IV. MODERN MATHEMATICAL PHYSICS

CONTENTS

GRAVITY, COSMOLOGY AND INTEGRABLE SYSTEMS

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1. One of the most important fields of the theoretical research is nowadays the theory of gravity and, especially, of black holes and cosmologies. The present observational data strongly suggest that Einstein's gravity should be somehow modified, one of the popular modifications being provided by superstring ideas. Mathematically, even the classical Einstein theory is enormously difficult and the superstring mathematics of the order of magnitude is more difficult. For this reason, realistic simplified models are of great value. Many useful models are obtained by dimensional reductions of higher dimensional (super)gravity theories. For example, Kaluza-type reductions with subsequent spherical or cylindrical reductions produce two-dimensional dilaton gravity models, and their further reductions give one-dimensional models describing static states, cosmologies and simple nonlinear waves. The simplest low-dimensional models may be classically integrable and have simple enough analytical solutions for their applications as the first approximation to non-integrable models. A class of such integrable models is considered in detail in [1].

In view of the mathematical problems of the string modifications of gravity, other, much simpler, modifications of gravity that affect only the gravitational sector (not touching other interactions) are also popular. One such modification has recently been proposed in [2] and is studied in [3] - [5]. It is based on Einstein's proposal (1923) to formulate the gravity theory in non - Riemannian space with a symmetric connection. Unlike H.Weyl, who earlier considered very special affine space, Einstein proposed to specify the space-time geometry by using the Hamilton principle to determine the connection coefficients from a *geometric Lagrangian* that is an arbitrary function of the generalized Ricci curvature tensor and of other fundamental tensors. A theory like this supplements the standard Einstein gravity with dark energy (the cosmological constant, in the first approximation) and a neutral massive (or tachyonic) vector field (*vecton*) (this interpretation of the Einstein generalized theory was given in [2], [3]). The concrete choice of the geometric Lagrangian determines further details of the theory, and the new physical theory can be described by an effective Lagrangian. The most natural geometric models then look similar to recently proposed brane models of cosmology usually derived from string theory. The simplest cosmological models derived in this approach are essentially non-integrable but approximately related to some completely integrable models.

2. In [1], concluding our investigation of integrable models related to gravity and cosmology we consider the most general class of the two-dimensional dilaton gravity theories with multi-exponential potentials and a subclass of these theories, in which the equations of motion reduce to the Toda and Liouville systems. It is shown that the parameters of the model should satisfy a certain constraint that is solved for a general multi-exponential model. It follows from the constraint that in gravity theories integrable Toda equations in general cannot appear without accompanying Liouville equations.

A complete explicit solution of general one-dimensional integrable Toda - Liouville systems describing static states, cosmologies and nonlinear waves is derived. The relation between the three solutions (SCW - triality) is a generalization of an earlier found duality between static states and cosmologies (SC - duality). For static states and cosmologies, we also propose

and study a more general one-dimensional Toda - Liouville model typically emerging in one-dimensional reductions of higher-dimensional gravity and supergravity theories.
3. In [2]-[5], we study possible generalizations of the affine gravity theory proposed by Einstein in three beautiful but forgotten papers (1923), the ideas of which were given a new interpretation and further development in [2]. After a study of possible generalizations of Einstein's approach we have found that the most interesting class of the effective theories based on Einstein's approach to affine generalizations of the gravity theory is given by the following effective Lagrangian:

$$L_{eff} = \sqrt{-g}\left[-2\Lambda\,[\det(\delta_i^j + \lambda f_i^j)]^{1/(D-2)} + R(g) + c_a\, g^{ij} a_i a_j\right], \tag{1}$$

