MODERN MATHEMATICAL PHYSICS: GRAVITY, SUPERSYMMETRY, INTEGRABILITY

CONTENTS

INTRODUCTION

The topics of main focus in the theme were:

- Supersymmetry and Superstrings;
- Quantum Groups and Integrable Systems;
- Quantum Gravity and Cosmology.

In the gravitation theory new solutions were constructed which describe extremal multicenter black holes. An important impact of vacuum Yang-Mills condensates on both the inflationary and the hot universe regimes was demonstrated. In the framework of the Galilean cosmology the new scaling solutions were discovered and investigated. An important problem of observational astronomy is the experimental detection of the black hole parameters. To this end, the interesting technique based ot the mechanism of shadows arising in the vicinity of super-massive black holes was proposed. The geometrical and topological properties of the full symmetric Toda systems were analyzed, and its equivalence to the Morse-Smale system was demonstrated. The development of the is proposed in the framework of the quantum field theory. On the basis of spectral summation method a rigorous derivation of the Lifshitz formula for the vacuum forces between material bodies was carried out. The quantum-mechanical systems of particles with extended worldline supersymmetry were studied. These investigations are relevant to a number of hot topics in modern theoretical physics, such as the AdS/CFT correspondence in diverse dimensions, the structure of supersymmetric integrable systems and their multiple relationships with N=4 super Yang-Mills theory and superstring theory. New one-dimensional systems with extended N=4 supersymmetry were constructed, including those containing couplings with external gauge fields. One of the most important results in this direction became the discovery of the fact that the necessary and sufficient conditions of the existence of N=4 supersymmetry in a wide class of models are equivalent to the famous Nahm equations.

Several results as well as full list of publications are presented below.

A.P. Isaev
A.S. Sorin

PSEUDOTORIC STRUCTURES AND NONSTANDARD LAGRANGIAN TORI

N.A. Tyurin
BLTP JINR

Abstract

We present a generalization of toric structures on compact symplectic manifolds called pseudotoric structure. We show that every toric manifold admits pseudotoric structures and then the construction of exotic Chekanov tori can be peformed in terms of pseudotoric structures.

Let (X, ω) be a symplectic manifold of real dimension $2n$ so it can be understood as the phase space of a classical mechanical system. Lagrangian geometry of X is focused on the questions about lagrangian submanifolds of X namely: which homology classes from $H_n(X, \mathbb{Z})$ can be realized by smooth lagrangian submanifolds; what are the topological types of these lagrangian submanifolds; classification up to lagrangian deformations of lagrangian submanifolds of the same topological type and homology class; classification up to Hamiltonian isotopy of lagrangian submanifolds of the same deformation type; unification of all lagrangian submanifolds in an appropriate category.

Recall that $S \subset X$ is lagrangian if the restriction $\omega|_S$ vanishes identically and real dimension of S is maximal, equal to n. Thus at least the homology class of S must be perpendicular to the cohomology class $[\omega]$. Two lagrangian submanifolds $S_0, S_1 \subset X$ are of the same deformation type if there is a family of lagrangina submanifolds $S_t, t \in [0, 1]$ which ends at S_0 and S_1. Hamiltonian isotopy of lagrangian submanifold $S_0 \subset X$ is given by a time dependent Hamiltonian function $H(x, t) : X \times \mathbb{R} \to \mathbb{R}$ which generates the flow ϕ^t_H, and $S_t = \phi^t_H(S_0)$ is the corresponding isotopy.

Toy example: dim $= 2$**.** Let Σ be a Riemann surface equipped with a symplectic form. Then since every loop is lagrangian (dimensional reason): every primitive homology class from $H_1(\Sigma, \mathbb{Z})$ is realizable by a smooth lagrangian submanifold; every smooth lagrangian submanifold is isomorphic to S^1; two loops from the same homology class are deformation equivalent; two loops are Hamiltonian isotopic if the symplectic area of the oriented film bounded by the loops is zero; the Fukaya category for a curve of any genus exists. Thus for this case the problem is completely solved!

But making one new step we face already highly nontrivial situation. Consider "the simplest and basic" 4- dimensional compact symplectic manifold — the projective plane $\mathbb{CP}^2$. Fot the projective plane we have that: since the cohomology group $H^2(\mathbb{CP}^2, \mathbb{Z}) = \mathbb{Z}$, any lagrangian submanifold must present trivial homology class; there are no lagrangian 2 - spheres (M. Gromov), Riemannian surfaces of genus $g > 1$ (M. Audin), Klein bottles (S. Nemirovsky and V. Shevchishin) — all these types are not realized by smooth lagrangian submanifolds of the projective plane; it was believed that well known Clifford tori are unique examples of lagrangian tori in $\mathbb{CP}^2$ since in 1996 Yu. Chekanov proposed a construction of lagrangian torus which is not Hamiltonian isotopic to a Clifford torus — and nobody knows are there other types of lagrangian tori; nevertheless certain constructions of appropriate categories exist (K. Fukaya, P. Seidel). Thus even for this basic case in dimension 4 the problem is not solved yet.

Why we are interested in lagrangian geometry? Lagrangian geometry is very important in Mathematical physics; f.e. several approaches to Geometric Quantization are based on Lagrangian geometry. In these approaches lagrangian submanifolds represent quantum states so an old idea of P.M. Dirac, stated that the phase space of classical mechanical system should contain the ingredients of a natural quantization procedure, is realized. Thus it is natural to study all possible states so it is reasonable to find all types of lagrangian submanifolds (see, f.e. [1]).

F.e. in ALAG - programme (abelian algebraic lagrangian geometry, see [2]) the Chekanov result ensures that the moduli space of half weighted Bohr - Sommerfeld lagrangian cycles of level 3, $\mathcal{B}^{hw,r}_{S,3}$, has at least two disjoint components, and may be in a future one will find certain connecting space with a tunneling effect between these components.

As well in a popular modern subject of Mathematical physics — Homological Mirror symmetry — one should try to describe all objects in the Fukaya category, so all types of nonisotopic lagrangian tori.

Well known Clifford tori in $\mathbb{CP}^2$ comes from the toric geometry: the projective plane carries two real Morse functions in involution with respect to the Poisson brackets induced by the Kahler form of the standard Fubini - Study metric. These functions can be explicitly expressed as:

$$f_1 = \frac{|z_1|^2 - |z_2|^2}{\sum_{i=0}^2 |z_i|^2}, f_2 = \frac{|z_0|^2 - |z_1|^2}{\sum_{i=0}^2 |z_i|^2}, \{f_1, f_2\}_\omega = 0$$

in homogeneous coordinates $[z_0 : z_1 : z_2]$; the degeneration set (where the algebraic independence is destroyed)

$$\Delta(f_1, f_2) = \{df_1 \wedge df_2 = 0\} \subset \mathbb{CP}^2$$

is formed by three lines $l_i, l_i = \{z_i = 0\}$; the action map $F = (f_1, f_2) : \mathbb{CP}^2 \to P_{\mathbb{CP}^2} \subset \mathbb{R}^2$ sends $\Delta(f_1, f_2)$ to the boundary component $\partial P_{\mathbb{CP}^2}$ of the convex polytop $P_{\mathbb{CP}^2}$ (in this case - a triangle), and the preimage of any inner point $p \in P_{\mathbb{CP}^2}$ is a smooth lagrangian torus, labeled by values of f_1, f_2. Thus the Clifford tori are just Liouville tori for this completely integrable system. And it is the standard picture for any toric manifold.

In 1996 Yu. Chekanov in [3] proposed the construction of exotic lagrangian tori by the first version to $\mathbb{R}^4$. The construction looks rather simple: fix a complex structure, so we have $\mathbb{C}^2$ with a coordinate system (z_1, z_2); choose a smooth contractible loop $\gamma \subset \mathbb{C}^*$ which lies in a half plane so $\mathrm{Re}\gamma > 0$; consider two - dimensional subset given in the coordinates by the explicit formula $(z_1, z_2) = (e^{i\phi}\gamma, e^{-i\phi}\gamma)$ — and it is a lagrangian torus! Note however that if the loop γ is not contractible, we get a torus which is equivalent to the standard one. Furthemore, since the projective plane without projective line $\mathbb{CP}^2 \backslash l$ is symplectomorphic to an open ball in $\mathbb{R}^4$ one implements the construction to the projective plane. Using certain special Hofer's capacity technique, Chekanov proved this torus is not equivalent to the standard one.

This exotic torus was called **the Chekanov torus**; the forthcoming paper by Yu. Chekanov and F. Schlenk contains the details how to construct these nonstandard tori in the projective space $\mathbb{CP}^n$ for certain n, the products $S^1 \times ... \times S^1$, and some other cases, see [4].

An alternative description of the Chekanov tori based on the notion of pseudotoric structure. We can produce the torus taking the pencil $\{Q_w\}$ such that $Q_w = \{z_1 z_2 = w\} \subset \mathbb{C}^2$ — one dimensional complex family of quadratic surfaces given in the coordinate system (z_1, z_2) by the quadratic equation which depends on complex parameter $w \in \mathbb{C}$. Then one takes real Morse function $F = |z_1|^2 - |z_2|^2$ and observes that the Hamiltonian vector field X_F of this function F preserves each quadric Q_w from the family. Then one fixes a smooth contractible loop $\gamma' \subset \mathbb{C}^*_w$ where $\mathbb{C}_w$ parameterizes our family $\{Q_w\}$. The choice of the value for our function F marks the level set on each quadratic surface which is a loop, so taking smooth loops $S_w = \{F = 0\} \cap Q_w$ on every quadratic surface $Q_w, w \in \gamma'$ and collecting all these loops along γ' one gets a torus:

$$T(\gamma') = \bigcup_{w \in \gamma'} S_w;$$

it is not hard to see, that we again get the Chekanov torus from the previous construction, if we put $\gamma = \sqrt{\gamma'}$.

Let's repeat the construction for the projective plane. To do this consider pencil of quadrics $\{Q_p\}$, $p \mapsto [\alpha : \beta] \subset \mathbb{CP}^1_{\alpha,\beta}$ where $Q_p = \{\alpha z_1 z_2 = \beta z_0^2\} \subset \mathbb{CP}^2$. Take real Morse function F explicitly given by the formula

$$F = \frac{|z_1|^2 - |z_2|^2}{\sum_{i=0}^2 |z_i|^2}$$

in homogenoues coordinates $[z_0 : z_1 : z_2]$. It can be checked directly that its Hamiltonian vector field X_F preserves each element of the pencil, so we can proceed as in the previous noncompact case. Let's choose a smooth contractible loop $\gamma \subset \mathbb{CP}^1_{\alpha,\beta} \backslash \{[1:0],[0:1]\}$ since the last points are covered by singular quadrics; then on each quadric $Q_p, p \in \gamma$ we can take the level set $S_p = \{F = 0\} \cap Q_p$, and this level set is a smooth loop. Then we collect the level sets S_p along the loop γ getting again a lagrangian torus $T(\gamma) = \bigcup_{p \in \gamma} S_p$. The point is that the resulting torus is exactly the Chekanov torus, given by the identification of symplectic ball in $\mathbb{R}^4$ and $\mathbb{CP}^2 \backslash$line. On the other hand if $\gamma \subset \mathbb{CP}^1_{\alpha,\beta}$ was taken non contractible then the resulting torus should be equivalent to a Clifford torus.

Therefore we get certain correspondence between the equivalence classes of lagrangian tori and the fundamental group of the punctured projective line $\pi_1(\mathbb{CP}^1_{\alpha,\beta} \backslash \{[0:1],[1:0]\})$ without the north and the south poles.

The construction scheme we've used above appears in the framework of **pseudotoric geometry** — certain generalization of the toric geometry. What is the difference between toric and pseudo toric considerations? We illustrate it on the ideal level by the following diagramme:

$$\begin{array}{c|c}
\mathbb{R} & \mathbb{C} \\
\text{real Morse function} \quad f & \text{Lefschetz pencil} \quad \{Q_p\} \\
\| & \| \\
f : X \to \mathbb{R} & \psi : X \backslash B \to \mathbb{CP}^1, Q_p = \overline{\psi^{-1}(p)} \\
\downarrow & \downarrow \\
\text{toric case} & \text{pseudtoric case} \\
(f_1, f_2) \quad \text{on} \quad \mathbb{CP}^2 & (f, \{Q_p\}) \quad \text{on} \quad \mathbb{CP}^2 \\
\text{such that} \quad \{f_1, f_2\}_\omega = 0 & \text{such that} \quad X_f \parallel Q_p \\
\uparrow & \uparrow \\
\text{standard commutation rel.} & \text{new commutation rel.}
\end{array}$$

A complex analog of a real Morse function is a Lefschetz pencil (this idea was discussed by V. Arnold, S. Donaldson¡ and many others), roughly speaking it is just a complex (or symplectic) map to the compactified complex space (classically – to the Riemann sphere). The question is how to relate the real data and new complex data, so what does it mean that a real function and a Lefschetz pencil commute? We propose the following new commutation relation: pencil $\{Q_p\}$ commutes with real function f if the Hamiltonian vector field X_f is parallel to each element Q_p of the pencil at each point. Geometrically (or dynamically) this means that the Hamiltonian flow generated by f preserves the "level sets" of the Lefschetz pencil — but it is exactly the same as for the real functions!

