

Transversity, Collins and Sivers Effects from COMPASS, HERMES and BELLE Data: New Global Analysis

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In collaboration with M. Anselmino, M. Boglione, U. D'Alesio,
F. Murgia, A. Kotzinian and C. Turk

Outline of this talk

- 1 Introduction
- 2 Collins effect in SIDIS and e^+e^- annihilation
 - The model for Collins FF and transversity
 - Description of the data & Predictions
- 3 Sivers effect in SIDIS
 - The model for the Sivers function
 - Description of the data & Predictions
- 4 Conclusions

The fundamental distributions of partons inside a nucleon

Unpolarised Distribution

$$f_1(x) \text{ or } q(x)$$



Distribution of unpolarised partons in an unpolarised nucleon.
Well known

Helicity Distribution

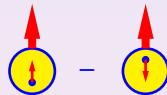
$$g_1(x) \text{ or } \Delta q(x)$$



Distribution of longitudinally polarised partons in a longitudinally polarised nucleon.
Known

Transversity Distribution

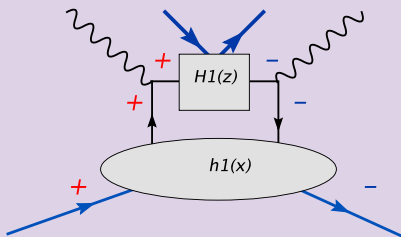
$$h_1(x) \text{ or } \Delta_T q(x)$$



Distribution of transversely polarised quarks in a transversely polarised nucleon.
Little known!
HERMES and COMPASS
first experimental measurements

Transversity in SIDIS

Transversity in Semi inclusive Deep Inelastic Scattering $IN \rightarrow l'hX$



Transversely polarised quark fragments into an unpolarised hadron:

$$D_{h/q^\uparrow}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{1}{2} S_{q'} \cdot (\hat{\mathbf{p}}_{q'} \times \hat{\mathbf{p}}_\perp) \Delta^N D_{h/q^\uparrow}(z, |\mathbf{p}_\perp|),$$

where \mathbf{p}_\perp is transverse momentum of produced hadron with respect to fragmenting quark \rightarrow **non-perturbative effect**.

Collins FF

Collins Fragmentation Function

There are two different notations for Collins FF:

$$D_{h/q\uparrow}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{1}{2} S_{q'} \cdot (\hat{\mathbf{p}}_{q'} \times \hat{\mathbf{p}}_\perp) \Delta^N D_{h/q\uparrow}(z, |\mathbf{p}_\perp|)$$

and

$$D_{h/q\uparrow}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{S_{q'} \cdot (\hat{\mathbf{p}}_{q'} \times \mathbf{p}_\perp)}{zM_\pi} H_1^{\perp q}(z, |\mathbf{p}_\perp|),$$

both $\Delta^N D_{h/q\uparrow}(z, |\mathbf{p}_\perp|)$ and $H_1^{\perp q}(z, |\mathbf{p}_\perp|)$ refer to Collins FF

Collins FF

Collins Fragmentation Function

There are two different notations for Collins FF:

$$D_{h/q^\uparrow}(z, \mathbf{p}_\perp) = D_{h/q}(z, |p_\perp|) + \frac{1}{2} S_{q'} \cdot (\hat{p}_{q'} \times \hat{\mathbf{p}}_\perp) \Delta^N D_{h/q^\uparrow}(z, |p_\perp|)$$

and

$$D_{h/q^\uparrow}(z, \mathbf{p}_\perp) = D_{h/q}(z, |p_\perp|) + \frac{S_{q'} \cdot (\hat{p}_{q'} \times \mathbf{p}_\perp)}{zM_\pi} H_1^{\perp q}(z, |p_\perp|),$$

Relation

$$\Delta^N D_{h/q^\uparrow}(z, |p_\perp|) = \frac{2|p_\perp|}{zM_\pi} H_1^{\perp q}(z, |p_\perp|).$$

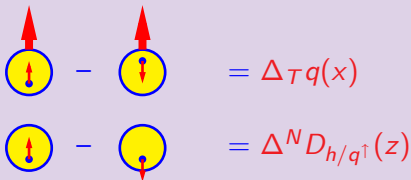
Trento conventions: A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Phys. Rev. **D70**, 117504 (2004).