which should be varied with respect to the metric and the vector field; λ and c_a are the parameters depending on D (Einstein's first model is obtained for $D = 4$, $\lambda = 1$, and $c_a = 1/6$; the mass squared of the vecton is proportional to $c_a \neq 0$ that depends on the affine geometry and is zero in the Riemannian case). When the vecton field is zero, we have the standard Einstein gravity with the cosmological constant Λ. Making the dimensional reduction from $D > 4$ to $D = 4$, we obtain the Lagrangian describing the vecton a_i, $f_{ij} \sim \partial_i a_j - \partial_j a_i$ and $(D-4)$ scalar fields a_k, $k = 4, .., D$.
The theory (1) is very complex, even at the classical level. Its spherically symmetric sector is described by the (1+1)-dimensional dilaton gravity coupled to one massive vector and to several scalar fields. If the mass of the vector field is zero and the scalar fields vanish, the *dilaton gravity is classically integrable with a rather general dependence of the Lagrangian on the massless Abelian gauge fields*, $X(\phi, f^2)$, where $f^2 = f_{ij}f^{ij}$ and ϕ is the dilaton field. Depending on the signs of *Lambda* and of c_a the vecton and the scalar particles may be normal particles, ghosts, or tachyons. This opens a way to many very different applications to modern theoretical cosmology.
If $\mu^2 \neq 0$, the theory is certainly *not integrable even with vanishing scalar fields*. It is also not easy to analytically construct its physically interesting approximate solutions. We have shown that the static solutions may have two horizons and that the cosmological solutions are, in general, non-isotropic. We studied some cosmological solutions for the $D = 3$ theory which is simply the Einstein gravity supplemented with the cosmological constant Λ and minimally coupled to the massive vector field a_i (for $D = 4$ this is the small λ approximation):

$$L_{eff} \;=\; \sqrt{-g}\left[R[g] - 2\Lambda - \kappa\left(\frac{1}{2}F_{ij}F^{ij} + \mu^2 A_i A^i\right)\right], \tag{2}$$

where $A_i \sim a_i$, $F_{ij} \sim f_{ij}$. This theory is very interesting and deserves further investigations.

In conclusion, we should like to note that the geometrical and dynamical models discussed here are not quite well understood, both conceptually and technically. Much work on them should be done before a realistic cosmological model could be constructed. In particular, one should study the relation between the geometry and dynamics discovered by Einstein. Possibly, one should try to find behind it some symmetry principles that are not yet understood. One should also study more general geometric Lagrangians. A very important task is to study non-integrable static and cosmological models produced by these theories.
We must once more clearly state that the generalization of gravity considered here has nothing to do with other matter fields. It does not suggest any unification of gravity with other forces of nature and with the standard matter. Its true meaning and its unexpected

relation to recent discoveries and ideas in cosmology is a real puzzle. Possibly, a role of this theory might be to replace the standard gravity inside the string theory, which did not yet completely succeed in giving a simple and natural explanation of dark energy, inflation, and dark matter.

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THERMODYNAMICS OF MINIMAL SURFACES AND ENTROPIC ORIGIN OF GRAVITY

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The fact that gravity is an emergent phenomenon dates back to ideas of the last century [1]. A renewal of interest in this point of view in the last years has been motivated by attempts to find a statistical explanation of the Bekenstein-Hawking entropy, see e.g. [2]. A possible source of the entropy is quantum correlations of underlying microscopic degrees of freedom across the black hole horizon.

By taking the black hole case as a guide a number of arguments was presented in [3] that the entanglement entropy of fundamental degrees of freedom lying in a constant time slice and spatially separated by a surface $\mathcal{B}$ is

$$S(\mathcal{B}) = \frac{\mathcal{A}(\mathcal{B})}{4G} \quad . \tag{3}$$

Here G is the Newton coupling and $\mathcal{A}$ is the area of $\mathcal{B}$. Thus, (3) has the Bekenstein-Hawking form. Equation (3) holds in the semiclassical approximation if the low-energy limit of the fundamental theory is the Einstein gravity.

For realistic condensed matter systems the entanglement entropy associated with spatial separation of the system is a non-trivial function of microscopic parameters. Its calculation is technically involved and model dependent. A remarkable consequence of (3) is that the entanglement entropy in quantum gravity may not depend on a microscopic content of the theory, it is determined solely in terms of geometrical characteristics of the surface and low-energy gravity couplings.

Another feature established in [3] is related to the shape of the separating surface. Since $S(\mathcal{B})$ includes contributions of all fundamental degrees of freedom, quantum fluctuations of the geometry should be taken into account in a consistent way. For static space-times this requires that $\mathcal{B}$ is minimal surface, i.e. a surface with the least area. A relevant physical example of a minimal surface is the intersection of a constant time slice and the horizon of a stationary black hole. Thus, the Bekenstein-Hawking entropy can be considered as a particular case of the entanglement entropy (3).