Leaving aside other speculative arguments, we summurize with the following

Definition ([5]): Pseudotoric structure on a compact symplectic manifold (X, ω_X) consists of

· **(real data)** $\{f_1, ..., f_k\}$ — algebraically independent almost everywhere real Morse functions in involution, $\{f_i, f_j\}_\omega = 0$;

· **(complex data)** family of compact symplectic $2k$ -dimensional submanifolds $\{Q_p\}, Q_p \subset X$, parameterized by a compact toric symplectic manifold $(Y, \omega_Y) \ni p$ (or, equivalently, a map with symplectic fibers

$$\psi : X \backslash B \to Y, \quad Q_p = \overline{\psi^{-1}(p)}, \quad B = \text{base set}$$

such that the following **commutation relations** hold:

· the Hamiltonian vector field X_{f_i} of each Morse function f_i from (*real data*) is parallel to each Q_p at each point (or, equivalently, each f_i preserves the fibers of ψ by the Hamiltonian action);

· for each smooth function $h \in C^\infty(Y, \mathbb{R})$ bi- vector field
$X_{\psi^* h} \wedge \nabla_\psi X_h \equiv 0$ — identically vanishes on $X \backslash B$.

In the last expression $X_{\psi^* h} \wedge \nabla_\psi X_h$ we take two vector fields for a function $h \in C^\infty(Y, \mathbb{R})$ taken on the base manifold Y, namely for the lifted function $\psi^* h \in C^\infty(X \backslash B, \mathbb{R})$ on $X \backslash B$ one takes the Hamiltonian vector field with respect to the symplectic form ω_X; on the other hand one takes the lift $\nabla_\psi X_h$ of the Hamiltonian vector field X_h defined by the symplectic form ω_Y on Y, and ∇_ψ is the symplectic connection defined by ψ since this map has symplectic fibers:

$$\nabla_\psi : \Gamma(TY) \to \Gamma(T(X \backslash B)).$$

The last condition looks too horrible but in practice one avoids all the difficulties, taking in mind the following remark: if X and Y are complex, $k = n-1$, and ψ is complex then the last commutation relation is automatically satisfied.

It's easy to see that the base set B of the family $\{Q_p\}$ must be contained by the degeneration locus $\Delta(f_1, ..., f_k) = \{df_1 \wedge ... \wedge df_k = 0\}$; the singular points of any fiber Q_p must be contained by the degeneration locus $\Delta(f_1, ..., f_k)$ as well; any fiber $Q_p = \psi^{-1}(p) \cup B$ endowed with restrictions $(f_1|_{Q_p}, ... f_k|_{Q_p})$ — is a completely integrable system (= toric (perhaps non smooth) symplectic manifold). Therefore pseudotoric structures supply us with the solutions of the following problem: for non completely integrable Hamiltonian system with the integrals $(f_1, ..., f_k)$ find toric leaves of the Hamiltonian action. And the point is that this would give a solution for thennon completely integrable system since for each initial point we can take the corresponding toric leave which contains this initial point and then using the "action - angle" coordinates on this leave we can write the corresponding solution.

The simplest (and the trivial) example of pseudotoric structure arises if one takes the direct product of two toric manifolds $Y_1 \times Y_2$. This structure is topologically trivial, as the product vector bundle. In analogy with the theory of vector bundles we introduce the following

Definition ([5]): *the number $n - k = \frac{1}{2} dim Y$ is called the rank of pseudotoric structure $(f_1, ..., f_k, \psi, Y)$.*

Clearly it is parallel to the notion of the rank of vector bundle.

If singular points of fiber Q_p lies in the base set B we say that the fiber Q_p is regular; if generic fiber Q_p is regular then we say that pseudotoric structure is regular; it's not hard to see that in the regular case the image $\psi(\Delta(f_1, ..., f_k) \backslash B) = D_{\text{sing}} \subset Y$ is a proper compact symplectic submanifold. This submanifold measures topological non triviality of the pseudotoric structure; this subset is empty if and only if the peudotoric structure is topologically trivial so it is the product of toric manifolds.

The main reason for the introduction of this new structure is the possibility to construct lagrangian fibrations on whole X starting with lagrangian fibrations on the base toric manifolds and using the toric nature of the fibers. If we choose a system $(h_1, ..., h_{n-k})$ of commuting moment maps on Y (since Y by the definition is toric) we get a lagrangian fibration on the base Y but at the same time we have the following

Theorem ([6]): *Choice of moment maps $(h_1, ..., h_{n-k})$ on the base Y of a regular pseudotoric structure $(f_1, ..., f_k, \psi, Y)$ on a given X defines a lagrangian foliation on X whose generic fiber is a smooth lagrangian torus.*

The dimensional reduction which happens on $\Delta(h_1, ..., h_{n-k}) \subset Y$ is reflected by the fact that the collection of fibers over $\Delta(h_1, ..., h_{n-k})$ must be cutted from X, and then the resting part $X \backslash (\bigcup_{p \in \Delta(h_1, ..., h_{n-k})} Q_p)$ carries lagrangian fibration. This lagrangian fibration is only generically smooth (so generic fiber is a smooth lagrangian torus), but the singular fibers have singularities which are not of generic type. The type of the singularities is controlled and can be described as follows. A Liouville torus in a completely integrable system carries periodic orbits and unbounded real lines (if we consider irrational motion along the torus). Our singular tori admit additionally trajectories of the separatrix type: take a periodic loop on a torus and contract it to a point — then the torus turns to be singular and instead of periodic loop one gets a stable point. This is the type of singularities which appear in our lagrangian fibration.

General scheme can be summarized by the following diagramme

$$
\begin{array}{lcccl}
(f_1, ..., f_k): & B & \longrightarrow & \partial P_{Q_p} & -\text{boundary component} \\
 & \cap & & \cap & \\
(f_1, ..., f_k): & X & \longrightarrow & P_{Q_p} & -\text{moment pol.for gen.fib.} \\
 & \psi \downarrow & & \times & \\
(h_1, ..., h_{n-k}): & Y & \longrightarrow & P_Y & -\text{moment pol.for base} \\
 & \cup & & \cup & \\
(h_1, ..., h_{n-k}): & D_{\text{sing}} & \longrightarrow & N & -\text{hypersurface in} P_Y
\end{array}
$$

— here singular lagrangian tori in the fibration are parameterized by certain "incidence cycle" appears from irregular singular fibers of the pseudotoric structure. Note that certain incidences exist: if we first consider our ambient manifold X with non complete set of integrals $(f_1, ..., f_k)$ then without any references to the complex data coming to the definition of the pseudotoric structure one can see the following. As it was shown by M. Atiyah, the image under the action map in $\mathbb{R}^k$ of whole X is a convex polytop P, but then every regular toric leave Q_p must have P as its convex polytop under the restricted action map $(f_1|_{Q_p}, ..., f_k|_{Q_p})$. Thus the type of the toric leave is visible under the action map on whole X. Furthemore, the base set B is contained by any leave thus it must be mapped to the boundary ∂P_{Q_p}. The boundary of any convex polytop is formed by the union of convex polytops so the base set itself is a union of toric submanifolds of smaller dimensions. For a singular leave $Q_p| \quad p \in D_{\text{Sing}}$ the polytop contains certain hypersurface which is the image of the singular set in the singular fiber. This hypersurface in P_{Q_p} is stable with respect to the contineous deformations, and summarizing under all $p \in D_{\text{Sing}}$ we get as subset $N_0 \subset P_{Q_p}$ of codimension at least 1. On the other hand the toric base Y one his its own action map which sends the divisor $D_{\text{Sing}} \subset Y$ to a hypersurface $N \subset P_Y$. As a big space which parameterizes our lagrangian fibration we has the direct product $P_{Q_p} \times P_Y$; and the singular lagrangian tori in our fibration correspond to certain subset of the direct product $N_0 \times N \subset P_{Q_p} \times P_Y$. This subset is our "incidence cycle".

Now we slightly generalize the discussion concerning not lagrangian fibrations but lagrangian tori. For this case we can say about possible lifting of lagrangian tori from the base manifold the of pseudotoric structure, namely

Theorem ([7]) *Let $(f_1, ..., f_k, \psi, Y)$ be a regular pseudotoric structure on a compact symplectic manifold X. Let $S \subset Y$ be a smooth lagrangian torus which doesn't intersect $D_{sing} \subset Y$. Then the choice of non critical values $(c_1, ...c_k)$ of $f_1, ..., f_k$ defines a smooth lagrangian torus $T(S, c_1, ..., c_k) \subset X$.*

Shortly, the proof of the theorem based on the same procedure we've applied above to construct the Chekanov tori. Taking a lagrangian torus on the base, we collect lagrangian tori from the toric fiber — it can be done simulteneously thanks to the global functions $f_1, ..., f_k$ — and the commutation relations from the very definition of pseudotoric structure ensure that the resulting figure is a lagrangian torus in X.

The last theorem shows that lagrangian tori from the base manifold after lifting could give different types of lagrangian tori in whole X. We can take the homology group $H_{n-k}(Y \backslash D_{\text{sing}}, \mathbb{Z})$ of the "punctured" base manifold and then attach to smooth lagrangian tori in the punctured base manifold different classes from the group. Conjecturelly the different classes from $H_{n-k}(Y \backslash D_{\text{sing}}, \mathbb{Z})$ can give different types of lagrangian tori in X. For example, it is true for the projective plane – as we've seen for Clifford and Chekanov tori in $\mathbb{CP}^2$ the following alternative appears:

$$H_1(\mathbb{CP}^1 \backslash ([1:0],[0:1]), \mathbb{Z}) \begin{array}{l} \nearrow \text{ Clifford type = primitive elem.} \\ \searrow \text{ Chekanov type = trivial elem.} \end{array}$$

But is it possible to construct such exotic lagrangian tori for any compact toric variety? The answer is an affirmative in view of the following

Theorem ([8]): *Any smooth compact toric symplectic manifold admits regular pseudotoric structure* $(f_1, ..., f_{n-1}, \psi, \mathbb{CP}^1)$ *of rank one. For this structure the singular divisor* $D_{sing} \subset \mathbb{CP}^1$ *consists of exactly two distinct points,* $p_N, p_S \subset \mathbb{CP}^1$*. The primitive and the trivial elements of* $H_1(\mathbb{CP}^1 \backslash (p_N \cup p_S), \mathbb{Z})$ *generates lagrangian tori of the standard type and of the Chekanov type respectively.*

The Theorem above which states the existence of pseudotoric structures on toric symplectic manifolds can be proved as follows ([8]). Let's take for a given toric X the set of commuting Morse moment maps $(f_1, ... f_n)$, which give the action map by "action coordinates" $F = (f_1, ..., f_n) : X \to P_X$ to convex moment polytop $P_X \subset \mathbb{R}^n$; then for the components D_i of the boundary divisor $D = F^{-1}(\partial P_X)$ one can find an integer combination $\sum \lambda_i D_i$ equals to zero. This sum can be rearranged to the form

$$\sum_{\lambda_i > 0} \lambda_i D_i = \sum_{\lambda_j < 0} |\lambda_j| D_j, \quad D_i \neq D_j;$$

therefore we have two divisors from the same linear system

$$D_+ = \sum_{\lambda_i > 0} \lambda_i D_i, \quad D_- = \sum_{\lambda_j < 0} |\lambda_j| D_j \in |\sum_{\lambda_i > 0} \lambda_i D_i|.$$

Then one takes the pencil $< D_+, D_- >$ spanned by two divisors $D_\pm$ with the base set $B = D_+ \cap D_-$, and it would be our pencil ψ from the definition of pseudotoric structure, and for generic point $p \in \mathbb{CP}^1$, $p \neq [1:0](\mapsto D_+)$, $[0:1](\mapsto D_-)$, the divisor $\overline{\psi^{-1}(p)} \subset X$ is smooth outside the base set B. The same linear combination $\sum \lambda_i D_i$ after substitution of linear forms l_i which correspond to D_i in $\mathbb{R}^n$ gives a linear relation on x_i — and this relation derive our real data $f'_1, ... f'_{n-1}$ from $f_1, ..., f_n$ just implying the corresponding linear condition.

