>11</sup>Start with  $k = 8$  and decrease it until the coefficient on the last included lag is significant.

<sup>12</sup>In a panel regression, this transformation is equivalent to adding time specific dummy variables that control for a single common time effect. However, this test is one such that real interest rate differential reverts to the cross-sectional mean, which is weaker than a standard test that requires a mean conversion to a fixed mean.

For the cross-sectional dependence, the following regression is considered,

$$\Delta\epsilon_{it} = \alpha_i + \beta_i\epsilon_{i,t-1} + e_{it} \quad (5)$$

where  $\Delta\epsilon_{it}$  is the first difference of the real interest rate for country  $i$  ( $i = 1, 2, \dots, N$ ) at time  $t$ , and  $e_{it}$  is an error term that is allowed to be serially correlated and has a single common factor structure,

$$e_{it} = \gamma_i f_t + \varepsilon_{it} \quad (6)$$

where  $f_t$  is an unobserved common factor,  $\gamma_i$  is the individual factor loading, and  $\varepsilon_{it}$  is a white noise idiosyncratic error.

For the other panel unit root test with cross-sectional dependence, we consider a test based on the  $t$  ratio of the least square estimate of  $\widehat{b}_i$  in the following cross-sectionally augmented Dickey-Fuller (CADF) regression for each cross-sectional unit, as suggested by Pesaran (2007),

$$\Delta\epsilon_{it} = \alpha_i + b_i\epsilon_{it-1} + c_i\bar{\epsilon}_{t-1} + \sum_{j=0}^{p_i} d_{ij}\Delta\bar{\epsilon}_{t-j} + \sum_{j=1}^{p_i} \delta_{ij}\Delta\epsilon_{i,t-j} + \eta_{it} \quad (7)$$

where  $\alpha_i$  is an individual specific effect or heterogeneous intercept,  $\bar{\epsilon}_t = \frac{1}{N} \sum_{i=1}^N \epsilon_{it}$ ,  $\Delta\bar{\epsilon}_t = \frac{1}{N} \sum_{i=1}^N \Delta\epsilon_{it}$ ,  $p_i$  is the lag length determined by Hall's general-to-specific method and  $\eta_{it}$  is the idiosyncratic disturbance which is assumed to be cross-sectionally independent. According to Pesaran (2007), the cross-sectional averages of  $\Delta\epsilon_{it}$  and  $\epsilon_{it-1}$  are included into (7) as a proxy for the unobserved common factor  $f_t$ . Under (5), the null hypothesis,  $H_0 : \beta_i = 0$ , for all  $i$  is tested against the heterogeneous alternative  $H_1 : \beta_1 < 0, \dots, \beta_{N_0} < 0, N_0 \leq N$  in the whole panel set. In line with IPS (2003), Pesaran proposes the CIPS test,

$$CIPS = \frac{1}{N} \sum_{i=1}^N CADF_i \quad (8)$$

where  $CADF_i$  is the CADF statistic for the  $i$ -th cross-sectional unit in (7). The distribution of the CIPS statistic is shown to be non-standard even for large  $N$ .

For both the IPS (2003) and the CIPS (2007), our statistical inferences are based on the non-parametric bootstrap methods. The data generating process (DGP) underlying the bootstrap is given by

$$\Delta\epsilon_{it} = \sum_{j=1}^{q_i} \delta_{ij}\Delta\epsilon_{it-j} + \omega_{it} \quad (9)$$

for  $i = 1, \dots, N$  where the contemporaneous error covariance matrix is  $\Sigma = E(\tilde{\omega}_t \tilde{\omega}_t')$ ,  $\tilde{\omega}_t = (\omega_{1t}, \omega_{2t}, \dots, \omega_{nt})'$ , and the lag length  $q_i$  is determined by Hall's general-to-specific method. Equation (9) asserts the null hypothesis that  $\epsilon_{it}$  is a driftless unit root process. The non-parametric bootstrap distribution applied to the IPS and CIPS is built as follows. First, we generate  $T - \max\{k_i\}$  innovation by random resampling with replacement of the fitted residuals. Second the initial values are obtained by block resampling as described by Berkowitz and Kilian (2000). Third, after dropping the first 50 pseudo-observations to avoid start up effect, we run the ADF and CADF regressions on the pseudo-data and obtain the  $\bar{t}$ . We do this 5000 times and the collection of 5000  $\bar{t}$  form the non-parametric bootstrap distributions of the test statistics from which p-values can be computed for the IPS and CIPS tests, respectively.

## 4. Empirical Results

We use end of period quarterly data of three-month Treasury bill rates, CPI, and PPI from 1974 Q1 to 2011 Q3 for eleven OECD countries including Belgium, Canada, France, Germany, Italy, Japan, New Zealand, Spain, Sweden, the United Kingdom, and the United States. Three-month Treasury bill rates are used to measure nominal interest rates because of their relatively fixed maturity across countries. International arbitrageurs have preference to use them to compare expected returns of their investment at home and abroad. CPI and PPI are used as proxies for the prices of non-tradable and tradable goods. All data are obtained from DataStream and the International Monetary Fund's International Financial Statistics. Data for some countries are not in the full sample due to data availability.<sup>13</sup> The inflation rates used to generate the ex post real interest rates in our empirical study are calculated by taking the actual inflation rates of the CPI and PPI from period  $t$  to period  $t+1$ . To test whether IT affects RIP, we classify countries based on if they have adopted IT. The countries that engage in IT in this study are New Zealand (1990), Canada (1991), Great Britain (1992), Sweden (1993), and Spain (1994), and they are summarized in Table 1. The data is divided into three groups: IT, NIT, and All countries. Furthermore, since many studies have pointed out the problem caused by choosing the U.S. as the base country, Germany is also considered as the base country in this paper.<sup>14</sup>

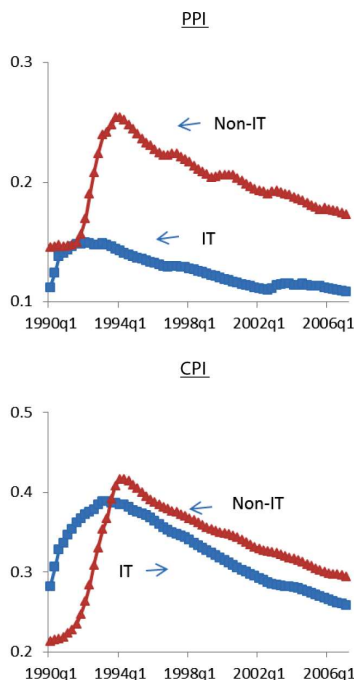


Figure 1. Long run variances of real interest rate differentials.

<sup>13</sup>Germany (1975Q3-2007Q3), New Zealand (1978Q1:2011Q3), Spain (1979Q1:2011Q3). For PPI panel, France is excluded from NIT group due to the availability of PPI. PPI of Italy (1981Q1:2011Q3) and Belgium (1980Q1:2011Q3) are not in full sample either.

<sup>14</sup>See Cumby and Mishkin (1986), Wu and Chen (1998), and Kim (2006) for more details.

Figure 1 shows long-run variance of real interest rate differentials with CPI and PPI for IT and NIT groups over 1990Q1 to 2007Q1. It provides a stylized comparison for the volatility of real interest rate differentials between the IT and NIT groups in the long run. Both the IT and NIT groups present a large drop in long-run variance of real interest rate differentials starting in the early 1990s, which could be explained by the mild and comfortable economic conditions during that period. However, the NIT group presents a larger long-run variance relative to the IT group as more countries begin to adopt IT policy. The stylized facts are consistent with the theoretical model proposed by Svensson (2000), which suggests that inflation targeting has stability effects on real interest rates in the long run.

**Table 1. Inflation Targeting Countries**

Countries	Year of Adoption	Prev. Anchor	Target Value	Target Variable
Canada	1991	None	2%+/-1%	CPI
New Zealand	1990	None	1% - 3%	CPI
Spain	1994	None	2%	CPI
Sweden	1993	Exchange Rate	2%+/-1%	CPI
UK	1992	Exchange Rate	2%+/-1%	CPI

Source: Petursson (2004), Mishkin and Schmidt (2007), Roger (2010) and central banks websites.

Table 2 presents results from tests for cross-sectional dependence. We first test if there exists cross-sectional dependence in our data using a general diagnostic test method proposed by Pesaran (2004). We estimated individual ADF( $k$ ) regressions for each country and computed pair-wise cross-sectional correlation coefficients of the residuals from these regressions in the panel.<sup>15</sup> As can be seen, the null of no cross-sectional dependence is strongly rejected for all cases.

**Table 2. Tests for Cross-Sectional Dependence**

	PPI			CPI		
	$CD_{IT}$	$CD_{ALL}$	$CD_{NIT}$	$CD_{IT}$	$CD_{ALL}$	$CD_{NIT}$
US	19.254 (0.000)	39.703 (0.000)	17.992 (0.000)	10.093 (0.000)	29.225 (0.000)	18.199 (0.000)
DM	7.235 (0.000)	14.637 (0.000)	4.590 (0.000)	8.158 (0.000)	21.689 (0.000)	11.126 (0.000)

Notes: All statistics are based on individual ADF regressions. Subscripts IT, ALL, and NIT represent Inflation Targeting (IT), All, and Non-Inflation Targeting (NIT) countries, respectively. Numbers in parentheses are p-values of CD statistics.

Table 3 reports results from IPS and CIPS tests as well as those from an IPS test after removing common time effects such as dollar specific effects across countries. The p-

<sup>15</sup>See Pesaran (2004) for details.

**Table 3. Real Interest Rate Differentials for PPI and CPI**

	IPS			IPS <sup>CT</sup>			CIPS		
	$\bar{t}_{IT}$	$\bar{t}_{ALL}$	$\bar{t}_{NIT}$	$\bar{t}_{IT}$	$\bar{t}_{ALL}$	$\bar{t}_{NIT}$	$\bar{t}_{IT}$	$\bar{t}_{ALL}$	$\bar{t}_{NIT}$
<b>PPI</b>									
US	-2.977 (0.002)	-2.666 (0.003)	-2.277 (0.077)	-3.079 (0.000)	-2.400 (0.001)	-1.887 (0.224)	-3.125 (0.000)	-2.356 (0.012)	-1.627 (0.326)
DM	-2.540 (0.004)	-2.153 (0.023)	-1.671 (0.373)	-2.918 (0.000)	-2.311 (0.001)	-1.869 (0.206)	-3.113 (0.001)	-2.531 (0.030)	-1.997 (0.301)
<b>CPI</b>									
US	-2.624 (0.004)	-2.235 (0.013)	-2.018 (0.144)	-2.893 (0.001)	-2.287 (0.003)	-1.932 (0.134)	-3.179 (0.001)	-2.243 (0.077)	-1.845 (0.389)
DM	-2.412 (0.016)	-2.039 (0.061)	-1.661 (0.383)	-2.911 (0.001)	-2.265 (0.005)	-2.014 (0.103)	-3.038 (0.002)	-2.254 (0.078)	-2.302 (0.057)

values reported in parentheses in Table 3 are taken from the non-parametric bootstraps.<sup>16</sup> The null hypothesis of non-stationary is rejected at the 5% significance level if the p-value is less than 5%. The empirical findings are summarized as follows.

Notes: Subscripts IT, ALL, and NIT represent Inflation Targeting (IT), all, and Non-Inflation Targeting (NIT) countries, respectively. IPS<sup>CT</sup> represents IPS without common time effects. Numbers in parentheses are p-values taken from the non-parametric bootstraps.

First, the IT group presents strongly supportive evidence of RIP with extremely low p-values under all cases and this finding is not sensitive to the choice of price indices, base currencies, or cross-sectional dependence. This implies that RIP holds better in countries under inflation targeting. The rejection rate of the non-stationary null is twelve out of twelve for the IT group and zero out of twelve for the NIT group. For the ALL group, the rejection rate is nine out of twelve. In addition, the bootstrapped p-values are generally smaller for the IT group than those for the NIT group, and those for the ALL group lie in between. This indicates a much higher possibility to reject the null for the IT group relative to the NIT and the ALL groups.

Second, except for the IT group, real interest rate differentials with PPI present a stronger mean reverting property than those with CPI. As shown by Froot and Rogoff (1995), PPI can represent a price index in terms of tradable goods while CPI can represent a price index in terms of relatively non-tradable goods. This finding is consistent with Dutton (1993) and Kim (2006) who find supportive evidence of RIP when interest rates are constructed in terms of tradable goods prices. For the NIT group, although none of the null hypotheses can be rejected, there are three out of six for which PPI has lower p-values than CPI. For the ALL group, this pattern is more obvious in that nine out of twelve cases can be rejected and among them six cases are with PPI. In addition, all cases with PPI show lower p-values than those with CPI.

Third, with the exception of the IT group, cross-sectional dependence matters for the validity of RIP. For the NIT group, only one case has a higher p-value from the IPS test than the CIPS test, implying that the probability to reject the null hypothesis under the CIPS test decreased. For the ALL group, this pattern is more obvious. In one out of four cases

<sup>16</sup>The lag length for each country is determined by Hall's (1994) general-to-specific method recommended by Campbell and Perron (1991). The maximum lag starts from 10.

we can reject the null hypothesis of stationarity from the IPS tests but the same is not true for the CIPS tests. In addition, the p-values of other three cases are higher. The results provide more empirical evidence to support O'Connell (1998) that international parities can be greatly affected by the assumption of cross-sectional dependence.

Finally, according to the results from the IPS tests with cross-sectional demeaned data, the IT group presents strong evidence for RIP, while the NIT group shows no evidence for RIP. The cross-sectional mean measures a single factor common time effect in the panel data and co-movements of real interest rate differentials in the pooled group are caused by a single common source of variance, which is the volatility of a base currency. After removal of the common time effect, information captured by demeaned data represents deviations from the cross-sectional mean rather than the long-run equilibrium mean. This is considered as a weak form for the validity of RIP. Compared to the results from the IPS tests in Table 2, the p-values in the IT, ALL, and NIT groups drop. When both the U.S. and Germany are used as the base countries, the supportive evidence for RIP is still obvious especially for the countries under IT macroeconomic policy.

## Conclusion

This paper investigates whether IT matters for RIP by employing panel unit root tests with and without cross-sectional dependence. The empirical results present a stronger mean reverting property of real interest rate differentials in countries under IT macroeconomic policy, implying that IT seems to play a significant role for RIP. The empirical evidence in this study is interesting and in line with Svensson (2000) and Mishkin and Schmidt-Hebbel (2007) in that IT does make a difference in the IT and NIT policy regimes by helping them achieve lower variability of inflation as well as the real interest rate at a long horizon. Furthermore, according to our empirical results, when it comes to RIP, IT is likely to outweigh other issues such as the choice of price indices, base currencies, and cross-sectional dependence. This paper provides a new perspective to explore studies related to international parity conditions.

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*Chapter 6*

# WITH STRINGS TOWARD SAFETY FUTURE ON FINANCIAL MARKETS

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## Abstract

Almost all known econometric models applied on a long term basis on financial forex market do not work sufficiently. The reason is that transaction costs and arbitrage opportunity are not included, as this does not simulate the real financial markets. Analyzes are not done on the non equidistance date but rather on the aggregate date, which is also not a real financial case. Almost all known prediction models are not stable for longer in treading on the financial forex market. In this chapter we would like to show a new way how to analyze and, moreover, forecast financial market. We utilize the projections of the real exchange rate dynamics onto the string-like topology. Our approach is inspired by the contemporary movements in the string theory. Inter-strings information transfer is analyzed as an analogy with dynamic of prices or currency at specified exchange rate options.

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**Keywords:** quantitative finance, string theory, trading strategy, financial forecasting, risk management

## 1. Introduction

We are currently in the process of transfer of modern physical ideas into the neighboring field called econophysics. The physical statistical view point has proved fruitful, namely, in the description of systems where many-body effects dominate. However, standard, accepted by physicists, bottom-up approaches are cumbersome or outright impossible to follow the

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behavior of the complex economic systems, where autonomous models encounter the intrinsic variability. We would like to transfer modern physics ideas into neighboring field called econophysics.

Digital economy is founded on data. In the chapter, we suggest and analyze statistical properties of heuristics based on the currency rate data which are arranged to mimic the topology of the basic variants of the physical strings and branes. Our primary motivation comes from the actual physical concepts [1, 2]; however, our realization differs from the original attempts in various significant details. The second aspect of our method is that it enables a transformation into a format which is useful for an analysis of a partial trend or relative fluctuations on the time scale window of interest.

As with most science problems, the representation of data is the key to efficient and effective solutions. The underlying link between our approach and the string theory may be seen in the switching from a local to a non-local form of the data description. This line passes from the single price to the multivalued collection of prices from the temporal neighborhood which we term here the string map. As we will see later, an important role in our considerations is played by the distance measure of the string maps. The idea of exploring the relationship between more intuitive geometric methods and financial data is not new. The discipline called the geometric data analysis includes many diverse examples of the conceptual schemes and theories grounded on the geometric representation and properties of data. Among them we can emphasize the tree network topology that exhibits usefulness in the studies of the world-trade network [3] and other network structures of the market constructed by means of inter-asset correlations [4, 5]. The multivariate statistical method called *cluster analysis* deals with data mapping onto representative subsets called *clusters* [6]. Here we work on the concept that is based on projection data into higher dimensional vectors in the sense of the work [7, 8]. Also, arguments based on the metrics are consistent with our efforts but not too obvious points in common with the original objectives of the nonlinear analysis.

The string theory development over the past 25 years achieved a high degree of popularity among physicists [9, 10]. The reason lies in its inherent ability to unify theories that come from diverse physical spheres. The prime instrument of the unification represents the concept of extra dimension. The side-product of theoretical efforts can be seen in the elimination of the ultraviolet divergences of Feynman diagrams. However, despite the considerable achievements, there is a lack of the experimental verification of the original string theory. In contrast, in the present work we exploit time-series which can build the family of the string motivated models of boundary-respecting maps. In a narrow sense, the purpose of the present data-driven study is to develop statistical techniques for the analysis of these objects.

Time series forecasting is a scientific field under continuous active development covering an extensive range of methods. Traditionally, linear methods and models are used. Despite their simplicity, linear methods often work well and may well provide an adequate approximation for the task at hand and are mathematically and practically convenient. However, the real life generating processes are often non-linear. This is particularly true for financial time series forecasting. Therefore the use of non-linear models is promising. Many observed financial time series exhibit features which cannot be explained by a linear model. We derive two models for predictions of EUR/USD prices on the forex market. This is the

first attempt for real application of the string theory in the field of finance, and not only in high energy physics, where it is established very well.

There are plenty of non-linear forecast models based on different approaches (e.g. GARCH [11], ARCH [12], ARMA [13], ARIMA [14] etc) used in financial time series forecasting. Currently, perhaps the most frequently used methods are based on Artificial Neural Networks (ANN, which covers a wide range of methods) and Support Vector Machines (SVM). A number of research articles compare ANN and SVM to each other and to other more traditional non-linear statistical methods. Tay and Cao ([15]) examined the feasibility of SVM in financial time series forecasting and compared it to a multilayer Back Propagation Neural Network (BPNN). They showed that SVM outperforms the BP neural network. Kamruzzaman and Sarker [16] modeled and predicted currency exchange rates using three ANN based models and a comparison was made with the ARIMA model. The results showed that all the ANN based models outperform the ARIMA model. Chen et al. [17] compared SVM and BPNN taking the auto-regressive model as a benchmark in forecasting the six major Asian stock markets. Again, both the SVM and BPNN outperformed the traditional models.

While the traditional ANN implements the empirical risk minimization principle, SVM implements the structural risk minimization ([18]). Structural risk minimization is an inductive principle for model selection used for learning from finite training data sets. It describes a general model of capacity control and provides a trade-off between hypothesis space complexity and the quality of fitting the training data (empirical error). For this reason SVM is often chosen as a benchmark to compare other non-linear models to. Also, there is a growing number of novel and hybrid approaches, combining the advantages of various methods using for example evolutionary optimization, methods of computational geometry and other techniques (e.g. [19], [20]).

In the present chapter we also exploits time series which can build the family of the string-motivated models of boundary-respecting maps. The purpose of the present data-driven study is to develop statistical techniques for the analysis of these objects and moreover for the utilization of such string models onto the forex market. Both of the string prediction models in this paper are built on the physical principle of the invariance in time series of the forex market. Founding of a stationary state in the time series of the market was studied in [21]

## 2. Data Analysis

First of all we need to mention some facts about data streams we analyzed. We analyze tick by tick data of EUR/USD, GBP/USD, USD/JPY, USD/CAD, USD/CHF major currency pairs from the OANDA market maker. We focussed on the three month period within three selected periods of 2009 which capture moments of the financial crisis. The streams are collected in such a way that each stream begins with Monday. More precisely, we selected periods denoted as Aug-Sep (from August 3rd. to September 7th.), Sep-Oct (Sep.7-Oct.5) and Oct-Nov (Sep.5-Nov.2). At first, the data sample has been decimated - only each 10th tick was considered. This delimits results to the scales larger than 10 ticks. The mean time

corresponding to the string length  $l_s$  in ticks is given by

$$T(l_s) = \langle t(\tau + l_s) - t(\tau) \rangle \simeq \frac{1}{\tau_{\text{up}} - \tau_{\text{dn}}} \sum_{\tau=\tau_{\text{dn}}}^{\tau_{\text{up}}} [t(\tau + l_s) - t(\tau)]. \quad (1)$$

### 3. One Dimensional Maps

By applying standard methodologies of detrending one may suggest to convert original series of the quotations of the mean currency exchange rate  $p(\tau)$  onto a series of returns defined by

$$\frac{p(\tau + h) - p(\tau)}{p(\tau + h)}, \quad (2)$$

where  $h$  denotes a tick lag between currency quotes  $p(\tau)$  and  $p(\tau + h)$ ,  $\tau$  is the index of the quote. The mean  $p(\tau) = (p_{\text{ask}}(\tau) + p_{\text{bid}}(\tau))/2$  is calculated from  $p_{\text{ask}}(\tau)$  and  $p_{\text{bid}}(\tau)$ .

In the spirit of the string theory it would be better to start with the 1-end-point open string map

$$P^{(1)}(\tau, h) = \frac{p(\tau + h) - p(\tau)}{p(\tau + h)}, \quad h \in \langle 0, l_s \rangle \quad (3)$$

where superscript (1) refers to the number of endpoints.

Later, we may use the notation  $P\{p\}$  which emphasizes the functional dependence upon the currency exchange rate  $\{p\}$ . It should also be noted that the use of  $P$  highlights the canonical formal correspondence between the *rate of return* and the internal *string momentum*.

Here the tick variable  $h$  may be interpreted as a variable which extends along the extra dimension limited by the string size  $l_s$ . A natural consequence of the transform, Eq.(3), is the fulfilment of the boundary condition

$$P^{(1)}(\tau, 0) = 0, \quad (4)$$

which holds for any tick coordinate  $\tau$ . Later on, we want to highlight effects of the rare events. For this purpose, we introduce a power-law  $q$ -deformed model

$$P_q^{(1)}(\tau, h) = f_q \left( \frac{p(\tau + h) - p(\tau)}{p(\tau + h)} \right), \quad h \in \langle 0, l_s \rangle \quad (5)$$

by means of the function

$$f_q(x) = \text{sign}(x) |x|^q, \quad q > 0. \quad (6)$$

The 1-end-point string has defined the origin, but it reflects the linear trend in  $p(\cdot)$  at the scale  $l_s$ . Therefore, the 1-end-point string map  $P_q^{(1)}(\cdot)$  may be understood as a  $q$ -deformed generalization of the *currency returns*. The illustration of the 1-end-point model is given in Fig.(1). The corresponding statistical characteristics displayed in Fig.(2) have been obtained on the basis of a statistical analysis discussed in section 2..



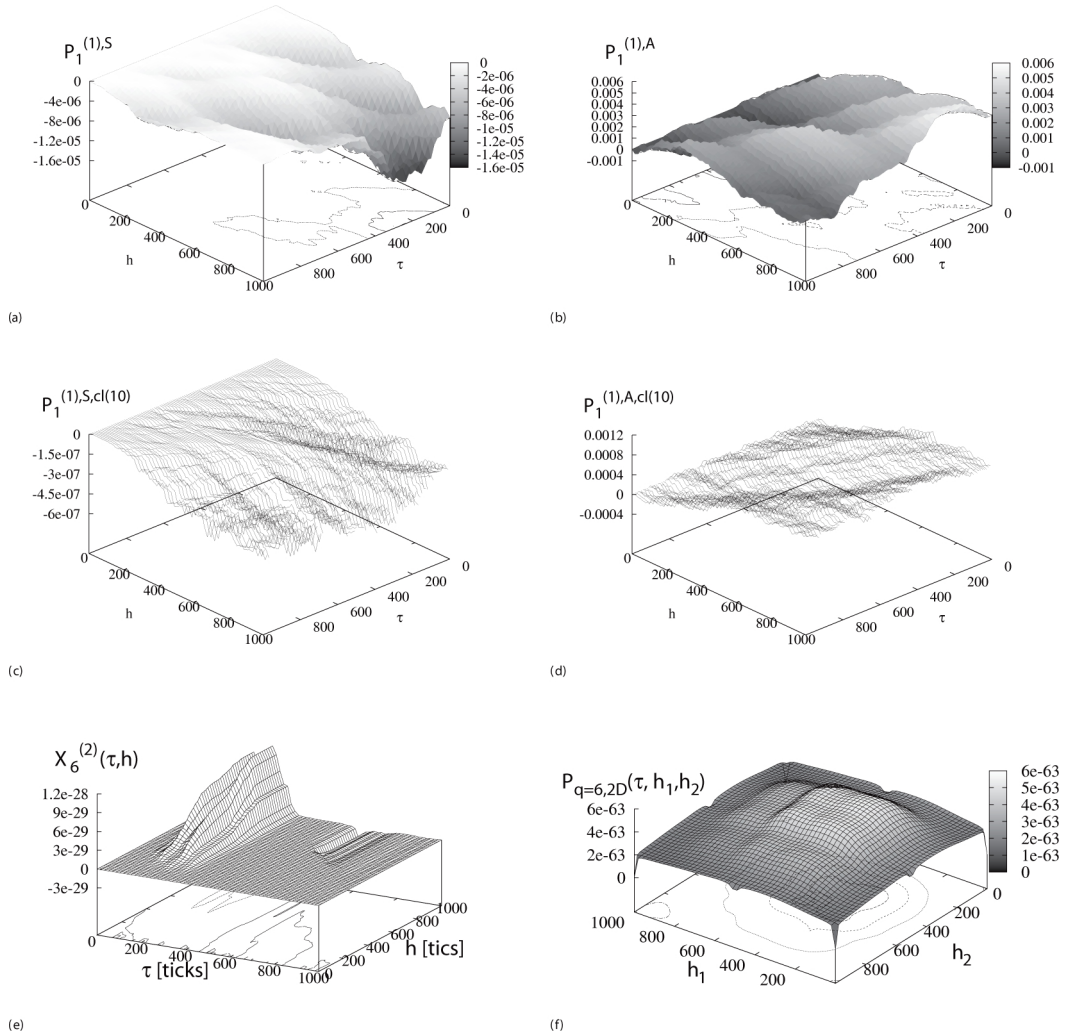


Figure 1. The illustrative examples of the currency data map for GBP/USD. The parts (a)-(d) constructed for date Fri, 31 Jul 2009 time interval 15:06:37 - 15:43:09 GMT. Time evolution of symmetric ( $P_{q=1}^{(1),S}$ ) and anti-symmetric ( $P^{(1),A}$ ) component of the 1-end-point string of size  $l_s = 1000$  calculated for  $q = 1$  (by means of Eq.(23)). In (c),(d) we see the same data mapped by means of the partially closed 1-end-point string ( $q = 1$ ) for  $N_m = 10$ , according to Eq.(31)). (e) The calculation carried out for the 2-end-point string for  $l_s = 1000$ ,  $q = 6$  at some instant. We see that conjugate variable  $X_{q=6}^{(2)}(\tau, h)$  satisfies the Neumann-type boundary conditions; (f) The instantaneous 2D-Brane state (date Fri, 31 Jul 2009 15:11:47 GMT) is computed according to Eq.(28).