The fact that $S(\mathcal{B})$ is a macroscopic quantity that obeys certain dynamical laws points to the similarity with thermodynamic entropy. A natural question is whether the entanglement on the fundamental level admits a form of thermodynamic laws.

The first step in this direction has been made in [3]. A calculation made there in the weak field approximation shows that a shift by a distance l of a test particle with a mass m out of the minimal surface results in the entropy change

$$\delta S(\mathcal{B}) = -\pi m \, l \quad . \tag{4}$$

Work needed to drag the particle by the background gravitational field is also proportional to l. One can relate the entropy change (4) and the work, the relation being an analog of the first law of thermodynamics. This also yields a local temperature on the surface (proportional to the product of the acceleration of a static observer near the surface and the normal vector to $\mathcal{B}$).

An intriguing hypothesis was suggested by E. Verlinde [4] in 2010 that gradients of the entropy of microscopic degrees of freedom in the underlying quantum gravity theory determine gradients of the gravitational field. The classical force of gravity can be interpreted as an entropic force, thus losing its fundamental nature. The hypothesis is based on a number of assumptions for so called ‘holographic screens’ which store information about fundamental microstates (’bits’) in such a way that a related entropy is proportional to the area of the screen. Interestingly, formula (4) for the entropy change was used by Verlinde as one of the postulates.
That is why deformations of minimal surfaces lying in constant time slices were studied in [5] for general static solutions to the Einstein equations. It was shown that formula (4) is remarkably universal and holds for such a general set up. This result provides: 1) strong support of the hypothesis [4] that gravity has an entropic origin, the minimal surfaces being a sort of holographic screens; 2) reduces by one the number of postulates of [4]; 3) suggests a definite physical interpretation for the entropy on the screen as the entropy of entanglement across the screen of quantum gravity states.

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'CUSP' SOLUTIONS IN GAUSS–BONNET GRAVITY

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Solutions with naked singularities are usually not in favor; therefore, subsequent investigations were produced only for the black hole branch. But now let us turn back to our motivation of adding higher order curvature corrections to the action. It is because of our belief in some fundamental quantum theory we add the GB term. Therefore, when we get a 'singularity' this just means that we moved outside the semi-classical limit with first-order corrections, but the 'real' quantum theory should deal with the growing curvature and energy density in some way. This is why the domain of initial conditions with high energy density should not be forgotten in favor of the domain of initial conditions with the event horizon.

The Gauss–Bonnet term provides a non-trivial contribution to the Einstein equations only in dimensions $D > 4$, while in $D = 4$ gravity one should include an interaction with the dilaton field to find some new dynamics (although the pure GB term in $D = 4$ can contribute, in some topological sense, modifying charges, for example).

One should mention that since the GB term is motivated by string theory, the corresponding calculations should be produced rather in the string frame than in the Einstein frame, and the transformation of the Gauss–Bonnet term between these frames generates a complicated additional term. It is known that not all solutions obtained in the Einstein frame can be reproduced in the string frame. Therefore, in [1] we produced two parallel calculations: for the usual Einstein-dilaton-Gauss–Bonnet action in order to extend some known results and for the stringy version of the action. The difference between two forms of the GB term is significant when dilaton field demonstrates the exponential growth.

We were searching for so-called 'cusp' solutions, treated as a mild singularity of the metrics, where the dilaton function and metric components are regular and non-vanishing, but their second (in a more general case even the first) derivatives possess singular behavior at some finite distance.

First, we have already mentioned the known singular solution with 1/2-th power expansion near the singularity. Such solutions can provide flat asymptotics, but also they can demonstrate from-cusp-to-cusp behavior, where the entire space can be split into several regions by cusps which are classically impenetrable. Then we showed the existence of a new singular solution to the Gauss–Bonnet dilatonic gravity, containing 1/3-th powers near the singular point. It cannot produce flat asymptotics and always spreads from one singularity to another. But it improves the inner region in the sense that one has flat space between cusps. Also, we found that the behavior of solutions in the case of classical and string versions of dilatonic gravity demonstrates a good qualitative coincidence.