In the original Chekanov paper not each lagrangian tori of different types are non isotopic but only the ones which admit an additional property. This property to be **monotone** is extremely important in both Geometric Qunatization and Mirror Symmetry investigations; it is stable with respect to the Hamiltonian deformations (and in Geometric Quantization it is well known as the Bohr – Sommerfeld condition). To impose this condition over the space of lagrangian tori first one should suppose additionaly that our given toric (X, ω_X) is monotone, this means that the cohomology class of the symplectic form is proportional to the canonical class of the associated almost complex structure on X: $K_X = k[\omega_X] \subset H^2(X, \mathbb{Z})$; so f.e. Fano varieties in algebraic geometry are monotone (but the simplest possible example is given by a simplectic vector space since the anticanonical bundle of this one is trivial). But non trivial case is rather wide and interesting: the projective spaces, the Grassmanians, the flag varieties — all are in this class.

Now, if the anticanonical class is proportional to the cohomology class of the symplectic form then it means that one can take an abelian connection a on the complex linear determinant bundle detTX whose curvature form F_a would be proportional to the symplectic form (and if our X is simply connected then this connection is unique up to gauge transformations). For any lagrangian submanifold $S \subset X$ the restriction of a to S gives a flat connection, and S is Bohr – Sommerfeld w.r.t. anticanonical class iff this restriction admits a covariantly constant section. The monotonicity condition is more special — certain homology class (which was called the universal Maslov class — it is defined to Bohr - Sommerfeld lagrangian submanifolds only) must be trivial.

Then the main conjecture we would like to propose in this circumstances is based on the following remark: if there is a standard monotone lagrangian torus then there exists a monotone lagrangian torus of the Chekanov type. And for the future work we have as the major aim the following

Main conjecture. *These monotone tori are not Hamiltonian isotopic.*

Remark, that for the projective plane the Conjecture is true.

At the end I would like to mention several applications of this generalized notion, pseudotoric structure.

Lots of methods in Mathematical Physics are invented and realized with great success in the case of toric varities. In Geometric Quantization (see, f.e., [9]) we know what one should do in the case when the phase space of classical mechanical system carries a real polarization, namely one takes the Bohr - Sommerfeld fibers and spann the Hilbert space. In Homological Mirror Symmetry (see, f.e., [10]) one takes the canonical fibration on lagrangian tori, counts the fibers with non trivial Floer cohomologies — and then builds on these fibers the corresponding Fukaya category. But all these methods can not be applied in non toric case since if one takes a non toric variety — nobody knows in general how to slice it on lagrangian fibers.

Pseudotoric structure on a symplectic manifold X gives way to apply these methods in more general setup. Indeed, the theorems above ensure that we can construct aslmost canonically certain lagrangian fibrations in the presence of pseudotoric structures. What is the difference with the "regular" toric case? It is in the appearence of singular lagrangian fibers. But as we've seen the types of singularities which appear in the fibers are very special: for example the notion of Bohr - Sommerfeld lagrangian cycle is still valid for these singular lagrangian tori without any modification. One hopes that the definition of the Floer cohomology can be modified as well. Then we can use the powerfull methods not only in toric geometry but in much more wider context — since there are compact symplectic manifolds which are not toric but nevertheless which admit pseudotoric structures. It is natural to call such a manifold **pseudotoric**: the examples are complex quadrics and certain complete intersections in $\mathbb{CP}^n$, the flag variety F^3 and conjecturelly certain complex Grasmmanians. And coming back to the main subject of the present talk we shoud say that in all these cases it is possible to construct lagrangian tori of different type using the pseudtoric structure.

Thus the natural problem arises: **which symplectic manifolds are pseudotoric?** Toric geometry itself concerns this question since in the framework of toric geometry one meets the problem with induced objects. F.e. if one takes the projectivization of the (co)tangent bundle of a toric variety – it is not longer toric (F^3 — the flag variety — is the projectivization of the tangent bundle of the "most toric" one — the projective plane) but it admits the Hamitonian action of an incomplete set of integrals lifted from the toric base. As well it could happen for certain moduli spaces over toric varieties. The construction of a pseudotoric structure is a solution of the toric leaves problem in general; and if this solution exists we can adopt the strong methods from toric geometry to this case.

As a byproduct we've touched a classical problem from mechanics — the study of non completely integrable systems. Again as we've seen the problem could be solved in the case when the phase space admits a psuodotoric structure. Then the solutions can be described in terms of the "action - angle" variables of the base manifold and the toric fibers. The difference with the completely integrable case is in the appearance of singular lagrangian tori of the Liouville type – and it is not problematic since it just means that some additional types of trajectories – separatrices – are presented in the story.

[1] N.A. Tyurin, *"Geometric quantization and algebraic lagrangian geometry"*, Surveys in Geometry and Number theory, LMS Lect. Note Ser. 338 (2007), pp. 279 - 318;

[2] A.L. Gorodentsev, A.N. Tyurin, *"Abelian lagrangian algebraic geometry"*, Izvestiya: Math., 65:3 (2001), pp. 437 -467;

[3] Yu.V. Chekanov, *"Lagrangian tori in symplectic vector space and global symplectomorphisms"*, Math. Zeitschrift, 223: 1 (1996), pp. 547 - 599;

[4] Yu. Chekanov, F. Schlenk, *"Notes on monotone lagrangian twist tori"*, arXiv:1003.5960;

[5] N.A. Tyurin, *"pseudotoric lagrangian fibrations of toric and non toric Fano varieties"*, Theor. Math. Physics, 162:3 (2010), pp. 307 -333;

[6] S.A. Belyov, N.A. Tyurin, *"On nontoric lagrangian fibrations of toric Fano varieties"*, Math. Notes, 87: 1 (2010), pp. 48 -59;

[7] S.A. Belyov, N.A. Tyurin, *"Liftings of lagrangian tori"*, Math. Notes, 91:5 (2012), pp. 784 - 786;

[8] S.A. Belyov, N.A. Tyurin *"Pseudotoric structures on toric symplectic varieties"*, Theor. Math. Physics, 175: 2 (2013), pp. 147 - 158;

[9] N. Woodhouse, *"Geometric Quantization"*, The Clarendon press Oxford University press, New York, 1980;

[10] K. Fukaya, Y.-G. Oh, H. Ohta, K. Ono, *"Lagrangian Floer theory on compact toric manifolds I"*, Duke. Math. J. 151 (2010), pp. 23-174.

HIDDEN SUPERSYMMETRY IN THE SUPER LANDAU MODELS

E.A. Ivanov and S.S. Sidorov

The name Landau model is of common use for any quantum-mechanical problem in which a charged particle moves over some manifold in the background of an external gauge field. Besides the original planar $2D$ Landau model [1], with the particle moving on a plane under the influence of uniform magnetic field orthogonal to the plane, much attention was paid to some its curved generalizations (see, e.g., [2, 3, 4, 5, 6]). The Landau problem and its generalizations constitute a theoretical basis of quantum Hall effect. Their most characteristic feature is the presence of Landau Levels (LL) in the energy spectrum, such that the gap between the Lowest Landau Level (LLL), and the excited LLs rapidly grows with intensification of the external gauge field. In the limit of strong external field only the LLL is relevant. In the lagrangian language, such a system is described by d=1 Wess-Zumino (or Chern-Simons) action, with the phase space being a non-commutative version of the original configuration space.

The Landau problems on the (2|2)-dimensional supersphere $SU(2|1)/U(1|1)$ and the (2|4)-dimensional superflag $SU(2|1)/[U(1)\times U(1)]$ as the simplest superextensions of the S^2 Haldane model [2] were considered in [7, 8, 9]. In order to better understand the common features of the super Landau models, the planar limits of these models (with the curved target supermanifolds becoming the (2|2)- and (2|4)-dimensional superplanes) were also studied [10, 11, 12, 13]. They are superextensions of the original Landau model and exhibit some surprising features.

First, the space of quantum states in these models involves ghosts, i.e. the states with negative norms, which seemingly leads to violation of unitarity. The planar super Landau models suggest a simple mechanism of evading the ghost problem. It was shown in [12] that all norms of states in the superplane models can be made non-negative at cost of introducing a proper metric operator in Hilbert space and so redefining the inner product.

The second feature closely related to the one just mentioned is that the passing to the new inner product makes manifest the hidden worldline $\mathcal{N}$=2 supersymmetry of the superplanar models, which so supply examples of $\mathcal{N}$=2 supersymmetric quantum mechanics. In the case of the curved $SU(2|1)/U(1|1)$ and $SU(2|1)/[U(1)\times U(1)]$ models, the hidden supersymmetry is associated with the non-abelian supergroup $SU(2|2)$.

In order to understand how general this phenomenon is, we recently constructed super Landau-type models associated with the pure fermionic cosets $SU(n|1)/U(n)$ and studied in detail the quantum theory for $n=2$ [14]. This study was a natural continuation of that performed in [15] where, as the corresponding Lagrangians, only Wess-Zumino terms on these supermanifolds were considered.

The Lagrangian we started with in [14] is the fermionic analog of the standard CP^n

sigma model Lagrangian

$$L = \frac{\dot{\bar{\xi}} \cdot \dot{\xi}}{1 - \xi \cdot \bar{\xi}} + \frac{(\dot{\bar{\xi}} \cdot \xi)(\dot{\xi} \cdot \bar{\xi})}{\left(1 - \xi \cdot \bar{\xi}\right)^2} - i\kappa \, \frac{\bar{\xi} \cdot \dot{\xi} - \dot{\bar{\xi}} \cdot \xi}{1 - \xi \cdot \bar{\xi}}, \tag{1}$$

where $\xi^i(t)$, $(i = 1, \ldots, n)$, are fermionic fields parametrizing the supermanifold $SU(n|1)/U(n)$. This Lagrangian is invariant under the transformations

$$\delta\xi^i = \epsilon^i + (\bar{\epsilon} \cdot \xi)\, \xi^i \,, \quad \text{and c.c.}\,, \tag{2}$$

which, together with the linearly realized subgroup $U(n)$, form the supergroup $SU(n|1)$. After quantization, because of the non-standard kinetic term for fermions in (1), *both* ξ^i and $\bar{\xi}_i$ become the coordinates of the wave functions. In the $SU(n)$-singlet sector of the full Hilbert space the number of independent wave functions is finite and equal to $n + 1$; there is present the same finite number of the LLs. The wave functions admit holomorphic or antiholomorphic representations, in which they depend on either ξ^i or $\bar{\xi}_i$. Due to the presence of the non-standard kinetic term for the fermions there are states with negative norms. However, for the $n = 2$ example we showed that, by introducing the appropriate metric operator, all norms can be made positive-definite, like in the $SU(2|1)/U(1|1)$ and $SU(2|1)/[U(1) \times U(1)]$ cases. We also found that at each LL the sets of wave functions belonging to some irrep of $SU(2|1)$ are also closed under an extended $SU(2|2)$ symmetry with respect to which they form the so called "short multiplets". This dynamical $SU(2|2)$ has the following structure relations

$$\begin{aligned}
&\{S^a_i, \bar{S}^j_b\} = \delta^a_b J^j_i - \delta^j_i \mathcal{J}^a_b + \delta^a_b \delta^j_i \left(2\kappa + \frac{\ell}{2} - \frac{1}{2}\right), \\
&\{S^a_i, S^b_j\} = \varepsilon_{ij}\varepsilon^{ab}\sqrt{C_2}\,, \qquad \{\bar{S}^i_a, \bar{S}^j_b\} = \varepsilon^{ij}\varepsilon_{ab}\sqrt{C_2}\,, \\
&[S^a_i, \mathcal{J}^c_b] = \delta^a_b S^c_i - \frac{1}{2}\delta^c_b S^a_i\,, \qquad [S^a_i, J^j_k] = \delta^j_i S^a_k - \frac{1}{2}\delta^j_k S^a_i\,.
\end{aligned} \tag{3}$$

Here, ℓ is the LL number, $C_2 = C_2(\ell)$ is the value of the quadratic Casimir of $SU(2|1)$ for the wave function at the given level and $\mathcal{J}^c_b$ are generators of some extra $SU(2)$ which is not a symmetry of the Lagrangian L. The generators $(S^1_i, \bar{S}^j_1, J^j_i)$ form the original $SU(2|1)$ symmetry. All generators are realized, e.g. in the holomorphic representation, by some differential operators acting on the set ξ^i. Thus the hidden symmetry at each LL is generated by the superalgebra $su(2|2)$ with the level-dependent central charges $2\kappa + \frac{\ell}{2} - \frac{1}{2}$ and $\sqrt{C_2(\ell)}$.

We also considered in [14] the planar limit of our quantum $SU(n|1)/U(n)$ model and found that at $n = 2$ the states are closed under some dynamical $su(2|2)$ which does not coincide with the hidden $su(2|2)$ of the $SU(2|1)/U(2)$ model. In particular, its central charges do not depend on the level ℓ.

So we have shown that the appearance of the hidden dynamical $SU(2|2)$ symmetry in the previously considered $SU(2|1)/U(1|1)$ and $SU(2|1)/[U(1) \times U(1)]$ quantum examples was not accidental; the extreme purely fermionic $SU(2|1)/U(2)$ model also exhibits the same hidden symmetry, though differently realized. It is interesting to study the issue of hidden supersymmetries in the super Landau models associated with the cosets of the higher-dimensional supergroups $SU(n|m)$ and in their appropriately defined planar limits.