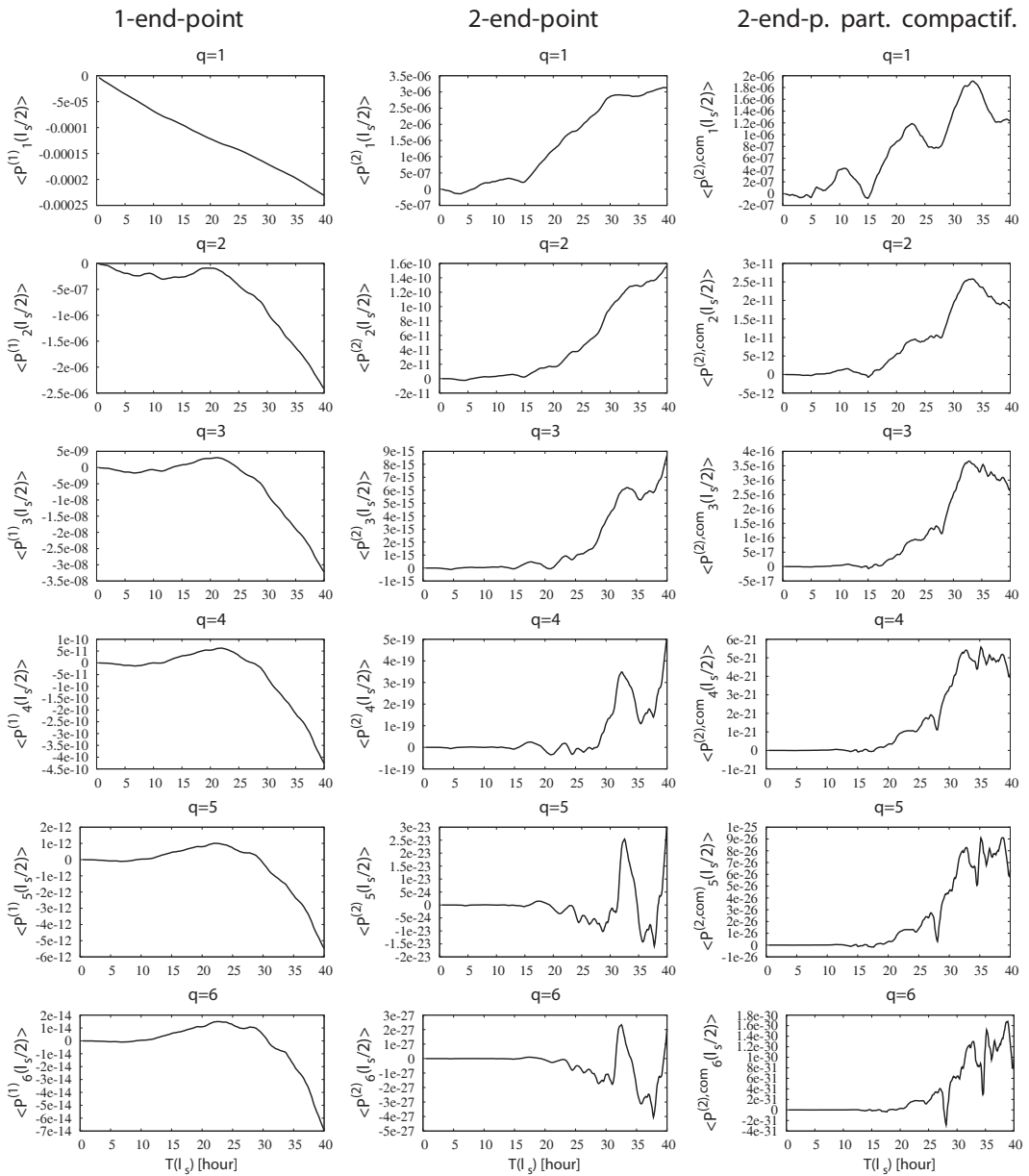


Figure 2. The variability in statistical characteristics caused by differences in topology and  $q$ . Calculated for the period Aug-Sep, GBP/USD currency. The model with  $q = 1$  has ability to reveal the currency long trend, on the other hand, the rare events are better visible for the 2-end-point string. The effect of the partial compactification with  $N_m = 4$  [see Eq.(31)] is demonstrated in the third column (again for the 2-end-point string).

Clearly, the situation with a long-term trend is partially corrected by fixing  $P_q^{(2)}(\tau, h)$  at  $h = l_s$ . The open string with two end points is introduced via the nonlinear map which combines information about trends of  $p$  at two sequential segments

$$P_q^{(2)}(\tau, h) = f_q \left( \left( \frac{p(\tau + h) - p(\tau)}{p(\tau + h)} \right) \left( \frac{p(\tau + l_s) - p(\tau + h)}{p(\tau + l_s)} \right) \right), \quad h \in \ll 0, l_s > . \quad (7)$$

The map is suggested to include boundary conditions of *Dirichlet type*

$$P_q^{(2)}(\tau, 0) = P_q(\tau, l_s) = 0, \quad \text{at all ticks } \tau . \quad (8)$$

In particular, the sign of  $P_q^{(2)}(\tau, h)$  comprises information about the behavior differences of  $p(\cdot)$  at three quotes  $(\tau, \tau + h, \tau + l_s)$ . The  $P_q^{(2)}(\tau, h) < 0$  occurs for trends of the different sign, whereas  $P_q^{(2)}(\tau, h) > 0$  indicates the match of the signs.

In addition to the variable  $P_q^{(2)}(\tau, h)$  we introduced the conjugate variable  $X_q^{(2)}(\tau)$  via the recurrent summation

$$X_q^{(2)}(\tau, h + 1) = X_q^{(2)}(\tau, h) + P_q^{(2)}(\tau, h - 1) [t(\tau + h) - t(\tau + h - 1)] \quad (9)$$

(here  $t(\cdot)$  stands for a time-stamp corresponding to the quotation index  $\tau$  in the argument). The above discrete form is suggested on the basis of the time-continuous Newton second law of motion  $\dot{X}_q^{(2)}(t, h) = P_q^{(2)}(t, h)$  (written here for a unit mass). The form is equivalent to the imposing of the quadratic kinetic energy term  $\frac{1}{2}(P_q^{(2)})^2$ . Thus, the Hamiltonian picture [10] can be reconstructed in the following way;

$$\mathcal{H} = \frac{1}{2} \sum_{h=0}^{l_s} \left[ (P_q^{(2)}(\tau, h))^2 - [\phi_{\text{ext}}(\tau, h + 1) - \phi_{\text{ext}}(\tau, h)] X_q^{(2)}(\tau, h) \right], \quad (10)$$

where  $\phi_{\text{ext}}(\tau, h)$  is the external field term which depends on the transform of the currency rate [see e.g. Eq.(7)]. We pass from the continuum to discrete theory by means of the functional form

$$\dot{P}_q^{(2)} = - \frac{\delta \mathcal{H}}{\delta X_q^{(2)}(h)} = \phi_{\text{ext}}(\tau, h + 1) - \phi_{\text{ext}}(\tau, h) = P_q^{(2)}(\tau, h + 1) - P_q^{(2)}(\tau, h), \quad (11)$$

where  $P_q^{(2)}(\tau, h)$  can be calibrated equal to  $\phi_{\text{ext}}(\tau, h)$ .

The discrete conjugate variable meets the *Neumann type* boundary conditions

$$X_q^{(2)}(\tau, 0) = X_q^{(2)}(\tau, 1), \quad X_q^{(2)}(\tau, l_s - 1) = X_q^{(2)}(\tau, l_s), \quad (12)$$

which is illustrated in Fig.(1)(d).

A more systematic way to obtain the 2-end-point string map represents the method of undetermined coefficients. The numerator of  $q = 1$  can be chosen in the functional polynomial form of degree 2 with coefficients  $\beta_0, \dots, \beta_5$  as follows:

$$P_{q=1, \text{Num}}^{(2)}(\tau, h) = \beta_0 p^2(\tau + h) + \beta_1 p^2(\tau) + \beta_2 p^2(\tau + l_s) + \beta_3 p(\tau)p(\tau + h) + \beta_4 p(\tau)p(\tau + l_s) + \beta_5 p(\tau + h)p(\tau + l_s). \quad (13)$$

Again, the Dirichlet conditions  $P_{q=1, \text{Num}}^{(2)}(\tau, 0) = P_{q=1, \text{Num}}^{(2)}(\tau, l_s) = 0$  yield  $P_{q=1, \text{Num}}^{(2)} = \beta_0(p(\tau) - p(\tau + h))(p(\tau + l_s) - p(\tau + h))$  with arbitrary  $\beta_0$ . The overlooked denominator part of fraction  $P_{q=1}^{(2)}$  then serves as a normalization factor.

Another interesting issue is the generalizing 1-end-point string to include the effect of many length scales

$$P_q^{(N_{l_s})}(\tau, h; \{l\}) = \prod_{i=1}^{N_{l_s}} f_q \left( \frac{p(\tau + l_i) - p(\tau + h)}{p(\tau + h)} \right), \quad (14)$$

which relies on the sequence  $\{l\} \equiv \{l_i, i = 1, \dots, N_{l_s}\}$ , including the end points ( $\min_{i=1, \dots, N_{l_s}} l_i$  and  $\max_{i=1, \dots, N_{l_s}} l_i$ ) as well as the  $N_{l_s} - 2$  interior node points that divide the string map into the sequence of unfixed segments of the non-uniform length (in general).

## 4. Spin as a Profit for Long

Discrete dynamical rules are implemented where the string state is sequentially transferred to the past and stored by means of instant replicas. In this model the  $m$ 'th string of replica system is described by the tuple

$$\left[ \{X_{k,s}^{(m)}(h)\}; m = 0, 1, \dots, M; h = 0, \dots, h_{\text{op}}; S^{(m)} \in \{-1, 1\} \right] \quad (15)$$

including string coordinates and additional one spin supplementary variable  $S^{(m)}$ . The meaning of the spin is the same as in particle physics where there are two possibilities for the spin orientation of particle [22]. Suppose the long position is opened at the quote  $h_{\text{op}}$  and closed at  $h_{\text{cl}} > h_{\text{op}}$ , then  $S^{(m)}$  describes profit when  $S^{(m)} = +1$  or loss when  $S^{(m)} = -1$ . In the case of buy order the sign of  $S^{(M)}$  can be deduced from the price change according  $S^{(M)} = \text{sgn}(x_b(h_{\text{cl}}) - x_a(h_{\text{op}}))$ .

The differences between string states can be measured by the string-string Hilbert  $L_p$ -distance as it follows

$$D_p^{(m,M)} = \left[ \frac{1}{d_X h_{\text{op}}} \sum_{h=0}^{h_{\text{op}}} \sum_{j=0}^{d_X} |X_j^{(M)}(h) - X_j^{(m)}(h)|^p \right]^{1/p}, \quad (16)$$

where  $m, M$  are string-replica indices. The fuzzy character of the prediction of the spin variable of  $M$ -th replica is described by means of

$$\bar{S}^{(M)} = \sum_{m=0}^{M_{\text{red}}} S^{(m)} w^{(m,M)}, \quad M_{\text{red}} = M - (h_{\text{cl}} - h_{\text{op}}), \quad (17)$$

which includes Boltzmann-like weights

$$w^{(m,M)} = \frac{\exp\left(-c_D D_p^{(m)} / \bar{D}_p^{(M)}\right)}{\sum_{m'=0}^{M_{\text{red}}} \exp\left(-c_D D_p^{(m')} / \bar{D}_p^{(M)}\right)}, \quad (18)$$

where the inter-replica distance is rescaled by the mean

$$\overline{D}_p^{(M)} = \frac{1}{M_{\text{red}} + 1} \sum_{m=0}^{M_{\text{red}}} D_p^{(m,M)}. \quad (19)$$

#### 4.1. Symbolic Dynamics and Inter-string Information Transfer

We postulated dynamics as ordered moves of the data. The moves originates from the initial string  $X_j^{(M)}(h)$  including transformation of data. The information then passes sequentially along copies  $X_j^{(m)}$  in the sence of decremented replica index  $m$  according to

$$\begin{aligned} X_j^{(M)}(h) &\leftarrow X_j(h), & S^{(M)} &\leftarrow \text{sgn}(x_b(h_{\text{cl}}) - x_a(h_{\text{op}})), \\ X_j^{(M-1)}(h) &\leftarrow X_j^{(M)}(h), & S^{(M-1)} &\leftarrow S^{(M)}, \\ &\dots & & \\ &\dots & & \\ X_j^{(1)}(h) &\leftarrow X_j^{(2)}(h), & S^{(1)} &\leftarrow S^{(2)}, \\ X_j^{(0)}(h) &\leftarrow X_j^{(1)}(h), & S^{(0)} &\leftarrow S^{(1)}. \end{aligned} \quad (20)$$

We see that information becomes lost at  $X_j^{(m=0)}$ . This metod could be useful for trading algorithym especially for selection of final trades.

### 5. Symmetry with Respect to $p(\cdot) \rightarrow 1/p(\cdot)$ Transform

The currency pairs can be separated into *direct* and *indirect* type. In a direct quote the *domestic* currency is the base currency, while the *foreign* currency is the quote currency. An *indirect* quote is just the opposite. Therefore, it would be interesting to take this symmetry into account. Hence, one can say that this two-fold division of the market network admits *duality symmetry*. Duality symmetries are some of the most interesting symmetries in physics. The term *duality* is used to refer to the relationship between two systems that have different descriptions but identical physics (identical trading operations).

Let us analyze the 1-end-point elementary string map when the currency changes from direct to indirect. The change can be formalized by means of the transformation

$$\hat{T}_{\text{id}} : P\{p(\cdot)\} \rightarrow \overline{P}\{p(\cdot)\} \equiv P\{1/p(\cdot)\}, \quad (21)$$

For the 1-end-point map model of the string, Eq.(5), we obtained

$$\hat{T}_{\text{id}} P_q^{(1)}(\tau, h) = \overline{P}_q^{(1)}(\tau, h) = f_q \left( \frac{p(\tau) - p(\tau + h)}{p(\tau)} \right). \quad (22)$$

Let us consider two-member space of maps  $V_P^{(1)} = \{P_q^{(1)}, \overline{P}_q^{(1)}\}$ . Important, we see that  $\hat{T}_{\text{id}}$  preserves the Dirichlet boundary conditions, in addition, the identity operator  $\hat{T}_{\text{id}}^2$  leaves the elements of  $V_P^{(1)}$  unchanged. The space  $V_P^{(1)}$  is closed under the left action of  $\hat{T}_{\text{id}}$ . These ideas are straightforward transferable to the 2-end-point string points.

Now we omit the notation details and proceed according to Eq.(21). The map  $P(\cdot)$  is decomposable into a sum of symmetric and antisymmetric parts

$$P^S = \frac{1}{2}(P + \bar{P}), \quad P^A = \frac{1}{2}(P - \bar{P}), \quad (23)$$

respectively. Due to of normalization by  $1/2$ , we get the projection properties

$$\hat{T}_{id}P^S = P^S, \quad \hat{T}_{id}P^A = -P^A. \quad (24)$$

To be more concrete, we choose  $q = 1$  and obtain

$$P_{q=1}^{(1),S} = 1 - \frac{1}{2} \left[ \frac{p(\tau)}{p(\tau+h)} + \frac{p(\tau+h)}{p(\tau)} \right], \quad P_{q=1}^{(1),A} = \frac{1}{2} \left[ \frac{p(\tau)}{p(\tau+h)} - \frac{p(\tau+h)}{p(\tau)} \right]. \quad (25)$$

and

$$P_1^{(2),A} = \frac{1}{2} \left[ \frac{p(\tau)}{p(\tau+l_s)} - \frac{p(\tau+l_s)}{p(\tau)} + \frac{p(\tau+h)}{p(\tau)} - \frac{p(\tau)}{p(\tau+h)} + \frac{p(\tau+l_s)}{p(\tau+h)} - \frac{p(\tau+h)}{p(\tau+l_s)} \right], \quad (26)$$

$$P_1^{(2),S} = 1 + \frac{1}{2} \left[ \frac{p(\tau+l_s)}{p(\tau)} + \frac{p(\tau)}{p(\tau+l_s)} - \frac{p(\tau)}{p(\tau+h)} - \frac{p(\tau+h)}{p(\tau)} - \frac{p(\tau+h)}{p(\tau+l_s)} - \frac{p(\tau+l_s)}{p(\tau+h)} \right].$$

We see that the  $P_{q=1}^{(1),S}$  and  $P_{q=1}^{(2),S}$  maps acquire formal signs of the systems with *T-dual symmetry* [2]. When the world described by the closed string of the radius  $R$  is indistinguishable from the world of the radius  $\propto 1/R$  for any  $R$ , the symmetry manifests itself by  $(R \pm \text{const.}/R)$  terms of the mass squared operator. The correspondence with our model becomes apparent one assumes that  $R$  corresponds to the ratio  $p(\tau)/p(\tau+h)$  in Eq.(25). However, we must also refer a reader to an apparently serious difference that in our model we do not consider for the moment the compact dimension. One can also find in the option price dynamics some real example of duality symmetry [23]. Concretely put-call duality which means "A call to buy foreign with domestic is equal to a put to sell domestic for foreign." Also most questions will not spell out what is domestic or foreign but let you decide what is the underlying asset and which is the strike asset.

### 5.1. $\hat{T}_{id}$ Transform under the Conditions of Bid-ask Spreads

Simply, the generalization can also be made with allowing for currency variables which appear as a consequence of the transaction costs [24]. The occurrence of ask-bid spread complicates the analysis in several ways. Instead of one price for each currency, the task requires the availability to two prices. The impact of ask-bid spread on the time-series properties has been studied within the elementary model [25].

Thus, for the purpose of a thorough and more realistic analysis of the market information, it seems straightforward to introduce generalized transform

$$\hat{T}_{id}^{ab}P\{p_{ask}(\cdot), p_{bid}(\cdot)\} = \bar{P}\{1/p_{bid}(\cdot), 1/p_{ask}(\cdot)\}, \quad (27)$$

which converts to Eq.(22) in the limit of vanishing spread.

## 6. Mapping to the Model of 2D Brane

Clearly, there is a possibility to go beyond a string model towards more complex maps including alternative spread-adjusted currency returns. This is extension of string theory into the higher dimensions from the string lines into the membranes called D-Branes [26]. Formally, the generalized mapping onto the 2D brane with the  $(h_1, h_2) \in \langle 0, l_s \rangle \times \langle 0, l_s \rangle$  coordinates which vary along two extra dimensions could be proposed in the following form:

$$P_{2D,q}(\tau, h_1, h_2) = f_q \left( \left( \frac{p_{ask}(\tau + h_1) - p_{ask}(\tau)}{p_{ask}(\tau + h_1)} \right) \left( \frac{p_{ask}(\tau + l_s) - p_{ask}(\tau + h_1)}{p_{ask}(\tau + l_s)} \right) \right) \quad (28)$$

$$\times \left( \frac{p_{bid}(\tau) - p_{bid}(\tau + h_2)}{p_{bid}(\tau)} \right) \left( \frac{p_{bid}(\tau + h_2) - p_{bid}(\tau + l_s)}{p_{bid}(\tau + h_2)} \right).$$

The map constituted by the combination of "bid" and "ask" quotes is constructed to satisfy the Dirichlet boundary conditions

$$P_{2D,q}(\tau, h_1, 0) = P_{2D,q}(\tau, h_1, l_s) = P_{2D,q}(\tau, 0, h_2) = P_{2D,q}(\tau, l_s, h_2). \quad (29)$$

In addition, the above construction, Eq.(28), has been chosen as an explicit example, where the action of  $\hat{T}_{id}^{ab}$  becomes equivalent to the permutation of coordinates

$$\hat{T}_{id}^{ab} P_{2D,q}(\tau, h_1, h_2) = P_{2D,q}(\tau, h_2, h_1). \quad (30)$$

Thus, the symmetry with respect to interchange of extra dimensions  $h_1, h_2$  can be achieved through  $P_{2D,q} + \hat{T}_{id}^{ab} P_{2D,q}$ . In a straightforward analogous manner one can get an antisymmetric combination  $P_{2D,q} - \hat{T}_{id}^{ab} P_{2D,q}$ . For a certain instant of time we proposed illustration which is depicted in Fig.(1)(b).

At the end of this subsection, we consider the next even simple example, where mixed boundary conditions take place. Now let the 2-end-point string be allowed to pass to the 1-end-point string by means of the homotopy  $P_{q_1, q_2}^{(1,2)}(\tau, h, \eta) = (1-\eta)P_{q_1}^{(1)}(\tau, h) + \eta P_{q_2}^{(2)}(\tau, h)$  driven by the parameter  $\eta$  which varies from 0 to 1. In fact, this model can be seen as a variant of the 2D brane with extra dimensions  $h$  and  $\eta$ .

### 6.1. Partial Compactification

In the frame of the string theory, the compactification attempts to ensure compatibility of the universe based on the four observable dimensions with twenty-six dimensions found in the theoretical model systems. From the standpoint of the problems considered here, the compactification may be viewed as an act of the information reduction of the original signal data, which makes the transformed signal periodic. Of course, it is not very favorable to close strings by the complete periodization of real input signals. Partial closure would be more interesting. This uses pre-mapping

$$\tilde{p}(\tau) = \frac{1}{N_m} \sum_{m=0}^{N_m-1} p(\tau + l_s m), \quad (31)$$

where the input of any open string (see e.g. Eq.(3), Eq.(7)) is made up partially compact.

Thus, data from the interval  $\langle \tau, \tau + l_s(N_m - 1) \rangle$  are being pressed to occupy "little space"  $h \in \langle 0, l_s \rangle$ . We see that as  $N_m$  increases, the deviations of  $\tilde{p}$  from the periodic signal become less pronounced. The idea is illustrated in Fig.(1)(c),(d). We see that the states are losing their original form (a),(b) are starting to create ripples.

For example, one might consider the construction of the  $(\tilde{D} + 1)$ -brane

$$f_q \left( \frac{p(\tau + h_0) - p(\tau)}{p(\tau + h_0)} \right) \prod_{j=1}^{\tilde{D}} f_q \left( \frac{\tilde{p}_j^{(\pm)}(\tau + h_j) - \tilde{p}_j^{(\pm)}(\tau)}{\tilde{p}_j^{(\pm)}(\tau + h_j)} \right) \quad (32)$$

maintained by combining  $(\tilde{D} + 1)$  1-end-point strings, where partial compactification in  $\tilde{D}$  extra dimensions is supposed. Of course, the construction introduces auxiliary variables  $\tilde{p}_j^{(\pm)}(\tau) = \sum_{m=0}^{N_m, j-1} p(\tau \pm m l_{s,j})$ .

## 7. Statistical Investigation of 2-end-point Strings

### 7.1. The Midpoint Information about String

In our present work, the strings and branes represent targets of physics-motivated maps which convert an originally dynamic range of currency data into the static frame. Of course, the data shaped by the string map have to be studied by the statistical methods. However, the question remains open about the selection of the most promising types of maps from the point of view of interpretation of their statistical response.

Many of the preliminary numerical experiments we performed indicating that the 2-end-point strings with a sufficiently high  $q$  (in this work we focus on  $q = 6$ , but other unexplored values may also be of special interest) yield interesting statistical information including focus on rare events. Unfortunately, there is difficult or impossible to be exhaustive in this aspect. Figure(3) shows how  $\langle P_6^{(2)}(\tau, h) \rangle$  and the corresponding dispersion  $\sigma_{P_6}$  change with a string length.

### 7.2. The Analysis of $P_q(l_s/2)$ Distributions

The complex trade fluctuation data can be characterized by their respective statistical moments. In the case of the string map the moments of the  $\xi$ th order can be naturally considered at the half length

$$\mu_{q,\xi} = \langle |P_q^{(N_i)}(\tau, l_s/2)|^{\xi/q} \rangle. \quad (33)$$

The comparison of the results obtained for the 1-end point and 2-end point strings is depicted in Fig.4. The remarkable difference in the amplitudes is caused by the manner of anchoring. The moments of longer strings are trivially larger.

### 7.3. Volatility vs. String Amplitude

The volatility as described here refers to the standard deviation of currency returns of a financial instrument within a specific time horizon described by the length  $l_s/2$ . The return volatility at the time scale  $l_s/2$  is defined by  $\sigma_r(l_s/2) = \sqrt{r_2(l_s/2) - r_1^2(l_s/2)}$  using



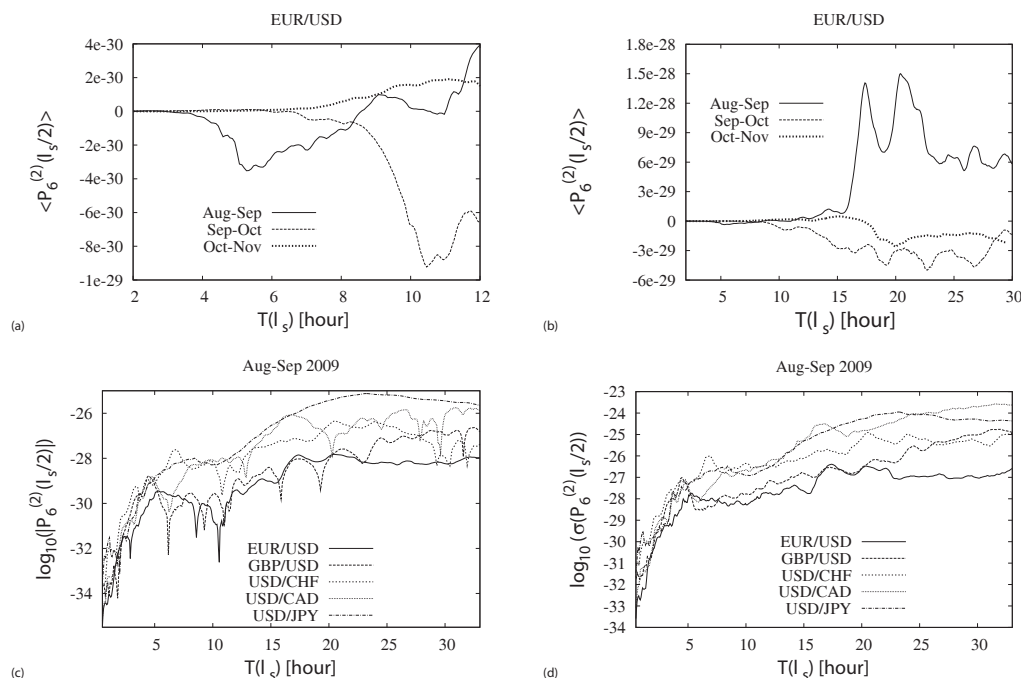


Figure 3. The illustrative calculations carried out for EUR/USD currency. Figure shows the parts (a),(b) which include a view of two different epochs (and their different details). We see the variability of the mean statistical characteristics of the 2-end-point open string. The part (b) turns in sign, but remarkable exceptional scales corresponding to the local maxima and minima remain the same. The string length is expressed in real-time units calculated by means of Eq.(1). In part (c) we present anomalies - peaks roughly common for different currencies. These picture of anomalies are supplemented by dispersions of  $P_q^{(2)}(l_s/2)$  (d).

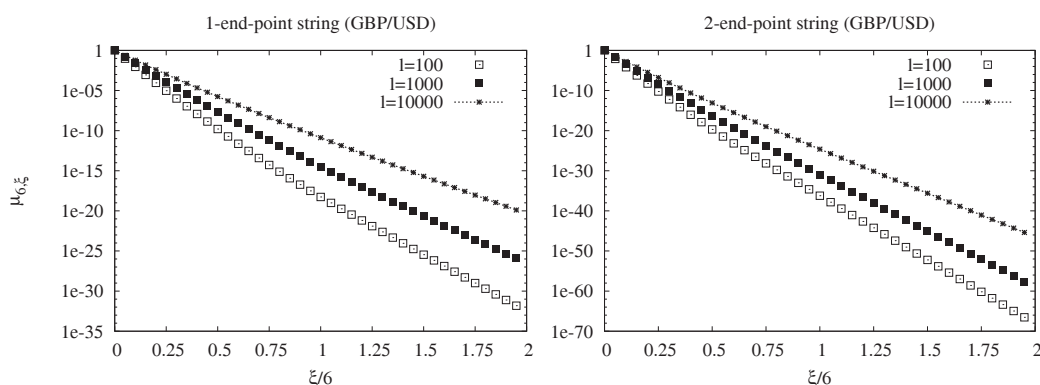


Figure 4. The mid-point fluctuations characterized by the statistical moments defined by Eq.(33). The calculations are carried out for GBP/USD currency rate, for Aug-Sep period, for different kinds of strings for several lengths. We see that fluctuations become more significant as the string size increases. In addition, one may observe the 2-end-point string to be more suppressive to the fluctuations.