Our investigation also sets up the question of quantum regularization of such singularities. There is some technique which allows one to deal with one spherically-symmetric singularity by constructing two quantum hamiltonians inside and outside the cusp. However, this cannot be done so easily in the case of (possibly infinite) series of different cusps.

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EINSTEIN-YANG-MILLS-HIGGS COSMOLOGY

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The cosmological models involving homogeneous and isotropic Yang-Mills (YM) fields were proposed recently as an alternative to scalar models of cosmic acceleration. There exists a unique $SU(2)$ YM configuration (generalizable to larger gauge groups) whose energy-momentum tensor is homogeneous and isotropic in space. It is parameterized by a single scalar field with a quatric potential. In the case of the closed universe the coupled YM – doublet Higgs system admits homogeneous and isotropic configurations, too.

In [1], we showed that coupled Yang-Mills-Higgs dynamics for closed FRW universe gives rise to new interesting evolution types which include transient regimes of cosmic acceleration, bounces and cyclic evolution. The presence of the YM component and Higgs phase rotation parameter changes substantially the dynamics of the universe at small scale factors. The interaction with the gauge field holds the Higgs field near the top of the potential, and the balance of accelerating and decelerating potentials of scalar and vector fields can freeze the scale factor. The phase rotation acts in reverse, making the dynamics of the system to be more similar to the scalar field with power-low potential, but its kinetic nature leads to the opposite sign in the potential at very small distances. Also, it increases the number of free parameters which can be useful in quantitative analysis.

One intriguing feature is possibility of an infinite sequence of cycles whose parameters change chaotically due to evolution of the YM component. This resembles the Multiverse models realized as a sequence of universes with different parameters in the ultralarge time scale.

The system of interacting Higgs and YM fields can be considered as a very natural candidate for the hybrid inflation scenario which is richer than standard slow roll inflation with a scalar field. A particular feature of the model is that due to the vector nature of the gauge field whose energy density depends on the scale factor the EYMH inflation also inherits this dependence. There can be large inflation in small universe, and a small inflation in a large universe. This looks quite similar to the current views on the evolution of the Universe with large initial inflation and slow late-time acceleration.

The conclusion is that while the pure Einstein-Yang-Mills (EYM) cosmology with the standard conformally invariant YM action gives rise to the hot universe, the Einstein-Yang-Mills-Higgs (EYMH) cosmology has a variety of regimes which include inflationary stages, bounces, and exhibit global cycling behavior reminiscent of the Multiverse developed in time.

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VIABLE MODELS OF MODIFIED GRAVITY

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1. Viable models of modified gravity which satisfy both local and cosmological tests are investigated. It is demonstrated that some versions of such highly non-linear models exhibit multiply de Sitter universe solutions which often appear in pairs, being one of them stable and the other unstable. It is explicitly shown that for some values of the parameters it is possible to find several de Sitter spaces (as a rule, numerically); one of them may serve for the inflationary stage, while the other can be used for the description of the dark energy epoch. The numerical evolution of the effective equation of state parameter is also presented, showing that these models can be considered as natural candidates for the unification of early-time inflation with late-time acceleration through dS critical points. Moreover, based on the de Sitter solutions, multiply SdS universes are constructed which might also appear at the (pre-)inflationary stage. Their thermodynamics is studied and free energies are compared [1].

2. The cosmological reconstruction is studied in the modified Gauss-Bonnet and F(R) gravities. Two alternative representations of the action (with and without auxiliary scalar) are considered. The approximate description of deceleration-acceleration transition cosmologies is reconstructed. It is shown that cosmological solution containing Big Bang and Big Rip singularities may be reconstructed only using the representation with the auxiliary field. The analytical description of the deceleration-acceleration transition cosmology in the modified Gauss-Bonnet gravity is demonstrated to be impossible at sufficiently general conditions [2].