In fact, in [16] we considered some new planar Landau model which is expected to be the planar limit of the super Landau model on the (2+2) dimensional projective superspace $SU(3|2)/U(2|2) \sim CP^{(2|2)}$. We started from the manifestly $\mathcal{N}=4$ supersymmetric worldline superfield formalism and finally, after passing to components and eliminating the relevant auxiliary fields, arrived at the following simple Lagrangian

$$L = |\dot{z}|^2 + |\dot{u}|^2 - i\kappa(\dot{z}\bar{z} - \dot{\bar{z}}z + \dot{u}\bar{u} - \dot{\bar{u}}u) + \dot{\zeta}\dot{\bar{\zeta}} + \dot{\xi}\dot{\bar{\xi}} - i\kappa(\dot{\zeta}\bar{\zeta} + \dot{\bar{\zeta}}\zeta + \dot{\xi}\bar{\xi} + \dot{\bar{\xi}}\xi), \tag{4}$$

where (z, u) are two complex bosonic fields and (ζ, ξ) are two complex fermionic ones. In the $SU(2)$ covariant notation, the same set of fields is represented by the real bosonic and fermionic quartets (f^{iA}, χ^{aA}), where $i, a = 1, 2$ are doublet indices of the two mutually commuting $SU(2)$ automorphism groups of the $\mathcal{N}=4, d=1$ Poincaré superalgebra, and A is the doublet index of an extra $SU(2)$ commuting with supersymmetry (the so called Pauli-Gürsey group $SU(2)_{PG}$). Despite the fact that the Lagrangian (4) looks like the sum of Lagrangians of two planar $\mathcal{N}=2$ models, it possesses a wide set of symmetries.

- First, it possesses the inhomogeneous target space supersymmetry

  $$(P_{iA}, \Pi_{aA}) \rtimes SU(2|2) = ISU(2|2), \tag{5}$$

  where the generators P_{iA}, Π_{aA} generate the so called "magnetic" super-translations acting as shifts of the involved bosonic and fermionic fields, while the $SU(2|2)$ part acts as the homogeneous rotations of the latter (mixing, in general, bosonic fields with the fermionic ones). So the full manifold of these fields (f^{iA}, χ^{aA}) can be identified with the supercoset $ISU(2|2)/SU(2|2)$.

- The worldline supersymmetry includes the default $\mathcal{N}=4$ supersymmetry, with the supercurrent

  $$S_{ia} = -\frac{i}{\sqrt{\kappa}}\, C^{AB}\dot{\chi}_{aA}\dot{f}_{iB}\,, \tag{6}$$

  where $C^{AB} = C^{BA}$ is a constant triplet breaking $SU(2)_{PG}$ down to $U(1)_{PG}$.

- Surprisingly, the Lagrangian (4) reveals one more, hidden, worldline $\mathcal{N}=4$ supersymmetry generated by the supercurrent

  $$\hat{S}_{ia} = \frac{i}{\sqrt{\kappa}}\, \dot{\chi}_{aA}\dot{f}_i^A\,. \tag{7}$$

- The anticommutator of these two different $\mathcal{N}$=4 supercharges is non-vanishing. In the quantum case:

  $$\{S^{ia}, \hat{S}^{jb}\} = 8i\kappa\left(\epsilon^{ab}\hat{T}^{ij} - \epsilon^{ij}\hat{T}^{ab}\right), \tag{8}$$

  where $\hat{T}^{ij}, \hat{T}^{ab}$ are $SU(2)$ generators related to those of two $SU(2)$ automorphism groups. Together with the standard anticommutators

  $$\{S^{ia}, S^{jb}\} = 2\epsilon^{ij}\epsilon^{ab}H_q\,, \quad \{\hat{S}^{ia}, \hat{S}^{jb}\} = 2\epsilon^{ij}\epsilon^{ab}H_q\,, \tag{9}$$

  they form the *worldline* supergroup $SU(2|2)_{dyn}$, in which the quantum hamiltonian H_q plays the role of the central charge.

The target space supergroup (5) and the worldline supergroup $SU(2|2)_{dyn}$ fully decouple in the appropriate basis. The quantum states are nicely distributed over the multiplets of the target space and the worldvolume supersymmetries. At each level N, the set of wave functions consists of the irreducible tensors of one $SU(2)$ automorphism group with the spins $s_1 = \frac{N}{2}$, $s_2 = \frac{N-1}{2}$ (entering twice) and $s_3 = \frac{N-2}{2}$, respectively. They form an irreducible representation of the worldline supersymmetry. Every such tensor also involves four independent components forming a multiplet of the target space supersymmetry. So the degeneracy of the N-th level is equal to

$$4[(2s_1 + 1) + 2(2s_2 + 1) + (2s_3 + 1)] = 16N\,. \tag{10}$$

The LLL states are singlets of the worldline $SU(2|2)_{dyn}$ supersymmetry, so the latter is not broken. Like in other planar super Landau models, there are states with the negative norms, but, once again, all norms can be made non-negative by introducing the appropriate metric operator in the full Hilbert space, with preserving the symmetry structure of the model.

The presence of hidden supersymmetries of the type $SU(m|n)$ (and proper contractions of the latter) in various Landau-type models with the target superspaces is an indication of the intimate relations of these models with integrable super spin-chain models, including those inspired by the quantum $\mathcal{N} = 4, d = 4$ super Yang-Mills theory, where the similar supergroup structures naturally appear (see, e.g., [17, 18, 19]). It is of interest to make these conjectured relationships manifest. There are also many other interesting directions for the future work along which the results of [14] and [16] could be further extended.

[1] L. Landau, *Diamagnetismus der Metalle*, Z.Phys. **64** (1930) 629.

[2] F.D.M. Haldane, *Fractional Quantization of the Hall Effect: A Hierarchy of Incompressible Quantum Fluid States*, Phys. Rev. Lett. **51** (1983) 605.

[3] S.-C. Zhang, J.-p. Hu, *A Four-dimensional generalization of the quantum Hall effect*, Science **294** (2001) 823, `arXiv:cond-mat/0110572`.

[4] D. Karabali, V.P. Nair, *Quantum Hall Effect in Higher Dimensions*, Nucl. Phys. B **641** (2002) 533, `arXiv:hep-th/0203264`.

[5] H. Elvang, J. Polchinski, *The Quantum Hall Effect on* R^4, `arXiv:hep-th/0209104`.

[6] K. Hasebe, *Hopf Maps, Lowest Landau Level, and Fuzzy Spheres*, SIGMA **6** (2010) 071, `arXiv:1009.1192 [hep-th]`.

[7] E. Ivanov, L. Mezincescu, P.K. Townsend, *Fuzzy* $CP^{(n|m)}$ *as a quantum superspace*, `arXiv:hep-th/0311159`.

[8] E. Ivanov, L. Mezincescu, P.K. Townsend, *A Super-Flag Landau Model*, In: "From Fields to Strings: Circumnavigating Theoretical Physics" eds. M. Shifman, A. Vainshtein, and J. Wheater, `arXiv:hep-th/0404108`.

[9] A. Beylin, T. Curtright, E. Ivanov, L. Mezincescu, P.K. Townsend, *Unitary spherical super-Landau models*, JHEP **0810** (2008) 069, `arXiv:0806.4716[hep-th]`.

[10] E. Ivanov, L. Mezincescu, P.K. Townsend, *Planar Super-Landau Models*, JHEP **0601** (2006) 143, `arXiv:hep-th/0510019`.

[11] K. Hasebe, *Quantum Hall Liquid on a Noncommutative Superplane*, Phys. Rev. D **72** (2005) 105017, `arXiv:hep-th/0503162`;

[12] T. Curtright, E. Ivanov, L. Mezincescu, P.K. Townsend, *Planar Super-Landau Models Revisited*, JHEP **0704** (2007) 020, arXiv:hep-th/0612300.

[13] A. Beylin, T. Curtright, E. Ivanov, L. Mezincescu, *Generalized N=2 Super Landau Models*, JHEP **1004** (2010) 091, arXiv:1003.0218 [hep-th].

[14] M. Goykhman, E. Ivanov, S. Sidorov, *Super Landau models on odd cosets*, Phys. Rev. D **87** (2013) 025026, arXiv:1208.3418 [hep-th].

[15] E. Ivanov, L. Mezincescu, A. Pashnev, and P. K. Townsend, *Odd coset quantum mechanics*, Phys. Lett. B **566** (2003) 175, arXiv:hep-th/0301241.

[16] V. Bychkov and E. Ivanov, *N=4 Supersymmetric Landau Models*, Nucl Phys. B **863** (2012) 33, arXiv:1202.4984 [hep-th].

[17] N. Beisert, *The Analytic Bethe Ansatz for a Chain with Centrally Extended* $su(2|2)$ *Symmetry*, J. Stat. Mech. **0701** (2007) P017, arXiv:nlin/0610017.

[18] N. Beisert, *The* $su(2|2)$ *Dynamic S-Matrix*, Adv. Theor. Math. Phys. **12** (2008) 945, arXiv:hep-th/0511082.

[19] G. Arutyunov, S. Frolov, *The S-matrix of String Bound States*, Nucl. Phys. B **804** (2008) 90, arXiv:0803.4323 [hep-th].

SUPERSYMMETRIC COMPONENT ACTIONS VIA COSET APPROACH

S. Krivonos and A. Sutulin

It is a well known fact that a domain wall spontaneously breaks the Poincaré invariance of the target space down to the symmetry group of the world volume subspace. This breaking results in the appearing of the Goldstone bosons associated with spontaneously broken symmetries. When we are dealing with the purely bosonic p-branes this information is enough to construct the corresponding action. In the case of the bosonic D-branes, which necessarily contain the gauge fields, is less clear, despite the knowledge of the explicit actions, etc. Fortunately, in the supersymmetric cases, where the supersymmetry is also partially spontaneously broken, the bosonic sector, which is the combination of Nambu-Goto and Born-Infeld actions, appears automatically.

¿From the mathematical point of view, the most appropriate approach to describe a partial breaking of Poincaré symmetry is the nonlinear realization (or coset) method [1] suitably modified for the cases of (supersymmetric) space-time symmetries in [2]. Schematically the coset approach works as follows. Splitting the generators of the target space D-dimensional Poincaré group, which is supposed to be spontaneously broken on the world volume down to the d-dimensional Poincaré subgroup, into the generators of unbroken $\{P, M\}$ and spontaneously broken $\{Z, K\}$ symmetries (the generators P and Z form D-dimensional translations, M generators span the $so(1, d-1)$ - Lorentz algebra on the world volume, while generators K belong to the coset $so(1, D-1)/so(1, d-1)$), one may realize all the transformations of D-dimensional Poincaré group by the left action on the coset element

$$g = e^{xP} e^{q(x)Z} e^{\Lambda(x)K}. \tag{1}$$

The spontaneous breaking of Z and K symmetries is reflected in the character of corresponding coset coordinates which are Goldstone fields $q(x)$ and $\Lambda(x)$ in the present case. All information about geometric properties is contained in the Cartan forms

$$g^{-1}dq = \Omega_P P + \Omega_M M + \Omega_Z Z + \Omega_K K, \tag{2}$$

which are transformed, except for Ω_M, homogeneously under an action of the symmetry group. Due to the general theorem [3] not all of the above Goldstone fields have to be treated as independent. In a given case the fields $\Lambda(x)$ can be covariantly expressed through x-derivatives of $q(x)$ by imposing the constraint

$$\Omega_Z = 0. \tag{3}$$

Thus, the model contains only the fields $q(x)$ as physical fields. The Cartan form Ω_P defines the vielbein E, which are d-bein in the present case, connecting the covariant world volume coordinate differentials Ω_P and the world volume coordinate differential dx as

$$\Omega_P = E \cdot dx. \tag{4}$$

Taking into account all these properties, it is possible immediately to write the invariant action

$$S = -\int d^d x + \int d^d x \ \det(E) \tag{5}$$

The trivial first term presented in (5) is needed to fulfill the condition $S_{q=0} = 0$. So, one concludes that the action (5) is just the static gauge form of the actions of $p = (D - d)$-branes.

The supersymmetric generalization of the coset approach involves new spinor generators Q and S which extend the D-dimensional Poicaré group to the supersymmetric one

$$\{Q, Q\} \sim P, \ \{S, S\} \sim P, \ \{Q, S\} \sim Z. \tag{6}$$

The most interesting cases are those when the Q supersymmetry is kept unbroken, while the S supersymmetry is supposed to be spontaneously broken. When all supersymmetries are considered as spontaneously broken, the corresponding action can be constructed similarly to the bosonic case, resulting in the some synthesis of Volkov-Akulov [4] and Nambu -Goto actions.

Another possibility is realized when a number of unbroken $\#Q$ and $\#S$ broken supersymmetries are equal to each other. In that case, so-called Partial Breaking of Global Supersymmetry takes place.

