Classical and quantum constraints in spin physics

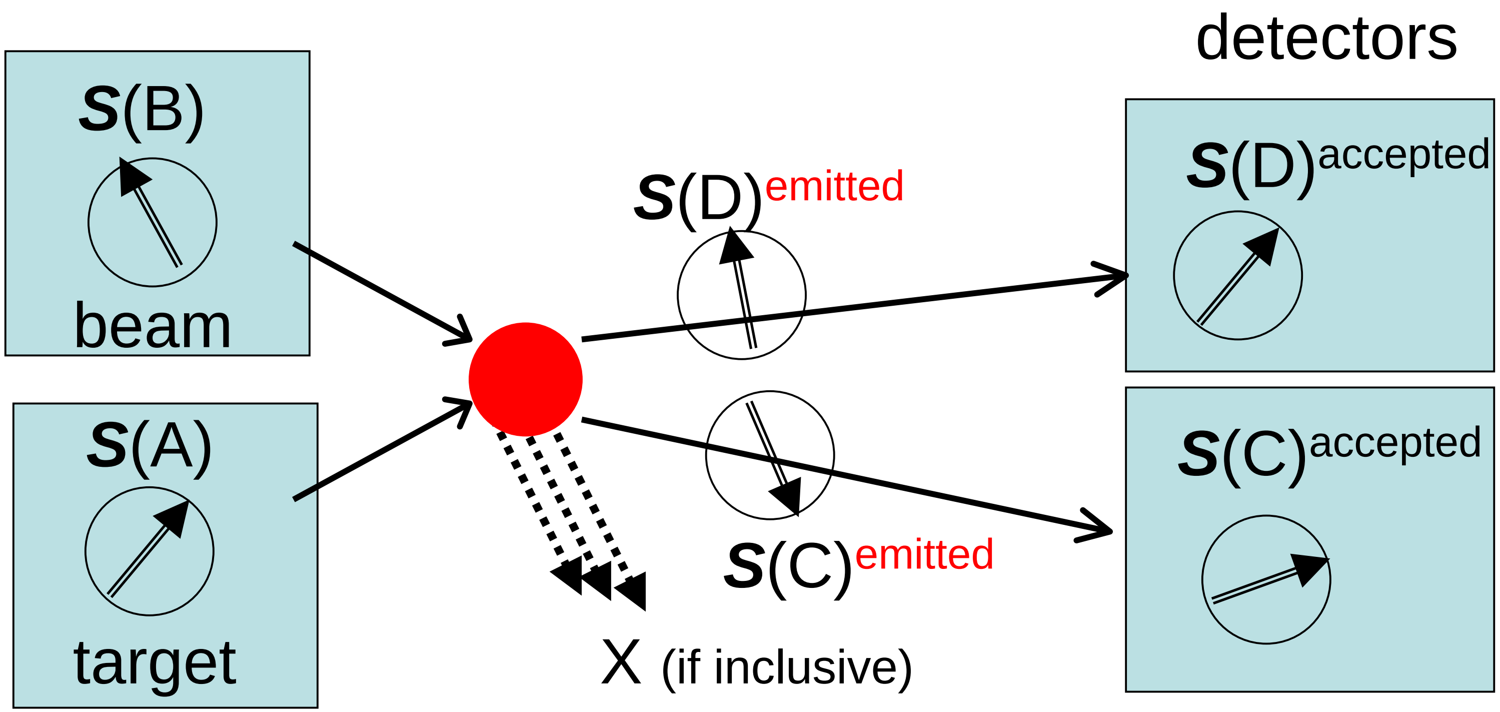
X.Artru

IPN-Lyon, France

- Symmetry constraints
- Positivity constraints
- Separability domains

Polarized experiment

A+B→C+D (+X)



emitted polarization ≠ *accepted* polarization

cross section and final polarization

(one-half spins) ***S*** =*polarization vector.* $|\mathbf{S}| \leq 1$

Polarized cross section :

$$d\sigma/\ d\Omega_{pol} = I_0\ F\{\ \mathbf{S}(A),\ \mathbf{S}(B),\ \mathbf{S}(\mathbf{C})^{acc}, \mathbf{S}(\mathbf{D})^{acc}\ \}$$