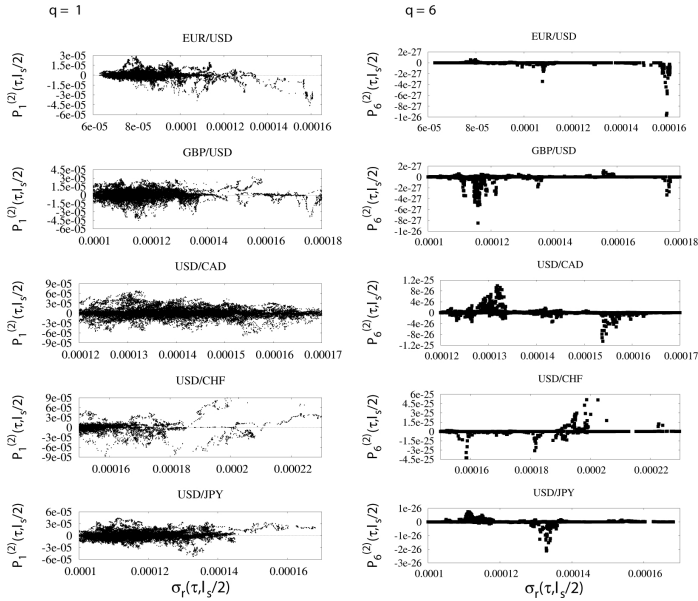


Figure 5. The scatterplot showing the relationship between the volatility  $\sigma_r(\tau, l_s/2)$  and the string amplitudes  $P_1^{(2)}(l_s/2)$  ( $q = 1$ ) and  $P_6^{(2)}(l_s/2)$  ( $q = 6$ ), respectively. The separating effect at high  $q$  is visible. The plot indicates conservation or brake of the price trend  $P_6^{(2)}(l_s/2)$  over the tick time  $\langle \tau, \tau + l_s \rangle$ . We see that the trend becomes coupled with the occurrence of specific isolated values of the volatility calculated for  $l_s = 10000$ ; period Aug-Sep.

$r_m(l_s) = \sum_{h=1}^{l_s/2} [(p(\tau+h) - p(\tau+h-1))/p(\tau+h)]^m$  for  $m = 1, 2$ . In Fig.(5), the rate of return volatility computed at the scale  $L = l_s/2$  demonstrates the linkage to the changes in the price trend represented by  $P_6^{(2)}(l_s/2)$ . Since the trend changes do not follow Gaussian distributions, we have used high  $q$  to analyze the impact of rare events. In Fig.(5), we show the identification of the semi-discrete levels of volatility by  $q = 6$ , while setting  $q = 1$  does not uncover common attributes. In the future interest one can compare our return volatility computed from the string amplitude with some volatility estimators or GARCH type of models [27, 28].

## 8. Learning of Buying and Selling Signals for Currency Pairs

In financial trading, position is a binding commitment to buy or sell a given amount of financial instruments. Open positions remain subject to fluctuations in the exchange rate. Open positions are closed by entering into a trade that takes the opposite position to the original trade. The net effect is to bring the total amount for currency pair back to zero. The bid price ( $p_{bid}(\tau)$ ) is always less than the ask price ( $p_{ask}(\tau)$ ) because brokers pay less than they receive for the same currency pair. The spread represents your cost to trade with broker. The currency pair  $p(\tau)$  indicates how much of the quote currency is required to purchase one unit of the base currency; particular currency, which comprises the physical aspects of a nations money supply. For example, EUR/USD = 1.5467 indicates that one

euro can buy 1.5467 US dollars.

### 8.1. Formalism of Trading Signals

Signals are produced by two indicators  $I^{(S)}, I^{(B)} \in \{0, 1\}$ . Signal for opening of sell position is encoded by  $I^{(S)} = 1$ , whereas  $I^{(B)} = 1$  signalizes open for buy

$$I^{(S)}(\tau, \Delta\tau) = 1_{p_{bid}(\tau) > p_{ask}(\tau + \Delta\tau)}, \quad (34)$$

$$I^{(B)}(\tau, \Delta\tau) = 1_{p_{ask}(\tau) < p_{bid}(\tau + \Delta\tau)}. \quad (35)$$

Here  $\tau$  is the timetick argument;  $p_{ask}(\tau + \Delta\tau)$  and  $p_{bid}(\tau + \Delta\tau)$  are future values of currency after the time  $\Delta\tau$ . The symbol  $1_{condit}$  is indicator function taking the value 1 when the condition *condit* is satisfied (*condit* = true), 0 when the condition is unsatisfied. In later analysis, we will distinguish symbols  $I^{(Y)}(\tau, \tau_M)$  from its prediction  $\hat{I}^{(Y)}(\tau, \tau_M)$ . The indicator is defined as

$$I^{(Y)}(\tau, \tau_M) = \max\{I^{(Y)}(\tau, 1), I^{(Y)}(\tau, 2), \dots, I^{(Y)}(\tau, \tau_M)\}, \quad (36)$$

where  $Y \in \{S, B\}$ ;  $\tau_M$  is the maximum waiting time to open order. The operator  $\max$  is used for the logical disjunction. It results in 1 whenever one or more of its particular indicators  $I^{(Y)}(\tau, \Delta t)$  is equal to 1. This is the process of transformation of data with the goal of highlighting useful information. We call strings as forms of data transformation to analyze sequences of events.

The organization of currency information  $p(\tau)$  represents exchange rate

$$p(\tau) = \frac{1}{2} (p_{ask}(\tau) + p_{bid}(\tau)), \quad (37)$$

or alternatively  $p(\tau) = \sqrt{p_{ask}(\tau)p_{bid}(\tau)}$ .

#### 8.1.1. String Maps

String map variants of the history influence models is defined as 1-end point string (with end-point  $h = 0$ )

$$P^{(1)}(\tau, h, Q) = 1 - \left( \frac{p(\tau - h)}{p(\tau)} \right)^Q, \quad (38)$$

$h = 0, 1, 2, \dots, l_s$ , where  $l_s$  is the string length.

Process of transferring continuous parameter of deformation  $Q$  into discrete counterparts is described

$$Q(j) = Q_{min} + (Q_{max} - Q_{min}) \frac{j}{NQ}, \quad j = 0, 1, \dots, NQ, \quad (39)$$

where 2-end-point string with end-points ( $h = 0, h = l_s$ ) is

$$P^{(2)}(\tau, h, Q) = \left[ 1 - \left( \frac{p(\tau - h)}{p(\tau)} \right)^Q \right] \left[ 1 - \left( \frac{p(\tau - l_s)}{p(\tau - h)} \right)^Q \right]. \quad (40)$$

## 8.2. Decision Making, Binary Classification on the Basis of Nonlinear Interconnected Strings

The architecture single layer forward can be defined as perception (binary classifier) with  $2 \times (N_Q + 1) \times (l_s + 1)$  input links and single output  $I^{(Y)}$ . The activation function is

$$\sigma(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}. \quad (41)$$

Suggested here prediction model combines the effects of different topology (1-end-point) vs. (2-end-point) strings, within them effects of the rare events controlled by the exponent  $Q$ , as well as the effect of tick delay ( $h$ )

$$\hat{I}^{(Y)}(\tau, \tau_M) = \sigma \left( \sum_{k=1,2} \sum_{h=0}^{l_s} \sum_{j=0}^{N_Q} W^{(Y)}(k, h, j) P^{(k)}(\tau, h, Q(j)) \right), \quad (42)$$

where  $W^{(Y)}(k, h, j)$  interconnections/weights of the group are subjected to the supervised learning technique which minimizes prediction error. One can choose e.g. the parameters  $\tau_M = 10$ ;  $N_Q = 12$ ;  $Q_{min} = -6$ ;  $Q_{max} = 6$ ;  $l_s = 50$ ;  $N_m = 10$ ; and by applying standard learning rules estimate the weights, analyze the relative importance of input units. Moreover examine the stability of the achieved results in the frame of the committee machine class, where several perceptrons are combined into a single response.

## 9. Correlation Function as Invariant

The meaning of invariant is that something does not change under transformation, such as some equations from one reference frame to another. We want to extend this idea also on the finance market, find some invariants in the finance data and utilize this as the prediction for the following prices. Unfortunately this model is able to define only one step prediction, see the definition below.

We suppose the invariant is in a form of correlation function

$$C_{(t, l_0)} = \sum_{h=l_0}^{h=l} w_h \left( 1 - \frac{p_{t-h}}{p_{t-1-h}} \right) \left( 1 - \frac{p_{t-1-h}}{p_{t-2-h}} \right), \quad (43)$$

with

$$w_h = \frac{e^{-h/\lambda}}{\sum_{h'=0}^l e^{-h'/\lambda}}, \quad (44)$$

including dependence on the time scale parameters  $l, l_0$  and  $\lambda$ . The relative weights satisfy automatically  $\sum_{h=0}^l w_h = 1$ .

A correlation function is a statistical correlation between random variables at two different points in our case the strings in time series. For simplicity as an example we used only one point strings equation (5) with parameter  $Q = 1$ . Ordinary the correlation function

is defined as  $C(\tau, l_0) = \langle P^1(\tau, l_0)P^1(\tau + 1, l_0) \rangle$ . We suppose the invariant in the form of the correlation function

$$C(\tau, l_0) = \sum_{h=l_0}^{h=l} W(h) \left( 1 - \frac{p(\tau - h)}{p(\tau - 1 - h)} \right) \left( 1 - \frac{p(\tau - 1 - h)}{p(\tau - 2 - h)} \right), \quad (45)$$

with weight  $W(h)$  defined above. We assume the condition of the invariance between close strings in  $\tau$  and at the next step  $\tau + 1$  in time series (It is the exact meaning of the one step prediction) in the form

$$C(\tau, l_0) = C(\tau + 1, l_0). \quad (46)$$

Now we want to find the exact expression for the one step prediction  $p(\tau + 1)$ . Therefore we evaluate one step correlation invariant equation (46) with initial condition  $l_0 = 0$

$$\begin{aligned} W(0) & \left( 1 - \frac{p(\tau)}{p(\tau - 1)} \right) \left( 1 - \frac{p(\tau - 1)}{p(\tau - 2)} \right) = \\ W(0) & \left( 1 - \frac{p(\tau + 1)}{p(\tau)} \right) \left( 1 - \frac{p(\tau)}{p(\tau - 1)} \right) + \\ W(1) & \left( 1 - \frac{p(\tau)}{p(\tau - 1)} \right) \left( 1 - \frac{p(\tau - 1)}{p(\tau - 2)} \right), \end{aligned} \quad (47)$$

which can be rewritten in the more compact form

$$C(\tau, 0) = W(0) \left( 1 - \frac{p(\tau + 1)}{p(\tau)} \right) \left( 1 - \frac{p(\tau)}{p(\tau - 1)} \right) + C(\tau + 1, 1) \quad (48)$$

and

$$\left( 1 - \frac{p(\tau + 1)}{p(\tau)} \right) = \frac{C(\tau, 0) - C(\tau + 1, 1)}{W(0) \left( 1 - \frac{p(\tau)}{p(\tau - 1)} \right)}. \quad (49)$$

We finally obtain the prediction

$$p(\tau + 1) = p(\tau) \left( 1 + \frac{C(\tau + 1, 1) - C(\tau, 0)}{W(0) \left( 1 - \frac{p(\tau)}{p(\tau - 1)} \right)} \right), \quad (50)$$

valid for  $p(\tau) \neq p(\tau - 1)$ . These are general definitions for the one step prediction correlation invariants. In the next section the similar equations can be found also for 2-end-point and 1-end-point mixed string modes with  $Q > 0$ .

## 9.1. Prediction Model Based on the String Invariants (PMBSI)

Now we want to take the above-mentioned ideas onto the string maps of finance data. We would like to utilize the power of the nonlinear string maps of finance data and establish some prediction models to predict the behavior of the market similarly as in the works [29, 30, 31]. We suggest the method where one string is continuously deformed into the other.

We analyze 1-end-point and 2-end-point mixed string models. The family of invariants is written using the parametrization

$$\begin{aligned}
 C(\tau, \Lambda) &= (1 - \eta_1)(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) & (51) \\
 &\times \left( 1 - \left[ \frac{p(\tau)}{p(\tau + h)} \right]^Q \right) \left( 1 - \left[ \frac{p(\tau + h)}{p(\tau + l_s)} \right]^Q \right) \\
 &+ \eta_1(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) \left( 1 - \left[ \frac{p(\tau)}{p(\tau + h)} \right]^Q \right) \\
 &+ \eta_2 \sum_{h=0}^{\Lambda} W(h) \left( 1 - \left[ \frac{p(\tau + h)}{p(\tau + l_s)} \right]^Q \right), & (52)
 \end{aligned}$$

where  $\eta_1 \in (-1, 1)$ ,  $\eta_2 \in (-1, 1)$  are variables (variables which we may call homotopy parameters),  $Q$  is a real valued parameter, and the weight  $W(h)$  is chosen in the bimodal single parameter form

$$W(h) = \begin{cases} 1 - W_0, & h \leq l_s/2, \\ W_0, & h > l_s/2. \end{cases} \quad (53)$$

We plan to express  $p(\tau + l_s)$  in terms of the auxiliary variables

$$A_1(\Lambda) = (1 - \eta_1)(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) \left( 1 - \left[ \frac{p(\tau)}{p(\tau + h)} \right]^Q \right), \quad (54)$$

$$A_2(\Lambda) = -(1 - \eta_1)(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) \left( 1 - \left[ \frac{p(\tau)}{p(\tau + h)} \right]^Q \right) p^Q(\tau + h), \quad (55)$$

$$A_3(\Lambda) = \eta_1(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) \left( 1 - \left[ \frac{p(\tau)}{p(\tau + h)} \right]^Q \right), \quad (56)$$

$$A_4(\Lambda) = \eta_2 \sum_{h=0}^{\Lambda} W(h), \quad (57)$$

$$A_5(\Lambda) = -\eta_2 \sum_{h=0}^{\Lambda} W(h) p^Q(\tau + h). \quad (58)$$

Thus the expected prediction form reads

$$\hat{p}(\tau_0 + l_{\text{pr}}) = \left[ \frac{A_2(\Lambda) + A_5(\Lambda)}{C(\tau_0 - l_s, \Lambda) - A_1(\Lambda) - A_3(\Lambda) - A_4(\Lambda)} \right]^{1/Q}, \quad (59)$$

where we use the notation  $\tau = \tau_0 + l_{\text{pr}} - l_s$ . The derivation is based on the invariance

$$C(\tau, l_s - l_{\text{pr}}) = C(\tau - l_{\text{pr}}, l_s - l_{\text{pr}}), \quad \Lambda = l_s - l_{\text{pr}}, \quad (60)$$

where  $l_{\text{pr}}$  denotes the prediction scale.

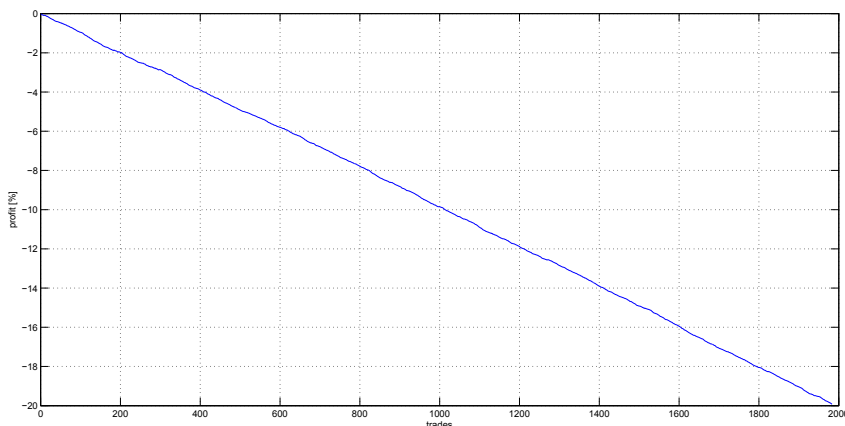


Figure 6. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on trades for one year period.

The model was tested for various sets of parameters  $l_s$ ,  $l_{pr}$ ,  $\eta_1$ ,  $\eta_2$ ,  $Q$  and the new parameter  $\epsilon$  which is defined as

$$\epsilon = |C(\tau, l_s - l_{pr}) - C(\tau - l_{pr}, l_s - l_{pr})| \quad (61)$$

and describes the level of invariance in real data. The best prediction (the best means that the model has the best ability to estimate the right price) is obtained by using the following values of parameters

$$l_s = 900, \quad l_{pr} = 1, \quad \eta_1 = 0, \quad \eta_2 = 0, \quad Q = 6, \quad \epsilon = 10^{-10}. \quad (62)$$

The graphical descriptions of prediction behavior of the model with and without transaction costs on the EUR/USD currency rate of the forex market are described in Figs 6-9. During a one year period the model lost around 20% of the initial money. It executed 1983 trades (Fig 6) where only 10 were suggested by the model (and earned money) and the rest of them were random (which can be clearly seen in Figs 8, and 9). The problem of this model is its prediction length (the parameter  $l_{pr}$ ), in this case it is one tick ahead. The price was predicted correctly in 48.57% of all cases (16201 in one year) and from these 48.57% or numerally 7869 cases only 0.13% or numerally 10 were suitable for trading. This small percentage is caused by the fact that the price does not change too often one tick ahead. One could try to raise the prediction length to find more suitable cases for trading. This is only partly successful because the rising parameter  $l_{pr}$  induces a loss of the prediction strength of the model. For example when  $l_{pr} = 2$  (two ticks ahead) the prediction strength decreases from around 50% to 15%.

The problem is that the invariant equation (46) is fulfilled only on the very short period of the time series due to the very chaotic nature of financial data behavior. Therefore the PMBSI is effective only on the one step prediction where there is very low probability

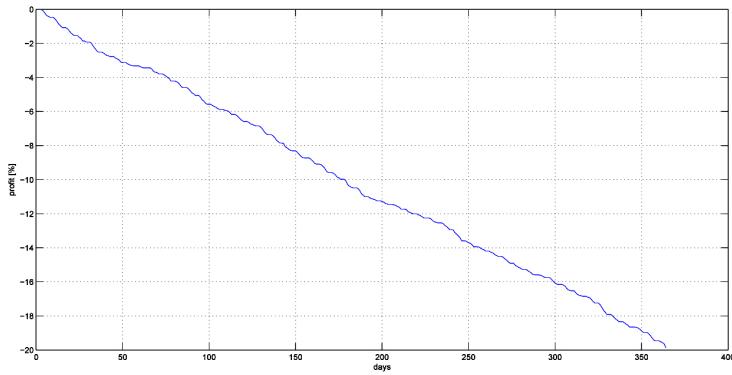


Figure 7. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on days for one year period.

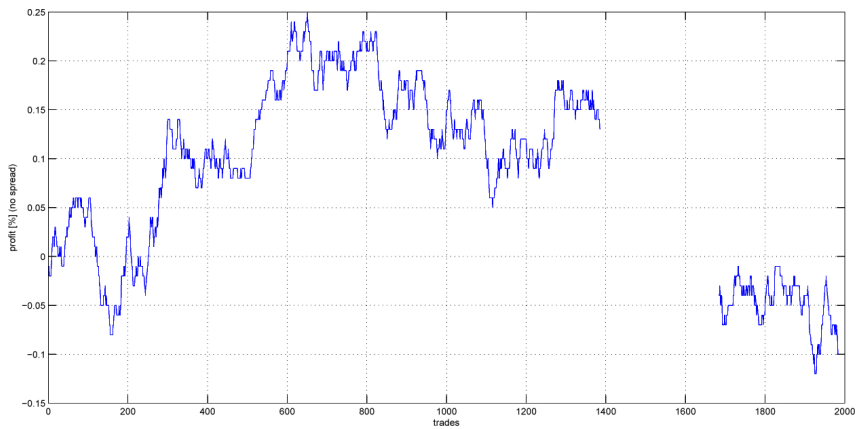


Figure 8. The profit of the model on the EUR/USD currency rate without transaction costs included dependence on trades for one year period.

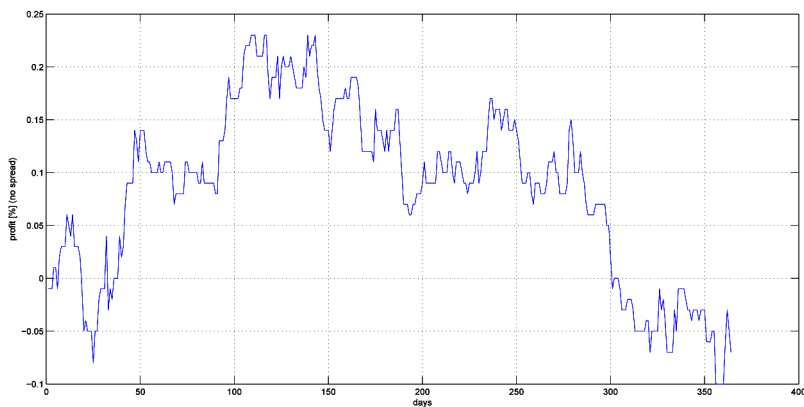


Figure 9. The profit of the model on the EUR/USD currency rate without transaction costs included dependence on days for one year period.



that time series change significantly. The situation, however, is different for more steps prediction where there is, on the contrary, a very high probability of big changes in time series to occur, and the following predictions have rather small efficiency in such cases. The only way how to establish better prediction also for more steps prediction is to choose the right weights equation (44). The right and optimized weights should considerably extend the interval where equation (46) is fulfilled. Therefore it is also our task in the future work.

## 9.2. Experimental Setup

The experiments were performed on two time series. The first series represented artificial data namely a single period of a sinusoid sampled by 51 regularly spaced samples. The second time series represented proprietary financial data sampled daily over a period of 1295 days. The performance of PMBSI was compared to SVM and to naive forecast. There were two error measures used, mean absolute error (MAE) and symmetric mean absolute percentage error (SMAPE) defined as follows:

$$MAE = \frac{1}{n} \sum_{t=1}^n |A_t - F_t|, \quad (63)$$

$$SMAPE = \frac{100}{n} \sum_{t=1}^n \frac{|A_t - F_t|}{0.5(|A_t| + |F_t|)}, \quad (64)$$

where  $n$  is the number of samples,  $A_t$  is the actual value and  $F_t$  is the forecast value. Each time series was divided into three subsets: training, evaluation and validation data. The time ordering of the data was maintained; the least recent data were used for training, while the more recent data were used to evaluate the performance of the particular model with the given parameters' setting. The best performing model on the evaluation set (in terms of MAE) was chosen and made to forecast for the validation data (the most recent) that were never used in the model optimization process. Experimental results on the evaluation and validation data are presented below. The parameters of the models were optimized by trying all combinations of parameters sampled from given ranges with a sufficient sampling rate. Naturally, this process is slow but it enabled us to get an image of the shape of the error surface corresponding to the given settings of parameters and ensured that local minima are explored. The above approach was used for both, PMBSI and SVM. The SVM models were constructed so that the present value and a certain number of the consecutive past values comprised the input to the model. The input vector corresponds to what will be referred to here as the *time window* with the length  $l_{tw}$  (representing the equivalent of the length of the string map  $l_s$  by PMBSI).

## 10. Comparison

There was a preliminary experimental analysis performed of the PMBSI method performed. The goal was to evaluate the prediction accuracy, generalization performance, convenience of the method in terms of the operator effort needed to prepare a working model, computational time and other aspects of the PMBSI method that may have become obvious during

the practical deployment. SVM was chosen as a benchmark. The experimental data comprised two sets: artificial data (a single period of a sinusoid) and real world data (financial, price development). We will provide a brief conclusion of the analysis here. Each time series was divided into three subsets for training, testing and validation. The results were calculated on the validation sets that have been entirely absent in the process of optimization of parameters.

The PMBSI predictor does not undergo a training process that is typical for ANN and SVM where a number of free parameters must be set (synaptic weights by ANN,  $\alpha$  coefficients by SVM). PMBSI features a similar set of weights ( $W$ ) but often very small and calculated analytically. The parameters to be optimized are only four:  $ls$ ,  $Q$ ,  $\eta_1$ ,  $\eta_2$ . This, clearly, is an advantage. On the other hand the optimal setting of the parameters is not easy to find as there are many local minima on the error surface. In this analysis the optimal setting was found by testing of all combinations of parameters from given ranges. Fig. 10 shows the Mean Absolute Error (MAE) of the 5-steps ahead forecast of the financial time series corresponding to various settings of  $ls$  and  $Q$  ( $\eta_1, \eta_2 = 0$ ). But the figure makes it also obvious that PMBSI's performance is approximately the same for a wide range of settings on this data.

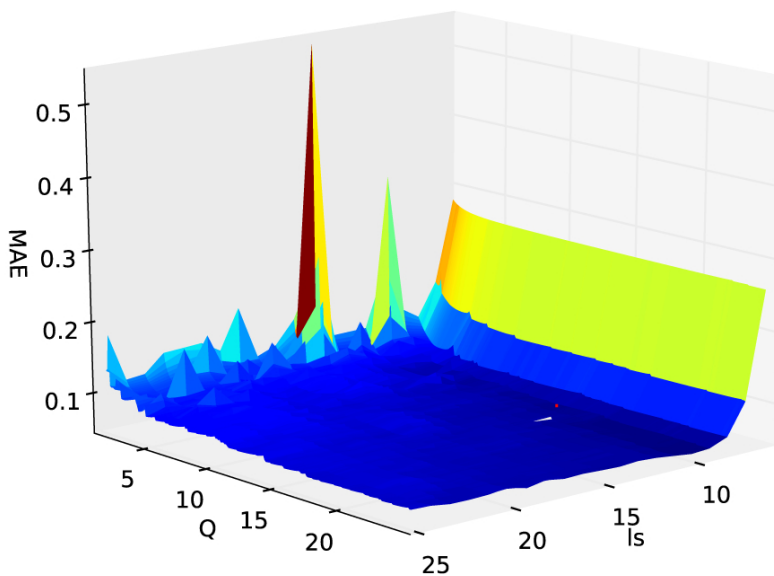


Figure 10. MAE corresponding to various settings of  $ls$  and  $Q$  on the financial data. The red dot is the global minimum of MAE.

For PMBSI to work the elements of time series must be non-zero otherwise the method will return *not a number* forecasts only. The input time series must then be modified by adding a constant and the forecast by subtracting the same constant. Even so the algorithm returned a *not a number* forecast in approx. 20% of the cases on the financial data. In such cases the last valid forecast was used. Due to reasons that are presently being evaluated the

**Table 1. Experimental results on artificial time series**

Method	$l_{pr}$	MAE eval	MAE valid	SMAPE valid
PMBSI	1	0.000973	<b>0.002968</b>	<b>8.838798</b>
	2	0.006947	0.034032	14.745538
	3	0.015995	0.161837	54.303315
Iterated PMBSI	1	-	-	-
	2	0.003436	0.011583	10.879313
	3	0.008015	0.028096	14.047025
SVM	1	0.011831	0.007723	10.060302
	2	0.012350	<b>0.007703</b>	<b>10.711573</b>
	3	0.012412	<b>0.007322</b>	<b>11.551324</b>
Naive forecast	1	-	0.077947	25.345352
	2	-	0.147725	34.918149
	3	-	0.207250	41.972591

**Table 2. Optimal PMBSI parameters**

$l_{pr}$	$l_s$	$Q$	$\eta_1$	$\eta_2$
1	2	0.30	0.80	-0.20
2	5	0.10	0.80	-0.60
3	8	0.10	0.80	-0.60

accuracy of PMBSI is matching and even outperforming SVM for a single step predictions but rapidly deteriorates for predictions of more steps ahead. Iterated prediction of several steps ahead using the single step PMBSI predictor improves the accuracy significantly. The sinusoid used for experiments was sampled by 51 points, the positive part of the wave was used for optimization of the parameters and the rest for validation (approx. 50-50 division). Fig. 11 shows the comparison of iterated versus the direct prediction using PMBSI. Table 1 shows the experimental results. The results of the best performing models are highlighted. The optimal  $l_{tw}$  for SVM was 3 for all predictions. Table 2 shows the optimal settings found for PMBSI. For  $l_{pr} = 1$  when PMBSI outperformed linear SVM the optimal length of the string map was shorter than the optimal time window for SVM; in the remaining cases it was significantly longer.