Collins effect

Collins effects $A \propto \sin(\phi_h + \phi_S)$

The azimuthal asymmetry arises due to modulation in fragmentation function, the Collins function $\Delta^N D_{h/q^\uparrow}(z, |p_\perp|)$ couples to transversity $\Delta_T q(x)$

$$A_N \sim \sin(\phi_h + \phi_S) \cdot \Delta_T q(x) \otimes \Delta^N D_{h/q^\uparrow}(z, |p_\perp|)$$



$$\begin{aligned} \text{Top row: } & \text{Yellow circle with blue dot and red arrow pointing up} - \text{Yellow circle with blue dot and red arrow pointing down} = \Delta_T q(x) \\ \text{Bottom row: } & \text{Yellow circle with blue dot and red arrow pointing up} - \text{Yellow circle with blue dot and red arrow pointing down} = \Delta^N D_{h/q^\uparrow}(z) \end{aligned}$$



J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

Collins effect

Collins effects $A \propto \sin(\phi_h + \phi_S)$

$$A_{UT}^{\sin(\phi_h + \phi_S)}(x, z) \sim \frac{\sum_q e_q^2 x \Delta_T q(x) \Delta^N D_{h/q^\uparrow}(z)}{\sum_q e_q^2 x f_q(x) D_{h/q}(z)},$$

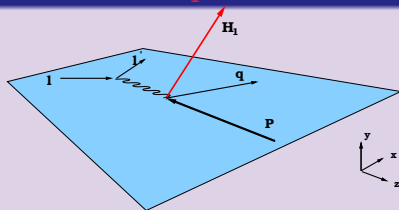
Positivity constraints :

$$|\Delta^N D_{h/q^\uparrow}(z, \mathbf{p}_\perp)| \leq 2D_{h/q}(z, \mathbf{p}_\perp)$$

Soffer bound :

$$|\Delta_T q(x)| \leq \frac{1}{2} [f_{q/p}(x) + \Delta q(x)]$$

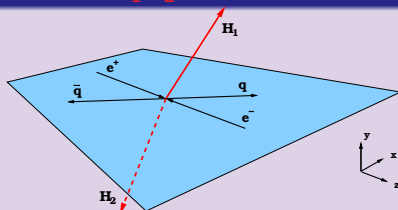
J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

SIDIS and e^+e^- annihilationSIDIS $IN \rightarrow l'H_1X$ 

Collins effect gives rise to azimuthal Single Spin Asymmetry

$$\begin{aligned}
 & \begin{array}{c} \uparrow \\ \text{red arrow} \\ \text{yellow circle} \\ \uparrow \\ \text{red arrow} \end{array} - \begin{array}{c} \uparrow \\ \text{red arrow} \\ \text{yellow circle} \\ \downarrow \\ \text{red arrow} \end{array} = \Delta_T q(x, Q^2) \\
 & \begin{array}{c} \uparrow \\ \text{red arrow} \\ \text{yellow circle} \\ \downarrow \\ \text{red arrow} \end{array} - \begin{array}{c} \downarrow \\ \text{red arrow} \\ \text{yellow circle} \\ \downarrow \\ \text{red arrow} \end{array} = \Delta^N D_{h/q\uparrow}(z, Q^2)
 \end{aligned}$$

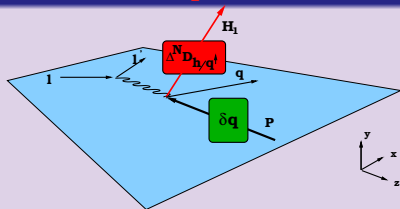
J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

 $e^+e^- \rightarrow H_1H_2X$ 

Collins effect gives rise to azimuthal asymmetry, q and \bar{q} Collins functions are present in the process:

$$\begin{aligned}
 & \Delta^N D_{h/q\uparrow}(z_1, Q^2) \\
 & \Delta^N D_{h/\bar{q}\uparrow}(z_2, Q^2)
 \end{aligned}$$