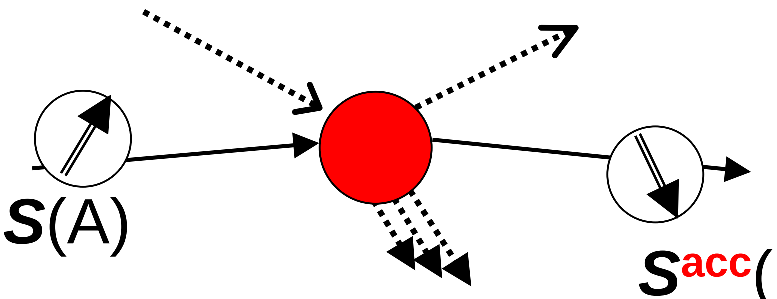
Emitted polarization of **C** as a function of ***S***(**A**) and ***S***(**B**)

$$\mathbf{S}(\mathbf{C})^{emit} = \frac{\nabla_{\mathbf{S}(\mathbf{C})^{acc}}\ F\{\ \mathbf{S}(A),\ \mathbf{S}(B),\ \mathbf{S}(\mathbf{C})^{acc}=0, \mathbf{S}(D)^{acc}=0\ \}}{F\{\ \mathbf{S}(A),\ \mathbf{S}(B),\ \mathbf{S}(\mathbf{C})^{acc}=0, \mathbf{S}(D)^{acc}=0\ \}}$$

Cartesian reaction parameters $(\mu|\nu)$ (or "*correlation*" parameters)

Exemple:

0+1/2 $\rightarrow$ 0+1/2



$$F\{S(A),S(B)\} = (0|0)$$
$$+ S_x(A)\,(x|0) + S_y(A)\,(y|0) + S_z(A)\,(z|0)$$
$$+ (0|x)\,S_x(B) + (0|y)\,S_y(B) + (0|y)\,S_y(B)$$
$$+ S_x(A)\,(x|x)\,S_x(B) + S_x(A)\,(x|y)\,S_y(B) + \ldots$$

$$= S_\mu(A)\,(\mu|\nu)\,S_\nu(B)$$

$S_\mu = (S_0\,,\,\boldsymbol{S})$ $\qquad S_0 = 1$ $\qquad (0|0) = 1$

Classical and quantum constraints for *parity*

- Assume that reaction has a *symmetry plane* (e.g. scattering plane in $2 \to 2$)

Parity + rotation $\Rightarrow$ *mirror reflection* Π about this plane (x,z)

$\Pi \; S_x \, \Pi^{-1} = - S_x$ "Π - odd"
$\Pi \; S_y \, \Pi^{-1} = + S_y$ "Π - even"
$\Pi \; S_z \, \Pi^{-1} = - S_z$ "Π - odd"

Classical parity rule: *all Π* ***- odd*** *observables vanish.*

Exemple : $\pi + N \to K + \Lambda$

$(x|0) = (z|0) = (x|y) = 0$, etc. ,

non-vanishing observables : $(y|0) \equiv A_N$, $(x|z)$, etc.

Quantum parity constraints

Parity conservation for *amplitudes*:

$$M = \Pi(B)\ M\ \Pi^{-1}(A)$$

or $$M^\dagger = \Pi(A)\ M^\dagger\ \Pi^{-1}(B)$$

Applying it to $(\mu|\nu) = \mathrm{Tr}\{ M\ \sigma_\mu\ M^\dagger\ \sigma_\nu \}$, one obtains:

- *Classical* parity when applying to *both* M and $M^\dagger$

- *Quantum* constraints when applying *only to* M *or to* $M^\dagger$

Exemple: $\pi + N \rightarrow K + \Lambda$

$$(y|y) = (0|0)\ ,$$

$$(0|y) = (y|0)\ .$$

Classical and quantum constraints for *positivity*

Classical positivity:

F{***S***(A),***S***(B),***S***(C),…. } is positive *for any set of polarization vectors* ***S***(A),***S***(B),***S***(C), etc.