## 11. Prediction Model Based on the Deviations from the Closed String/Pattern Form (PMBCS)

For the next trading strategy we want to define some real values of the string sequences. Therefore we define the momentum which acquired values from the interval (0, 1). The

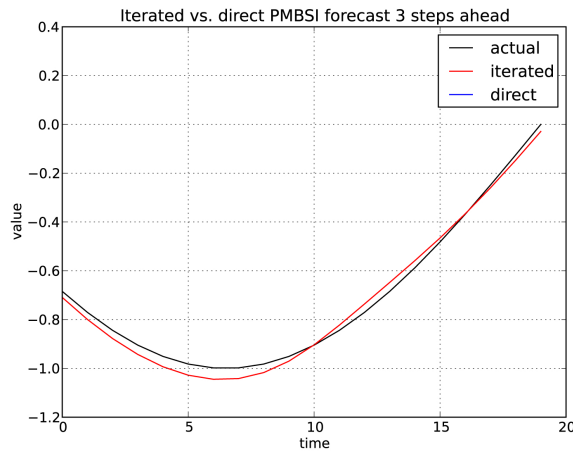


Figure 11. Iterated and direct prediction using PMBSI on artificial data.

momentum  $M$  is not strictly invariant as in the previous model of the time series in its basic definition. It is a trading strategy to find such a place in the forex time series market where  $M$  is exactly invariant or almost invariant and we can predict increasing or decreasing of prices with higher efficiency. For example our predictor somewhere in the time series has 55% of efficiency to predict the movement of price but in the invariant place of our trading strategy where Eqs. (61), and (65) are almost invariant the efficiency of our predictor increased to 80%. Therefore the idea to find the invariant in time series plays a crucial role in our trading strategy but one still needs to find an appropriate expression for such a prediction.

To study the deviations from the benchmark string sequence we define momentum as

$$M_{(l_s, m; Q, \varphi)} = \left( \frac{1}{l_s + 1} \sum_{h=0}^{l_s} \left| \frac{p(\tau + h) - p_{\min}(\tau)}{p_{\max}(\tau) - p_{\min}(\tau)} - \frac{1}{2} \left( 1 + \cos \left[ \frac{2\pi mh}{l_s + 1} + \varphi \right] \right) \right|^Q \right)^{1/Q} \quad (65)$$

where

$$p_{\text{stand}}(\tau; h; l_s) = \frac{p(\tau + h) - p_{\min}(\tau; l_s)}{p_{\max}(\tau; l_s) - p_{\min}(\tau; l_s)}, \quad p_{\text{stand}} \in (0, 1),$$

and

$$p_{\max}(\tau; h; l_s) = \max_{h \in \{0, 1, 2, \dots, l_s\}} p(\tau + h), \quad p_{\min}(\tau; h; l_s) = \min_{h \in \{0, 1, 2, \dots, l_s\}} p(\tau + h),$$

and  $\varphi$  is a phase of periodic function. The momentum defined above takes the values from the interval  $M_{(l_s, m; Q, \varphi)} \in (0, 1)$ . The periodic function  $\cos(\varphi)$  in the definition of equation (65) could be substituted by other types of mathematical functions. The results with different kinds of functions could be different.

### 11.1. Elementary Trading Strategy Based on the Probability Density Function of $M$

The purpose is to take advantage of it whenever the market conditions are favorable. As in the previous model we are detrending forex data into the one dimensional topological object "strings"

with different parameters. The trading strategy is based on the description of rate curve intervals by one value called the moment of the string. These moments are statistically processed and some interesting values of moments are found. The values directly affect the opening and closing of trade positions. The algorithm works in two complementary phases. The first phase consists of looking for "good" values of moments followed by second phase which uses results from the first phase and opening/closing of trade positions occur. Simultaneously the first phase is looking for new "good" values of moments.

Risk is moderated by a number of allowed trades that the algorithm can open during a certain period. Also it is moderated by two parameters which affect the selection of suitable moments for trading. The maximum number of trades is 10 per hour. The algorithm is tested on various periods of historical data. The number and period of simultaneously opened trades are monitored all the time.

The first set of parameters describes the moment (simple scalar function of several variables from the interval  $(0,1)$ ). The first set consists of these parameters: length of moment string (number of ticks or time period), quotient or exponent of moment, frequency of moment function, and phase shift of moment function. The second set of parameters controls trading strategy and consists of these variables: maximum number of simultaneously opened trades, skewness of moments distribution and Sharpe ratio of closed trades. As soon as the algorithm calculates the value of the moment and finds out that the value is "good", then it immediately carries out an appropriate command.

The risk of the algorithm is governed by the second set of parameters and can vary from zero (low risk but also low or zero number of trades) to the boundary values controlled by the model parameters. These boundary values are unlimited but could be easily affected by the skewness and Sharpe ratio. These parameters can limit loss to certain value with accuracy  $\pm 2$  percent but also limit overall profit significantly if low risk is desired.

An arbitrage opportunity is taking advantage of the occurrence of a difference in distribution. Opportunity is measured by *Kullback-Leibler* divergence

$$D_{KL} = \sum_{j(\text{bins})} \text{pdf}(M^+(j)) \log \left( \frac{\text{pdf}(M^+(j))}{\text{pdf}(M^-(j))} \right) \quad (66)$$

where larger  $D_{KL}$  means better opportunities ( $D_{KL} > D_{\text{threshold}}$ ) e.g. when  $D_{KL} > D_{\text{threshold}}$  it means the buying of Euro against USD could be more profitable. Statistical significance means the smaller the statistics accumulated into bins  $\text{pdf}(M^+(j))$ ,  $\text{pdf}(M^-(j))$ , the higher is the risk ( $M$  from the selected range should be widespread). The meaning of pdf in the definition of equation above is the probability density function.

More generally we can construct the series of  $(l_s + 1)$  price ticks  $[p(\tau), p(\tau + 1), \dots, p(\tau + l_s)]$  which are transformed into a single representative real value  $M(\tau + l_s)$ . Nearly stationary series of  $M(\tau + l_s)$  yield statistics which can be split into: branch where  $M$  is linked with future uptrend/downtrend and branch where  $M$  is linked with future profit/loss taking into account transaction costs. Accumulation of  $\text{pdf}(M_{\text{long}}^{+-})$  means (profit+ / loss-) or  $\text{pdf}(M_{\text{short}}^{+-})$  (profit+ / loss-).  $M^+$  in equation (66) describes when equation (65) brings profit and  $M^-$  loss.

As in the previous section the model was again tested for various sets of free parameters  $l_s$ ,  $h$ ,  $Q$ ,  $\varphi$ . This model can make "more-tick" predictions (in tests it varies from 100 to 5000 ticks). Therefore it is much more successful than the previous model. It is able to make a final profit of around 160% but this huge profit precedes a fall down of 140% of the initial state. It is important to emphasize that all profits mentioned here and below are achieved by using leverage (borrowing money) from 1 to 10. The reason for leverage is the fact that the model could simultaneously open up to 10 positions (one position means one trade i.e. one pair of buy-sell transactions). If one decides not to use any leverage the final profit decreases 10 times. On the other hand, with using the leverage 1 to 20 the final profit doubles itself. Of course, the use of higher leverages is riskier as dropdowns are also higher. There is, for example, in Fig. 12 a dropdown approx. 6% around 600 trades. With

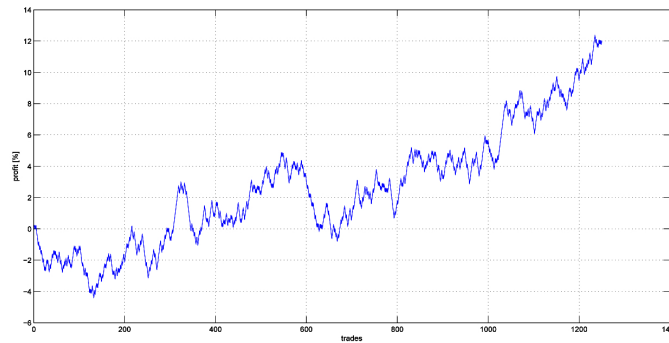


Figure 12. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on trades for one year period.

the use of leverage 1 to 20 this dropdown rises to 12%.

128000 combinations of model's parameters have been calculated. Figures 12-15 describe some interesting cases of the prediction behavior of the model with the transaction cost included on the EUR/USD currency rate of the forex market. Figures 12, and 13 describe the model (one set of parameters) under conditions that the fall down must not be higher than 5%. The best profit achieved in this case is 12%.

In order to sort out the best combinations of parameters it is helpful to use the statistical quantity called the Sharpe ratio. The Sharpe ratio is a measure of the excess return per unit of risk in a trading strategy and is defined as

$$S = \frac{E(R - R_f)}{\sigma}, \quad (67)$$

where  $R$  is the asset return,  $R_f$  is the return on a benchmark asset (risk free),  $E(R - R_f)$  is the mean value of the excess of the asset return over the benchmark return, and  $\sigma$  is the standard deviation of the excess of the asset return. You mention the Sharpe ratio Eq. (67). The values of the Sharpe ratio for the best fit are e.g. for Fig. 15 it is the value 1.896 and for Fig. 16 it is the value 1.953, where as a reference profit we choose a bank with 5% profit.

Figure 14 shows the case where the Sharpe ratio has the highest value from all sets of the calculated parameters. One year profit is around 26% and the maximum loss is slightly over 5%. Figure 15 describes the case requiring a high value of Sharpe ratio and with the aim to gain profit of over 50%.

There exist sufficiently enough cases with high Sharpe ratio which leads to enhancement of the model to create self-education model. This enhancement takes some ticks of data, finds out the best case of parameters (high Sharpe ratio and also high profit) and starts trading with these parameters for some period. Meanwhile, the trading with previously found parameters model is looking for a new best combination of parameters. Figure 16 describes this self-education model where parameters are not chosen and the model itself finds the best one from the financial data and is subsequently looking for the best values for the next trading strategy.

## 12. Conclusion

We shown that the string theory may motivate the adoption of the nonlinear techniques of the data analysis with a minimum impact of justification parameters. The numerical study recovered interesting fundamental statistical properties of the maps from the data onto string-like objects. The remarkable deviations from the features known under the notion of the efficiently organized market have been observed, namely, for high values of the deformation parameter  $q$ .

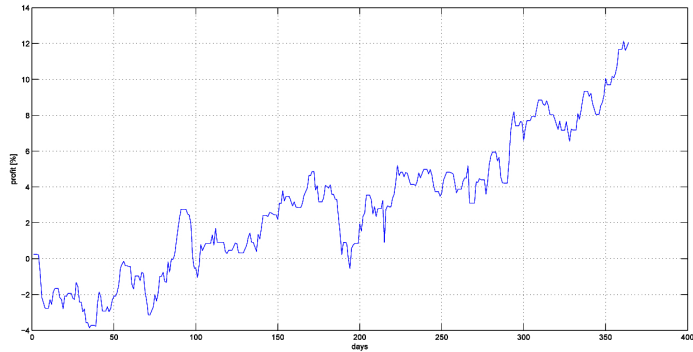


Figure 13. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on days for one year period.

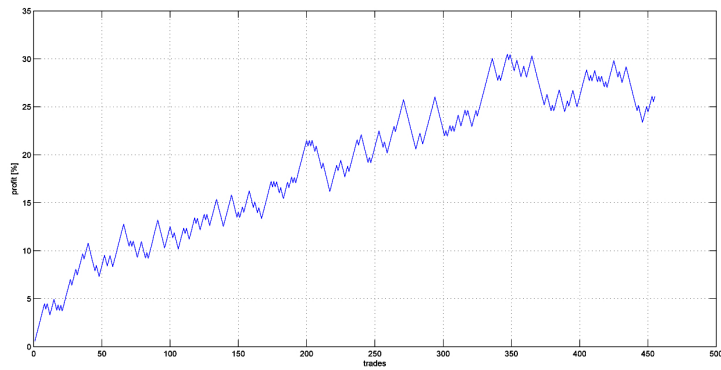


Figure 14. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on trades for one year period.

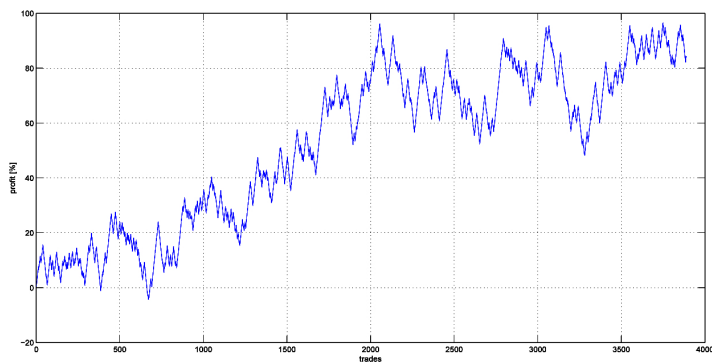


Figure 15. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on trades for one year period.

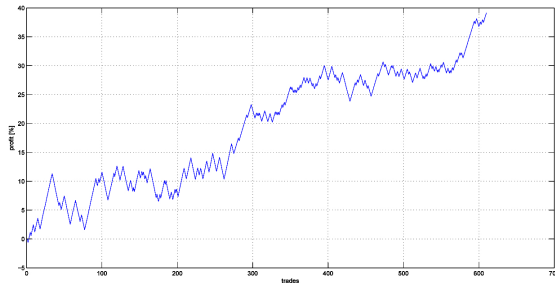


Figure 16. The profit of the self education model on the EUR/USD currency rate with transaction costs included dependence on trades for one year period.

The numerical analysis of the intra-string statistics was supplied qualitatively by the toy models of the maps of the exponential and periodic data inputs. Most of the numerical investigations have been obtained for the open topology; however, we described briefly the ways to partial compactification. The data structures can also be mapped by means of the curled dimension which arises as a sum of periodic data contributions. The idea of the compactified strings can be realized as well by the application of the inverse Fourier transform of the original signal. The interesting and also challenging task represents finding of link between string map and log-periodic behaviour of speculative bubbles of the stock market indices [32, 33]. It would be also interesting to examine R/S analysis of the Hurst exponents [34, 35] for the case of finite strings instead of the usual point prices. The full string dynamics analyzes with different currency on financial market was already published in [36].

The study of string averages exhibited occurrences of the anomalies at the time scales proportional to the string length. We showed that global and common market timescales can be extracted by looking at the changes in the currencies. The extensions of the string models of branes including ask/bid spread were discussed. The membrane 2d-brane approach could be also helpful e.g. for computations of the volatility surface in option pricing [37]. We studied the relationship between the arbitrage opportunities and string statistics. We showed that extraction of the valuable information about the arbitrage opportunities on given currency could be studied by means of the correlation sum which reflected the details of the occupancy of phase-space by differently polarized strings and branes.

We have 5 free parameters in our prediction models. We have also tried out-of-sample tests, however, only using small data samples. We have not encountered "overfitting" due to the fact that parameters are stable enough within our string theory approach to produce profit even if we slightly change them. For all computations in the second model we are taking bid-offer spreads into account. We are calculating with real values of bid-offer spreads from historical data and it is dependent on where we are simulating on Oanda or Icap etc. A number of trades per day varies from 2 to 15 depending on fit strategy.

We established two different string prediction models to predict the behaviour of forex financial market. The first model PMBSI is based on the correlation function as an invariant and the second one PMBCS is an application based on the deviations from the closed string/pattern form. The financial market invariants could be some other form of definition of scaling laws found in [38] We found the difference between these two approaches. The first model cannot predict the behavior of the forex market with good efficiency in comparison with the second one which, moreover, is able to make relevant profit per year. From the results described we can conclude that the invariant model as one step price prediction is not sufficient for big dynamic changes of the current prices on the finance market. As can be seen in Figs. 8,9 when the transaction costs are switched off the model has some tendency to make a profit or at least preserve fortune. It means that it could also be useful but for other kinds of data, where the dynamics of changes are slower, e.g. for energetic [39] or seismographic data [40] with longer periods of changes. Finally the PBMSI in the form presented in



this paper should be applicable with good efficiency only to other kinds of data with smaller chaotic behavior in comparison with financial data.

Moreover PMBSI is a method under development. Unlike SVM or ANN, at this stage PMBSI does not require a training process optimizing a large number of parameters. The experimental results indicate that PMBSI can match or outperform SVM in one step ahead forecasts. Also, it has been shown that finding optimal settings for PMBSI may be difficult but the method's performance does not vary much for a wide range of different settings. Besides the further testing of PMBSI we consider that fast methods for optimization of parameters must be developed. Because of the character of the error surface we have chosen to use evolutionary optimization as the method of choice. After a fast and successful parameters' optimization method is developed optimization of the weighting parameters (Eqs. (44), and (50)) will be included into the evolutionary process.

The profit per year from the second prediction model was obtained from approximately 15 % and more depending on the parameter set from the data we have chosen. This model is established efficiently on the finance market and could be useful to predict future prices for the trading strategy.

Of course the model still needs to be tested further. With the flow of new financial data the model can be more optimized and also, it could become resistant to a crisis. The presented models are universal and could also be used for predictions of other kind of stochastic data. The self-educated models presented in Fig. 16 are very useful because they are able to find on their own the best parameter set from data. These models could also be very helpful for portfolio optimization and financial risk management in the banking sector. Finally we very much hope that the presented approach will be very interesting and useful for a broad spectrum of people working on the financial market.

For another application of string approach, we sketched some hierarchical model of algorithmic chemistry from string atoms to string molecules as a method of adaptive boosting. Discrete dynamical rules are implemented where string state is sequentially transferred to the past and stored by means of instant replicas as was developed in Section 4. We defined a spin of strings which could detect a long-run profit where a fuzzy character of the prediction of the spin variable of N-th replica can be investigated. Finally inter-strings information transfer can be analyzed as an analogy with dynamic of prices or currency at a specified exchange rate options.

### 13. List of Terms

- String theory: is an active research framework in particle physics where particle are rather 1-dimensional oscillating open or closed lines "strings"
- Brane theory: are membranes of different dimensionality from a one dimensional membrane which is in fact a string lines, including 2, 3 or more dimensional membranes
- Extra dimension: string theory predicts extra dimensions, in classical string theory the number of dimensions is not fixed by any consistency criterion
- Conjugate variable: are pairs of variables mathematically defined in such a way that they become Fourier transform duals of one-another
- T-duality: is a symmetry of quantum field theories with differing classical descriptions, of which the relationship between small and large distances in various string theories is a special case
- Compact dimension: is curled up in itself in very small Planck length and the fact that the dimension is smaller than the smallest particle means that it cannot be observed by conventional means
- Regge slope parameter: was introduced in the quantum theory of string, and its relation to the string tension involves
- Spin in quantum mechanics: is an intrinsic form of angular momentum carried by elementary particles, composite particles (hadrons), and atomic nuclei

• Gâteaux derivative: or directional derivative is often used to formalize the functional derivative commonly used in the calculus of variations and physics

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*Chapter 7*

## **A GENERAL APPROACH TO RISK DISCLOSURE FOR RETAIL INVESTORS**

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### **ABSTRACT**

This study aims to improve the insufficient communication of risks in financial markets, particularly in the retail sector. The communication of risks is usually based on only one number (mostly related to volatility) or on an endless list of risks. We suggest a tool for communicating investment risks that combines four types of risk, is simple and easy to understand, and takes investors' time perspective into consideration. We believe that the simplicity of this communication tool renders it particularly effective in explaining and communicating risks to retail clients.

### **1. INTRODUCTION: CURRENT PROBLEMS IN RISK COMMUNICATION TO RETAIL CLIENTS**

In recent decades, we have observed two interdependent trends in financial markets: increasing attention to risk matters, on the one hand, and the advancing independence and activity of retail investors and an increasing recognition of the necessity to effectively communicate risks to them, on the other hand.

Risks related to financial instruments have recently come under particular scrutiny. Famously, Alan Greenspan stated that 'the underpricing of risk worldwide' played a major role in the origins of the last economic crisis (Skidelsky 2009, p.3). The risk management

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profession now eagerly discusses new risks and risk concepts. However, the discussion primarily centers on the internal regulations of banks and on adjusting various guidelines and requirements while tightening control of their applications. However, this concentration of the debate around bank regulations widely ignores the second trend we mentioned: the increased involvement of retail investors in financial markets. Nijman and Werker (2006) highlight the following developments that stress the necessity to shift attention to this financial market segment within the risk debate. Retail investors are offered a steadily increasing number of investment products; those products become more and more complicated and require a sophisticated comprehension of their nature. Additionally, individuals are forced to make an increasing number of financial decisions, which are often ill-informed due to the profound misunderstanding of risky instruments. The U.S. Securities and Exchange Commission (2012) examined financial literacy among investors and found “that investors have a weak grasp of elementary financial concepts” (p. iii). Risk is a crucial concept that is often poorly understood but is considered by retail investors as indispensable to their ability to make informed financial decisions. The limited literacy that “normal citizens” display in risk issues, even those “normal citizens” who are highly educated (e.g., law and medical students), is confirmed by recent findings in behavioral finance (e.g., Giegerenzer 2002).

The disinformation plaguing investors and the miscommunication of risks to them in the investment industry are additional crucial issues that must be addressed to avoid misleading and harming investors (Service 2000). This miscommunication takes two forms: either too many risks with too many details are disclosed or the complexity of risk issues is reduced to one number – volatility (price fluctuations).

The website [retailinvestor.org](http://retailinvestor.org), which is dedicated to educating retail investors on relevant financial issues, is one example: this website lists and explains 13 (thirteen) different risk kinds. Similar examples can be found in the information sheets of banks and brokerage firms. However, the most prevalent method for communicating investment risks is to reduce various types of risk to just one number – volatility. Financial products are usually divided into two categories: “high risk” instruments, such as equities, and “low risk” instruments, such as bonds (Service 2000). Underlying this division of risk categories is the “high return-high risk” formula suggested by the Modern Portfolio Theory (Markowitz 1952). According to this theory, risk (or volatility, also referred to as *vola*) is understood as a variance in annual returns or, more simply, as price fluctuation (the more volatile the price, the more risky the asset). A closely related risk concept – *beta* – represents the correlated volatility of an asset in relation to the market, which is measured by a representative index. An asset with  $\beta > 1$  fluctuates more than the market. Therefore, this asset is riskier.  $\beta < 1$  implies lesser risk. Beta measures systematic risk originating in the Capital Asset Pricing model suggested by Sharpe (1964). Volatility measures became established risk measures after they were integrated with the legal requirements regulating the manner in which risk in the financial markets should be communicated. For example, the Committee of European Securities Regulators (2010) proposed the methodology for calculating a unified risk indicator for investment funds. This Synthetic Risk and Reward Indicator (SRRI) is based solely on past volatility and is an obligatory part of the Key Investor Information Document (KIID), which is the approved and standardized method of informing mutual fund investors in Europe about the risks of investment funds.

The focus on price risk understood as volatility, however, misleads investors. The following aspects of this misleading confusion should be discussed: neglect of all other types

of risk; misrepresentation of investment opportunities; concentration on the riskiness of particular *asset classes* and on diversification as a key to risk reduction; and neglect of the time perspective of investors in the risk disclosure.

### 1.1. Neglect of All Other Types of Risk

As risk concepts, volatility and beta have become confusing and dissatisfactory for investors. First, the explicit focus on vola and beta as risk measures implies concentration on just one particular type of risk, namely, price risk, and neglects other important types of risk. Consider, for example, the following quote from Warren Buffet: “The riskiness of an investment is not measured by beta (a Wall Street term encompassing volatility and often used in measuring risk) but rather by the probability – the reasoned probability – of that investment causing its owner a loss of purchasing power over his contemplated holding period. Assets can fluctuate greatly in price and not be risky as long as they are reasonably certain to deliver an increased purchasing power over their holding period. And as we will see, a non-fluctuating asset can be laden with risk” (Buffet 2012, p. 16).

Obviously, volatility is not a suitable measure of risk because it represents the effort to encapsulate various risks in a single reference number (investment risk is generally equated with price risk). However, there are other parameters that influence the riskiness of an asset, e.g., creditworthiness (and more generally, the potential change of the fundamental situation) of a company, which determines the possibility of default, liquidity of the asset and inflation. Those risks should be clearly differentiated and presented to retail clients. The following example illustrates how misleading a risk assessment based on volatility can be.

Consider the German medtech company Fresenius Medical Care (FMC). Until the company’s share price collapsed on July 1, 2013 after a profit warning on its US-business stemming from the Obama administration’s introduction of price caps on particular medical services, FMC was generally considered a “defensive” investment (Wall Street online 2012) due to the low volatility of the share price. Obviously, the price volatility did not reflect significant operating risks related to the company: high indebtedness and dependence on one market – the USA – which is exposed to strict government supervision and is therefore vulnerable to regulatory changes. Those threats to company’s value explain why ratings agencies (Standard & Poor's BB+, Moody's Ba1) do not consider FMC Bonds “investment grade”. Interestingly, for many equity analysts who derive their risk assessments with regard to only volatility, FMC continues to be a defensive investment.

If we compare the FMC with BASF, the leading German chemicals group, we see that BASF shares appear much riskier if we take into consideration only price risk. The risk parameters, which are relevant according to modern portfolio theory, are much lower for Fresenius (beta = 0.3; volatility = 19%) than for BASF (beta 1.15; volatility 22.5%), as calculated for 250 days on July 12, 2013.

BASF, however, is a financially solid and broadly diversified company that is also the industry leader in sustainability issues. Its credit rating is very good (Standard & Poor's A+, Moody's A1). Based on the valuation measure PE, BASF shares are much cheaper than FMC shares (based on 2012 earnings, BASF had a PE of 12.5 on July 12, 2013, while FMC had a PE of 18.3 on the same day). Furthermore, with a trading volume averaging €220 m daily, the share is much more liquid than the FMC share (€50 m daily on average). Thus, when

considering only volatility, the BASF share seems more risky than the FMC share. Based on other risk assessments, BASF is less risky and a more favorable investment.

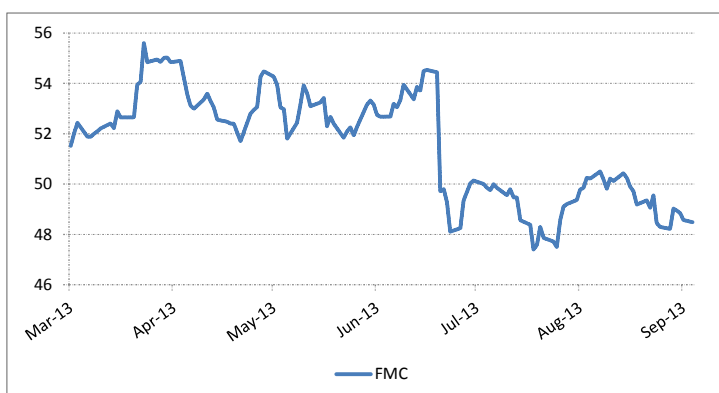


Figure 1: Shareprice development of FMC (Source: Deutsche Börse AG)

A similar argument applies to investment funds. The famous example is the hedge fund LTCM. This fund conducted highly leveraged arbitrage operations in financial markets (which allowed for stable returns during “normal” times) when market forces caused fundamentally unjustified spreads to narrow. LTCM enjoyed high returns combined with low volatility, which appeared favorable if one considers only volatility as a risk measure. The dependence on high leverage, however, caused a significant operational risk. Additionally, the size of the holdings was becoming large relative to their trading turnovers, increasing liquidity risk. During the Emerging Markets crises of 1998, financial markets experienced stress that widened spreads dramatically, causing the liquidity of LTCM’s positions to evaporate. The high leverage and the inability to sell assets caused LTCM to collapse within weeks. This seemingly low-risk fund immediately lost all of its value (Lowenstein 2000). Thus, if the risk of an investment is assessed only on the smoothness of its price increase, the investment may seem to be low-risk after price appreciation in a speculative bubble (that is about to burst).

## 1.2. Misrepresentation of Investment Opportunities

Moreover, the focus on volatility as a risk measure disregards the fact that downside price movements can present opportunities to buy assets at attractive prices, thereby increasing long-term returns. For example, Buffet (2010, p.3) wrote in a shareholder letter that “in the face of widespread pessimism about our economy”, his Berkshire Hathaway company was enthusiastic enough to have invested \$6 billion, including the takeover of the railway company BSNF.

It should be stressed that in general the relationship between investment returns and risks postulated by the modern portfolio theory is doubtful and unsupported by empirical findings. For example, O’Shaughnessy (2011) tested different investment strategies, combining various stocks to explore their performance characteristics. He found that no sensible relationship between volatility and performance could be proven for well-performing strategies. However,



low-performance strategies seem to be connected with high volatility (risk), contradictory to the initial statement of modern portfolio theory, which postulates that taking higher risks should create higher returns.