In the supersymmetric case all symmetries can be realized by group elements acting on the coset element

$$g = e^{xP} e^{\theta Q} e^{q(x,\theta) Z} e^{\psi(x,\theta) S} e^{\Lambda(x,\theta) K}. \tag{7}$$

The main novel feature of the supersymmetric coset (7) is the appearance of the Goldstone superfields

$$q(x, \theta), \quad \psi(x, \theta), \quad \Lambda(x, \theta)$$

which depend on the coordinates of the world volume superspace $\{x, \theta\}$. The rest of the coset approach machinery works in the same manner: one may construct the Cartan forms (2) for the coset (7) (which will contain the new forms Ω_Q and Ω_S), one may find the supersymmetric d-bein and corresponding bosonic ∇_P and spinor ∇_Q covariant derivatives, etc. One may even write the proper generalizations of the covariant constraints (3) as

$$\Omega_Z = 0, \quad \Omega_S| = 0, \tag{8}$$

where $|$ means the $d\theta$-projection of the form (see e.g. [5] and references therein).

Unfortunately, this similarity between purely bosonic and supersymmetric cases is not complete due to the existence of the following important new features of theories with partial breaking of global supersymmetry:

- In contrast with the bosonic case, not all of the physical fields appear among the parameters of the coset. Nevertheless, *it is true* that the *all physical bosonic components* can be found in the quantity $\nabla_Q \psi|$.

- The supersymmetric generalization (8) of the bosonic kinematic constraints (3) in most cases contains not only kinematic conditions, but also dynamic superfield equations of motion. Moreover, in many cases it is unknown how to split these constraints into kinematical and dynamical ones.
- But the most unpleasant feature of the supersymmetric cases is that the standard methods of nonlinear realizations fail to construct the superfield action! The main reason for this is simple: all that we have at hands are the covariant Cartan forms, which we can construct the superfield invariants from, while the superspace Lagrangian is not invariant. Instead it is shifted by the full spinor derivatives under unbroken and/or broken supersymmetries.

Therefore, all that we can do until now, within the supersymmetric coset approach, is

- to find the transformation properties of the superfields and construct the covariant derivatives
- to find the superfield equations of motion and/or covariant variants of irreducibility constraints.

That is why during recent years some new methods to construct the actions (in terms of superfield or in terms of physical components) have been proposed. Among them one should mention the construction of the linear realization of partially broken supersymmetry [6], [7], [8], [9] and reduction from higher dimensional supersymmetric D-brane action [10] to lower dimensions [11].

In a recent work we did one further step in the application of the supersymmetric coset approach, by demonstrating how on-shell component actions can be constructed within it. This construction is so simple that it can be schematically formulated just here.

The main idea is to start with the Ansatz for the action manifestly invariant with respect to *spontaneously broken supersymmetry*. Funny enough, it is rather easy to do, due to the following properties:

- in the parametrization of the coset element (7) the superspace coordinates θ do not transform under broken supersymmetry. Thus, all components of superfields transform *independently*,
- the covariant derivatives ∇_P and ∇_Q are invariant under broken supersymmetry. Therefore, the bosonic physical components which are contained in $\nabla_Q \psi|$ can be treated as "matter fields" with respect to broken supersymmetry,
- all physical fermionic components are just $\theta = 0$ projections of the superfield $\psi(x, \theta)$ and these components transform as the fermions of the Volkov-Akulov model [4] with respect to broken supersymmetry.

The immediate consequence of these facts is the conclusion that the physical fermionic components can enter the component on-shell action through the determinant of the d-bein E constructed with the help of the Cartan form Ω_P in the limit $\theta = 0$, or through the space-time ∇_P derivatives of the "matter fields", only. Thus, the most general Ansatz for the on-shell component action, which is invariant with respect to spontaneously broken supersymmetry, has the form

$$S = \int d^d x - \int d^d x \det(E) \mathcal{F}(\nabla_Q \psi|, \nabla_P q|). \qquad (9)$$

The explicit form of the function $\mathcal{F}$ can be fixed by two additional requirements

1. The action (9) should have a proper bosonic limit, which is known in almost all interesting cases.
2. The action (9) in the linear limit should possess a linear version of unbroken supersymmetry, i.e. it should be just a sum of the kinetic terms for all bosonic and fermionic components with the relative coefficients fixed by unbroken supersymmetry.

These conditions completely fix the component action.

The main results, obtained in this paper by using a method of nonlinear realizations, are the construction of the component actions for $N=1$, $D=4$ supermembrane and for supersymmetric $D2$ brane which are written in terms of geometric objects such as $\det(E)$ and covariant derivatives. In the first case the component action reads

$$S = \int d^3 x \left[2 - \det(E) \left(1 + \sqrt{1 - \frac{1}{2} \nabla^{ab} q \nabla_{ab} q} \right) \right], \qquad (10)$$

where the explicit expression for $\det(E)$ has the from

$$\det(E) = 1 + \frac{1}{2} \psi^a \nabla_{ab} \psi^b - \frac{1}{16} \psi^d \psi_d \, \nabla^{ab} \psi^c \nabla_{ab} \psi_c. \qquad (11)$$

In the case of supersymmetric $D2$ brane one obtains

$$S = 2 \int d^3 x \left[1 - \det(E) \frac{1}{1 - 2\lambda^2} \right] = \int d^3 x \left[2 - \det(E) \left(1 + \sqrt{1 + 8\widetilde{F}^2} \right) \right] \qquad (12)$$

where the field strength $\tilde{F}^{ab} = \frac{\lambda_{ab}}{1-2\lambda^2}$ is introduced, and has at the bosonic limit the structure of the standard Born-Infeld action for $D2$ brane.

It should be clear that the extremely simple form of the component actions is achieved due to the quite special choice of the physical components: all of them are fields of the nonlinear realization. This is in a dramatic contrast with the superfield approach, in which the main objects are the (super)fields of the linearly realized broken supersymmetry [6, 8, 9]. Of course, it is preferable to have the superfield actions, but their very nice superspace forms become very complicated after passing to the components. Moreover, in such component actions it is a very nontrivial task to select some geometric objects and structures. In this respect, our construction looks as an alternative one, and the component form and on-shell character of our actions is the price we have to pay for their simplicity and clear geometric meaning.

[1] S.R. Coleman, J. Wess, B. Zumino,
Structure of phenomenological Lagrangians. 1,
Phys.Rev. 177 (1969) 2239,
Structure of phenomenological Lagrangians. 2,
Phys.Rev. 177 (1969) 2247.

[2] D.V. Volkov,
Phenomenological Lagrangians,
Sov.J.Part.Nucl. **4**(1973) 3;
V.I. Ogievetsky,
"Nonlinear realizations of internal and space-time symmetries",
In Proceedings of the Xth Winter School of Theoretical Physics in Karpacz, Vol.1, p.117, 1974.

[3] E.A. Ivanov, V.I. Ogievetsky,
The Inverse Higgs Phenomenon in Nonlinear Realizations,
Teor. Mat. Fiz. **25** (1975) 164.

[4] D.V. Volkov and V.P. Akulov,
Possible universal neutrino interaction,
JETP Lett. **16**(1972 438;
Is the neutrino a Goldstone particle?
Phys. Lett. **B46**(1973) 109.

[5] S. Bellucci, E. Ivanov, S. Krivonos,
Superbranes and Super Born-Infeld Theories from Nonlinear Realizations,
Nucl.Phys.Proc.Suppl. **102** (2001) 26; `arXiv:hep-th/0103136`.

[6] J. Bagger, A. Galperin,
New Goldstone multiplet for partially broken supersymmetry,
Phys.Rev. **D55** (1997) 1091; `arXiv:hep-th/9608177`.

[7] M. Rocek, A.A. Tseytlin,
Partial breaking of global D=4 supersymmetry, constrained superfields, and 3-brane actions,
Phys.Rev. **D59** (1999) 106001; `arXiv:hep-th/9811232`.

[8] E. Ivanov, S. Krivonos,
N=1, D=4 supermembrane in the coset approach,
Phys.Lett **B453** (1999) 237, `arXiv:hep-th/9901003`.

[9] S. Bellucci, E. Ivanov, S. Krivonos,
Towards the complete N=2 superfield Born-Infeld action with partially broken N=4 supersymmetry,
Phys.Rev. **D64** (2001) 025014; `arXiv:hep-th/0101195`.

[10] M. Aganagic, C. Popescu, J.H. Schwarz,
Gauge-Invariant and Gauge-Fixed D-Bane Actions,
Nucl.Phys. **B495** (1997) 99, `arXiv:hep-th/9612080`.

[11] E. Bergshoeff, F. Coomans, R. Kallosh, C.S. Shahbazi, A. Van Proeyen, *Dirac-Born-Infeld-Volkov-Akulov and Deformation of Supersymmetry*, arXiv:1303.566[hep-th].

VECTOR FIELDS IN COSMOLOGY

E.A. Davydov

The modern challenge in cosmology is to find the mechanism for the inflation and for the present accelerated expansion. This almost implies the fundamental modification of gravity theory, or particle physics, or both. For example, one can introduce new (usually scalar) fields, sometimes with rather specific properties, or consider the modified gravity: theories with higher order curvature corrections, $F(R)$ gravity, non-minimal coupling, affine theory of gravity e.t.c. But the ocean of models can hardly be verified with our observational facilities. That's why one should focus either on the models which just slightly modify the existing theories, or choose the most fundamental ones, which can provide a new insight.

The theories with scalar fields allow one to construct simple and useful models, but they often imply a significant modification of particle physics, and are usually introduced 'ad hoc', without the fundamental reasoning. It may appear that the vector fields, which are very well studied in particle physics, may play their role in cosmology as well, but this possibility is studied very poorly up to now. Also we would like to mention that the fundamental modifications of gravity may also provide the effective theory with new vector degrees of freedom, not scalar ones. The well-known example of the Kaluza–Klein dimensional reduction may be supplemented by the affine generalization of gravity.

The vector models in cosmology have to solve the problem of diluting of the vector component whose amplitude is scaled out as $1/a$ with large and/or rapidly growing a, being the scale factor of the universe. This usually implies the construction of some dynamical 'scalars' within the vector theory. The corresponding vector-to-scalar effective lagrangians often inherit a specific coupling of these scalars to metric functions. And the naive Abelian vector model does not provide the isotropic configuration, and the conformal symmetry provides only the equation of state $p = \epsilon/3$.

In [1], we based our vector model on the well-known fact that Yang–Mills configuration with the $SU(2)$ gauge symmetry has three vector potentials A^a_μ which in the case of the FRW metrics allow the homogeneous and isotropic configuration. Indeed, in the Abelian case, the anisotropy comes from the stress-energy tensor components, proportional to E_iE_j, B_iB_j, where E_i, B_i are the 'electric' and 'magnetic' parts of the field tensor. But in the non-Abelian case, one has to take traces which vanish for the anisotropic components.

Next we considered the pure YM theory described by the action $S = \int L\sqrt{-g}d^4x$, where the lagrangian $L(\mathcal{F}, \mathcal{G})$ depends arbitrarily on two YM invariants

$$\mathcal{F} = -F^a_{\mu\nu}F^{a\mu\nu}/2, \quad \mathcal{G} = -\tilde{F}^{a\mu\nu}F^a_{\mu\nu}/4, \quad \tilde{F}^{a\mu\nu} = \frac{\epsilon^{\mu\nu\lambda\tau}F^a_{\lambda\tau}}{2\sqrt{-g}}. \tag{1}$$

We focused on the dependence of the lagrangian on the pseudoscalar $\mathcal{G}$. In gauge theories the linear in $\mathcal{G}$ term is induced by instantons and is called *theta-term*. We assumed, however, a more general dependence of L on $\mathcal{G}$, motivated, e.g. by vacuum polarization. The contribution coming from $\mathcal{G}$ was found to produce the energy density and pressure with the ratio $w = \varepsilon/p = -1$, except the linear case, which does not contribute to the Friedmann equations.

The next task was to choose the model in which the standard linear $\mathcal{F}$-term, does not overwhelm the contribution of the non-linear, hence higher order $\mathcal{G}$-term which decreases faster in the inflationary scenario. We found that it is sufficient to choose the configuration which is linear in energy density of $\mathcal{G}$-term, while the Lagrangian remains non-linear:

$$L = \frac{\mathcal{F}}{2} - \theta\,\mathcal{G}\ln\mathcal{G}. \tag{2}$$

The logarithmic dependence of this kind may come from the quantum corrections.

The slow-roll approximation ensures that the inflationary dynamics is allowed for the universe filled by both electric and magnetic fields slightly below Plank values. The inflation ends when the magnetic field increases up to the Plank value and dominates in the universe. The sufficient number of e-folds gained during the inflation is above sixty, as it follows from the current observational data. In this model, it can be achieved for the value of parameter θ being of the order of ten.