3. Cosmological dynamics is considered in the generalized modified gravity theory with the $R\Box R$ term added to the action of the form $R + R^N$. Influence of the $R\Box R$ term on the known solutions of modified gravity is described. We show that in a particular case of $N = 3$ these two non-Einstein terms are equally important on power-law solutions. These solutions and their stability have been studied using a dynamical system approach. Some results for $N \neq 3$ (including stability of de Sitter solution in the theory under investigation) have been found using other methods [3].

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MATHEMATICAL PRINCIPLES UNIFYING GENERAL RELATIVITY AND QUANTUM MECHANICS

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The goal of accomplished studies is the unification of General Relativity and Quantum Mechanics into unified complete theory – General Quantum Mechanics. New concepts of physical space, time, gravitational field, internal symmetry, and spin are needed to solve this most urgent problem of modern physics and to provide, for the first time, understanding of a general structure of reality. The main laws of General Quantum Mechanics are formulated in terms of the equations for physical fields that are derived from the first principles and, hence, these equations are not phenomenological but fundamental ones. The set of equations in question involves equations of gravidynamics, equations of spinstatics and spindynamics, describing the whole spectrum of spin phenomena, equations of general electromagnetic field that deal with visible and hidden light (the so called dark matter). Characteristic solutions of these equations are found. General Quantum Mechanics is an integral structure in which geometry, symmetries, and fields are tightly connected and kept inseparable providing the uniqueness of the theory and the adequate solution of the most difficult conceptual problems.

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$\mathcal{N}$=4 SUPERSYMMETRIC MECHANICS WITH GAUGE FIELDS

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Supersymmetric quantum mechanics (SQM) [1] is the simplest ($d = 1$) supersymmetric theory. It has plenty of applications in various domains.
Of special interest are SQM models with extended $\mathcal{N} > 2, d = 1$ supersymmetries. An efficient tool of dealing with extended supersymmetries is the *Harmonic Superspace* (HSS) method [2]. Recently, the $d = 1$ version of the HSS approach [3] was applied to construct $\mathcal{N} = 4$ SQM models with the Lorentz-force type couplings to the external gauge fields, i.e. couplings of the form $A_m(x(t))\dot{x}^m(t)$. SQM models of this kind provide superextensions of the Landau problem (see, e.g., [4]) and give quantum-mechanical realizations of Hopf maps (see, e.g., [5]), etc.
Until recently, only $\mathcal{N} = 4$ superextensions of the couplings to *abelian* background gauge fields were known [3]. The coupling to *non-abelian* gauge backgrounds was constructed in [6, 7, 8] (see also [9]). This construction essentially exploits the *semi-dynamical* (or *spin*) supermultiplet $(\mathbf{4}, \mathbf{4}, \mathbf{0})$ firstly introduced in [10] to set up a new $\mathcal{N} = 4$ superextension of the renowned multi-particle Calogero model. Bosonic fields of the spin multiplet are described by the U(1) gauged Wess-Zumino $d = 1$ action and, after quantization, yield generators of the gauge group SU(2). A key role is also played by the manifestly $\mathcal{N} = 4$ supersymmetric $d = 1$ gauging procedure [11]. It turns out that the requirement of off-shell $\mathcal{N} = 4$ supersymmetry forces the external gauge potential to be (a) *self-dual* and (b) satisfying the $4D$ 't Hooft ansatz or its $3D$ reduction.
The relevant superfield action consists of the three pieces:

$$S = \int dt d^4\theta du\, R_{kin}(q^{+a}, q^{-b}, u) - i\frac{k}{2}\int du d\zeta^{(-2)}\, V^{++} - \frac{1}{2}\int du d\zeta^{(-2)}\, K(q^{+a}, u) v^+ \bar{v}^+, \quad (5)$$