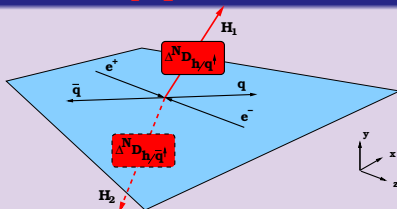
D. Boer, R. Jacob and P. J. Mulders *Nucl. Phys.* **B504** (1997) 345

SIDIS and e^+e^- annihilationSIDIS $IN \rightarrow l'H_1X$ 

Cross Section $\sim \sin(\phi_H + \phi_S) \cdot$
 $\Delta_{Tq}(x, Q^2) \otimes \Delta^N D_{h/q^\uparrow}(z, Q^2)$

?

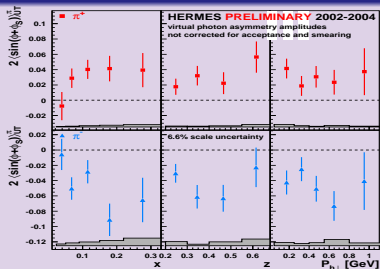
$\Delta_{Tq}(x, Q^2) \neq 0 ?$
 $\Delta^N D_{h/q^\uparrow}(z, Q^2) \neq 0 ?$

 $e^+e^- \rightarrow H_1H_2X$ 

Cross Section $\sim \cos(\phi_{H_1} + \phi_{H_2}) \cdot$
 $\Delta^N D_{h/q^\uparrow}(z_1) \otimes \Delta^N D_{h/\bar{q}^\uparrow}(z_2)$

?

$\Delta^N D_{h/q^\uparrow}(z_1, Q^2) \neq 0 ?$
 $\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2) \neq 0 ?$

SIDIS and e^+e^- annihilationSIDIS $IN \rightarrow l'H_1X$ 

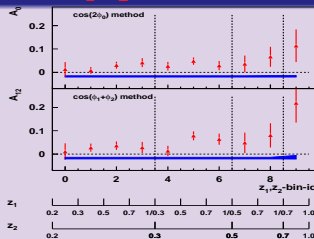
HERMES, **proton** target,
 $p_{lab} = 27.5$ (GeV)

HERMES

$$\Delta_T q(x, Q^2) \neq 0!$$

$$\Delta^N D_{h/q^\uparrow}(z, Q^2) \neq 0!$$

HERMES Collaboration, A. Airapetian
et al. *Phys. Rev. Lett.* **94** 94 (2005) 012002

 $e^+e^- \rightarrow H_1 H_2 X$ 

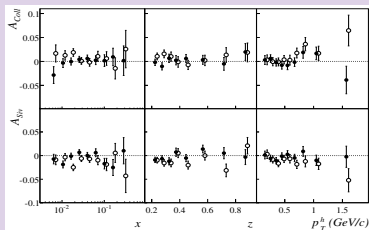
BELLE, $\sqrt{s} = 10.52$ (GeV),

BELLE

$$\Delta^N D_{h/q^\uparrow}(z_1, Q^2) \neq 0!$$

$$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2) \neq 0!$$

Belle Collaboration,
K. Abe *et al.*, *Phys. Rev. Lett.* **96**(2006)232002

SIDIS and e^+e^- annihilationSIDIS $IN \rightarrow l'H_1X$ 

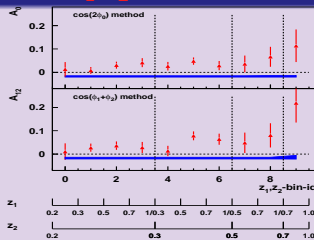
COMPASS, **deuteron** target
 $p_{lab} = 160$ (GeV)

COMPASS

$$\Delta T q(x, Q^2) \neq 0 ?$$

$$\Delta^N D_{h/q\uparrow}(z, Q^2) \neq 0 ?$$

COMPASS Collaboration, E. S. Ageev *et al.*,
 Nucl. Phys. **B765**, 31 (2007).