The high energy physics does not suit as well for the explanation of the dark energy problem. The answer can come from the geometry. In [2], we explored the effective models in the context of the affine generalization of gravity. For a simplified three-dimensional case we found that the effective theory may contain a non-trivial interaction of the Kaluza-Klein and affine 'vecton' degrees of freedom. Moreover, it allows the scalar representation of the Higgs-like doublet (ϕ, ψ) with the non-linear four-order potential:

$$V = 2\Lambda e^{\gamma} + \left[\lambda^2\Lambda\phi^2 + m^2\psi^2 + e^{-2\gamma}(Z + \psi\phi)^2\right] e^{-\gamma}, \tag{3}$$

where γ is a dilaton field coming from the dimensional reduction and λ, Λ, m, Z are the parameters of the theory. As is known, this kind of potentials may be used to model the accelerating expansion as well, and the parameters of the model come form the geometry, not from the particle physics, hence they may explain the 'smallness' of the present acceleration expressed via the effective Λ-term.

We would like to emphasize that due to the space-time symmetries arising in most practical cases, vector models can be treated, as a matter of fact, as some scalar-dilaton theories. This should allow one to work out a universal approach to the investigation of both vector and scalar theories. The physically motivated theories containing vector fields, supported by the well-developed methods within the scope of scalar models, should provide, as we hope, the answers to the open questions in cosmology, like the inflation and present accelerated expansion.

[1] D. V. Gal'tsov and E. A. Davydov,"Yang-Mills condensates in cosmology," Int. J. Mod. Phys. Conf. Ser. **14**, (2012) 316 [arXiv:1112.2943 [hep-th]].

[2] E. A. Davydov, "Vector fields in cosmology," AIP Conf.Proc. **1444** (2011) 125 arXiv:1112.3289 [hep-th]].

EXOPLANETARY SEARCHES WITH GRAVITATIONAL MICROLENSING

A.F. Zakharov

In paper [1], the discovery potential for exoplanet searches with gravitational microlensing was represented. It was pointed out that the technique gives an opportunity to discover light exoplanets at great distances from host stars, for instance exoplanets with a solid surface the near the so-called "snow line" where there is an opportunity to have water in a liquid phase.

In paper [2], polarization signatures of exoplanet existence in gravitational microlens systems were considered. Gravitational microlensing, when finite size source effects are relevant, provides a unique tool for the study of the source star stellar atmospheres through an enhancement of a characteristic polarization signal. This is due to the differential magnification induced during the crossing of the source star. A specific set of reported highly magnified, both single and binary exoplanetary systems, microlensing events towards the Galactic bulge was considered and the expected polarization signal was evaluated. To this purpose, several polarization models were considered which apply to different types of source stars: hot, late type main sequence and cool giants. As a result, the polarization signal P was computed, which goes up to P=0.04 percent for late type stars and up to a few per cent for cool giants, depending on the underlying physical polarization processes and atmosphere model parameters. Given a I band magnitude at maximum magnification of about 12, and a typical duration of the polarization signal up to 1 day, it was concluded that the currently available technology, in particular the polarimeter in FORS2 on the VLT, potentially may allow the detection of such signals. This observational programme may take advantage of the currently available surveys plus follow up strategy already routinely used for microlensing monitoring towards the Galactic bulge (aimed at the detection of exoplanets). In particular, this allows one to predict in advance for which events and at which exact time the observing resources may be focused to make intensive polarization measurements.

[1] A.F. Zakharov, Exoplanet searches with gravitational microlensing, *Physics – Uspekhi*, **54** (10), 17 (2011).

[2] G. Ingrosso, S. Calchi Novati, F. de Paolis, P. Jetzer, A.A. Nucita, F. Strafella, A. Zakharov, Polarization in microlensing towards the Galactic bulge, *Monthly Notices of Royal Astronomical Society*, **426**, 1496 (2012).

TESTS OF GRAVITATIONAL THEORIES AND ASTROPHYSICS

A.F. Zakharov

In paper [1], some possible observational signatures of R^n gravity at Galactic scales and how these signatures could be used for constraining this type of R^n gravity were studied. For that purpose, two body simulations in the R^n gravity potential were performed and the obtained trajectories of S2-like stars around the Galactic center, as well as the resulting parameter space of the R^n gravity potential were analyzed. The constraints on Rn gravity were discussed which can be obtained from the observations of orbits of S2-like stars with the present and next generations of large telescopes. Comparisons between the theoretical results and observations were made. The results show that the most probable value for the parameter r_c in the R^n gravity potential in the case of S2-like stars is around 100 AU, while the universal parameter β is close to 0.01. Also, the gravity potential induces the precession of S2-like star orbits in the opposite direction with respect to general relativity; therefore, such a behavior of orbits is qualitatively similar to the behavior of Newtonian orbits with a bulk distribution of matter (including a stellar cluster and dark matter distributions).

In paper [2], ASTROD I is based on the 2010 proposal submitted for the ESA call for class-M mission proposals, and is a sequel and an update to the previous paper [Experimental Astronomy 23 (2009) 491-527; designated as Paper I] which was based on our last proposal submitted for the 2007 ESA call. In this paper, we present our orbit selection with one Venus swing-by together with orbit simulation. In Paper I, the orbit choice is with two Venus swing-bys. The present choice takes shorter time (about 250 days) to reach the opposite side of the Sun. A preliminary design of the optical bench was also presented and elaborated on the solar physics goals with the radiation monitor payload. Telescope size, trade-offs of drag-free sensitivities, thermal issues was discussed and an outlook was presented. ASTROD I is a planned interplanetary space mission with multiple goals. The primary aims are: to test General Relativity with an improvement in sensitivity of over 3 orders of magnitude, improving our understanding of gravity and aiding the development of a new quantum gravity theory; to measure key solar system parameters with increased accuracy, advancing solar physics and our knowledge of the solar system; and to measure the time rate of change of the gravitational constant with an order of magnitude improvement and the anomalous Pioneer acceleration, thereby probing dark matter and dark energy gravitationally. It is envisaged as the first in a series of ASTROD missions. ASTROD I will consist of one spacecraft carrying a telescope, four lasers, two event timers and a clock. Two-way, two-wavelength laser pulse ranging will be used between the spacecraft in a solar orbit and deep space laser stations on Earth, to achieve the ASTROD I goals.

In paper [3], shadow formation around supermassive black holes were simulated. Due to enormous progress in observational facilities and techniques of data analysis researchers approach an opportunity to measure shapes and sizes of the shadows at least for the closest supermassive black hole at the Galactic Center. Measurements of the shadow sizes around the black holes can help to evaluate parameters of black hole metric. Theories with extra dimensions (RandallSundrum II braneworld approach, for instance) admit astrophysical objects (supermassive black holes, in particular) which are rather different from the standard ones. Different tests were proposed to discover signatures of extra dimensions in supermassive black holes since the gravitational field may be different from the standard one in the general relativity (GR) approach. In particular, gravitational lensing features are different for alternative gravity theories with extra dimensions and general relativity. Therefore, there is an opportunity to find signatures of extra dimensions in supermassive black holes. It was shown how measurements of the shadow sizes can put constraints on parameters of black hole in spacetime with extra dimensions.

In paper [4] a formation of holes in gravitationally lensed systems is given. Enormous progress is made in developing observational facilities. As a result, there are new opportunities to observe structures at sub-mas resolution. To explore gravitationally lensed systems, we simulate radio-lobe images distorted by microlensing. We show that the positions of holes in lensed images may indicate the positions of microlens groups or overdensities.

[1] D. Borka, P. Jovanovic, V. Borka Jovanovic and A. F. Zakharov, Constraints on R^n gravity from precession of orbits of S2-like stars, *Physical Reviews D*, **85**, 124004 (2012).

[2] C. Braxmaier, H. Dittus, B. Foulon, E. Goklu, C. Grimani, J. Guo, S. Herrmann, C. Lammerzahl, W.-T. Ni, A. Peters, B. Rievers, E. Samain, H. Selig, D. Shaul, D. Svehla, P. Touboul, G. Wang, A.-M. Wu, A. F. Zakharov, Astrodynamical Space Test of Relativity using Optical Devices I (ASTROD I) - A class-M fundamental physics mission proposal for Cosmic Vision 2015-2025: 2010 Update, *Experimental Astronomy*, **34**, 181 (2012).

[3] A. F. Zakharov, F. de Paolis, G. Ingrosso, A.A. Nucita, Shadows as a tool to evaluate black hole parameters and a dimension of spacetime, *New Astronomy Reviews*, **56**, 64 (2012).

[4] A. F. Zakharov, S. Simić, L. Č. Popović, and P. Jovanović, Evaluation of microlens distributions in gravitationally lensed systems based on accurate radio observations, in Advancing the Physics of Cosmic Distances Proceedings IAU Symposium No. 289, 2012 Richard de Grijs, ed., Cambridge University Press, p. 437, (2013).

LIFSHITZ FORMULA BY A SPECTRAL SUMMATION METHOD

V.V. Nesterenko and I.G. Pirozhenko

The Lifshitz formula was derived by making use of the spectral summation method, which is a mathematically rigorous simultaneous application of both the mode-by-mode summation technique and scattering formalism. The contributions to the Casimir energy of electromagnetic excitations of different types (surface modes, waveguide modes, and photonic modes) were clearly retraced. A correct transition to imaginary frequencies was accomplished with allowance for all the peculiarities of the frequency equations, and pertinent scattering data in the complex ω plane was solved completely. Some subtleties and vague points in previous derivations of the Lifshitz formula were elucidated.

1. V. V. Nesterenko, I. G. Pirozhenko, "Lifshitz formula by a spectral summation method" Phys. Rev. A 86, 5, 052503, 2012.

LIST OF PUBLICATIONS

JOURNAL PUBLICATIONS

1. R. Aouane, V.G. Bornyakov, E.M. Ilgenfritz, V.K. Mitrjushkin, M. Muller-Preussker and A. Sternbeck, “Landau gauge gluon and ghost propagators at finite temperature from quenched lattice QCD” Phys. Rev. D85, 034501 (2012).

2. A. B. Arbuzov, B. M. Barbashov, V. N. Pervushin, A. Borowiec, A. F. Zakharov, Strong Gravitation Waves in Terms of Maurer - Cartan Forms, *Physics of Atomic Nuclei,* **74**, 832 (2011).

3. A. B. Arbuzov, B. M. Barbashov, R. G. Nazmitdinov, V. N. Pervushin, A. Borowiec, K. N. Pichugin, and A. F. Zakharov, Universe as a Representation of Affine and Conformal Symmetries, *Physics of Particles and Nuclei Letters,* **8**, 187 (2011).

4. S. Belliard, S. Pakuliak, E. Ragoucy and N.A. Slavnov “Highest coefficient of scalar products in $SU(3)$-invariant integrable models“
J. Stat. Mech. (2012) P09003.

5. S. Bellucci, N. Kozyrev, S. Krivonos, A. Sutulin, “N=4 Chiral Supermultiplet Interacting with A Magnetic Field”, Phys. Rev. D 85 (2012) 065024 (e-Print Archive: arXiv:1112.0763 [hep-th]).

6. S. Bellucci, S. Krivonos, A. Sutulin, “SU(2) reductions in N=4 multidimensional supersymmetric mechanics”, J. Phys. A 45 (2012) 125402 (e-Print Archive: arXiv:1006.0376 [hep-th]).

7. S. Bellucci, S. Krivonos, A. Sutulin, “N=4, d=1 Supersymmetric Hyper-Kaehler Sigma Models with Isospin Variables”, JHEP 1102 (2011) 038 (e-Print Archive: arXiv:1012.2250 [hep-th]).

8. S. Bellucci, S. Krivonos, A. Sutulin, “N=4, d=1 Supersymmetric Hyper-Kaehler Sigma Models and Non-Abelian Monopole Background”, Acta Polytechnica 51 (2011) 13 (e-Print Archive: arXiv:1006.4770 [hep-th]).

9. S. Bellucci, S. Krivonos, “N=2 supersymmetric particle near extreme Kerr throat”, JHEP 1110 (2011) 014 (e-Print Archive: arXiv:1106.4453 [hep-th]).

10. S. Bellucci, S. Krivonos, A. Sutulin, “CP^n supersymmetric mechanics in U(n) background gauge fields”, Phys. Rev. D84 (2011) 065033 (e-Print Archive: arXiv:1106.2435 [hep-th]).

11. V.G. Bornyakov, V.K. Mitrjushkin, “SU(2) lattice gluon propagators at finite temperatures in the deep infrared region and Gribov copy effects” Phys.Rev. D84 (2011) 094503.

12. V.G. Bornyakov, V.K. Mitrjushkin and R.N. Rogalyov, “Gluon Propagators in 3D $SU(2)$ Theory and Effects of Gribov Copies” Phys.Rev. D86 (2012) 114503.

13. V.G. Bornyakov, V.K. Mitrjushkin, “Lattice QCD gluon propagators near transition temperature” Int. J. Mod. Phys. A27, 1250050 (2012).