where v, $\bar{v}$ are analytic harmonic $\mathcal{N} = 4$ superfields representing the off-shell spin multiplet. They obey the constraints $(D^{++} + iV^{++})v^+ = (D^{++} - iV^{++})\bar{v}^+ = 0$, where D^{++} is the harmonic derivative preserving Grassmann $\mathcal{N} = 4$ analyticity and the analytic superfield V^{++} gauges U(1) symmetry realized on $v, \bar{v}$. The independent bosonic fields in v, $\bar{v}$ form the SU(2) doublet $\varphi^\alpha, \bar{\varphi}_\alpha$, with $\alpha = 1, 2$ being the index of the automorphism SU(2) acting on the harmonic variables $u^\pm_\alpha$, $u^{+\alpha}u^-_\alpha = 1$. The "coordinate" analytic superfields $q^{+a}, a = 1, 2$, satisfy the harmonic constraints $D^{++}q^{+a} = 0$ and carry the $(\mathbf{4}, \mathbf{4}, \mathbf{0})$ off-shell representation. In particular, the bosonic fields enter q^{+a} as $q^{+a} = x^{\alpha a}(t)u^+_\alpha + \ldots$. The further details can be found in [7, 8].
The first two pieces in (5) represent a sigma-model type interaction of $x^{\alpha a}$ and the one-dimensional "*Fayet-Iliopoulos*" term, respectively. The third term is most important. It produces a generalized *WZ* term:

$$\int dt \left[i\bar{\varphi}^\alpha(\dot{\varphi}_\alpha + iB\varphi_\alpha) - \frac{1}{2}\bar{\varphi}^\beta\varphi_\gamma \left(\mathcal{A}_{\alpha a}\right)_\beta^{\ \gamma}\dot{x}^{\alpha a} + \text{fermions without } \partial_t \right], \quad (6)$$

where

$$\left(\mathcal{A}_{\alpha b}\right)_\beta^{\ \gamma} = \frac{i}{h}\left(\varepsilon_{\alpha\beta}\,\partial^\gamma_{\ b}h - \frac{1}{2}\delta^\gamma_\beta\,\partial_{\alpha b}h\right), \quad h(x) = \int du\, K(x^{+a}, u^\pm_\beta), \ \ \Box h(x) = 0\,.$$

The background SU(2) gauge field $\mathcal{A}_{\alpha b}$ is self-dual, $\mathcal{F}_{\alpha\beta} = 0$. It precisely matches with the general *'t Hooft* ansatz for $4D$ self-dual SU(2) gauge fields:

$$(\mathcal{A}_{\alpha b})_{\gamma}^{\beta} \Rightarrow (\mathcal{A}_{\mu})_{\gamma}^{\beta} = \frac{1}{2}\mathcal{A}_{\mu}^{i}(\sigma_i)_{\gamma}^{\beta} \quad \mathcal{A}_{\mu}^{i} = -\bar{\eta}_{\mu\nu}^{i}\partial_{\nu}\ln h(x)\,.$$

Finally, let me list some properties and consequences of this new class of off-shell $\mathcal{N} = 4$ SQM models.

- Our non-abelian SQM construction solves the long-lasting problem of setting up $\mathcal{N} = 4$ SQM model with *Yang* monopole [12] as a background.
- After the target space dimensional reduction $4D \to 3D$ one obtains $\mathcal{N} = 4$ SQM with couplings to the external $3D$ non-abelian monopole fields which satisfy the static version of the 't Hooft ansatz. This is a non-abelian generalization of the class of $\mathcal{N} = 4$ SQM models pioneered in [13].
- Our construction is limited to the 't Hoft ansatz and to the gauge group SU(2). Surprisingly, the *on-shell* actions, with all auxiliary fields eliminated, admit a direct extension to SU(N) and general self-dual backgrounds [6] - [8]. It is still an open question how to derive these models from an *off-shell* superfield approach.
- A further analysis of the new class of $\mathcal{N} = 4$ SQM models will mostly be concentrated on their possible applications in superextensions of the Landau problem and higher-dimensional quantum Hall effect, as well as in supersymmetric black-hole stuff. Also, it is an ambitious task to extend this consideration to $\mathcal{N} = 8, d = 1$ supersymmetry.

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NEW N=4 SUPERCONFORMAL MECHANICS OF n PARTICLES

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It proved surprisingly difficult to construct N=4 superconformal mechanics for more than three particles. The Hamiltonian (or the action) of this type of models is determined by two scalar prepotentials, F and U which are functions of the bosonic particle coordinates $\{x^i\}$ and obey two nonlinear differential equations, namely, the celebrated WDVV equation for F and a Killing-type equation for U in the F background. On top of this, conformal invariance imposes some homogeneity conditions on U' and F'''. Each solution to all equations produces a consistent many-particle model.