 $e^+e^- \rightarrow H_1 H_2 X$ 

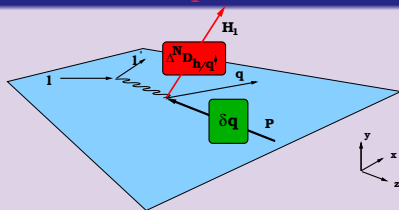
BELLE, $\sqrt{s} = 10.52$ (GeV),

BELLE

$$\Delta^N D_{h/q\uparrow}(z_1, Q^2) \neq 0 !$$

$$\Delta^N D_{h/\bar{q}\uparrow}(z_2, Q^2) \neq 0 !$$

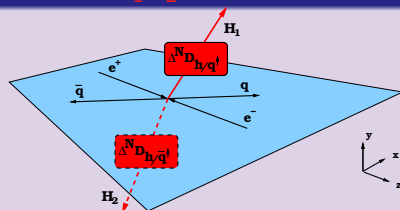
Belle Collaboration,
 K. Abe *et al.*, *Phys. Rev. Lett.* **96**(2006)232002

SIDIS and e^+e^- annihilationSIDIS $IN \rightarrow l' H_1 X$ 

?

Are HERMES and COMPASS data compatible?

Fit **HERMES** & **BELLE** and check if we describe **COMPASS** data.

 $e^+e^- \rightarrow H_1 H_2 X$ 

?

$\Delta^N D_{h/q'}^{SIDIS}(z) = \Delta^N D_{h/q'}^{e^+e^-}(z)$?

Fit simultaneously **HERMES**, **COMPASS** and **BELLE** data sets.

Unpolarised distribution and fragmentation functions.

$f_{q/p}(x, k_\perp)$ and $D_{h/q}(z, p_\perp)$ TMD distribution and fragmentation functions are used.

We assume the k_\perp and p_\perp dependences to be factorized in a Gaussian form

$$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2 / \langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$\langle k_\perp^2 \rangle = 0.25 \text{ (GeV}^2\text{)}$$

$$\langle p_\perp^2 \rangle = 0.2 \text{ (GeV}^2\text{)}$$

M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin,
Phys. Rev. **D71**, 074006 (2005).

Unpolarised distribution and fragmentation functions.

$f_{q/p}(x, k_\perp)$ and $D_{h/q}(z, p_\perp)$ TMD distribution and fragmentation functions are used.

We assume the k_\perp and p_\perp dependences to be factorized in a Gaussian form

Distribution functions:

$f_{q/p}(x)$ GRV LO 1998

M. Gluck, E. Reya, and A. Vogt, Eur. Phys. J. **C5**, 461 (1998).

Fragmentation functions:

$D_{h/q}(z)$ Kretzer

S. Kretzer, Phys. Rev. **D62**, 054001 (2000).

Collins function

Model for Collins FF

$\Delta^N D_{h/q\uparrow}(z, |p_\perp|) \implies$ we use factorization of z and p_\perp
and Gaussian dependence on p_\perp

$$\Delta^N D_{h/q\uparrow}(z, p_\perp) = 2 \mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp) \frac{e^{-p_\perp^2 / \langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle},$$

with

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma + \delta)(\gamma + \delta)}{\gamma \delta \delta}$$

$$h(p_\perp) = \sqrt{2} e \frac{p_\perp}{M} e^{-p_\perp^2 / M^2},$$

where N_q^C , γ , δ , and M are parameters.

Collins function

Model for Collins FF

$\Delta^N D_{h/q\uparrow}(z, |p_\perp|) \implies$ we use factorization of z and p_\perp
and Gaussian dependence on p_\perp

$$\Delta^N D_{h/q\uparrow}(z, p_\perp) = 2 \mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp) \frac{e^{-p_\perp^2 / \langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle},$$

with

$$\begin{aligned} \mathcal{N}_q^C(z) &\leq 1 \\ h(p_\perp) &\leq 1 \end{aligned}$$

positivity constraint $|\Delta^N D_{h/q\uparrow}(z, \mathbf{p}_\perp)| \leq 2D_{h/q}(z, \mathbf{p}_\perp)$ is fulfilled.