Similar findings were delivered by Robeco Asset Management (van Vliet 2012). This company's research demonstrates that volatile stocks significantly underperformed stocks with medium and low volatility in the USA between 1931 and 2009. However, volatile stocks outperformed in periods of increasing market euphoria, such as during the bull markets of the 1940s, 1950s and 1990s.

Therefore, the focus on price risk, understood as volatility, can mislead long-term investors. The perceived connection between high risk and high return is particularly confusing and may have caused investors to buy more volatile assets than was compatible with their investment objectives.

### **1.3. Concentrating on the Riskiness of Particular Asset Classes and on Diversification as a Key to Risk Reduction**

Concentrating on the riskiness (volatility) of particular *asset classes* and diversification as *the* method for reducing risk is also problematic. First, the correlations between and within asset classes increased during the last financial crisis, so that diversification failed (Authers 2009, p. 25).

Second, volatility as a risk measure is applicable only to liquid and transparent asset classes, which are traded on the stock exchange (equities, bonds, etc.). Illiquid and intransparent asset classes, such as real estate and private equities (although they often constitute significant portions of investors' portfolios today), are not exposed to significant price fluctuations simply because they are not frequently traded. This holds more generally for all infrequently traded securities (e.g., some small cap stocks). However, those assets are more risky because they are not transparent and illiquid. Furthermore, transaction costs for illiquid assets are significantly higher, which dents returns and creates additional risks. Thus, such peculiarities of individual financial instruments should be integrated into risk assessment, which should focus on individual securities within asset classes.

### **1.4. Neglect of Investors' Time Perspective in the Risk Disclosure**

Furthermore, the focus on volatility implies neglecting the time perspective of investors. Warren Buffet rejects volatility measures as misleading, because for him - as a typical long-term investor - other risk factors are more relevant. At the same time, for a short-term trader who aims to benefit from market fluctuations, a risk measure based on volatility may deliver a suitable instrument for risk assessment.

Service's analysis (2000) of a sample of brochures from eight superannuation providers (approximately 43% of the Australian retail superannuation market) demonstrated that only one provider took the influence of investment time horizon into consideration: "By regarding risk as the likelihood that your investment needs will not be met, we see that risk is not an inherent quality of an asset, but rather depends on the investment needs that you want the asset to satisfy".

Thus, Service (p. 5) concludes, “the consumer’s time horizon should be part of any risk definition”. Indeed, the relevance of single risk measures depends strongly on the investor’s time perspective. The concentration on price risk can be (and often is) justified by the short-term orientation of the majority of professional investors in modern financial markets. However, many retail investors who often invest surplus cash to save for retirement or their children’s education have longer time horizons. For these investors, the relevance of the price risk is lower, while operational and inflation risk become more important (remember, again, the Buffet quote and the FMC example above).

In what follows, we present our suggestion for a risk evaluation system for individual securities. This system brings together the various types of risk, is simple and easily understood, and takes time perspective into consideration. Due to its simplicity, we consider this system particularly suited to explaining and communicating risk to retail clients.

## 2. THE RISK DISCLOSURE SYSTEM FOR RETAIL INVESTORS

If volatility, or what we call price risk, is insufficient to represent the risks of a particular security, which risks should be considered? As we already mentioned, some sources provide endless lists of various risks and their very detailed descriptions. This approach is of little help, particularly if we keep in mind the low financial literacy of retail investors. However, there are cases when just a few risks are presented as crucial. We refer to International Financial Reporting Standards Statement No.7, Financial Instruments: Disclosures (IFRS 7), which was developed to guide risk disclosure of financial instruments by companies. This guideline focuses on three major risks: credit risk, liquidity risk and market risk. We took this standard as orientation, adjusted it for inflation risk and the clear graphical representation of risks.

More specifically, while deciding which risks should be taken into account, we considered the factors that influence investment results. We focused on the threats that can prevent a client from meeting his or her investment goals.

As is generally known, investors can benefit from their investments in two main ways. First, investors can benefit from the favorable development of the fundamental situation of the issuer behind the financial instrument, i.e., the issuer operates successfully, creates *value* and gives the value back to investor in the form of various cash flow payments (dividends, interest rates, etc.). Second, the fundamental value of the issuer is constantly (re-)evaluated by financial markets so that investors can benefit or suffer from the development of the market price of the financial instrument. Thus, risks are related to threats to cash flow and to price changes.

Keeping this in mind, we decided to include four risks in our integrated risk evaluation system (Figure 1):

- *Threats to future cash flows: operational risk* is the risk of unfavorable change in the fundamental situation (substance) of the issuer behind the financial instrument (as broadly related to the credit risk in IFRS7), and *inflation risk* is the danger of a loss of purchasing power of the invested money and received payments due to increases in the general price level.

- *Threats to the evaluation of the financial instrument: price risk* (market risk in IFRS7) is the risk of unfavorable change in the security price due to market sentiment, and *liquidity risk* is the risk that, in the absence of sufficient demand, an investment cannot be sold or only sold with delays, high expenses or steep value discounts.

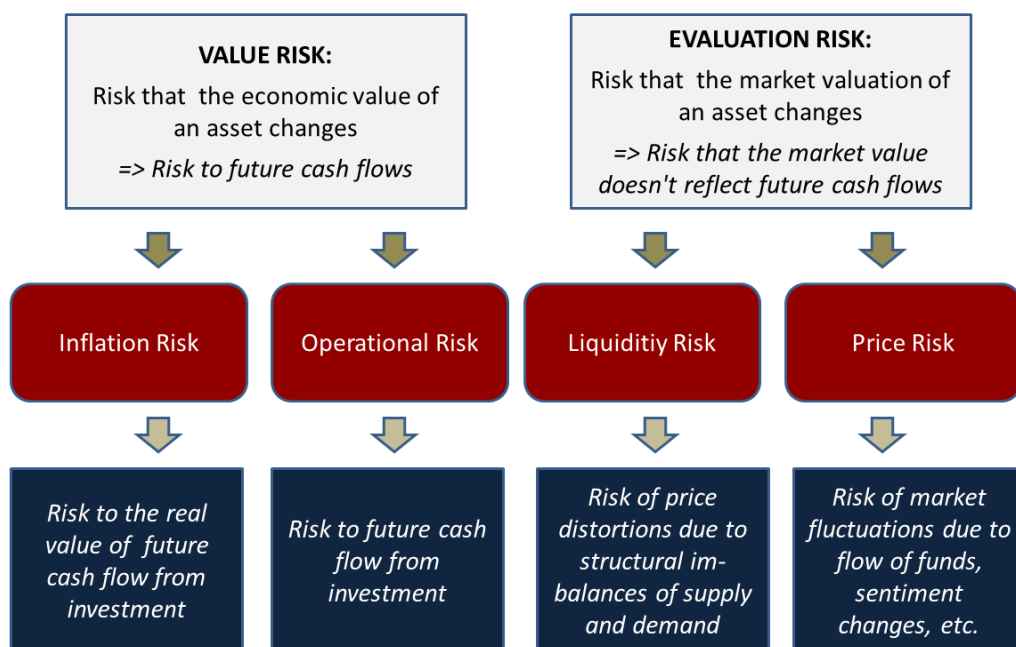


Figure 2. Overview of risk categories.

Now, we clarify these risks with the following example (Figure 2).

The left graph shows the aggregated development of nominal earnings of the companies included in the S&P 500 index (US-equities).

The operational risk represents the threat that the future development of these earnings will change unfavorably. Inflation risk means that the real economic value of this earnings stream changes through inflation.

The right graph shows how the earnings are evaluated by the market. It is obvious that the evaluation of the earnings fluctuates more strongly than the earnings themselves. On average, these earnings are evaluated with a P/E of 16. In periods of pessimism (e.g., the oil crisis of the 1970s), valuation tends to be much lower. In periods of extreme optimism (e.g., during the dotcom boom in 2000), valuations tend to be much higher. Additionally, valuations fluctuate quite significantly in the short term, as the zig-zag pattern of annual fluctuations shows.

In the case of the S&P 500, liquidity risk as a threat to evaluation is less relevant because the index consists of the most frequently traded shares in the USA. However, this type of liquidity risk may be more important for single components of the S&P 500.

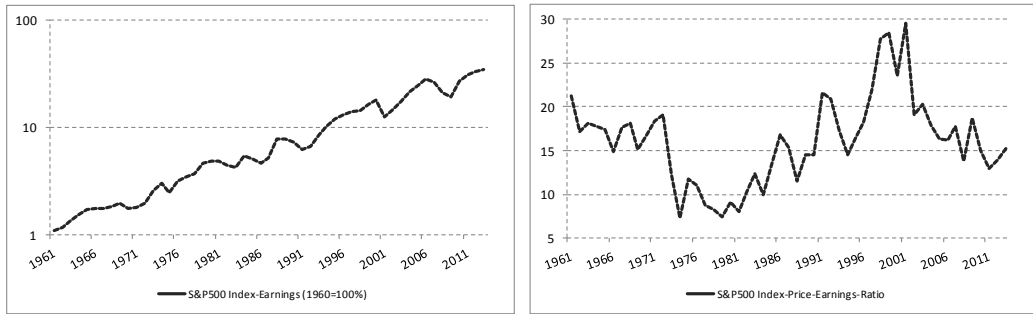


Figure 3. S&P 500: earnings and evaluation in 1961-2011. (Source: <http://www.longrundata.com/spearnings.php>)

Now, let us explain how we evaluate risk. First of all, we do it in a way that can be comprehended by non-professionals. To avoid incomprehensibility, we developed a system that values risk similar to the system used to determine school marks. In the first step, we applied the German system with marks from 1 to 6, but our methodology can easily be adjusted to the systems of assigning marks in other countries. We award the following risk marks:

- Mark 1: *Very good* (minimum risk) (comparable in the United Kingdom: A grade; France: 18-20; Spain 9-10, Italy & Netherlands: 10; Switzerland: 6).
- Mark 2: *Good* (practically no risk; increased risk only under unlikely and extreme circumstances) (comparable in the United Kingdom: B grade; France: 15-17.9; Spain 7-8, Italy & Netherlands: 8-9; Switzerland: 5).
- Mark 3: *Adequate* (normally low risk; increased risk under extreme circumstances) (comparable in the United Kingdom: C grade; France: 12-14.9; Spain 6, Italy & Netherlands: 7; Switzerland: 4).
- Mark 4: *Sufficient* (normally high risk; highly increased risk under extreme circumstances) (comparable in the United Kingdom: D grade; France: 10-11.9; Spain 5, Italy & Netherlands: 6; Switzerland: 4).
- Mark 5: *Deficient* (high risk) (comparable in the United Kingdom: E grade; France: 6-9.9; Spain 3-4, Italy & Netherlands: 3-5; Switzerland: 2).
- Mark 6: *Insufficient* (irresponsibly high risk) (comparable in the United Kingdom: F grade; France: 0-5.9; Spain & Italy 0-2; & Netherlands: 1-2; Switzerland: 1).

We evaluate every investment with regard to four risk categories, awarding a school mark for every risk type. Clear criteria for evaluating each type of risk are defined. Note that each of the sub-criteria is assessed on a single basis. The final risk note is derived from the worst outcome for a single sub-criterion. Thus, within a set of criteria, a bad outcome cannot be compensated for by good ones with respect to other criteria. The final risk note serves as a *knock-out criterion* for the entire group. The evaluation results for each criterion are subsequently aggregated into a school mark. This process allows the evaluation of different risks for different financial instruments with the same methodology and makes the risks comparable.

In the following sub-section, we list particular risk criteria and explain how they are translated into school marks.

## 2.1. Operational Risk

Operational risk is one of the most straightforward risks among the four risk types discussed in this paper. Operational risk relates to the fundamental analysis of the issuers of the financial instruments. Good orientation is provided by the ratings agencies (which are not very transparent in the details of their rating processes but seem to use broadly similar approaches). The ratings agencies use a list of fundamental characteristics of issuers, and we included some of these characteristics in our evaluation of operational risk. To evaluate operational risk, we used the following criteria:

- *At company level*: Debt (on and off the balance sheet); free cash flow; product variety; barriers to market entry; management quality; dependence on technology change.
- *With government bonds*: Budget deficits; shadow budgets; current account; development of the monetary system; demography; sustainability of the social security system.

Operational risk is measured by the percentage of a potential sustained nominal decline in value in the following 5-10 years. This allows the awarding of marks and ranking assets as follows:

- Mark 1	$x < 0,2\%$	very safe
- Mark 2	$0,2\% < x < 2\%$	safe
- Mark 3	$2\% < x < 12\%$	relatively safe
- Mark 4	$12\% < x < 25\%$	speculative
- Mark 5	$25\% < x < 50\%$	very speculative
- Mark 6	$> 50\%$	irresponsibly speculative

## 2.2. Inflation Risk

Inflation risk is, on the contrary, one of the most difficult risks to evaluate. Its importance for retail investments is often neglected and was only “discovered” as a standalone risk relatively recently (Kibble&Prentice 2010; JP Morgan 2012). Defining and calculating this risk is difficult because “the drivers – and effects – of inflation change from one period to the next as policy responses to inflation evolve. At the same time, we can never entirely predict how different asset classes will react to inflation in the future” (JP Morgan 2012, p. 8). Nevertheless, to adhere to our school mark approach and evaluate the impact of inflation on investment performance, we assess different factors that influence the real value of future cash flows. Therefore, we assess how inflation may affect the value of the investment measured in money terms (e.g., through accounting standards) and how the future cash flows

of an investment can be adapted to future inflation (e.g., through flexible payments). We suggest using the following criteria to assess inflation risk: duration of investment, price or payment alignment clauses, capital intensity, interest sensitivity and price elasticity of demand. We then translate our assessment into school marks:

- Mark 1            Inflationary pressures can be passed on completely.
- Mark 2            Inflationary pressures can be passed on mostly.
- Mark 3            Inflationary pressures can be passed on partly. Real value losses are possible with inflation above 5% p.a.
- Mark 4            Inflationary pressures can be passed on slightly. Real value losses are possible with inflation above 3% p.a.
- Mark 5            Inflationary pressures cannot be passed on. Real value losses are possible with inflation above 2%.
- Mark 6            Real value losses are possible even with very low inflation rates or deflation.

### 2.3. Price Risk

To evaluate price risk, we use traditional volatility measures, such as standard deviation, betas for different periods and the maximum drawdown. Note that price risk is inevitably evaluated in the context of an asset class because price fluctuations are not directly comparable due to the use of different trading methods. We rank the assets as follows:

- Mark 1            No price fluctuations.
- Mark 2            Low price fluctuations.
- Mark 3            Bonds: occasionally high price fluctuations. Equities: relatively low price fluctuations.
- Mark 4            Bonds: high price fluctuations. Equities: normal price fluctuations.
- Mark 5            Relatively high price fluctuations.
- Mark 6            Extreme price fluctuations.

### 2.4. Liquidity Risk

Liquidity risk is not easily observed. However, newer studies recognize the importance of liquidity risk for asset performance (Amihud 2002, Pastor and Stambanugh 2003, Amihud et al. 2012).

The quintessence of this research is that, to measure liquidity and liquidity risk, one must rely on proxy measures, such as bid-ask spreads (Benston and Hagermann 1974), trading volume, daily price and various measures of market depth. We agree with Keynes, who noted that “liquidity [...] is not defined or measured as an absolute standard but on a scale [our school mark system – authors] which incorporates key elements of volume, time and transaction costs” (as cited in Lhabitant 2008). We chose the following criteria for liquidity risk: average and bottom levels of stock market turnover, bid-ask spreads and transaction costs. Liquidity is assessed by evaluating the trading conditions.

- 
- Mark 1 Always tradable, with very high turnover and no significant market impact.
  - Mark 2 Mostly tradable, with high turnover. A small market impact may be measurable.
  - Mark 3 Smaller order sizes generally tradable without problems. Larger orders will have a moderate market impact.
  - Mark 4 Generally tradable, short-term liquidity squeezes possible. Larger orders may create a significant market impact.
  - Mark 5 Only occasionally trading. Midterm liquidity squeeze possible.
  - Mark 6 Not tradable or only tradable with very high bid ask spreads.

Similar to a school report, which summarizes achievements and shortcomings over a school year, we prepared a risk report that contains all of the relevant risk characteristics of a stock, bond, fund or other financial instrument. One decisive novelty of our approach is the concise graphical presentation of risk in one picture, which should enhance communication with retail clients.

We now illustrate our risk evaluation system with concrete examples.

*Example 1:* For a corporate bond with 9-year maturity and high solvency, such as the BASF SE MTN v. 2012(2022) (ISIN DE000A1R0XG3), the following risk evaluation would arise:

- *Operational risk:* Mark 2 => the payback of the bond is practically certain. However, under very extreme circumstances, a low remaining risk exists.
- *Price risk:* Mark 2 => only low price fluctuations should be feared.
- *Liquidity risk:* Mark 3 => in general, there is a good trading volume, but not always.
- *Inflation risk:* Mark 5 => with a current yield of only 2% over the long-term horizon, a real depreciation is likely, with accelerating inflation rates.

*Example 2:* In regard to the shares of a leading technology company, such as Apple, Inc. (ISIN US0378331005), a different picture appears:

- *Operational risk:* Mark 5 => although the company is a market leader with a solid balance sheet, it could be strongly endangered by technological change.
- *Price risk:* Mark 5 => short-term movements of prices depend strongly on market sentiment. Therefore, very high stock price fluctuations are to be expected.
- *Liquidity risk:* Mark 1 => normally very high stock market turnover.
- *Inflation risk:* Mark 2 => sale prices develop regardless of general price levels. Low capital intensity avoids the negative balance sheet effects of inflation.

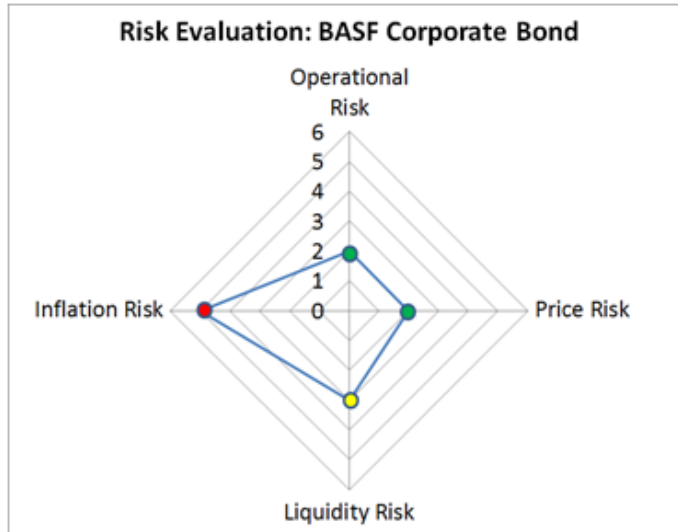


Figure 4. Risk Evaluation: BASF Corporate Bond.

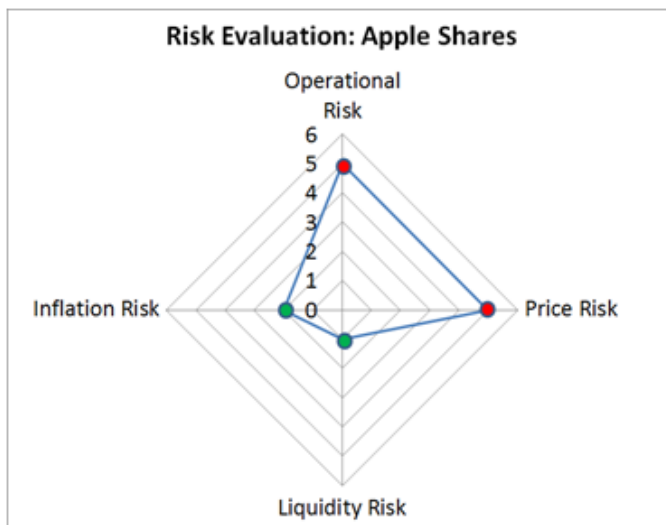


Figure 5. Risk Evaluation: Apple Shares.

We will now discuss the relevance of investors' time perspective.

### 3. CONSIDERING INVESTORS' TIME PERSPECTIVE

The crucial component of communicating risk to retail customers is establishing a relationship between risk evaluations, investment objectives and the time horizon of the investor. For example, as follows from the aforementioned quote, price risk is largely irrelevant to Warren Buffet – the declared long-term investor – as well as to a private client who saves for his retirement. This irrelevance stems from the fact that price fluctuations may be temporary. Additionally, liquidity risk is less importance because the private client does



not feel pressured to sell assets at a specific point in time. The most crucial point for long-term investors is that the investment returns more at the end of the investment period, adjusted for inflation. Thus, operational risk as a threat to fundamental substance and future cash flow is much more important than price risk or liquidity risk.

The picture is different in the case of a client who trades privately on a daily basis to exploit market inefficiencies. Such a trader would be strongly concerned about liquidity and daily price changes. The long-term-oriented operational and inflation risks are less important.

We summarize the relationship between various risk types and time horizons in the following table (Figure 5). This table can be used as a tool to communicate with clients.

Investment horizon	very short-term less than 1 Day	short-term between 1 Day and 0,5 years	Medium-term between 0,5 and 5 years	long-term between 5 and 10 years	very long-term more than 10 years
Liquidity risk	very important	important	rarely important	unimportant	unimportant
Price risk	important	very important	important	rarely important	unimportant
Operational risk	unimportant	rarely important	important	very important	very important
Inflation risk	unimportant	unimportant	rarely important	important	very important

Figure 6. The relationship between investors' time horizons and types of risk.

Thus, depending on the time perspective, investors can rank and weigh the relevant risks.

## CONCLUSION

This study contributes to the improvement of the insufficient communication of risks in financial markets. In particular, we argue for upgrading the standards of risk communication with retail investors. We are aware that the urgent need for better communication of risk is not easy to meet. For example, the CFA institute (2011) conducted a study on *professional users'* satisfaction with financial instrument risk disclosures by companies. Although those companies report according to the established International Financial Reporting Standards Statement No. 7, Financial Instruments: Disclosures (IFRS 7), the users of the companies' reports complain that risk estimations are difficult to understand, vague, inconsistent and not directly comparable. Note that we are discussing the international standards that were developed to disclose and communicate the risks of financial instruments to trained professionals. Yet, those norms are unsatisfactory. Thus, developing similar standards for communicating risk to retail investors who lack specific knowledge and understanding would be even more challenging. However, we believe that the risk disclosure tool suggested in this paper can contribute to shifting the risk communication paradigm in the retail sector due to its relative simplicity, consistency and clear graphical representation. The *integrated* risk communication tool we present is better than common risk disclosure methods using just one number (volatility) and methods using large, incomprehensible lists of risks.

However, in this paper, we have only laid the groundwork for improving the communication of risks to investors. This topic must be further developed. For example, more research on the determinants of liquidity risk and inflation risk is necessary. Additionally, the interconnectedness between particular risk types – an issue we ignored to keep our

communication concept simple – should be considered. This interconnectedness is crucial and must be integrated into the concept.

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*Chapter 8*

## THE LAW AND REGULATION OF CHINESE MERGERS AND ACQUISITIONS: THE TAKEOVER MEASURES

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### ABSTRACT

Relatively little is known in western countries about the legal framework under which M&A activities are conducted in China. Yet, a detailed understanding of the rules and regulations that govern this area is essential if the research agenda on Chinese M&A activities which is now emerging in the western literature is to be properly conducted. We provide a detailed summary of the laws and regulations as they affect the conduct of M&A activities in China. We begin with a brief summary of China's main stock exchanges and their listing requirements. We then outline some of the unique characteristics of Chinese capital markets – in particular the distinction between “A” shares and “B” shares issued by Chinese firms. The Takeover Measures – which is the principal law regulating M&A activities in China - is then considered. Here our focus is on the mandated bid rules, the disclosure of substantial shareholdings, the tender offer rules and the defence mechanisms which may be used in Chinese M&A activities. We conclude by identifying some of the important issues that are likely to affect the legal framework surrounding Chinese M&A activities in the years ahead.

### INTRODUCTION

China's admission to the World Trade Organisation (WTO) in 2001 and its generally vibrant economy has meant that Chinese merger and acquisition (M&A) activities have increased considerably over the last decade (Fei, 2004). The Chinese government has responded to this increased volume of M&A activities by establishing a legal framework which, on the one hand, is in line with best international practice but also, meets the unique political and socio-economic conditions that have characterised the People's Republic of

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China since its formation in 1949. Moreover, an ever increasing number of foreign firms have been attracted by the favourable investment climate that has been put into place by the Chinese government. Many of these foreign firms have chosen to implement their investment activities through takeovers and mergers with local Chinese firms. Unfortunately, in western countries relatively little is known about the legal framework that governs Chinese M&A activities. Yet a detailed understanding of the rules and regulations that govern this area is essential if the research agenda on Chinese M&A activities which is now emerging in the western literature (Fei, 2004; Prasad, 2004; Wei, 2006) is to be properly conducted. Given this, our purpose in this paper is to provide a detailed summary of the laws and regulations as they affect M&A activities in the People's Republic of China. Our expectation is that this will be of interest and assistance to those who undertake research on the economic motivations for and economic consequences of Chinese M&A activities as well as those who might be contemplating some kind of active participation in the Chinese capital market.

Our analysis begins with a detailed consideration of the "General Principles of Measures for the Administration of the Takeovers of Listed Companies", which we will henceforth refer to as the "Takeover Measures". This law came into force in 2002 under the auspices of the China Securities Regulatory Commission (CSRC), which is China's principal securities market regulator.

However, the Takeover Measures, 2002 have since been revised in 2006, 2008, 2011 and also most recently, in February 2012 by the CSRC in order to fill gaps, loopholes and other deficiencies which experience has shown existed in the laws and regulations covering M&A activities in China. The Takeover Measures also aim to make Chinese laws in the M&A area more compatible with best international practice. Furthermore, in order to address the anti-trust issues associated with M&A activities, the Standing Committee of the Tenth National People's Congress of China promulgated a new Anti-Monopoly Law which came into force on 1 August 2008. However, the complexities of the Takeover Measures and the Anti-Monopoly Law are such that it is not possible to deal with both pieces of legislation in a single summary paper like this. Given this, it is the Takeover Measures which will be the principal focus of this article and we delay a detailed consideration of the Anti-Monopoly Law to a follow up article.

The remainder of the paper is structured as follows. Section 2 provides a brief summary of China's main stock exchanges and their listing requirements. Next, section 3 summarises some unique characteristics of Chinese capital markets – in particular, the distinction between "A" shares and "B" shares that are issued by Chinese firms. With rare exceptions only domestic Chinese nationals can purchase A shares whilst foreign investors are normally restricted to purchasing B shares. Moreover, A shares are comprised of state-owned shares, legal person shares and public individual shares - but only public individual shares can be traded on the stock exchange. In section 4 we summarise the legal provisions of the Takeover Measures with occasional reference, where necessary, to the Anti-Monopoly Law. Our consideration of the Takeover Measures centres principally on the mandated bid rules, the disclosure of substantial shareholdings, the tender offer rules and the defence mechanisms which may be used in Chinese M&A activities.

Finally, section 5 provides our summary conclusions and identifies some important issues that are likely to affect the legal framework surrounding Chinese M&A activities in the years ahead.

## CHINESE STOCK EXCHANGES

In recent years China's economy has been gradually transformed from the centrally-planned economy that was introduced in 1949 to a market-orientated socialist economy. The movement towards a market-orientated socialist economy began in 1978 when the Chinese government implemented a reform programme which encouraged the formation of private rural enterprises and businesses, lifted many restrictions on foreign trade and investment, abolished controls over the prices of some basic commodities and outputs, and boosted investment in industrial production and the education of its workforce. As part of the reform process, in 1981 China's State Council<sup>1</sup> created a national bond market by issuing national treasury bonds for the first time. Subsequent to this, several other kinds of national bonds were issued; for example, those issued by the Ministry of Finance to fund key construction projects. However, the new bond market only satisfied the liquidity requirements of central government, leaving the needs of private and many state-owned enterprises unaddressed. Hence, in order to solve the financial difficulties faced by private and state-owned enterprises, the People's Bank of China (PBC) authorised the establishment of two nationwide stock exchanges; namely, the Shanghai Stock Exchange which began operations in 1990 and the Shenzhen Stock Exchange which began operations in 1991.