In one dimension (time), four supercharges imply invariance under the exceptional superalgebra $D(2,1;\alpha)$, for some value of the real parameter α. For the special cases of $D(2,1;0) \simeq su(1,1|2)$ and $D(2,1;1) \simeq D(2,1;-\frac{1}{2}) \simeq osp(4|2)$ some results were obtained in [1, 2]. On the one hand, by gauging the U(n) isometry of matrix superfield models, one can construct U(2) spin-extended mechanics for arbitrary values of α [1, 3]. However, the particle coordinates parametrize a non-flat target space, except for $\alpha{=}-\frac{1}{2}$, i.e. for the $osp(4|2)$ case. On the other hand, for α=0 a superspace approach produced an alternative formulation which allowed for the construction of a few nontrivial four-particle solutions [2].

In our recent paper [4], by adding to the particle coordinates and their superpartners a single harmonic variable (parametrizing a two-sphere), we have overcome the technical barrier for constructing $\mathcal{N}$=4 superconformal mechanics models with more than three particles. The structure equations determining the two prepotentials F and U admit simple solutions based on deformed root systems, for an arbitrary number of particles and for the superconformal symmetry algebra $D(2,1;\alpha)$ at any value of α. We have restricted ourselves to permutation-invariant prepotentials and performed a numerical survey of all permutation-symmetric (deformed) root configurations, with and without translation invariance.

In each moduli space of WDVV solutions F based on a deformed A_n or BCD_n root system, we have identified a permutation-invariant one-parameter (t) subfamily. It turns out that the solutions of the Killing-type equation for the second prepotential U in the background of a given WDVV solution F live on a curve $h_n(t,\alpha) = 0$ in the (t,α) plane. The $A_n(t{=}\infty)$ model, built on the roots and fundamental weights of A_{n-1}, is degenerate but distinguished by its translation invariance. Since $h_n(\infty, -\frac{1}{2}) = 0$, this solution exists only for the $osp(4|2)$ case. Also, its bosonic potential is not of Calogero-type. All other solutions lack translation invariance. Of course, one may reinterpret their variables as *relative* particle coordinates and add the center of mass to reclaim translation invariance, but permutation symmetry will usually be lost in the new variables.

An exception occurs at n=3 because of the $A_3 \simeq D_3$ isometry. Inside the F_3 family (with parameter t) of WDVV solutions (a reduction of the F_4 family), we have identified two curves $h_3^{(1,2)}(t,\alpha) = 0$ and one isolated point $(\hat{t},\hat{\alpha})$ for U solutions. The corresponding models can all be lifted to translation-invariant four-particle systems. For the n>3 exceptional root systems (F_4 and E_n and reductions thereof) and also for some super root system (a reduction of AB_4), only sporadic solutions for particular values of α and without translation invariance occur.

Obviously, lacking is geometrical understanding of the ‘zoo’ of solutions. It would be nice to find *sufficient* conditions on α or on (t,α) for the existence of U solutions in a given

F background. This may become more transparent if the requirement of permutation symmetry is dropped, so that further (F, U) solutions can be revealed. Although this requirement is physically reasonable (and this is only for the full translation-invariant system), it is mathematically unnatural. Perhaps a superspace reformulation of our models will shed more light on this question.

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PSEUDO TORIC GEOMETRY

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Geometric Quantization and Mirror Symmetry are very important and quite popular subjects in modern mathematical physics which are deeply related. The relationships and connections are believed to exist in general, but at the same time, they have been found, approved and explicitly described in several particular cases only while in a number of other cases they are intuitively established as artefacts and have nowadays certain "assertoric"status. Both the subjects are based on geometric ideas and constructions realizing an old poetic slogan of S.S. Chern "Physics and geometry are of one family...": Geometric Quantization studies the intrinsic geometry of the phase space of classical mechanical system, and Mirror Symmetry states that complex geometry is dual to symplectic geometry. The basic and firm geometric situation, where Geometric Quantization is parallel to Mirror Symmetry, and the methods of both the subjects work perfectly and unobstructed is given by toric geometry.