(subject to the conditions |***S***| ≤ 1)

Example: $1/2 + 1/2 \rightarrow X$

if F{ ***S***(A),***S***(B) } = 1+ **c** ***S***(A).***S***(B) (isotropic case)

then $-1 \leq \mathbf{c} \leq +1$.

Quantum constraints for positivity

1) case $1/2 + 1/2 \rightarrow X$

cross section = I_0 F{***S***(A),***S***(B)} , with

F{***S***(A),***S***(B)} = $(\mu\nu)\ S_\mu(A)\ S_\nu(B)$

$$\Downarrow$$

Cross section ***matrix***

$R_{A+B} = (\mu\nu)\ \sigma_\mu(A) \otimes \sigma_\nu(B)$ $(\sigma_0 = I)$

Quantum positivity: R is ***semi-positive*** (like a density matrix)

$$\langle \Psi_{A+B} | R | \Psi_{A+B} \rangle \geq 0$$

If Ψ_{A+B} is ***separable*** one obtains only ***classical*** positivity. In the isotropic example,

$$\langle \Psi_A \otimes \Psi_B | 1 + \mathbf{c}\, \sigma_i(A) \otimes \sigma_i(B) | \Psi_A \otimes \Psi_B \rangle \geq 0$$

$$\Rightarrow \qquad \mathbf{-1 \leq c \leq 1}$$

If Ψ_{A+B} is ***entangled*** $(\neq \Psi_A \otimes \Psi_B)$ one obtains a ***quantum positivity*** *constraint* :

$$\mathbf{-1 \leq c \leq 1/3}$$

which is more severe

Usefull rule:

"fully anti-parallel spins (c=-1) are allowed"

"fully parallel spins (c=+1) are forbiden"

The entangled state which which imposes $c \leq 1/3$ is the *spin singlet :*

$$\Psi_{A+B} = 2^{-1/2} \left(|\uparrow\downarrow\rangle - |\uparrow\downarrow\rangle \right)$$

$$\Rightarrow \langle \sigma_i(A) \otimes \sigma_i(B) \rangle = -3$$

$$\Rightarrow R\{ \sigma(A), \sigma(B) \} = 1 - 3c$$

$\Rightarrow$ $R>0$ for $c \leq 1/3$

Initial entangled states are not easy to prepare, but not impossible.

Exemple: $e^+e^- \rightarrow 2\,\gamma$'s, when e^+e^- form a *para-positrium.*

Quantum positivity constraints.

2) case $1/2 + 0 \to 1/2 + X$

Polarized cross section

$\sim F\{ \boldsymbol{S}(A), \boldsymbol{S}(B) \} = (\mu|\nu)\; S_{\mu}(A)\, S_{\nu}(B)$

$$\Downarrow$$

cross section matrix (again ***semi-positive***)

$R_{A\text{-}B}\{ \sigma(A), \sigma(B) \} = (\mu|\nu)\, \sigma_{\mu}(A) \otimes \sigma_{\nu}(B)$

Note the *transposition* in σ (B), related to the *crossing* from the $1/2 + 1/2 \to X$ case.