Initially, a variety of organisations, including the PBC, the State Council, the Ministry of Finance and local government bore responsibility for regulating these two stock exchanges. But the need for a different regulatory framework became clear after a number of regulatory failures of which the "810 incident" on the Shenzhen Stock Exchange is probably the most prominent example. This incident occurred on 10 August, 1992 when some 700,000 "would be" investors packed into the Shenzhen Stock Exchange to subscribe for a new issue of bonds by the Chinese Government. The prescribed five million subscription forms were used up within a few hours. Violent rioting resulted, as it became clear to investors that PBC officials had corrupted the process of handing out and collecting the subscription forms for the bond issue. The government restored order by distributing another five million subscription forms the next day. The incident, to a large extent, was caused by the fact that too many organisations claimed regulatory authority over the Chinese securities markets and their operations.

It was almost inevitable that a regulatory framework like this would lead to confusion and corruption – as indeed it did (Walter and Howie, 2003). Incidents like this left the State Council with little choice but to remove the ambiguity which had arisen in the regulation and administration of China's securities markets. Consequently, in 1992 the State Council created the China Security Regulatory Commission (CSRC). However, it took the CSRC until 1997 to rest regulatory control of securities markets in China away from the PBC, the Ministry of Finance and other government agencies and thereby assume effective control as the sole regulator of Chinese capital markets. Under China's Securities Law, the CSRC has "authority to implement a centralised and unified regulation of the nationwide securities market in order to ensure their lawful operation"<sup>2</sup>. The CSRC's powers include responsibility for regulating and supervising the issue of securities, as well as the investigation and imposition of penalties

<sup>1</sup> The State Council is the chief administrative authority of China. It is chaired by the Premier of China and includes the heads of each government department and the heads of the more important government agencies.

<sup>2</sup> See [http://en.wikipedia.org/wiki/China\\_Securities\\_Regulatory\\_Commission](http://en.wikipedia.org/wiki/China_Securities_Regulatory_Commission).

for, “illegal activities related to securities and futures”<sup>3</sup>. Its role is broadly similar to that of the Securities and Exchange Commission (SEC) in the United States. By February, 2011 the Shanghai and Shenzhen Stock Exchanges had more than 2,100 listed firms between them with a combined market capitalisation in excess of \$US4.0 trillion<sup>4</sup>.

Firms applying to list their shares on the Shanghai Stock Exchange must conform with its listing requirements which are largely based on the “Securities Law of the People’s Republic of China” and the “Company Law of the People’s Republic of China”<sup>5</sup>. When a firm plans a public issue of shares for the first time it must seek the approval of the CSRC. Once the CSRC has approved a public issue of shares then the affected firm may apply to have its shares listed on the Shanghai Stock Exchange. A second requirement is that after the public issue of shares the firm’s total share capital must not be less than ¥50 million Yuan. Moreover, the firm must have been in business for more than 3 years and have been profitable over the last three consecutive years before making an application to the Shanghai Stock Exchange for a listing.

In the case of former large and medium sized state-owned enterprises re-established as private or public firms in accordance with the “Securities Law of the People’s Republic of China” and “Company Law of the People’s Republic of China”, the profitability requirement can be calculated consecutively; that is, profits from the period when the firm was state-owned can be included as a component of the three year profitability calculation. There must also be at least 1,000 individual shareholders whose investment in the shares of the firm exceeds ¥1,000 Yuan.

Furthermore, publicly offered shares (that is, state-owned shares, legal person shares, public individual shares, B shares, etc.) must be more than 25% of the firm’s total share capital. When a firm’s total share capital exceeds ¥400 million Yuan, the minimum percentage of shares that must be issued to the public is reduced from 25% to 10%. Finally, the firm must not have been involved in any major illegal activities or false accounting practices in the three years prior to its listing on the Shanghai Stock Exchange.

The Shenzhen Stock Exchange is the smaller of the two mainland Chinese stock exchanges. However, its listing requirements are nonetheless broadly similar to those of the Shanghai Stock Exchange<sup>6</sup>. In particular, when a firm plans a public issue of shares for the first time it must seek the approval of the CSRC. Once the CSRC has approved a public issue of shares then the affected firm may apply to have its shares listed on the Shenzhen Stock Exchange.

Public listing on the Shenzhen Stock Exchange is only available to firms with an issued share capital in excess of ¥30 million Yuan. The comparable figure on the Shanghai Stock Exchange is ¥50 million Yuan and so it is not surprising that there is a preponderance of small and medium sized firms listed on the Shenzhen Stock Exchange. Furthermore, publicly offered shares must be more than 25% of the firm’s total share capital<sup>7</sup>. Finally, firms listing on the Shenzhen Stock Exchange must have a good credit record for the three years prior to

<sup>3</sup> See [http://en.wikipedia.org/wiki/China\\_Securities\\_Regulatory\\_Commission](http://en.wikipedia.org/wiki/China_Securities_Regulatory_Commission).

<sup>4</sup> According to Wikipedia, by February 2011, there were 900 firms listed on the Shanghai Stock Exchange with a market capitalization of US\$2.7 trillion. Likewise, by February 2011, there were 1,211 firms listed on the Shenzhen Stock Exchange with a market capitalization of US\$1.3 trillion.

<sup>5</sup> See the official website of Shanghai Stock Exchange: [www.sse.com.cn](http://www.sse.com.cn).

<sup>6</sup> See <http://www.szse.cn/main/en/aboutsse/listingqualifications/>, the official website of Shenzhen Stock Exchange.

<sup>7</sup> There is a provision for firms with share capital in excess of RMB 400 million to reduce this figure as in the case of the Shanghai Stock Exchange.



listing. This latter requirement also applies for firms listing on the Shanghai Stock Exchange, although it is stated in a slightly different way.

## UNIQUE CHARACTERISTICS OF CHINESE CAPITAL MARKETS

China's currency, the Chinese Yuan, is not completely and freely convertible into foreign currencies. This is because the Chinese government has implemented a policy which restricts the amount of Chinese Yuan that can leave the country in order to preserve the nation's foreign currency reserves. This policy has had a stabilising effect on the rate at which the Chinese Yuan trades against most foreign currencies and this in turn has created a degree of certainty for firms and other organisations which operate in export and/or import-oriented markets. However, this policy of restricted trading in the Chinese Yuan means that a distinction has had to be drawn between foreign investors and investors who are Chinese nationals. Chinese nationals will normally purchase "A" shares which are shares whose principal (that is, prices) and dividends are denominated in the Chinese Yuan and which are exclusively traded on the stock market in terms of the Yuan. Foreign investors usually have only very limited access to A shares. However, foreign investors (including investors from Taiwan, Hong Kong and Macao) who wish to invest in mainland Chinese firms will normally do so by purchasing so called "B" shares. Trading on the stock market in B shares occurs in either the US dollar or the Hong Kong dollar. Foreign investors who buy and sell B shares must commission an authorised Chinese securities institution to deal with the transaction. The authorised institutions may then enter into proxy agreements with approved securities institutions outside of China in buying and selling B shares. Dividends, bonuses and trading earnings from B shares may be remitted outside of China after the deduction of relevant taxes (Campbell, 2006). Individual foreign investors are allowed to hold up to a maximum of 25% of the B shares on issue for a given Chinese firm with the proviso that the foreign investors' ownership of B shares does not give them overall control of the firm (Chakravaty, Sarkar and Wu, 1998)<sup>8</sup>. In summary, A shares are the main body of shares traded on the Shanghai and Shenzhen stock exchanges; B shares account for less than 1% (in terms of market capitalisation) of all shares traded on these two stock exchanges (Huang, 2005). This in turn means that B shares normally have only a minor impact on the trading activities of the two mainland Chinese stock exchanges.

A unique feature of A shares issued by Chinese firms is that they cannot always be traded on the stock exchange. In fact, A shares are classified into three groups which are "state-owned" shares, "legal person" shares and "public individual" shares in terms of the strictly

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<sup>8</sup> Since China's admission to the WTO in February, 2001 Chinese domestic nationals have been able to purchase and trade B shares. However, B shares are still denominated and traded in either the U.S dollar or the Hong Kong dollar and thus Chinese nationals must have access to these currencies if they are to purchase B shares. Moreover, since December, 2002 Qualified Foreign Institutional Investors (QFII) have been allowed to purchase A shares. QFII's include overseas asset management institutions, insurance companies, securities companies and other asset management institutions approved by the CSRC and who have been granted a foreign exchange quota by the State Administration of Foreign Exchange (SAFE) to buy A shares. Interestingly, foreign banks are not included in the definition of QFII. A QFII can only invest in A shares, not B shares. Also, shares held by each QFII in one listed firm should not exceed 10% of the total shares outstanding for that firm and the total shares held by all QFIIs in one listed firm cannot exceed 20% of the total shares outstanding for that firm.

defined groups of shareholdings in China. State-owned shares are those owned by the state, including the central government and local authorities. Legal person shares are those held by domestic legal enterprises and institutions (but not individuals) such as state-private mixed enterprises and non-bank financial institutions (Qi, Wu and Zhang, 2000). An important point that needs to be made here, however, is that only public individual shares are freely tradable on the two mainland Chinese stock exchanges; that is, state-owned shares and legal person shares cannot be traded on these exchanges. Furthermore, non-tradable A shares (that is, state-owned shares and legal person shares) account for a majority of the A shares issued by most listed firms. Li and Zhang (2007) quote statistics which show that in 2004, Chinese firms had ¥712 Yuan billion of A shares on issue. However, ¥454.3 Yuan billion or 64% of these A shares were non-tradable. In particular, state-owned shares accounted for 74% of the non-tradable shares or slightly less than half the A shares issued by Chinese firms. The tradability restrictions which apply to state-owned shares and legal person shares can act as a deterrent for M&A activities and hence, the overall allocative efficiency of the Chinese economy. The only way that non-tradable shares can be transferred is through a private takeover agreement. The laws and regulations relating to private takeover agreements are summarised in a subsequent section of this paper.

The principal reason for the existence of such a large proportion of non-tradable A shares is to prevent state-owned assets from falling into the hands of private or foreign parties. In other words, if state-owned shares were allowed to be transferred to private or foreign owners, then the socialist economic principles upon which the Chinese political system is founded might be compromised. It also guards against the possibility of fraud and misappropriation by private firms and individuals. However, we have previously observed how the existence of a significant block of non-tradable shares might be detrimental to the long run development and health of the Chinese economy. In particular, it leads to a divergence in the values of the traded as against the non-traded A shares and weakens the stock market's price discovery function. This in turn leads to a lowering of allocative efficiency in the Chinese economy as a whole. The problems caused by this dichotomy between traded and non-traded A shares had become so acute that in 2000, the Chinese government began implementing a reform programme under which it eventually aims to remove all restrictions in the trading of state-owned shares (Jin and Yu, 2009).

To begin with, the reform programme had only limited success. However, in April, 2005 the CSRC issued a new plan for shareholding structure reform called 'Guquan Fenzhi Gaige', under which market-based processes are gradually being implemented for the transfer of share ownership rather than the government-imposed processes which had prevailed up until that time. Under the Guquan Fenzhi Gaige, representatives of the group of shareholders with tradable A shares (that is, public shareholders) agree terms and conditions for the conversion of non-tradable A shares into tradable A shares with representatives of the group of shareholders who hold the non-tradable A shares. These terms and conditions not only include the rate at which the non-tradable shares are to be converted into tradable shares but also, any other forms of compensation which are to be paid to the previously existing tradable shareholders. Since the non-tradable shareholders are granted a new and valuable trading privilege, the Guquan Fenzhi Gaige measures allow the compensation given to previously existing tradable shareholders to take a variety of forms, including the issue of new tradable shares, cash payouts and the issue of new warrants, etc. The rate at which non-tradable shares are converted into tradable shares varies from one firm to another because the terms are

absolutely negotiable between the holders of the non-tradable shares and the holders of tradable (that is, public) shares.

In addition, the Guquan Fenzhi Gaige measures stipulate that a certain proportion of the non-tradable shares which are converted into tradable shares cannot be sold in the first few years after an agreement has been reached about their conversion into tradable shares. Firms that have successfully been through the process of converting their non-tradable shares into tradable shares use the prefix ‘G’ as part of their stock exchange listing names. By April 2007, 1,290 companies (representing some 96% of all firms listed on the Shanghai and Shenzhen stock exchanges) had begun negotiations under the Guquan Fenzhi Gaige measures to convert non-tradable shares into tradable shares. However, most of the formal agreements reached in this area have stipulated that non-tradable shares which have been converted into tradable shares cannot be traded on the stock exchange for a period of at least 5 years (and in many instances much longer) after conversion. This means that the supply of tradable shares is unlikely to increase in any substantial way for many years to come (Cooper, 2008).

## CHINA’S TAKEOVER LEGAL REGIME

### Framework and Overview of China’s Takeover Laws

We have previously noted how the merger of the local securities regulatory authorities with the CSRC in 1997 has meant that the CSRC has assumed exclusive authority for the regulation of securities markets in China. There are now two main laws regulating the M&A activities of listed firms in China. The first is the “Securities Law of the People’s Republic of China (PRC)”, which came into force in 1999. The stated objectives of the Securities Law is to regulate the issuance, sale and purchase of securities, to protect the lawful rights and interests of investors, safeguard the public interest, enhance economic order and promote the growth of the socialist market economy in China<sup>9</sup>. Hence, the Securities Law covers a wide range of regulatory activities, including the public listing of securities and stock exchange regulation, on-going disclosure of information by listed firms, prohibited trading acts and the regulation of mergers and acquisitions by publicly listed firms, etc. The Securities Law is comprised of twelve chapters, only one of which - chapter 4 - contains provisions relating to M&A activities. Even in chapter 4, however, the Securities Law lays down only very general provisions relating to M&A activities. More detailed regulatory provisions have been promulgated by the CSRC and are to be found in the second important law alluded to earlier; namely, the Takeover Measures. The Takeover Measures sets up the most comprehensive legal framework to date for the M&A activities of Chinese listed firms.

### General Principles of Measures for the Administration of the Takeovers of Listed Companies

As previously noted, the Takeover Measures which were promulgated by the CSRC in 2002 and revised in 2006, 2008, 2011 and February 2012, is the main and most important law

<sup>9</sup> Refer to Article 1 of the Securities Law of People’s Republic of China.

associated with the regulation of M&A activities for Chinese firms. The Takeover Measures aim to regulate takeovers of listed firms and the related alteration of share entitlements, protect the legitimate rights and interests of listed firms and investors, maintain the order and efficient operation of securities markets and promote the optimum distribution of resources throughout the Chinese economy. Moreover, protecting the interests of investors has a very high priority in the Takeover Measures. The Takeover Measures emphasise that M&A activities shall be conducted in light of the principles of openness (Gong Kai), fairness (Gong Ping) and equity (Gong Zheng)<sup>10</sup>. It is these principles which underscore the requirement of the Takeover Measures that the information disclosed by firms involved in M&A activities shall be truthful, accurate and complete and must not contain any false record, misleading statement or significant omissions<sup>11</sup>.

Article 4 of the takeover Measures stipulates that takeovers involving foreign investors must have the approval of the related Department of State; this will normally be the CSRC but there will be circumstances in which the approval of other government instrumentalities will also be required. For example, in 2008 the Ministry of Commerce (MOFCOM) blocked the U.S. Coca-Cola Company from mounting a successful takeover bid for the Chinese fruit giant, Huiyuan Juice Group Ltd on the grounds that it would have infringed provisions of the Chinese Anti-Monopoly Law. Furthermore, the Huiyuan Juice Group is a famous national Chinese brand closely linked with the Chinese people who would not approve of a well known domestic national brand like this falling into the hands of foreign owners. Finally, Article 4 also provides that foreign investors must be subject to Chinese laws and regulations and obey the judicial and arbitral system of China.

Under some circumstances bidding firms are prohibited from takeover activities. Hence, under Article 6(1) of the Takeover Measures a bidding firm will be prevented from using the takeover procedures to acquire another firm if, in the opinion of the CSRC, it has been in a continuous state of high indebtedness (literally, “large debts”) and has a history of not being able to meet its debts as they fall due for payment (literally, “has not paid off its due debts”). However, the Takeover Measures, 2006 are silent as to what is meant by a continuous state of high indebtedness and has not paid its debts as they fall due. Secondly, under Article 6(2) of the Takeover Measures if a bidding firm has committed a major illegal act or has been suspected of being involved in a major illegal activity during the 3 years preceding the proposed takeover, then the CSRC may prohibit the bidding firm from conducting any M&A activities.

Similarly, under Article 6(3) if the bidding firm has committed any serious credit-breaking acts in the securities market during the 3 years preceding the takeover, then the CSRC may also prohibit the bidding firm from conducting any M&A activities. There are also a few other circumstances under which the CSRC can refuse to sanction takeover activities by personal individuals. For example, under Article 147 of the Chinese Company Law, a person who is without or has limited capacity of civil conduct or a person who has a criminal conviction within 3 years prior to the takeover date will be prohibited by the CRSC from participating in any takeover activities. As we note above there are several other circumstances under which the CSRC will refuse to sanction takeover activities by individuals or firms; further details are to be found in the “Securities Law of the People’s

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<sup>10</sup> See Article 3 of the Takeover Measures, 2006.

<sup>11</sup> See Article 3 of the Takeover Measures, 2006.

Republic of China”, the “Company Law of the People’s Republic of China” and the “Takeovers Measures”.

## **Mandatory Bid Rules**

A mandatory bid rule sits at the heart of China’s takeover laws. Thus, under Article 61 of the Securities Law and Articles 23, 24, 25 and 83 of the Takeover Measures an investor who by himself or who in conjunction with other “concerted parties” (Yizhi Xingdongren) controls 30% or more of the equity shares of a listed target firm is required to make either a general or partial tender offer for the remaining shares in the affected target firm. Article 83 of the Takeover Measures defines “concerted parties” as those with whom the primary investor(s) is (are) acting in concert by means of private agreement or any other arrangement in order to boost their joint voting power in the target firm. Thus, the mandatory bid rule will apply to an investor himself or who in conjunction with other “concerted parties” jointly controls 30% or more of the shares in the target firm not only by means of co-jointly acquiring shares, but also by investment relationship, agreement, partnership cooperation, joint venture, simultaneously acting as directors, etc. Hence, not only the acquirer’s own shareholdings, but also the shareholdings of its concerted parties acting in concert will be counted when calculating an investor’s shareholding in a target firm.

Here it is important to note that the mandatory bid rule provides protection for shareholders of the target firm by ensuring that the control premium paid by the acquiring firm is equitably shared amongst all the shareholders of the target firm. But on the other hand, this kind of protection may come at the expense of the contestability of takeovers since the cost of the takeover may rise and some potential bidders may be dissuaded from being involved in the takeover process because of it. However, under certain circumstances the CSRC can exempt bidding firms and concerted parties from the mandatory tender provisions of the Takeover Measures. The exact conditions under which the exemption applies are given detailed consideration in a subsequent section of this paper.

## **Disclosure of Substantial Shareholdings**

Article 13 of the Takeover Measures taken in conjunction with Article 86 of the Securities Law requires the disclosure of substantial shareholdings in listed target firms (5% or more of the equity stock) and is meant to provide the market with an early warning of possible takeover activity. Article 22 of the Takeover Measures provides that a substantial shareholding in a target firm shall include not only the shares registered under the acquiring firm’s name but also shares held in conjunction with other concerted parties as well as those shares not registered under the acquiring firm’s name but for which the voting rights are actually controlled by the given acquiring firm.

Articles 13, 14 and 15 of the Takeover Measures require that if an acquiring firm coupled with its concerted parties come to hold 5% of the shares of a listed target firm by means of transactions on the stock exchange, transfer agreement, administrative transfer, implementation of court ruling, inheritance or donation, etc., then they must disclose their position to the market by submitting a written report which summarises the information

specified in Article 16 of the Takeover Measures. The written report must be submitted to the Head Office of the CSRC in Beijing as well as to the relevant stock exchange. The acquiring firm must also send a copy of the written report to the CSRC representative office in the locality of the target firm (hereinafter referred to as the representative office) and at the same time, formally notify the target firm that it has submitted a report to the CSRC and the stock exchange. The acquiring firm must also make a formal announcement to the general public within three business days of the date on which the substantial shareholding is acquired. Furthermore, the acquiring firm cannot continue to buy or sell any shares in the target firm until it has satisfied the provisions of Articles 13, 14 and 15 of the Takeover Measures; that is, until the market has been fully informed of its substantial shareholding in the target firm. Equally, Articles 13 and 14 of the Takeover Measures provide that if an acquiring firm along with its concerted parties increase or decrease their shareholding in the listed target firm by 5% by means of transactions in the stock exchange or transfer agreements, etc. (that is, by 5% to 10%, 10% to 15% and so on) they again must send a copy of the written report specified by Article 16 of the Takeover Measures to the CSRC and the stock exchange and they must also notify the target firm and the general public. During the disclosure period and for two days thereafter, the acquiring firm cannot buy or sell any further shares in the target firm.

Here it is important to note that there are two categories of disclosure report for substantial shareholdings under Article 16 of the Takeover Measures. Specifically, if the acquiring firm and their concerted parties are not the largest shareholder or the actual controlling shareholder of the listed target firm and their collective shareholding in the target firm is in excess of 5% but less than 20% of the shares on issue, then only the simplified report as specified in Article 16 is required. This simplified report includes the names and domiciles of the acquiring firm and their concerted parties, the timing and the method used by the acquiring firm and their concerted parties to acquire their shareholding in the target firm and a brief summary of the shares in the target firm purchased and sold on the stock exchange by the acquiring firm and their concerted parties over the previous six months.

The second category is when the substantial shareholding of the acquiring firm and their concerted parties exceeds 20% but is less than 30% of the total issued shares of the listed target firm. In this circumstance Article 17 of the Takeover Measures requires that a detailed report must be submitted to the relevant stock exchange and the Head Office of the CSRC in Beijing. A copy of the report must also be filed with the CSRC representative office. Also the target firm and the general public must also be notified within three business days from the date when the variation in the substantial shareholding occurs. The detailed report includes the information specified for the simplified report under Article 16 as well as a structural chart of the relationship between the shareholdings of the acquiring firm and their concerted parties in the target firm and the important transactions which have occurred between the acquiring firm and their concerted parties and the target firm over the previous two years. The detailed report must also provide a summary of the financing arrangements used to acquire the additional shares in the target firm. Furthermore, where inter-industry competition exists between the acquiring firm and the target firm, the detailed report must indicate what arrangements have been made to maintain the operational independence of the target firm.

The substantial shareholding disclosure threshold and regulations in China are broadly similar to those which apply in most advanced industrialised countries. In determining the threshold at which the market and other participants must be informed of a substantial shareholding, regulators must strike a balance across a variety of competing considerations.

For example, lower thresholds provide more protection for the shareholders of the target firm. Against this lower thresholds will make it difficult for the acquiring firm to obtain the foothold necessary to launch a successful takeover bid. It will also more than likely increase the price which the acquiring firm will have to pay in order to mount a successful takeover bid.

## **Tender Offer Rules**

As previously noted, Articles, 23, 24, 25 and 83 of the Takeover Measures provide that an acquiring firm which by itself or in conjunction with other concerted parties controls 30% or more of the equity shares issued by a listed target firm must make either a general or partial tender offer for the remaining shares in the target firm. A general tender offer is an offer made to all shareholders in the target firm to acquire the shares that the acquiring firm does not presently own. Thus, if the acquiring firm owns 30% of the shares in the target firm it will make a general tender offer to acquire the remaining 70% of shares on issue. A partial tender offer is an offer made to all the shareholders of the target firm for a fractional part of the shares not already held; subject to the requirement that the minimum tender offer must be for at least 5% of the shares in the target firm. Thus, if the acquiring firm owns 30% of the target firm's shares, the minimum partial tender offer will be to acquire 5% of the total issued capital thereby increasing the acquiring firm's interest in the target firm from 30% to 35% of the total issued shares.

The partial tender offer, which was not available before 2006 (only general tender offers existed prior to this date) represents a significant improvement in comparison to the takeover regulations which were previously in force as it effectively provides more flexibility for potential acquiring firms and thus reduces the transaction costs associated with takeovers. It is also important to note that Article 62 of the Takeover Measures provides that under certain circumstances acquiring firms may be exempted by the CSRC from the mandated tender offer regulations. The specific circumstances under which acquiring firms can apply for exemption are given detailed consideration in a later section of this paper.

Articles 36 of the Takeover Measures specifies that the acquiring firm may pay the consideration for a takeover in cash, securities, a combination of cash and securities or any other lawful mode of consideration<sup>12</sup>. Moreover, if the consideration for the takeover is to be paid in cash there must be a public announcement to that effect and the acquiring firm must deposit not less than 20% of the total amount of the takeover consideration as a performance guarantee with a bank designated by the China Securities Depository and Clearing Corporation.

On the other hand, Article 36 provides that if the acquiring firm pays the takeover consideration by means of securities, then it must ensure that the audited financial statements of the issuer of the said securities as well as a valuation report on the affected securities are made available to the target firm's shareholders. If the securities are not listed on the stock exchange, then Article 36 of the Takeover Measures also provides that the shareholders of the target firm must be offered a cash alternative to the securities.

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<sup>12</sup> Here it is important to note that prior to 2006 the consideration for all takeovers had to be in cash. This often caused difficulties for acquirers in the financing of takeovers.

It is also important to note that Article 35 of the Takeover Measures places a lower limit on the offer price which an acquiring firm can make for a listed target firm's shares. Specifically, the price the acquiring firm pays under a tender offer must not be less than the maximum price the acquiring firm has paid for any of the shares of the target firm over the six months preceding the announcement of the tender offer. Article 35 also provides that if the offer price is below the arithmetic average of the daily weighted average prices during the thirty trading days prior to the announcement of the tender offer, then a financial consultant must be hired by the acquiring firm to produce a report that covers issues such as whether there has been manipulation of stock prices, whether the acquiring firm has failed to disclose its concerted parties, whether there has been any other arrangement for the acquiring firm to obtain shares in the target firm during the previous six months and finally, whether the offer price is "reasonable" taking into account all of the circumstances and events surrounding the acquisition process.

Under article 28 of the Takeover Measures if the shares of the listed target firm are purchased by means of a tender offer, then the acquiring firm shall employ a financial consultant who must submit a written report (hereinafter referred to as the tender offer report) to the head office of the CSRC in Beijing as well as to the relevant stock exchange. The financial consultant must also send a copy of the tender offer report to the local representative office of the CSRC and inform the target firm about the pending tender offer. Article 29 of the Takeover Measures provides that the tender offer report should include the names and domicile of the acquiring firm and their concerted parties and the number of shares they hold in the target firm, the price and quantity of the shares to be purchased, details of the way the tender offer is to be financed, whether arrangements have been made to encourage intra-industry competition and the important transactions that have occurred between the acquiring firm and their concerted parties and the target firm over the previous two years. If in the opinion of the CSRC the tender offer report does not infringe any laws, administrative regulations and other related legal provisions then the tender offer report must be made available to the public fifteen days after its initial filing with the CSRC.

The Takeover Measures also place specific reporting and other responsibilities on the directors of the listed target firm. In particular, Article 32 of the Takeover Measures provides that the board of directors of the target firm must make an investigation into the capacity, credit status and purpose of the takeover by the acquiring firm and provide an analysis of the terms and conditions surrounding the tender offer. Moreover, the board of directors of the target firm must make a recommendation about whether or not the shareholders of the target firm should accept the tender offer and they must also hire an independent financial consultant to provide a professional opinion about each of the above issues. Within 20 days of the acquiring firm's tender offer report being made available to the public, the target firm must submit a report to the CSRC which summarises all of the above information. The CSRC will then make a decision about whether to allow the proposed takeover offer to proceed.

Moreover, Article 37 of the Takeover Measures provides that the term stipulated for acceptance of the tender offer should not be less than 30 days and not be more than 60 days from the date of the tender offer is made, except where there is a contested offer. The CSRC has adjudged that this period allows shareholders of the listed target firm sufficient time to make a rational decision about whether to accept the tender offer without prejudicing the interests of the acquiring firm. Under Article 42 of the Takeover Measures shareholders of the listed target firm who accept the tender offer must entrust a securities firm to go through the



related procedures for preliminary acceptance of the tender offer. The securities firm must apply to the China Securities Depository and Clearing Corporation for temporary custody of the shares under the preliminarily accepted tender offer. Shares under temporary custody of the China Securities Depository and Clearing Corporation are held in escrow.