14. D. Borka, P. Jovanovic, V. Borka Jovanovic and A. F. Zakharov, Constraints on R^n gravity from precession of orbits of S2-like stars, *Physical Reviews D*, **85**, 124004 (2012).

15. C. Braxmaier, H. Dittus, B. Foulon, E. Goklu, C. Grimani, J. Guo, S. Herrmann, C. Lammerzahl, W.-T. Ni, A. Peters, B. Rievers, E. Samain, H. Selig, D. Shaul, D. Svehla, P. Touboul, G. Wang, A.-M. Wu, A. F. Zakharov, Astrodynamical Space Test of Relativity using Optical Devices I (ASTROD I) - A class-M fundamental physics mission proposal for Cosmic Vision 2015-2025: 2010 Update, *Experimental Astronomy*, **34**, 181 (2012).

16. I.L. Buchbinder, E.A. Ivanov, I.B. Samsonov, “Superconformal N=3 SYM Low-Energy Effective Action”, JHEP 1201 (2012) 001 (e-Print Archive: arXiv:1111.4145 [hep-th]).

17. V. Bychkov, E. Ivanov, “N=4 Supersymmetric Landau Models”, Nucl. Phys. B 863 (2012) 33 (e-print Archive: arXiv:1202.4984 [hep-th]).

18. D.J. Cirilo-Lombardo, “On unified field theories, dynamical torsion and geometrical models: II”, Phys. Part. Nucl. Lett. 8 (2011) 507.

19. D.J. Cirilo-Lombardo, “Unified field theoretical models from generalized affine geometries II”, Int. J. Theor. Phys. 50 (2011) 1699.

20. F. Delduc, E. Ivanov, “Geometry and Harmonic Superspace: Some Recent Progress”, Phys. Part. Nucl. 43 (2012) 562 (e-Print Archive: arXiv:1201.3794 [hep-th]).

21. F. Delduc, E. Ivanov, “N = 4 mechanics of general (4, 4, 0) multiplets”, Nucl. Phys. B855 (2012) 815 (e-Print Archive: arXiv:1107.1429 [hep-th]).

22. E. A. Davydov, “Vector fields in cosmology,” AIP Conf.Proc. **1444** (2011) 125 [arXiv:1112.3289 [hep-th]].

23. M. Goykhman, E. Ivanov, S. Sidorov, “Super Landau Models on Odd Cosets”, Phys. Rev. D87 (2013) 025026, JINR-E2-2012-89 (e-Print Archive: arXiv:1208.3418 [hep-th]).

24. M. Goykhman, E. Ivanov, “Worldsheet supersymmetry in pohlmeyer-reduced superstrings”, J. Phys. Conf. Ser. 343 (2012) 012047.

25. M. Goykhman, E. Ivanov, “Worldsheet Supersymmetry of Pohlmeyer-Reduced $AdS_n \times S^n$ Superstrings”, JHEP 1109 (2011) 078 (e-Print Archive: arXiv:1104.0706 [hep-th]).

26. M. Goykhman, E. Ivanov, “Hidden supersymmetry in Pohlmeyer-reduced form of AdS(n) x S(n) superstrings”, Int. J. Mod. Phys. Conf. Ser. 13 (2012) 98.

27. S. Fedoruk, E. Ivanov, O. Lechtenfeld, “Nahm equations in supersymmetric mechanics”, JHEP 1206 (2012) 147 (e-Print Archive: arXiv:1204.4474 [hep-th]).

28. S.A. Fedoruk, E.A. Ivanov, A.V. Smilga, “Real and complex supersymmetric d=1 sigma models with torsions”, Int. J. Mod. Phys. A 27 (2012) 1250146 (e-Print Archive: arXiv:1204.4105 [hep-th]).

29. S. Fedoruk, E. Ivanov, O. Lechtenfeld, “Superconformal Mechanics”, J.Phys. A45 (2012) 173001 (e-Print Archive: arXiv:1112.1947 [hep-th]).

30. S. Fedoruk, P. Kosinski, J. Lukierski, P. Maslanka, “Nonrelativistic counterparts of twistors and the realizations of Galilean conformal algebra”, Phys. Lett. B699 (2011) 129 (e-Print Archive: arXiv:1012.0480 [hep-th]).

31. S. Fedoruk, J. Lukierski, “The algebraic structure of Galilean superconformal symmetries”, Phys. Rev. D84 (2011) 065002 (e-Print Archive: arXiv:1105.3444 [math-ph]).

32. S. Fedoruk, J. Lukierski, “New particle model in extended space-time and covariantization of planar Landau dynamics” Phys. Lett. B 718 (2012) 646 (e-Print Archive: arXiv:1207.5683 [hep-th]).

33. F. Ferrari, M. Piatek, “On a singular Fredholm-type integral equation arising in N=2 super Yang-Mills theories”, Phys.Lett. B 718 (2013) 1142 (e-Print Archive: arXiv:1202.5135 [hep-th]).

34. F. Ferrari, M. Piatek, “Liouville theory, N=2 gauge theories and accessory parameters”, JHEP 1205 (2012) 025 (e-Print Archive: arXiv:1202.2149 [hep-th]).

35. F. Ferrari, J. Paturej, M. Piatek, T.A. Vilgis, “Dynamics of two topologically entangled chains”, J. Math. Phys. 52 (2011) 043301 (e-Print Archive: arXiv:1103.3359 [cond-mat.stat-mech]).

36. P.P. Fiziev, D.R. Staicova, “Solving Systems of Transcendental Equations Involving the Heun Functions”
American Journal of Computational Mathematics, 2012, 2, 95-105.

37. P.P. Fiziev, D.V. Shirkov, “The (2 + 1)-dimensional axial universessolutions tothe Einstein equations, dimensional reduction pointsand KleinFockGordon waves”
J. Phys. A: Math. Theor. 45 (2012) 055205 (15pp)

38. P. Fre and A. S. Sorin, Extremal Multicenter Black Holes: Nilpotent Orbits and Tits Satake Universality Classes, JHEP **1301** (2013) 003 [arXiv:1205.1233 [hep-th]].

39. P. Fre, A. S. Sorin and M. Trigiante, Integrability of Supergravity Black Holes and New Tensor Classifiers of Regular and Nilpotent Orbits, JHEP **1204** (2012) 015[arXiv:1103.0848 [hep-th]].

40. D. V. Gal’tsov and E. A. Davydov,“Yang-Mills condensates in cosmology,” Int. J. Mod. Phys. Conf. Ser. **14**, (2012) 316 [arXiv:1112.2943 [hep-th]].

41. E.A. Ivanov, "Harmonic Superfields in N=4 Supersymmetric Quantum Mechanics", SIGMA 7 (2011) 015 (e-Print Archive: arXiv:1102.2288 [hep-th]).

42. S. Fedoruk, E. Ivanov, J. Lukierski, "Galilean Conformal Mechanics from Nonlinear Realizations", Phys. Rev. D83 (2011) 085013 (e-Print Archive: arXiv:1101.1658 [hep-th]).

43. E.A. Ivanov, "Generalized Landau models with N=2 worldline supersymmetry", Phys. Atom. Nucl. 75 (2012) 1227-1233.

44. E. Ivanov, "N=4 supersymmetric quantum mechanics and harmonics", Int. J. Geom. Meth. Mod. Phys. 09 (2012) 1261006.

45. E.A. Ivanov, A.V. Smilga, "Dirac Operator on Complex Manifolds and Supersymmetric Quantum Mechanics", Int. J. Mod. Phys. A 27 (2012) 1230024 (e-Print Archive: arXiv:1012.2069 [hep-th]).

46. G. Ingrosso, S. Calchi Novati, F. de Paolis, P. Jetzer, A.A. Nucita, F. Strafella, A. Zakharov, Polarization in microlensing towards the Galactic bulge, *Monthly Notices of Royal Astronomical Society*, **426**, 1496 (2012).

47. S. Krivonos, O. Lechtenfeld, "Many-particle mechanics with D(2,1:alpha) superconformal symmetry", JHEP 1102 (2011) 042 (e-Print Archive: arXiv:1012.4639 [hep-th]).

48. V. V. Nesterenko and I. G. Pirozhenko, "Vacuum energy in conical space with additional boundary conditions" Classical and Quantum Gravity, ISSN:0264-9381, eISSN:1361-6382, :IOP Publishing Limited, 28, 17, 175020 (1-26), 2011.

49. V. V. Nesterenko, I. G. Pirozhenko, "Lifshitz formula by a spectral summation method" Phys. Rev. A 86, 5, 052503, 2012.

50. V. V. Nesterenko, I. G. Pirozhenko, "Conic Singularity of the Space-Time and Cosmic Censorship Hypothesis", JINR News, 2, 9-10, ISSN:0134-4811, 2012.

51. P. Tretyakov. Scaling Solutions in Galileon Cosmology. Grav. Cosmol. 18 (1), 93-95 (2012).

52. V. N. Pervushin, A. F. Zakharov, Supernovae Type Ia and Cosmological Models, *JINR News*, 2012, N 1, p. 10.

53. V. N. Pervushin, A. B. Arbuzov, B. M. Barbashov, R. G. Nazmitdinov, A. Borowiec, K. N. Pichugin, and A. F. Zakharov, The General Relativity with Conformal Units, *Physics of Particles and Nuclei*, **43**, (5), 682 (2012).

54. V. N. Pervushin, A. B. Arbuzov, B.M. Barbashov, R.G. Nazmitdinov, A. Borowiec, K. N. Pichugin, A. F. Zakharov, Conformal and affine Hamiltonian dynamics of general relativity, *General Relativity and Gravitation*, **44**, 2745 (2012).

55. G. Torricelli, I. Pirozhenko, S. Thornton, A. Lambrecht, and C. Binns, "Casimir force between a metal and a semimetal" Europhysics Letters (IOP Publishing), ISSN:0295-5075, eISSN:1286-4854, :IOP Publishing Limited, 93, 51001, 2011.

56. M. Piatek, "Classical conformal blocks from TBA for the elliptic Calogero-Moser system", JHEP 1106 (2011) 050 (e-Print Archive: arXiv:1102.5403 [hep-th]).

57. A. R. Pietrykowski, "Interacting Scalar Fields in the Context of Effective Quantum Gravity", Phys. Rev. D 87 (2013) 024026 (e-Print Archive: arXiv:1210.0507 [hep-th]).

58. N.A.Tyurin, "The Chekanov tori and pseudotoric structures", Russian Mathematical Surveys, 66:1.

59. N.A.Tyurin, "Lagrangian fibration construction on the flag variety F3", Theor. math. phys.,166:2.

60. N.A.Tyurin, "Nonstandard Lagrangian tori and pseudotoric structures", Theoretical and Mathematical Physics(2012), 171:2.

61. N.A.Tyurin, S.A.Belev, "Lifts of Lagrangian tori", Mathematical Notes, 2012, 91:5.

62. A.F. Zakharov, Exoplanet searches with gravitational microlensing, *Physics – Uspekhi*, **54** (10), 17 (2011).

63. A.F. Zakharov, V.N. Pervushin, Conformal cosmological model and SNe Ia data, *Physics of Atomic Nuclei*, **75**, 1418 (2012).

64. A. F. Zakharov, F. de Paolis, G. Ingrosso, A.A. Nucita, Shadows as a tool to evaluate black hole parameters and a dimension of spacetime, *New Astronomy Reviews*, **56**, 64 (2012).

PREPRINTS AND DATA BASES

1. S. Bellucci, N. Kozyrev, S. Krivonos, A. Sutulin, "Symmetries of N=4 supersymmetric CP(n) mechanics", e-Print Archive: arXiv:1206.0175 [hep-th].

2. S. Bellucci, S. Krivonos, A. Shcherbakov, A. Sutulin, "On the road to N=2 supersymmetric Born-Infeld action", e-Print Archive: arXiv:1212.1902 [hep-th].

3. S. Bellucci, S. Krivonos, A. Nersessian, V. Yeghikyan, "Isospin particle systems on quaternionic projective spaces", e-Print Archive: arXiv:1212.1663 [hep-th].

4. Yu.B. Chernyakov, G.I. Sharygin, A.S. Sorin, "Bruhat Order in Full Symmetric Toda System", arXiv:1212.4803.

5. S. Fedoruk, J. Lukierski, "New spinorial particle model in tensorial space-time and interacting higher spin fields", e-Print Archive: arXiv:1210.1506 [hep-th].

6. F. Ferrari, M. Piatek, “On a path integral representation of the Nekrasov instanton partition function and its Nekrasov–Shatashvili limit”, e-Print Archive: arXiv:1212.6787 [hep-th].

7. P. Fiziev, “Withholding Potentials, Absence of Ghosts and Relationshipbetween Minimal Dilatonic Gravity and $f(R)$ Theories“ arXiv:1209.2695.

8. P. Fre, A.S. Sorin, M. Trigiante, ”Black Hole Nilpotent Orbits and Tits Satake Universality Classes”, arXiv:1107.5986 [hep-th], pgs. 1 - 65 (2011).