Quantum positivity. Case $1/2+0 \to 1/2 + X$ (continued)

Example: $F\{ \boldsymbol{S}(A),\boldsymbol{S}(B) \} = 1+ \mathbf{d}\, \boldsymbol{S}(A).\boldsymbol{S}(B)$

$$\Downarrow$$

$$R\{ \sigma(A), \sigma(B) \} = 1+ \mathbf{d}\, \boldsymbol{\sigma}_i(A) \otimes \sigma_i(B)$$

Classical positivity : $-1 \leq \mathbf{d} \leq 1$

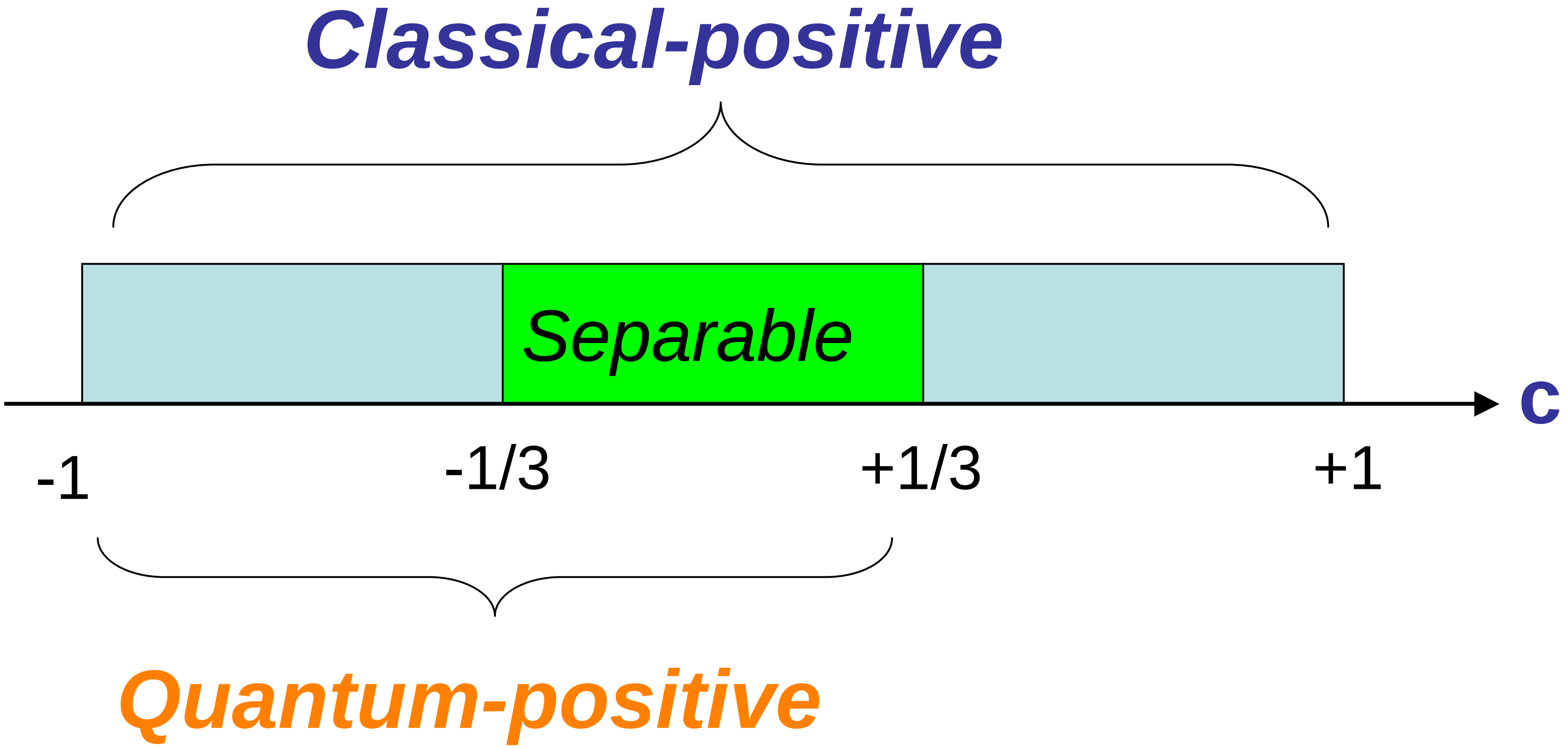
Quantum positivity : $1/3 \leq \mathbf{d} \leq 1$