Toric geometry or geometry of toric varieties is rooted in the theory of integrable systems; by the very definition a toric variety is a closure of a complex torus $(\mathbb{C}^*)^n$ and thus, it admits a set of moment maps $(f_1, ..., f_n)$, real functions in involution, so a complete set of integrals. Since we are interested in the case of classical mechanical systems with compact phase space, each toric variety X corresponds via the action map

$$F = (f_1, ..., f_n) : X \to \mathbb{R}^n$$

to a convex compact polytop $P_X = F(X) \subset \mathbb{R}^n$, and the fibers of F by the Loiville theorem are lagrangian tori. The corresponding lagrangian fibration on X is called canonical. The Bohr - Sommerfeld fibers of this fibration represent the quantum states; more generally in Geometric Quantization any Bohr - Sommerfeld torus can be understood as a quantum state. Then the Hamiltonian isotopies on X can be understood as quantum symmetries and it follows that a pure mathematical problem becomes a problem of Geometric Quantization, namely, the classification problem for Bohr - Sommerfeld lagrangian tori in X.

Even recently one trusted that the standard fiber is a unique class of Bohr - Sommerfeld lagrangian tori up to Hamiltonian isotopies in a toric variety before Yu. Chekanov found ([1]) an exotic monotone (= Bohr - Sommerfeld in our language) lagrangian torus in $\mathbb{R}^4$; then ([2]) this example was multiplied to a range of exotic tori in $\mathbb{R}^{2n}$, the projective spaces $\mathbb{CP}^n$ and the products $\mathbb{CP}^1 \times \mathbb{CP}^1 ... \times \mathbb{CP}^1$. At the same time, it was established ([3], [4]) that these exotic Chekanov tori can be defined in terms of *pseudo toric structures*, introduced by the author and S. Belyov in [5],[6]. Roughly, a pseudo toric structure (of rank 1) on a symplectic manifold X of real dimension $2n$ is polydata $(f_1, ..., f_{n-1}, \psi)$, where $\{f_i\}$, the "classical"piece of the data, is a set of commuting functions and ψ, the "complex"part, is a Lefschetz pencil ("complex analogy of Morse function according to V.I. Arnold [7] and S. Donaldson [8]). This ψ by definition gives a family of symplectic submanifolds D_z of X parameterized by the projective line $\mathbb{CP}^1$, and the commutation relation for f_i and ψ reads as follows: the Hamiltonian flow $\phi^t_{f_i}$ preserves each fiber D_z for any $i = 1, ..., n-1$. Thus, each fiber D_z endowed with the restrictions $\{f_i|_{D_z}\}$ is itself a completely integrable system. However, the fibers are different: there appear several singular fibers (or if each fiber is singular — more singular than a generic one) which mark a chain of distinguished points $z_1, ..., z_m \in \mathbb{CP}^1$. Primitive elements of the fundamental group $\pi_1(\mathbb{CP}^1 \backslash \{z_i\})$ represent different classes of Bohr - Sommerfeld lagrangian tori. The main conjecture we plan to work on in the near future

states that these classes are nonequivalent under the Hamiltonian isotopies. At the same time, in a forthcoming paper [9] we show that any smooth toric variety admits pseudo toric structures, and, therefore, summing up all these facts we expect the existence of exotic Bohr - Sommerfeld lagrangian tori of the Chekanov type in any toric variety.
Leaving aside the case of non toric varieties which nevertheless admit pseudo toric structures, see [10], I would like to conclude this note with the following remark. The construction of pseudo toric polydata on a given X can be repeated several times: since each fiber of ψ is itself a toric variety, we can apply the procedure uniformally to each D_z "pulverizing"our X into smaller pieces and cancelling an integral from the complete set $\{f_i\}$ at every step. At the end, we would get the following picture: our X with one function H is sliced by rational Riemann surfaces Σ_Z parameterized by complex $n-1$ - dimensional space $L \ni Z$ such that the Hamiltonian flow ϕ^t_H preserves each Σ_Z. Then real solutions of the classical mechanical system with the phase space X and the Hamiltonian H can be found easily as solutions of systems with one degree of freedom. This approach looks like certain new conception of integrability which can specify the classical Liouville integrability as the spectral curve setting it does.

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