9. E.A. Ivanov, B.M. Zupnik, “Bispinor Auxiliary Fields in Duality-Invariant Electrodynamics Revisited”, e-Print Archive: arXiv:1212.6637 [hep-th].

10. N. Kozyrev, S. Krivonos, O. Lechtenfeld, “N=2 supersymmetric $S^2 -> CP^3 -> S^4$ fibration viewed as superparticle mechanics”, e-Print Archive: arXiv:1210.4587 [hep-th].

11. I. B. Pestov, ”Geometrical theory of Fundamental Interactions. Foundations of Unified Physics”, Communication of JINR, P2-2012-140, Dubna (2012).

12. I. B. Pestov, ”Geometrical theory of Fundamental Interactions. Gravidynamics”, Communication of JINR, P2-2012-141, Dubna (2012).

13. I. B. Pestov, ”Geometrical theory of Fundamental Interactions. Spinstatics and Spindynamics”, Communication of JINR, P2-2012-142, Dubna (2012).

14. I. B. Pestov, ”Geometrical theory of Fundamental Interactions. Generalized Electromagnetic Field”, Communication of JINR, P2-2012-143, Dubna (2012).

15. V. V. Nesterenko, I. G. Pirozhenko, ”Lifshitz formula by spectral summation method”, arXiv:1112.2599 [quant-ph].

16. I. Pirozhenko, M. Bordag, “On the Casimir repulsion in sphere-plate geometry”, arXiv:1302.5290 [quant-ph

17. D.Staicova, P. Fiziev, “New results for electromagnetic quasinormal modes of black holes” arXiv:1112.0310.

II. PARTICIPATION IN CONFERENCES

- “N=4 supersymmetric quantum mechanics with semi-dynamical supermultiplets”, S. Fedoruk, Invited Talk at The International Workshop “Supersymmetry in Integrable Systems - SIS’11”, 01-04 August 2011, Hannover, Germany.

- “Abelian lagrangian geometry: from geometric quantization to mirror symmetry”,N.A. Tyurin Proceedings of 36th National conference on Theor. Physics, Vietnham, 36 (2011).

- “Galilean conformal and superconformal symmetries: algebraic structures and dynamical realizations”, S. Fedoruk, Invited Talk at The Seventh International Conference “Quantum Theory and Symmetries (QTS-7)”, 7-13 August, 2011, Prague, Czech Republic.

- “2dCFT/Gauge/Bethe correspondence”, M. Piatek, Invited Talk at The International Workshop “Supersymmetries and Quantum Symmetries (SQS?2011)”, July 18-23, 2011, Dubna, Russia.

- “Classical limit of Liouville Theory, N=2 Gauge Theories and Calogero-Moser systems”, M. Piatek, Invited Talk at The International Workshop “Stringtheory/2011”, April 15-17, 2011, Warsaw, Poland.

- “Classical Conformal Blocks, Twisted Superpotentials, Yang’s Functionals and Uniformization of the 4-punctured Sphere”, M. Piatek, Invited Talk at the Workshop “Branes and Bethe Ansatz in Supersymmetric Gauge Theories”, March 21-25, 2011, Simons Center for Geometry And Physics, Stony Brook State University of New York, USA.

- “General N=4 superfield mechanics of the (4,4,0) multiplet”, E. Ivanov, Invited Talk at The International Workshop “Supersymmetry in Integrable Systems - SIS’11”, 01-04 August 2011, Hannover, Germany.

- “Worldsheet supersymmetry in Pohlmeyer-reduced superstrings”, E. Ivanov, Invited Talk at The Seventh International Conference “Quantum Theory and Symmetries (QTS-7)”, 7-13 August, 2011, Prague, Czech Republic.

- “N=4 supersymmetric mechanics: harmonic superspace as the universal tool of model-building”, E. Ivanov, Invited Talk at The XV International Conference “Symmetry Methods in Physics (SYMPHYS-XV)”, Dedicated to memory of A.N. Sissakian, July 12-29, 2011, Dubna & Yerevan, Russia & Armenia.

- “CP(n) supersymmetric mechanics in U(n) background gauge fields”, A. Sutulin, Invited Talk at The International Workshop “Supersymmetry in Integrable Systems - SIS’11”, 01-04 August 2011, Hannover, Germany.

- “Generalized nonlinear chiral supermultiplets”, A. Sutulin, Invited Talk at The Seventh International Conference “Quantum Theory and Symmetries (QTS-7)”, 7-13 August, 2011, Prague, Czech Republic.

- “Many-particle mechanics with D(2,1;alpha) superconformal symmetry”, S. Krivonos, Invited Talk at The International Workshop “Supersymmetry in Integrable Systems - SIS’11”, 01-04 August 2011, Hannover, Germany.

- “CP(n) supersymmetric mechanics”, S. Krivonos, Invited Talk at The Seventh International Conference “Quantum Theory and Symmetries (QTS-7)”, 7-13 August, 2011, Prague, Czech Republic.

- “CP(n) supersymmetric mechanics in U(n) background gauge fields”, S. Krivonos, Invited Talk at The XV International Conference “Symmetry Methods in Physics (SYMPHYS-XV)”, Dedicated to memory of A.N. Sissakian, July 12-29, 2011, Dubna & Yerevan, Russia & Armenia.

- “Supersymmetric mechanics with non-Abelian gauge fields”, S. Krivonos, Invited Talk at The International Conference “Recent Advances in Quantum Field and String Theory”, September 26-30, 2011, Tbilisi, Georgia.

- “CP(n) Supersymmetric Mechanics in U(n) Background Gauge Fields”, S. Krivonos, Invited Talk at The International Conference “Advances of Quantum Field Theory”, October 4-7, 2011, Dubna, Russia.

- “Spin, confinement, localization and supersymmetry”, “Bargmann-Wigner generalizations of the relativistic wave equations, arbitrary spin and the interaction problem”, “On generalized affine geometries, dynamical torsion and spin”, D.-J. Cirilo Lombardo, Talks at the A.I. Akhiezer Memorial Conference: “QED and Statistical Physics”, August 29 September 02, 2011, Kharkov, Ukraina.

- “Noncommutative structures from generalized affine geometries”, D.-J. Cirilo Lombardo, Invited Talk at The Seventh International Conference “Quantum Theory and Symmetries (QTS-7)”, 7-13 August, 2011, Prague, Czech Republic.

- “N=4 mechanics with spin variables and Nahm equations”, S. Fedoruk, Invited Talk at XX International Colloquium ”Integrable Systems and Quantum Symmetries” June 17 - 23, 2012, Prague, Czech Republic.

- “Gauged N=4 supersymmetric mechanics with spin supermultiplets”, S. Fedoruk, Invited Talk at the XXIX International Colloquium on Group-Theoretical Methods in Physics, August 20-26, 2012, Chern Institute of Mathematics, Tianjin, China.

- “N=4 supersymmetric mechanics and Nahm equations”, S. Fedoruk, Invited Talk at the Armenia-Dubna Workshop on Problems of integrable (supersymmetric) systems December 24 - 25, Dubna, Russia.

- “N=2 supersymmetric $S^2 -> CP^3 -> S^4$ fibration viewed as superparticle mechanics”, S. Krivonos, Invited Talk at the XX-th International Conference on Integrable Systems and quantum symmetries Prague, Chech Rep., June 17-23, 2012.

- “HP(n) sigma-model and instanton”, S. Krivonos, Invited Talk at 3rd workshop on Supersymmetry in Integrable Systems Yerevan, Armenia, August 27-30, 2012.

- “On the road to N=2 supersymmetric Born-Infeld theory”, S. Krivonos, Talk at the Round Table 5, France - Italy - Russia “Frontiers of Mathematical Physics”, December 16-18, 2012, Dubna.

- “N=2 supersymmetric fibration viewed as supersymmetric mechanics”, N. Kozyrev, Invited Talk at 3rd workshop on Supersymmetry in Integrable Systems Yerevan, Armenia, August 27-30, 2012.

- “N=2 Supersymmetric mechanics on HP(n) with Sp(n+1) symmetry”, N. Kozyrev, Invited Talk at the Armenia-Dubna Workshop on Problems of integrable (supersymmetric) systems, December 24 - 25, Dubna, Russia.
- “N=4 Supersymmetric mechanics on CP(n) and its symmetries”, A. Sutulin, Invited Talk at XX International Colloquium ”Integrable Systems and Quantum Symmetries” June 17 - 23, 2012, Prague, Czech Republic.
- “Symmetries of N=4 supersymmetric CP(n) mechanics”, A. Sutulin, Invited Talk at 3rd workshop on Supersymmetry in Integrable Systems Yerevan, Armenia, August 27-30, 2012.
- “On the road to N=2 supersymmetric Born-Infeld theory”, A. Sutulin, Talk at the Round Table 5, France - Italy - Russia “Frontiers of Mathematical Physics”, December 16-18, 2012, Dubna.
- “Superfield methods in three-dimensional supergravities”, B. Zupnik, Invited Talk at the Ginzburg conference, Moscow, FIAN, May 28 - June 2, 2012.
- “Nonabelian duality as a symmetry of auxiliary interaction”, B. Zupnik, Invited Talk at the Round Table 5, France - Italy - Russia “Frontiers of Mathematical Physics”, December 16-18, 2012, Dubna.
- “Super Landau Models on odd cosets”, S. Sidorov, Invited Talk at the Armenia-Dubna Workshop on Problems of integrable (supersymmetric) systems, December 24 - 25, Dubna, Russia.
- “Bispinor auxiliary fields in duality invariant electrodynamics”, E. Ivanov, Invited Talk at the Round Table 5, France - Italy - Russia “Frontiers of Mathematical Physics”, December 16-18, 2012, Dubna.
- “One-dimensional models from harmonic superspace”, E. Ivanov, Invited Talk at the Armenia-Dubna Workshop on Problems of integrable (supersymmetric) systems, December 24 - 25, Dubna, Russia.
- “Supersymmetric Mechanics with Spin Multiplets and Nahm Equations”, E. Ivanov, Invited Talk at the Ginzburg conference, Moscow, FIAN, May 28 - June 2, 2012.
- “Super Landau models with both world-line and target supersymmetries: N=2 and N=4 examples”, E. Ivanov, Invited Talk at the conference “Quantum Field Theory and Gravity (QFTG-12)”, Tomsk, July 31 August 4, 2012.
- “N=4 supersymmetric Landau model”, E. Ivanov, Invited Talk at the workshop “Classical and Quantum Integrable Systems”, Dubna, January 23-27, 2012.
- “Landau-type models with worldline N=4 supersymmetry”, E. Ivanov, Invited Talk at the 17th International Seminar on High Energy Physics “Quarks-2012”, Yaroslavl, Russia, 4-10 June, 2012.

- “Liouville theory, N=2 gauge theories and accessory parameters”, M. Piatek, Invited Talk at II Workshop on Geometric Correspondences of Gauge Theories, September 17-21, 2012, SISSA-Trieste, Italy.

- “Liouville theory, N=2 gauge theories and accessory parameters”, M. Piatek, Invited Talk at XXXI Workshop on Geometric Methods in Physics, June 24-30, 2012, Bialowieza, Poland.

- I.B. Pestov, “Mathematical Principles Unifying General Relativity and Quantum Mechanics.” Proceedings of the XX International Baldin Seminar on High Energy Physics Problems. Relativistic Nuclear Physics and Quantum Chromodynamics. V.I, p.47-52, E1,2-2011-121, Dubna, 2011.

- I.B.Pestov, ”The Unified Geometrical Theory of Microworld and Macroworld.” XLVIII All-Russia Conference on Problems of Physics of Particles, Physics of Plasma and the Condensed Matter,Optoelectronics. Dedicated to the 100th anniversary of Professor Ya.P. Terletsky. People’s Frendship University Press, 2012, p.159-162.

- A. F. Zakharov, S. Simić, L. Č. Popović, and P. Jovanović, Evaluation of microlens distributions in gravitationally lensed systems based on accurate radio observations, in Advancing the Physics of Cosmic Distances Proceedings IAU Symposium No. 289, 2012 Richard de Grijs, ed., Cambridge University Press, p. 437, (2013).

- A. F. Zakharov, F. de Paolis, G. Ingrosso, A.A. Nucita, Shadows as a tool to evaluate black hole parameters and a dimension of spacetime, New Astronomy Reviews, **56**, 64 (2012).

- I. G. Pirozhenko , “Fluctuation induced quantum interactions in the background of bodies with nontrivial dielectric (or magnetic) responce”, The 37th National Conference on Theoretical Physics, Institute of Physics, Vietnam Academy of Science and Technology, Cua Lo, Vietnam.

- I. G. Pirozhenko , “On the vacuum energy between a sphere and a plane at finite temperature”, X-th Conference on Quantum field theory under the influence of external conditions, Uni. Zaragoza and Centro de Ciencias de Benasque Pedro Pasqual, Benasque, Spain.