"full spin transmission (d=+1) is allowed"

"full spin reversal (d=-1) is forbiden"

Different domains for R_{A+B} $(1/2 + 1/2 \to X)$

One-parameter case: $R_{A+B} = 1 + c\, \sigma_i(A) \otimes \sigma_i(B)$



Three-parameter case:

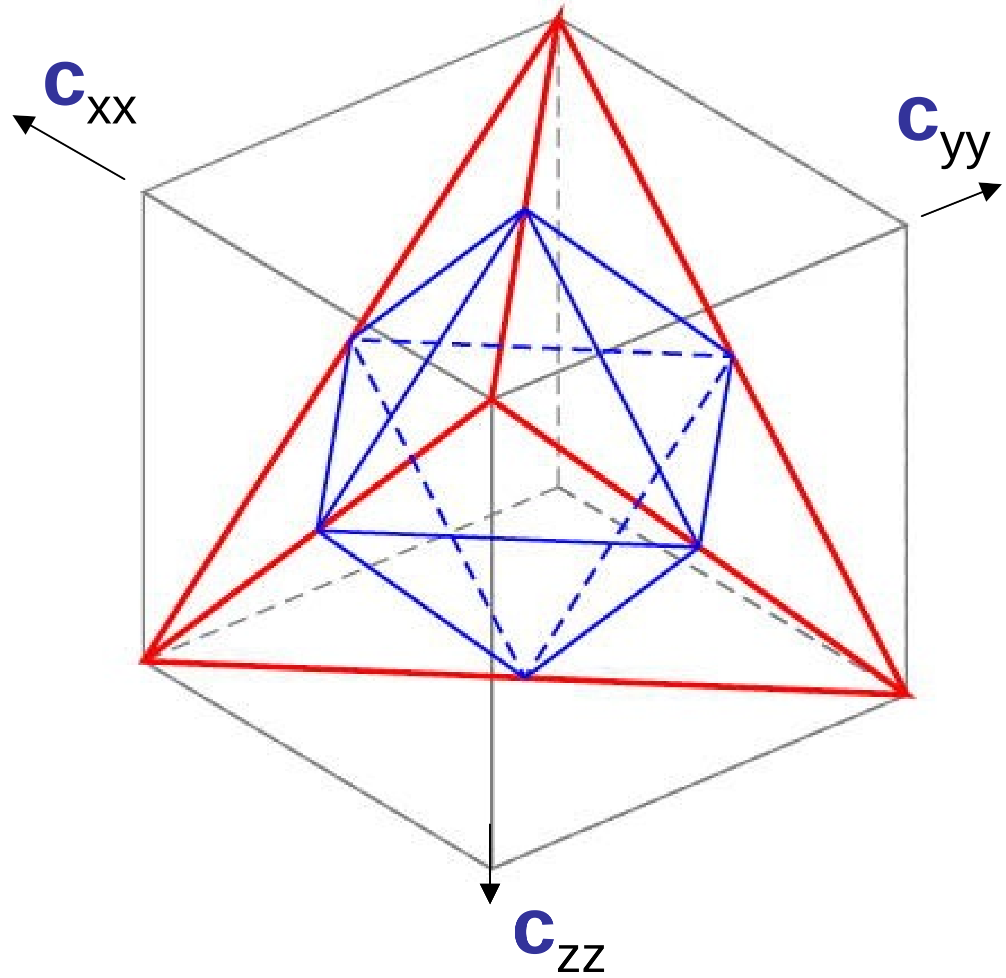
$R_{A+B} =$

$1 + \mathbf{c}_{xx}\ \sigma_x(A)\ \sigma_x(B)\ \mathbf{c}_{xx} + \mathbf{c}_{yy}\ \sigma_y(A)\ \sigma_y(B) + \mathbf{c}_{zz}\ \sigma_z(A)\ \sigma_z(B)$