Transversity

$$\Delta_T q(x, \mathbf{k}_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) [f_{q/p}(x) + \Delta q(x)] \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T},$$

where

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha + \beta)^{(\alpha + \beta)}}{\alpha^\alpha \beta^\beta},$$

N_q^T , α , β and $\langle k_\perp^2 \rangle_T$ are parameters.

$$\mathcal{N}_q^T(x) \leq 1$$

thus Soffer bound

$$|\Delta_T q(x)| \leq \frac{1}{2} [f_{q/p}(x) + \Delta q(x)]$$

is fulfilled.

Description of $A_{UT}^{\sin(\phi_h+\phi_S)}$

We use HERMES and COMPASS data sets on $A_{UT}^{\sin(\phi_h+\phi_S)}$ in the fitting procedure, we use one of the two sets of data from BELLE corresponding to either $\cos(\varphi_1 + \varphi_2)$ or $\cos(2\varphi_0)$ extraction method.

Favored and unfavored fragmentation functions are defined as follows:

$$D^{fav}(z) \equiv D^{u \rightarrow \pi^+}(z) = D^{d \rightarrow \pi^-}(z) = D^{\bar{u} \rightarrow \pi^-}(z) = D^{\bar{d} \rightarrow \pi^+}(z)$$

$$D^{unfav}(z) \equiv D^{u \rightarrow \pi^-}(z) = D^{d \rightarrow \pi^+}(z) = D^{\bar{u} \rightarrow \pi^+}(z) = D^{\bar{d} \rightarrow \pi^-}(z)$$

HERMES Collaboration, L. Pappalardo *et al.*, in the proceedings of the XIV International Workshop on Deep Inelastic Scattering, Tsukuba city, Japan, April 20th - April 24th. (2006).

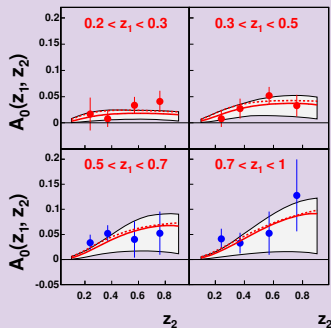
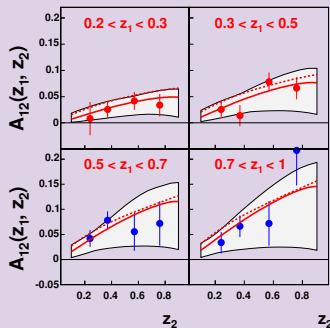
COMPASS Collaboration, E. S. Ageev *et al.*, Nucl. Phys. **B765**, 31 (2007).

Belle Collaboration, R. Seidl *et al.*, Phys. Rev. Lett. **96**, 232002 (2006).

Description of the data Anselmino *et al* Phys.Rev.D75:054032,2007Table: FIT I $\cos(\varphi_1 + \varphi_2)$ and FIT II $\cos(\varphi_0)$ are within 1σ

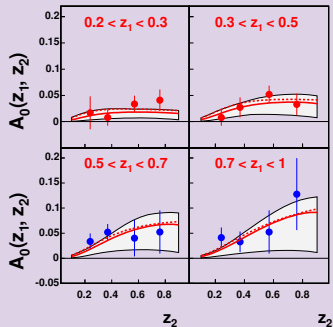
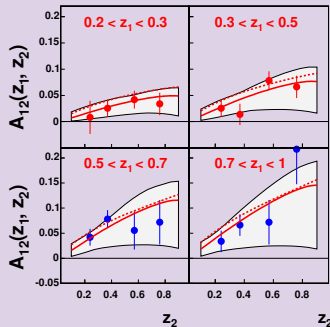
Transversity						
FIT I	N_u^T	=	0.48 ± 0.09	N_d^T	=	-0.62 ± 0.30
FIT II	N_u^T	=	0.42 ± 0.09	N_d^T	=	-0.53 ± 0.28
FIT I	α	=	1.14 ± 0.68	β	=	4.74 ± 5.45
FIT II	α	=	1.20 ± 0.83	β	=	5.09 ± 5.87
Collins FF						
FIT I	N_{fav}^C	=	0.35 ± 0.16	N_{unf}^C	=	-0.85 ± 0.36
FIT II	N_{fav}^C	=	0.41 ± 0.10	N_{unf}^C	=	-0.99 ± 1.24
FIT I	γ	=	1.14 ± 0.38	δ	=	0.14 ± 0.36
FIT II	γ	=	0.81 ± 0.40	δ	=	0.02 ± 0.37
FIT I	M^2	=	$0.70 \pm 0.65 \text{ GeV}^2$			
FIT II	M^2	=	$0.88 \pm 1.15 \text{ GeV}^2$			