The Takeover Measures also pay particular attention to the interests of minority shareholders after the takeover has been completed. If the tender offer expires and the acquiring firm has sufficient acceptances (normally at least 75 percent of all outstanding shares) then the acquiring firm may initiate proceedings to delist the target firm. In this circumstance, Article 44 of the Takeover Measures provides that the remaining shareholders in the target firm have the right to enforce the sale of their shares on the same terms and conditions as shareholders who have accepted the tender offer before the expiration date. This means that the remaining minority shareholders are protected from a “freeze-out” takeover on terms and conditions less favourable than the shareholders who have already accepted the tender offer before the expiration date.

## Defence Mechanisms

In an earlier section<sup>13</sup> we note how in April, 2005, the CSRC issued the shareholding structure reform called ‘Guquan Fenzhi Gaige’. Guquan Fenzhi Gaige required shareholders with tradable A shares in a particular firm to agree terms and conditions under which the non-tradable A shares in that firm will be converted into tradable A shares. It will be recalled that prior to 2005 the large majority of A shares were non-tradable and were mainly held by stated-owned controlled entities. This in turn made tender offers and hostile takeovers extraordinarily difficult. However, the gradual conversion of non-tradable shares into tradable shares after 2005 has facilitated an expansion in M&A activities with a consequent increase in the number of tender offers and hostile takeovers. This in turn has required that significant reforms be made to the takeover defence measures available to Chinese target firms. In response to this, the CSRC has incorporated some important improvements into the defence mechanisms available to listed target firms under the Takeover Measures.

Firstly, Article 8 of the Takeover Measures provides that when the board of directors of a listed target firm implement defensive measures against a potential takeover they must do so in such a way as to satisfy the fiduciary duties owed to the target firm and its shareholders. In particular, the defensive measures should be beneficial to the target firm and its shareholders and must not pose an inappropriate obstacle to the attempted takeover. Furthermore, the board of directors of the target firm must not provide financial assistance either directly or indirectly to the acquiring firm by making use of the target firm’s resources and nor may they damage the legitimate rights and interests of the target firm and its shareholders.

Secondly, under Article 33 of the Takeover Measures once the acquiring firm has filed the provisional tender offer documents with the CSRC and before the completion of the tender offer, the board of directors of the listed target firm must not take any defensive measures which might have a significant effect on the composition or value of the target firm’s assets, its liabilities, other entitlements or its business performance. In other words, when the board of directors of the target firm become aware of the pending tender offer they

<sup>13</sup> As in the section entitled “Unique Characteristics of Chinese Capital Markets”.

must not dispose of any of the target firm's assets, make any significant external investments or adjust in any way the main business of the target firm or give guarantees or loans on behalf of the target firm without the approval of the shareholders in general meeting. This requirement prevents the target firm from initiating activities which might frustrate the acquiring firm in its efforts to consummate the tender offer and also, from implementing any other activities which may not be in the best interests of the shareholders of the target firm. In other words, it implies that the catalogue of defensive measures taken by the target firm in the takeover is determined by the shareholders, and not the directors, which is quite similar to the "shareholder-based" model which underscores the City Code on Takeovers and Mergers in the United Kingdom.

### **Private Agreement Takeovers**

The socialist principles upon which the Chinese state is founded has meant that there are certain strategic industries which must remain predominantly under government control. In these industries whilst some shares may be traded by private individuals, most shares will remain under the control of the government and will not be available for trading on the stock exchange. As a result of these factors it is occasionally the case that it is impracticable for prospective acquiring firms to make tender offers for firms which operate in industries that are of strategic importance to the socialist principles upon which the Chinese state is founded. In such circumstances the only way a prospective acquirer can make a takeover offer for the target firm is to reach a private agreement with the Chinese government. Here, the Takeover Measures lay down detailed rules governing the way in which an agreement for takeover is to be reached between the prospective acquiring firm and the non-tradable shareholders.

First, if an acquiring firm intends to reach an agreement to purchase more than 30% of the issued shares of the target firm, then the shares that exceed the aforementioned 30% threshold must be acquired by means of a tender offer unless the acquiring firm applies for an exemption under Article 61 of the Takeover Measures. The specific circumstances under which acquiring firms can apply for exemption are given detailed treatment in the next section of this paper.

Second, the period between the signing of the agreement and the transfer of the related shares is called the transitional period. Article 52 of the Takeover Measures provides that it is only in exceptional circumstances that the acquiring firm can change the composition of the board of directors of the target firm during the transitional period. However, in such exceptional circumstances the directors from the acquiring firm must not exceed one third of the total number of all directors of the target firm. Furthermore, article 52 also provides that the target firm must not give any guarantee (financial or otherwise) to the acquiring firm or any of its concerted parties during the transitional period. In addition, unless the target firm is experiencing serious financial difficulties, it must not raise capital by the public issue of shares, conduct significant purchases or sales of assets or involve itself in any major investment or any other affiliated transactions with the acquiring firm or its affiliated parties during the transitional period.

Third, where there is a controlling shareholder of the target firm who transfers their shareholdings to the acquiring firm by means of agreement, then an investigation as to the capacity, credit status and the purpose of the takeover by the acquiring firm must be

conducted by an independent financial consultant appointed under the provisions of Article 50 of the Takeover Measures. The information obtained from the investigation must be reported to the CSRC. Moreover, if the controlling shareholder or any of its concerted parties has not paid off its debts to the target firm, or has not removed any guarantees that the target firm has provided for its debts, or is associated with any other circumstances that may damage the interests of the acquiring firm, then under Article 53 of the Takeover Measures the board of directors of the acquiring firm must disclose the aforementioned circumstances and also take effective measures to protect the interests of its shareholders. These two provisions of the Takeover Measures are designed to protect the shareholders of the acquiring firm from any conflicts of interest that may influence the motives of the controlling shareholder of the target firm.

Finally, under Articles 54 and 55 of the Takeover Measures both the acquiring firm and the target firm involved in the takeover must appoint a securities firm to apply to the China Securities Depository and Clearing Corporation for temporary custody of the shares to be transferred under the takeover agreement. They must also deposit the consideration for the purchase of the shares in the bank designated by the China Securities Depository and Clearing Corporation. Moreover, in accordance with the business operation rules of the stock exchange and the China Securities Depository and Clearing Corporation, after the related parties have agreed to go through with the takeover, the shares are removed from the temporary custody of the securities firm and transferred to the acquiring firm. The target firm shareholders then receive the consideration deposited with the bank designated by the China Securities Depository and Clearing Corporation.

### **Application of Exemption**

We have previously noted<sup>14</sup> that under the Takeover Measures an acquiring firm that controls 30% or more of the equity shares of a listed target firm is required to make either a general or partial tender offer for the shares that it does not already hold in the target firm. However, there are circumstances under which acquiring firms are able to apply to the CSRC for an exemption from the requirement to make a tender offer. Here Article 62 of the Takeover Measures provides that an acquiring firm may apply for an exemption from the requirement to make either a general or partial tender offer if the acquiring firm and the target firm can prove that the transfer of shares would not affect the ultimate overall control of the target firm<sup>15</sup>. Article 62 of the Takeover Measures also provides that an acquiring firm may apply for an exemption from the requirement to make a tender offer if the target firm is suffering from serious financial difficulties and the acquiring firm undertakes not to transfer the shareholdings and entitlements already gained in the target firm to a third party for at least 3 years. Unfortunately, Article 62 was largely silent about what constituted “serious financial

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<sup>14</sup> As in the sub-section entitled “Mandatory Bid Rules”.

<sup>15</sup> In February, 2012 the CSRC amended Articles 62 and 63 of the Takeover Measures so that an acquiring firm which already holds more than 50% of the shares issued by the target firm can be exempted from filing an exemption application to the CSRC. Also, the amended Articles 62 and 63 provide for the creation of an expert advisory committee which will give advice, accounting and asset appraisals for firms involved in acquisitions. This amendment is designed to reduce the Chinese government’s role in approving takeovers and instead allow professionals to have a greater input into the takeover process.

difficulties” and this caused a great deal of confusion for both the CSRC and firms which sought to apply for an exemption under this provision of the Takeover Measures. Given this in January, 2011 the CSRC promulgated an amendment to Article 62 which provides that a listed target firm shall be considered to be in serious financial difficulties if any of the following conditions apply:

- (1) it has recorded losses for the most recent two consecutive years;
- (2) the listing of its shares has been suspended as the company has recorded losses for three consecutive years;
- (3) the book value of its shareholders’ equity is negative at the end of the most recent year, or
- (4) it reported a loss in the most recent year and its main line of business has been terminated for at least six months.

### **Financial Consultant**

We have previously noted<sup>16</sup> how acquiring and target firms involved in takeovers are required to appoint a professional financial consultant. The Takeover Measures impose detailed obligations and responsibilities on the affected financial consultants. Article 66 of the Takeover Measures provides that the financial consultant appointed by the acquiring firm must issue a “financial consultation” report that includes the following information:

- (1) The purposes for the takeover as given by the acquiring firm;
- (2) An analysis of the financial status and credit status of the acquiring firm, including an assessment of whether the acquiring firm has access to the financial resources and managerial expertise necessary to successfully implement the takeover;
- (3) If the acquiring firm pays the consideration for the takeover in securities then the financial consultant must assess the liquidity of the securities offered as consideration for the takeover;
- (4) If there is intra-industry competition between the acquiring and target firms, the financial consultant must evaluate the arrangements which have been made to maintain the operational independence of the target firm, and
- (5) Whether the original controlling shareholder of the target firm has paid off its debts to the target firm and has removed any guarantees that the target firm has provided for the original controlling shareholder’s debts.

Likewise, Article 67 of the Takeover Measures provides that the target firm must appoint a financial consultant who must also issue a financial report. The report for the target firm must include the following information:

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<sup>16</sup> As in the sub-section entitled “General Principles of Measures for the Administration of the Takeovers of Listed Companies” and the sub-section entitled “Tender Offer Rules”.

- (1) Whether or not the acquiring firm has the financial capacity to purchase the target firm and whether the acquiring firm intends to use the assets or capital obtained from the target firm to finance the takeover;
- (2) If a tender offer is involved the financial consultant must evaluate whether the takeover price fully reflects the value of the target firm. The financial consultant must also make an explicit recommendation about whether the shareholders of the target firm should accept the tender offer;
- (3) If the acquiring firm pays the consideration for the takeover in securities then the financial consultant must conduct a valuation analysis of the said securities and recommend whether the offer made by the acquiring firm should be accepted by the target firm's shareholders, and
- (4) If the takeover involves a management buy-out, the financial consultant must make an assessment of the price set for the management buy-out, the method of payment, the sources of financing for the buy-out and the associated repayment plans.

## SUMMARY AND CONCLUSION

China is now the second largest economy in the world and an ever increasing number of foreign firms have been attracted by the favourable investment climate that has been put into place by the Chinese government. Many of these foreign firms have chosen to implement their investment activities through takeovers and/or mergers with local Chinese firms (Fei, 2004). However, relatively little is known outside of China about the legal framework under which M&A activities are conducted in China. This paper seeks to fill this gap in the literature by providing a summary of the laws and regulations that govern investment activities in the M&A area in mainland China. Our expectation is that this will be of particular interest to those who undertake research on the economic motivations for and economic consequences of, M&A activities in China. Our summary of the laws in this area will also be of importance to those who are considering or who are actively involved with merger and/or acquisition activities either directly or as a concerted party with local Chinese firms.

We commence our summary of the laws and regulations governing M&A activities in China with a brief consideration of the development of China's two mainland stock exchanges, their listing requirements as well as their distinctive characteristics. Probably the most important distinguishing characteristic of the mainland Chinese stock markets is that traded shares are comprised of A shares and B shares. With rare exceptions only domestic Chinese citizens can hold A shares whilst foreign investors are generally restricted to holding B shares. Furthermore, A shares are classified into three categories; namely, state-owned shares, legal person shares and public individual shares. Only public individual shares are freely tradable on the two mainland Chinese stock exchanges. However, in April, 2005, the Chinese government implemented the Shareholding Structure Reform - Guquan Fenzhi Gaige - under which non-tradable A shares will gradually be converted into tradable shares.

The most important laws and regulations governing M&A activities in China are the Takeover Measures and the Anti-Monopoly Law. The Takeover Measures cover such areas as the mandated bid rules, tender offer rules, the disclosure of substantial shareholdings and the

defence mechanisms which may be mounted against takeovers and mergers, etc. The CSRC is China's principal securities market regulator and is responsible for ensuring that all investment activities conducted on the two mainland stock exchanges conform to relevant Chinese laws, including the Takeover Measures. The role of the CSRC is broadly similar to that of the Securities and Exchange Commission (SEC) in the United States. The Anti-Monopoly Law details the mandatory pre-merger and acquisition notification process, the investigation procedures that are to be followed by the Ministry of Commerce (MOFCOM) and other government agencies and the procedures MOFCOM must follow for promulgating its decisions, etc.

We conclude our summary of the laws governing M&A activities in China with the observation that although the laws in this area are now more compatible with best international practice than at any time in the history of the People's Republic of China, they are still far from perfect. In particular, the Takeover Measures are a melange of the M&A laws that have been enacted in several different western countries. For example, the defence mechanisms summarised in Article 33 of the Takeover Measures are grounded in the "shareholder-based" model which underscores the City Code on Takeovers and Mergers in the United Kingdom. In contrast, the "due diligence" responsibilities of the directors of a target firm involved in a defence mechanism under Article 8 of the Takeover Measures are grounded in the "director-based" model which underscores the Delaware General Corporation Law in the United States (Huang, 2005). The potential inconsistencies which arise from the differing legislative origins on which the Takeover Measures are based have the potential to cause considerable confusion in M&A law and practice in China.

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*Chapter 9*

# A QUANTITATIVE MODEL OF SPECULATIVE ATTACK: GAME COMPLETE ANALYSIS AND POSSIBLE NORMATIVE DEFENSES

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## ABSTRACT

We analyze a kind of speculative-attack on government-bond markets, through Game Theory, applying the *Complete Analysis of a Differentiable Game* (introduced by David Carfi). We propose a way to stabilize government bonds prices in European “periphery”. We’ll focus on two economic-operators: an Investment Bank, the Speculator, and the European Central Bank. The Speculator, our first player, can influence the market and gain by creating arbitrage opportunities, as the crisis of Euro-bond has shown. In the model, European Central Bank, our second player, purchases government-bonds to stabilize their price and the normative-authority introduces a Tobin-tax on financial transactions preventing only “extra-profits” of speculation.

**Keywords:** speculative attacks, government bonds, complete analysis of a differentiable game, Tobin tax

## 1. SPECULATIVE ATTACKS IN OUR CONTEXT

In our paper, the term "speculative attack" means the sudden demand of foreign currency that the Central Bank has to face in order to rebalance supply and demand of its own currency. It is important to realize that this situation is caused by exogenous factors; it isn't related to the performance of the current "fundamentals of national economy", but to the

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expectation of changes in the exchange rate. Usually, speculative attacks affect the currencies of the countries that adhere to a fixed exchange rate regime. This happens because the fixed parity is a reference point for the speculators who bet more easily in the only direction (devaluation) of the exchange rate.

A speculative attack is determined by a well-organized and strong speculator's action. Through the theory of speculative attacks we try to explain the mechanism by which agents coordinate the demand of a strong currency. There exist other models of speculative attacks. The attack in our case is conducted through instruments that allow to create a misalignment between the current and the future market's price for government bonds issued by "peripheral" countries, and it is focused against economies that have structural weaknesses. This makes the weak countries increasingly vulnerable until these countries are forced to take extreme measures to avoid possible default.

## 2. DESCRIPTION OF THE STRATEGIC INTERACTION

The game  $G$  we propose takes place in two phases. In the first phase, the Speculator (first player) may decide to short sell bonds, in order to achieve future earnings, betting on the depreciation of the bonds, or to intervene not in the market. In the second phase, occurring immediately after the first, the ECB may decide to intervene in the market buying the same titles in order to limit the depreciation on them and to ensure the issuing States to raise the necessary funds to meet the requirement for national resources. Alternatively, if ECB considers the fluctuations in the valuations titles as normal trend, it can simply check the regularity.

Let's start with our model.

- 1) We assume there is a unit stock  $U$  of government bonds (usually, a unit  $U$  costs 1000 euro).
- 2)  $M$  represents the amount of government bonds issued, at a certain time, by a State, given in integer multiples of  $U$ .
- 3) We consider that the interests on the bonds should be paid in advance, with an anticipated rate interest  $d$ .
- 4) If  $d$  is the rate of interest at which the State sells the bonds, the conventional price at which the agents buy or short-sell the unit amount  $U$  of bonds is given by
- 5)  $P(d) = 100(1 - d)$ .
- 6) In our game, "short selling" is the financial transaction involving the sale of bonds without having the property of them, hoping to buy them back later at a lower price, thus realizing a profit. This operation is then implemented with the aim of obtaining a profit following a downtrend movement in prices of titles priced in the financial market.
- 7) The value  $P_t$  at time  $t$  of a government bond is related with its present value  $P_0$  by the linear relation  $P_t = P_0(1 + i)^t$ , where  $i$  is the risk-free interest on the financial market.

## 2.1. Strategies

Our first player is a Speculator who may choose to short-sell government bonds, in order to cause a depreciation of bonds itself and to achieve profit.

The Speculator may choose among the strategies  $x \in E := [0,1]$ , any  $x$  in  $E$  denotes a percentage of bonds  $M$ , short-sold for the operation.

- By strategy  $x = 0$  the Speculator doesn't make any financial transaction;
- By strategy  $x > 0$  the Speculator short-sells an amount  $xM$  of bonds;
- By strategy  $x = 1$  the Speculator short-sells the entire quantity  $M$ .

On the other hand, our second player, the European Central Bank, operates on government bond market in consequence of the intervention of the first player, choosing a strategy:

$$y \in F := [0,1]$$

where  $y$  indicates the percentage of bonds  $M$  that the ECB may buy.

- by strategy  $y = 0$ , the ECB doesn't intervene in the market;
- by  $y > 0$ , ECB buys the amount of government bonds  $yM$ ;
- by strategy  $y = 1$ , the ECB buys the maximum amount of bonds ( $M$ ).

We illustrate below the bi-strategy space:  $E$  times  $F$ .

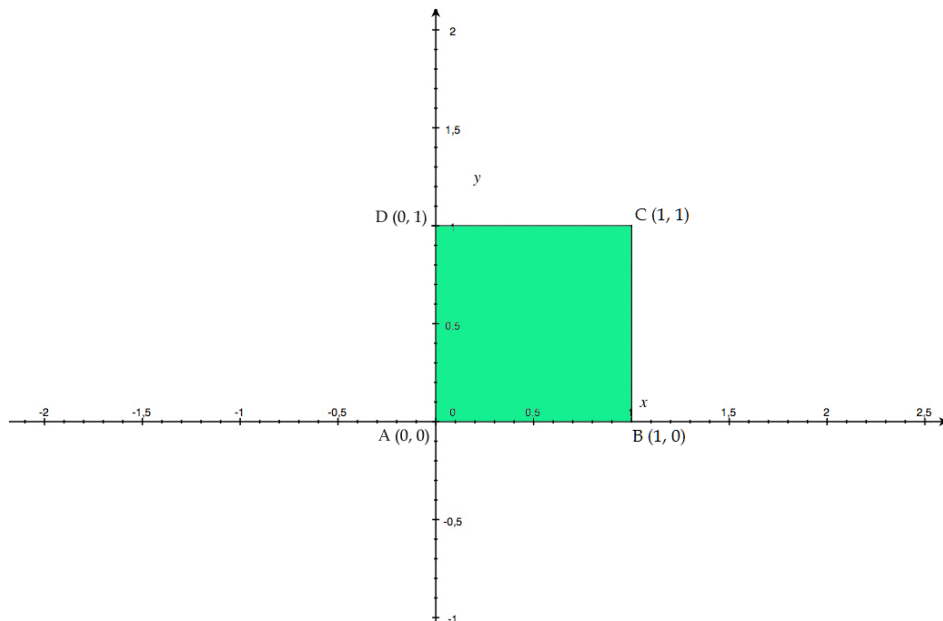


Figure 1. The bi-strategy space.

## 2.2. The Speculator's Payoff Function

The Speculator's payoff function is the function giving the gain of the first player at time 1. It is defined by the gain deriving from short-selling an amount  $xM$  of bonds at 0 minus the loss deriving from the purchase of the bonds themselves at time 1. At any bi-action  $(x,y)$ , the value  $f_1(x,y)$  is the quantity  $xM$  multiplied by the difference

$$P_0(1 + i) - P_1(x, y),$$

between the price at 0 (capitalized at 1 by the risk-free rate  $i$ ) and the unit price  $P_1(x, y)$  of the bonds at time 1, this latter price depends on both the actions  $(x,y)$  chosen by the two players within time 0 and time 1.

### 2.2.1. Tobin Tax

We propose, in order to regulate speculations and to offer profit to the ECB and to the States in economic troubles, to introduce - by regulatory authorities - a tax on the speculative financial transactions (a Tobin Tax) that affects the gain of the Speculator, so that *the speculator can't take unfair advantage from the oscillation of the title price that he has caused*, note that our tax will not inhibit at all the possibility of achieving profits but it will allow to fairly re-distribute the profits to ESB and States.

### Assumption

We impose the following Tobin tax:

$$T(x,y) := a yM (x - x^2),$$

where the term  $x^2$  reduces the incidence of the tax and  $a$  is a certain coefficient.

*The Speculator's payoff function* is so defined by:

$$f_1(x, y) = xM [P_0(1 + i) - a (b + y - x)] - a yM (x - x^2)$$

where:

- 1)  $xM$  is the amount of bonds that the Speculator short sells at time 0, corresponding to the strategy  $x$ ;
- 2)  $P_0(1 + i)$  is the price of the title at time 0 capitalized for a period, where  $(1 + i)$  is the capitalization factor at the free risk rate  $i$ ;
- 3)  $P_1(x, y) := a (b + y - x)$  is the price of the title at time 1, virtually calculated as new intersection between the demand curve and the supply curve of the government bonds. This value depends on a coefficient  $a$ , which indicates the marginal incidence of the strategies of the two players on the equilibrium price, and on a coefficient  $b$ , which is an index depending on the amount of government bonds offered by the state;
- 4)  $T(x,y) = a yM (x - x^2)$  is the tax on financial transactions.

The Speculator payoff function is so defined by:

$$f_1(x, y) = xM [P_0(1 + i) - P_1(x, y)] - a yM (x - x^2)$$

where we impose:

$$P_1(x, y) = a(b + y - x).$$

From

$$f_1(x, y) = xM [P_0(1 + i) - a(b + y - x)] - a yM (x - x^2),$$

setting  $M = 1$ ,  $a = 1$ ,  $b = 1$  and  $P_0 = 1$  (we consider 100 as a unit), we have:

$$f_1(x, y) = x^2(1 + y) - 2xy + ix.$$

### Remark

If the Speculator achieves profits with certainty, then necessarily we deduce:

$$P_0 > a(b + y - x),$$

so, assuming the ECB chooses the worst possible strategy for Speculator (i.e.  $y = 1$ ) we have:

$$P_0 > a(b + 1 - x).$$

The first player's best reply to  $y = 1$  is  $x = 1$ , it follows that  $P_0 > ab$  (or at least  $P_0 = ab$ ).

### Theorem

The relation  $P_0 > ab$  (or at least  $P_0 = ab$ ) is a necessary condition to the gain of the Speculator.

We shall not consider the case  $P_0 < ab$ , in which the Speculator has no convenience to operate in the bond's market.

## 2.3. ECB Payoff Function

The ECB payoff function, which is the function that represents the gain of the ECB, is determined by the product of the quantity of titles bought, which is:

$$yM$$

multiplied by the difference between the price at time 1 of the title given by  $P_1(x, y)$  and the price of the title capitalized at time 0, represented by the value  $P_0(1 + i)$ , then:

$$P_1(x, y) - P_0(1 + i)$$

add to this the value of tax on transactions, which we assume goes to increase the reserves of the ECB, which is represented by the following equation:

$$ayM(x - x^2).$$

The ECB payoff function is given by:

$$f_2(x, y) = yM [P_1(x, y) - P_0(1 + i)] + ayM(x - x^2)$$

where:

- 1)  $yM$  is the amount of titles that the ECB buys to stabilize the price;
- 2)  $P_1(x, y) = a(b + y - x + x^2)$  is the price at time 1 of the bond, based on the new intersection between the demand curve and the offer of titles. We note that the value of the title, in addition to depend on the coefficients  $a$  and  $b$ , depend on the strategy put in place by the Speculator, represented by  $x$ , and by  $x^2$  which causes the variation in the application of titles can't be perfectly compensated, at the price level, to the corresponding action of the ECB;
- 3)  $P_0(1 + i)$  is the price at time 0 after capitalization by  $(1 + i)$ ;
- 4)  $ayM(x - x^2)$  is the tax on financial transactions.

The ECB payoff function is:

$$f_2(x, y) = yM [P_1(x, y) - P_0(1 + i)] + ayM(x - x^2)$$

where:

$$P_1(x, y) = a(b + y - x + x^2)$$

and so, we obtain

$$f_2(x, y) = yM [a(b + y - x + x^2) - P_0(1 + i)] - ayM(x - x^2)$$

recalling our choices for the constants -  $M = 1$ ,  $a = 1$ ,  $b = 1$  and  $P_0 = 1$  - we have:

$$\begin{aligned} f_2(x, y) &= y [1 + y - x + x^2 - 1 - i] + y(x - x^2) = \\ &= y^2 - xy + x^2y - iy + xy - x^2y \\ &= y(y - i). \end{aligned}$$

Thus, the second player payoff function is:

$$f_2(x, y) = y(y - i).$$

### 3. GAME STUDY: PAYOFF SPACE

In this section we face the more technical part of the Complete Analysis: the complete knowledge of the payoff space. This knowledge requires several steps.

#### 3.1. Critical Space

Our game  $G$  is structured as it follows: strategy set and payoff function of the first player are

$$E = [0, 1] \quad f_1(x, y) = x^2(1 + y) - 2xy + ix.$$

strategy set and payoff function of the second player are

$$F = [0, 1] \quad f_2(x, y) = y(y - i).$$

Since  $G$  is a non-linear game, we should determine the points of the critical zone, which belong to the bi-strategy space. We calculate the Jacobian determinant and find the point at which it is equal to 0, to determine the game critical area.

The gradients of  $f_1$  and  $f_2$  are defined by:

$$\text{grad } f_1(x, y) = (2x(1 + y) - 2y + i, x^2 - 2x),$$

$$\text{grad } f_2(x, y) = (0, 2y - i).$$

The Jacobian determinant is:

$$\det J_f(x, y) = (2x(1 + y) - 2y + i)(2y - i).$$

Therefore the critical space of this game is:

$$Z := \{(x, y) : (2x(y + 1) - 2y + i)(2y - i) = 0\}.$$

The determinant is equal to 0 when  $(2y - i) = 0$ , i.e.  $y = i/2$ , or when

$$2x(y + 1) - 2y + i = 0, \text{ i.e. } x = (2y - i)/(2y + 2).$$

The critical zone is shown in the following figure.

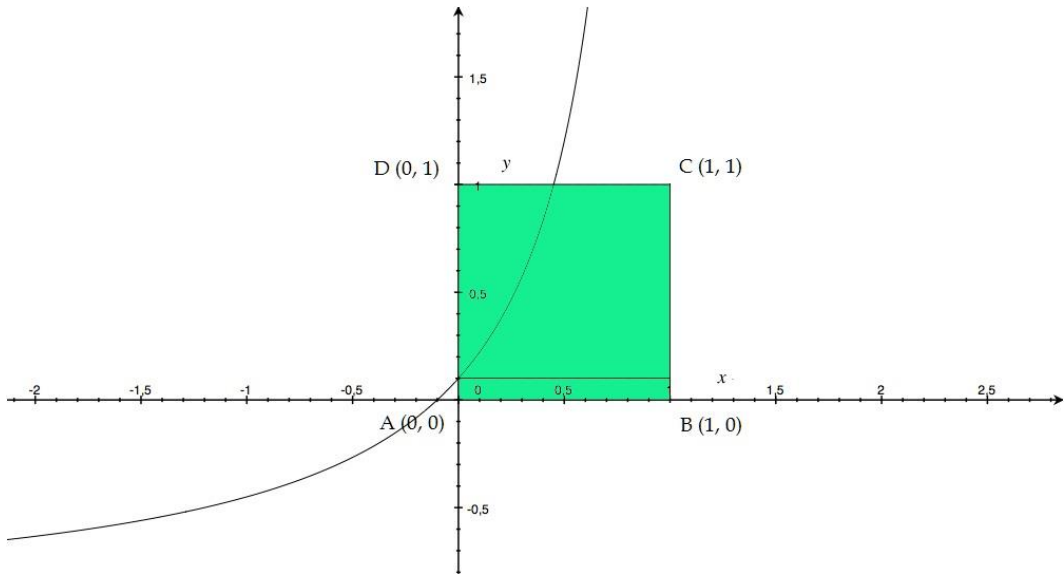


Figure 2. The game critical space.