(cube) classical positivity

(tetraedron) quantum positivity

(octaedron) separability



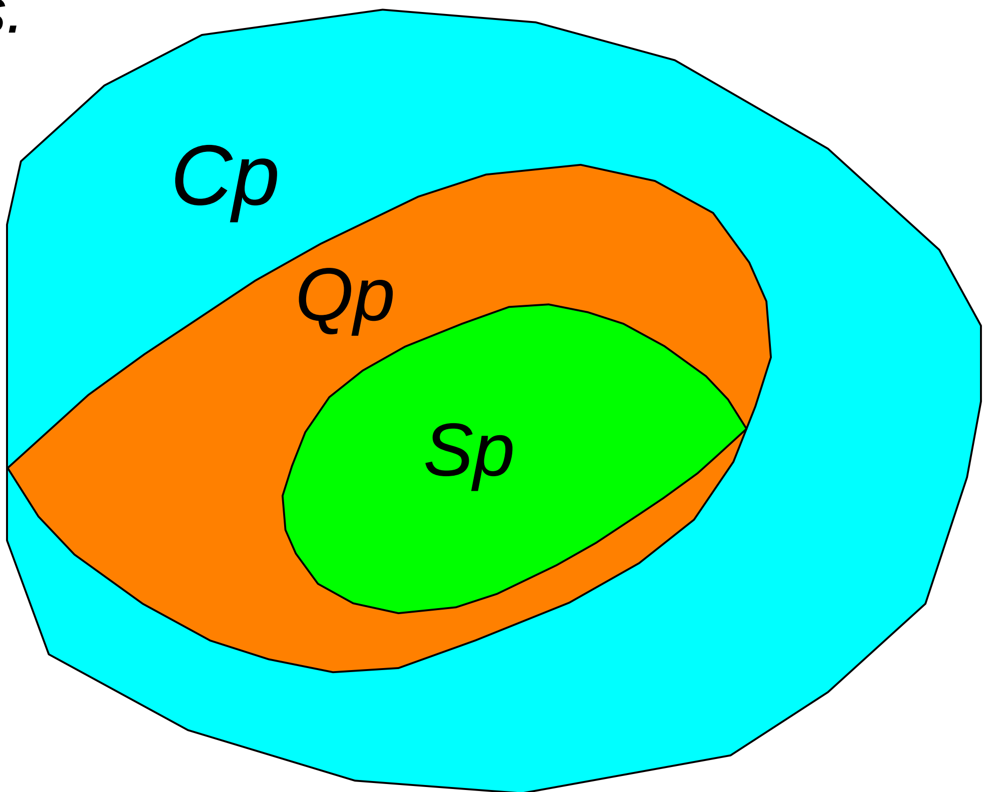
Duality [***classical*** positivity] $\leftrightarrow$ [***separability***]

ρ = initial density matrix ; R = cross section matrix

Cross section: $\sigma \sim \mathrm{Tr}\,(\rho R)$
R is ***acceptable*** by ρ if $\mathrm{Tr}(\rho R) \geq 0$.
This is the case for $\rho \in$ *Sp* and R $\in$ *Cp*:
Cp and *Sp* are *dual domains.*

Qp is self-dual

Sp , *Qp* and *Cp*
are *convex.*
Sp $\subset$ *Qp* $\subset$ *Cp*



Conclusions

- Quantum constraints are stronger than classical constraints for :

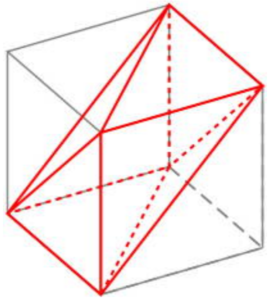
  - discrete symmetries like parity, time reversal, charge conjugation, identical particles.

  - positivity

- the classical positivity domain is dual to the separability domain

Additional remark: some particles are unpolarized, quantum constraints become weaker or disappear. The same is true for inclusive reactions.

THANK YOU !



Classical positivity, quantum positivity and separabilty domains