### 3.2. Payoff Space

To graph the payoff space  $f(E \times F)$  we transform all the sides of the bi-strategy rectangle and the critical space  $Z$ , by the function  $f$ .

1) The Segment  $[A, B]$  is the set of All Bi-Strategies  $(x, y)$  Satisfying the Following Conditions:

$$y = 0 \quad \text{and} \quad x \in [0, 1].$$

Computing the image of the generic point  $(x, 0)$ , we have:

$$f(x, 0) = (x^2 + ix, 0).$$

Then, in the payoff universe, we have the parametric curve:

$$(X(x) = x^2 + ix \ \& \ Y = 0) \ (x \text{ in } E).$$

Since  $x \in [0, 1]$  and the first payoff  $X(x)$  is increasing in our range, the image of segment  $[A, B]$  is the part of abscissa-axis between 0 and  $1 + i$ .

We assume the risk-free interest rate as  $i = 20\%$ , the transformation will be the segment  $[A', B']$  between 0 and 1.2.

2) The Segment  $[B, C]$  is the Set of All Bi-Strategies  $(x, y)$  Satisfying the Conditions

$$y \in [0, 1] \quad \text{and} \quad x = 1.$$



Calculating the image of the generic point  $(1, y)$ , we have

$$f(1, y) = (1 + i - y, y(y - i)).$$

Then we have the parametric curve:

$$(X(y) = 1 + i - y \ \& \ Y(y) = y(y - i)) \ (y \text{ in } F).$$

We have:

$$y = 1 + i - X$$

and consequently

$$Y = (1 + i - X)(1 - X) = X^2 - (2 + i)X + 1 + i,$$

i.e. a parabola segment with vertex  $(1 + i / 2, -i^2 / 4)$  between the extremes

$$f(B) = f(1, 0) = (1 + i, 0),$$

$$f(C) = f(1, 1) = (i, 1 - i).$$

Thus, for  $i = 20\%$ , we have:  $B' = (1.2, 0)$  and  $C' = (0.2, 0.8)$ .

- 3) The Segment  $[C, D]$  is the Set of All Bi-Strategies  $(x, y)$  Satisfying the Conditions  $y = 1$  and  $x \in [0, 1]$ .

Computing the image of the generic point  $(x, 1)$ , we have

$$f(x, 1) = (2x^2 + (i - 2)x, 1 - i).$$

Then we have the parametric curve:

$$(X(x) = 2x^2 + (i - 2)x \ \& \ Y = 1 - i) \ (x \text{ in } E).$$

We consider the minimum of the function  $X$ , attained at  $x = 1/2 - i/4$ , that is

$$X_{\min} = \min(X) = 2(1/2 - i/4)^2 + (i - 2)(1/4 - i/4) = -(2 - i)^2/8.$$

For  $i := 0.2$ , we have  $X = -(9/5)^2/8 = -0.405$ .

- 4) The Segment  $[A, D]$  is the Set of All Bi-Strategies  $(x, y)$  Satisfying the Conditions  $y \in [0, 1]$  and  $x = 0$ .

Computing the image of generic point  $(0, y)$ , we have:

$$f(0, y) = (0, y(y - i)).$$

Then we have the parametric curve

$$(X = 0 \text{ et } Y = y(y - i)) \text{ (} y \text{ in } F).$$

The function  $Y$  has minimum value  $-i^2/4$  and goes up to  $1 - i$ . For interest rate  $i = 20\%$ , we have that the transformation is the segment  $[A', D']$ , where the function  $Y$  ranges from  $-0.001$  to  $0.8$ .

Finally we must transform the game critical space

$$x = (2y - i)/(2y + 2) \text{ and } y = i/2.$$

After considering the parametric curve

$$(X(y) = f_1((2y - i)/(2y + 2), y) \text{ et } Y(y) = f_2((2y - i)/(2y + 2), y)) \text{ (} y \text{ in } F),$$

we use a software to obtain its representation. Below you find the complete representation.

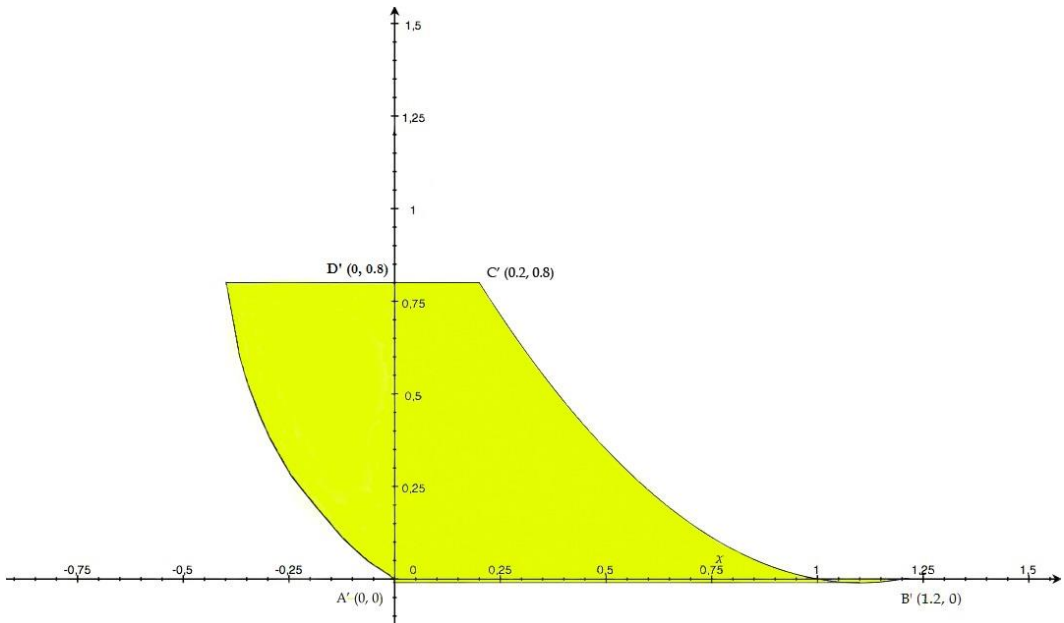


Figure 3. The game payoff space.

### 4. GAME STUDY: BEHAVIORAL ANALYSIS

In this section we proceed with the complete analysis of the players possible behaviors.

### 4.1. Extremes and Pareto Analysis

The supremum of the game  $G$ , that is the bi-gain  $(1 + i, 1 - i)$ , it is a shadow maximum because it doesn't belong to the payoff space. We calculate the game infimum by minimizing the function

$$f_1(x, y) = x^2(1 + y) + x(i - 2y)$$

in bi-strategy space starting from the sides of the domain: on  $[A, B]$  we have  $f_1(x, y) = ix + x^2$  and the minimum is 0; on  $[A, D]$  we have  $f_1(x, y) = 0$ , so the minimum is 0; on  $[B, C]$  we have  $f_1(x, y) = 6/5 - y$  and the minimum is  $1/5$ ; on  $[C, D]$  we have  $f_1(x, y) = 2x^2 - (9/5)x$ , the minimum point is  $9/20$  and the minimum is  $-0.405$ .

The game infimum is the bi-gain  $(-(2 - i)^2/8, -i^2/4)$ , it is a shadow minimum because it doesn't belong to the payoff space.

*The Pareto maximal boundary of the payoff space* is the curve from  $B'$  to  $(1,0)$  and from  $(1,0)$  to  $C'$ , lying on the topological boundary of the payoff-space.

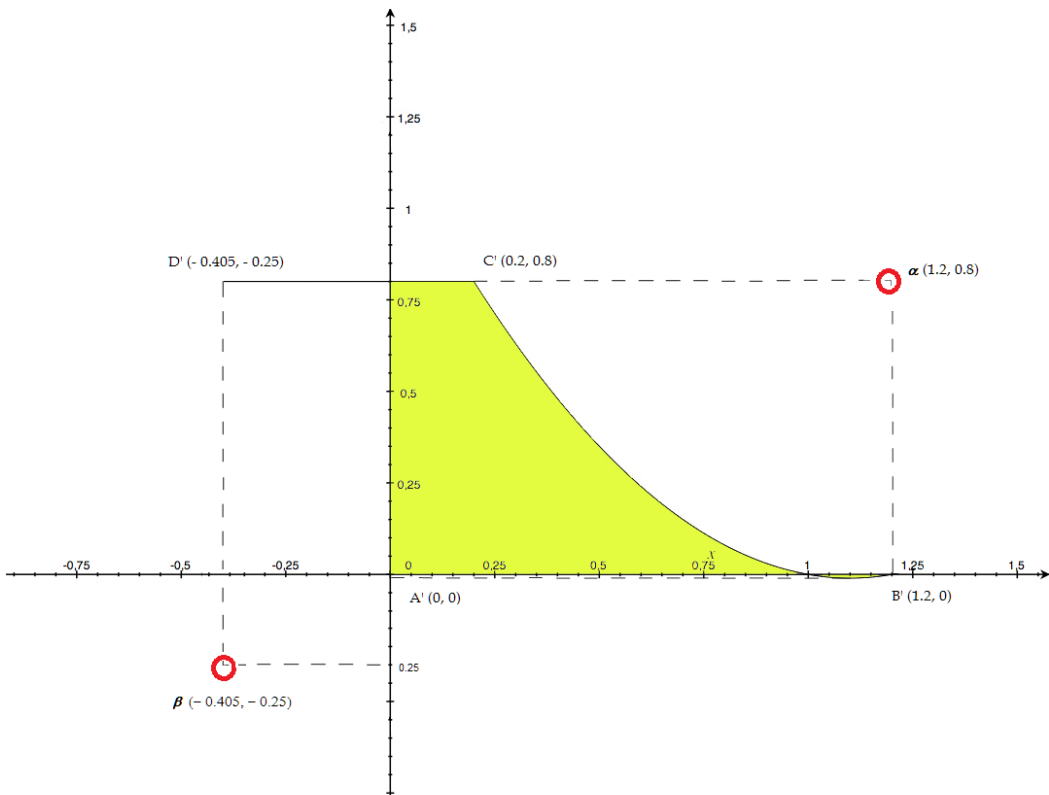


Figure 4 Game extremes.

## 4.2. Nash Equilibria

We calculate, as usual, the best reply correspondence by maximizing, for each player  $j$ , the payoff function  $f_j$ , after fixing every possible strategy of the other player  $3 - j$ . The Speculator's best reply correspondence is defined by:

$$B_1 : F \rightarrow E : y \rightarrow \max (E, f_1(., y)),$$

where  $\max (E, f_1(., y))$  is the set of all the Speculator's strategies maximizing the section  $f_1(., y)$ .

On the other hand, the ECB's best reply correspondence is defined by:

$$B_2 : E \rightarrow F : x \rightarrow \max (F, f_2(x, .)),$$

where  $\max (F, f_2(x, .))$ , is the set of all the ECB's strategies maximizing the section  $f_2(x, .)$ .

### *Speculator's Best Reply*

Recalling that:

$$f_1(x, y) = x^2(1 + y) - 2xy + ix,$$

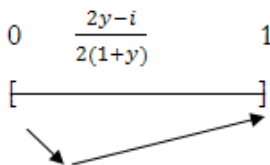
we have:

$$D_1 f_1(x, y) = 2x(1 + y) - 2y + i > 0 \text{ iff } x > x^* := (2y - i)/(2 + 2y).$$

Note, first of all, that  $x^*$  can assume the maximum value of 1, indeed:

$$x^* = (2y - i)/(2 + 2y) < 1$$

is equivalent to  $2y - i < 2 + 2y$ , that is always true; note, on the contrary that  $x^*$  is positive if and only if  $y > i/2$ . When  $y < i/2$ , the section  $f_1(., y)$  is increasing on  $E$ , and the maximum point (best reply) is 1. When  $y = i/2$  the situation is similar. When  $y > i/2$  the situation is described by the below graph:



Concluding, since

$$f_1(1, y) = (1 + i) - y > 0 = f_1(0, y),$$

(the rate of interest  $i$  is strictly positive) we have:

$$B_1(y) = 1 \quad \text{if} \quad y < i/2,$$

$$B_1(y) = 1 \quad \text{if} \quad y = i/2,$$

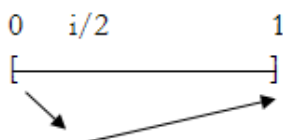
$$B_1(y) = 1 \quad \text{if} \quad y > i/2.$$

Let us continue with the second player:

$$f_2(x, y) = y(y - i)$$

we have:

$$D_2 f_2(x, y) = 2y - i > 0 \text{ iff } y > i/2;$$



$$B_2(x) = F \quad \text{if} \quad x > 0.$$

In red you see the graph of  $B_1$  and in blue that of  $B_2$ , we have:

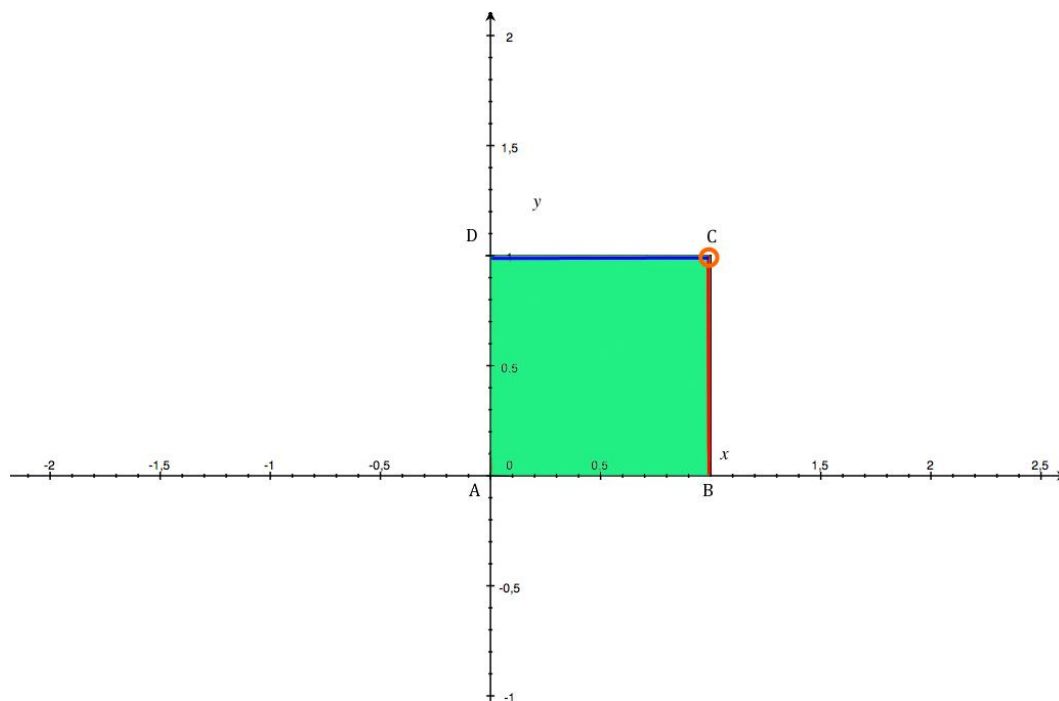


Figure 5 Nash Equilibrium.

The set of Nash equilibria, which derive from the intersection of best reply two graphs is:

$$\text{Eq}(B_1, B_2) = \{(1, 1)\}.$$

The Nash equilibrium can be considered good because it is on the Pareto maximal boundary. This means that if the two players only think about profit, and decide to choose the selfish strategy to obtain the maximum payout possible, will arrive on the Pareto maximal boundary. Selfishness, in this case, is convenient.

**Remark**

Note that  $f_1(0, y) = 0$  and  $f_1(1, y) = (1 + i) - y$ , so  $1 + i - y > 0$  if and only if  $y < 1 + i$  (this is always true when  $i > 0$ ). When  $i$  is not necessarily positive, we have:

$$B_1(y) = 1 \quad \text{if} \quad y < 1 + i$$

$$B_1(y) = 0 \quad \text{if} \quad y > 1 + i$$

$$B_1(y) = \{0, 1\} \quad \text{if} \quad y = 1 + i.$$

In this latter case, we have not anymore a dominant strategy for the Speculator.

**Remark (Dominant Strategy for the First Player)**

The dominant strategy of the first player is 1. We see immediately this from the form of the best reply correspondence. We could see it also directly. Indeed, note that  $f(1, y) > f(x, y)$ , for  $x \in [0, 1]$  and for  $y$  in  $[0, 1]$ . Indeed:

$$(1 + i) - y > (i - 2y)x + x^2(1 + y)$$

is equivalent to

$$(i - 2y) + (1 + y) > (i - 2y)x + x^2(1 + y),$$

that is always true.

### 4.3. Defensive Phase

Suppose that both players are cautious, fearful or risk averse and that they choose the strategies that allow them to minimize their losses, in this case, they shall adopt a *defensive behavior*.

**Conservative Value of a Player**

It is the maximum of his worst gain function:

$$v_1^\# = \sup_E f_1^\#,$$

where  $f_1^\#$  is the Speculator worst gain function and it is given by:

$$f_1^\#(x) = \inf_{y \in F} f_1(x, y)$$

recalling that:

$$f_1(x, y) = x^2(1 + y) - 2xy + ix,$$

we have:

$$f_1^\#(x) = \inf_{y \in F} (x^2(1 + y) - 2xy + ix).$$

In order to minimize the worst gain function, consider the derivative:

$$D_2 f_1(x, y) = -2x + x^2 = x(x - 2).$$

Since  $x$  lies in  $[0, 1]$ , we have  $D_2 f_1 < 0$ ; so the minimum point of the section  $f_1(x, \cdot)$  is  $y = 1$ , from this we have:

$$f_1^\#(x) = (i - 2)x + 2x^2.$$

Now, let us calculate the conservative value  $v_1^\# = \sup_{x \in E} f_1^\#$ . We have to maximize the function  $f_1^\#$ , it is increasing after  $(2 - i)/4$ , hence the function  $f_1^\#$  have as a minimum point the strategy  $(2 - i)/4$ ; so, in the domain  $[0, 1]$  the point maximizing the function is 1, consequently the strategy  $x^\# = 1$ ; so that  $v_1^\# = i$ . Now, let us calculate  $v_2^\# = \sup_F f_2^\#$ , where  $f_2^\#$  is the ECB worst-gain function and it is given by

$$f_2^\#(y) = \inf_{x \in E} f_2(x, y),$$

for every  $y$  in  $F$ . Recalling that:

$$f_2(x, y) = y(y - i),$$

we have

$$f_2^\#(y) = \inf_{x \in E} (y(y - i)) = y(y - i),$$

it follows that:

$$v_2^\# = \sup_{y \in F} (y(y - i)).$$

The worst gain function of the second player is increasing from the point  $i/2$ , so, in  $F = [0, 1]$ , the maximizing strategy is 1, consequently the unique conservative strategy for the ECB is  $y^\# = 1$  and the conservative value  $v_2^\#$  is equal to  $1 - i$ . Resuming, we have  $v^\# = (v_1^\#, v_2^\#) = (i, 1 - i)$ .

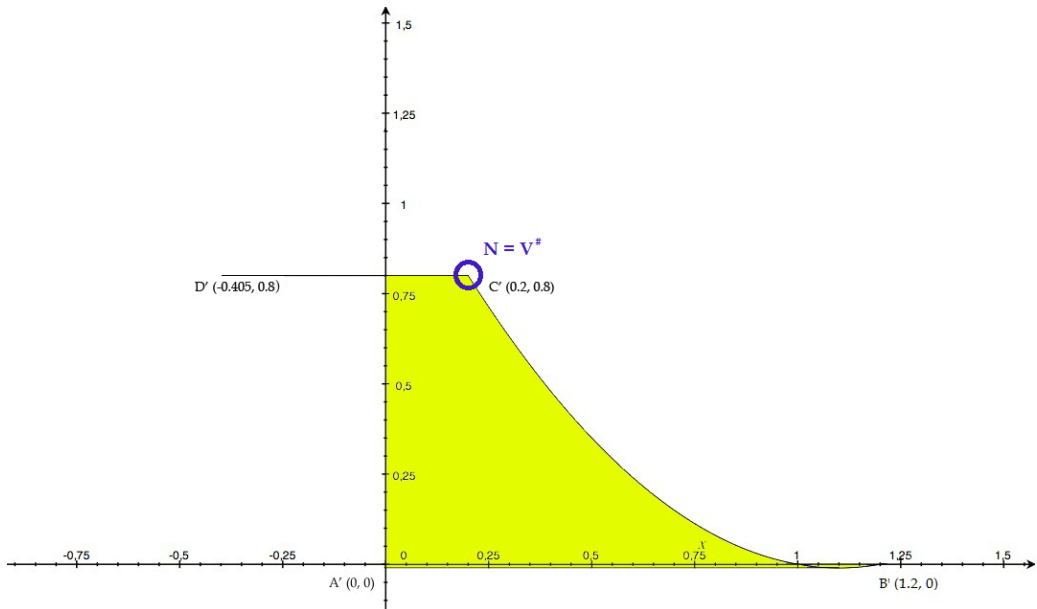


Figure 6. Nash Equilibrium and  $v^\#$ .

The **conservative cross** is given by the bi-strategy  $(x^\#, y^\#)$ , which is the point C. If the Speculator and the ECB decide to defend themselves from the strategies of the other they arrive to the conservative cross and then, in the payoff space, at the point C', which belongs to the Pareto maximal boundary. Moreover, the point C' is the Nash equilibrium payoff of our game.

The game G is inessential, i.e., the conservative bi-value belongs to the Pareto maximal boundary; moreover, if the players choose to maximize their gains or whether they choose for a defensive behavior, they arrive on the same payoff C'.

#### 4.4. Offensive Phase

If one player wants to ruin the other one, he would choose the strategy that, for any strategy of the other player, maximizes the loss of the other. In this case, we consider the worst offensive correspondence.

The ECB worst offensive correspondence versus the Speculator is the correspondence

$$O_2: E \rightarrow F : x \rightarrow \min (F, f_1(x, \cdot))$$

where  $\min (F, f_1(x, \cdot))$  is the set of ECB's strategies minimizing the section  $f_1(x, \cdot)$ . The Speculator worst offensive correspondence is:

$$O_1 : F \rightarrow E : y \rightarrow \min (E, f_2(\cdot, y)),$$



where  $\min (E, f_2(.,y))$  is the set of Speculator's strategies minimizing the section  $f_2(.,y)$ .

Recalling that:

$$f_1(x, y) = x^2(1 + y) - 2xy + ix$$

we have:

$$O_2(x) = F \text{ if and only if } x = 0$$

$$O_2(x) = 1 \text{ if and only if } x > 0.$$

Recalling that  $f_2(x, y) = y(y - i)$ , we obtain:

$$O_1(y) = E, \text{ for every } y \text{ in } [0, 1].$$

The set of offensive equilibria is the union of  $[A,D]$  with  $[D,C]$ .

## 5. COOPERATIVE SOLUTION

We may, now, consider a solution leading to the maximization of profit for both players, a cooperative agreement between the ECB and the Speculator.

One way would be to share the maximum collective profit, by determining the maximum of the collective gain function:

$$g(X, Y) = X + Y$$

in the payoff space of game  $G$ . Let  $V := \max_s g$ , which is attained at the point  $B'$ , which is the only bi-gain belonging to the line:

$$X + Y = 1 + i$$

and to the payoff space.

Once we have obtained the payoff  $B'$ , with a binding agreement, we need to calculate  $K'$  as the intersection between the line of maximum collective profit and the line passing through the Nash Equilibrium payoff  $C'$  and the sup of the collective Pareto boundary beyond (greater than)  $C'$ . In other terms, we have suggested a Kalai-Smorodinsky solution of the bargaining problem in which the decisional constraint is the Pareto boundary of maximum collective utility and the threat point is the payoff  $C' = v^\#$ .

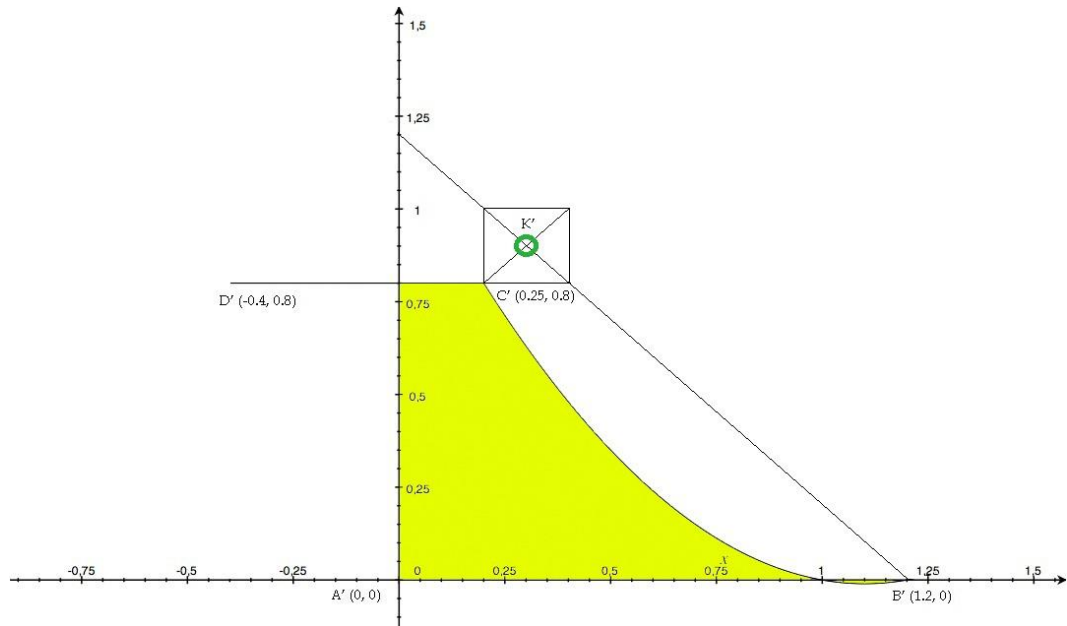


Figure 7. Kalai Smorodinsky solution.

## DISCUSSION AND CONCLUSION

### The Problem

For speculative activity we mean an activity of trading whose main purpose is not the purchase of goods for consumption or the earn from the production; rather, the main purpose is to make money on the difference expected between the purchase price and the selling price. For example, in the dynamics of currency crises the central bank loses resources in favor of the speculators who bet on the change of the value of the national currency in terms of foreign currency. The positions of economists about the problem of speculation are far from appear well defined.

### The Model and its Positive Features

We have modeled a particular trading activity, which is beneficial to the functioning of the market, instead of leading to the loss of public resources in favor of private operators: this positive effect derives from the introduction of a particular non-linear Tobin tax. Moreover, we show a financial situation that could be considered in agreement with the thought of the *Nobel Prize Milton Friedman*, who argues that speculation, if destabilizing is not profitable. That is, if speculators can earn and make profits, their activities must necessarily be stabilizing. Friedman says that speculation is inherently stabilizing, in fact, destabilizing speculation is "naturally" removed from the perfect market functioning since it determines losses and becomes unsustainable.

## Solutions of the Model

In the case-study of our analysis, we observe that the effects of the speculative attack, thanks to the Tobin tax, are positive for the community. On the contrary, very often, subjects acting to speculate in the financial markets, manage large masses of private savings and to achieve the maximum profits creates the same for large imbalances in the market. The result of these operations is the erosion of public resources in the form of central bank reserves and rapid economic changes in large areas of society.

- We have a dominant unique Nash equilibrium (coinciding with the unique conservative cross) in which the speculator gains and speculates as much as he can, but in which a consistent part of the possible gain guaranteed by the financial interaction is taken by the ECB.

- We have also found a bargaining solution that allows to gain all the possible maximum collective gain and that allows to share fairly this gain in such a way that again a consistent part of the maximum gain guaranteed by the financial interaction is taken by the ECB.

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