Description of BELLE data

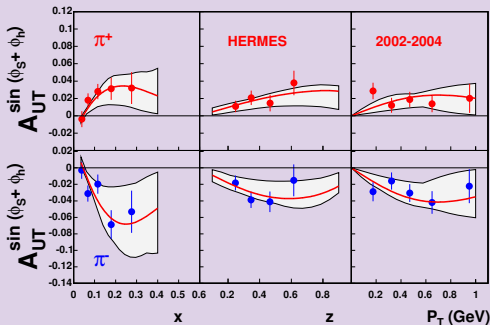
BELLE $\cos(\varphi_0)$ BELLE $\cos(\varphi_1 + \varphi_2)$ 

Solid line corresponds to FIT II, dashed line corresponds to FIT I

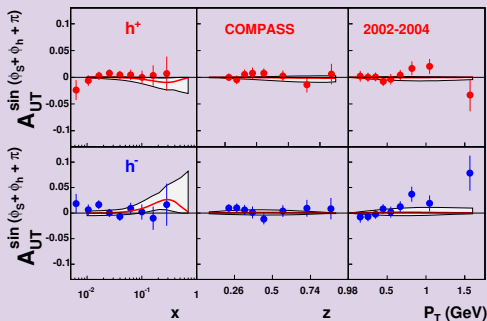
Description of BELLE data

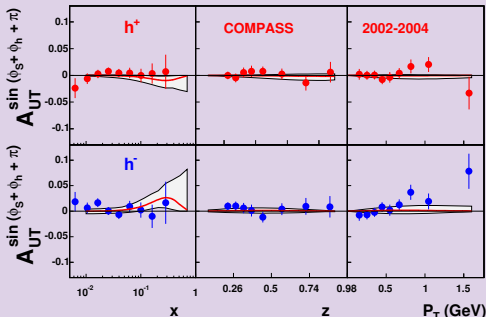
BELLE $\cos(\varphi_0)$ BELLE $\cos(\varphi_1 + \varphi_2)$ 

FIT I and FIT II are compatible

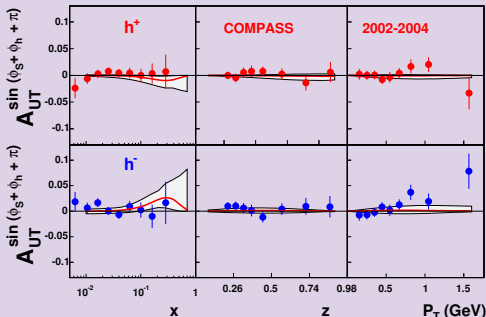
Description of HERMES data $A_{UT}^{\sin(\phi_h+\phi_S)}$ HERMES $A_{UT}^{\sin(\phi_h+\phi_S)}$ $ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.

HERMES Collaboration, L. Pappalardo *et al.*, in the proceedings of the XIV International Workshop on Deep Inelastic Scattering, Tsukuba city, Japan, April 20th - April 24th. (2006).

Description of COMPASS data $A_{UT}^{\sin(\phi_h+\phi_S+\pi)}$ COMPASS $A_{UT}^{\sin(\phi_h+\phi_S+\pi)}$ $\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.COMPASS Collaboration, E. S. Ageev *et al.*, Nucl. Phys. **B765**, 31 (2007).

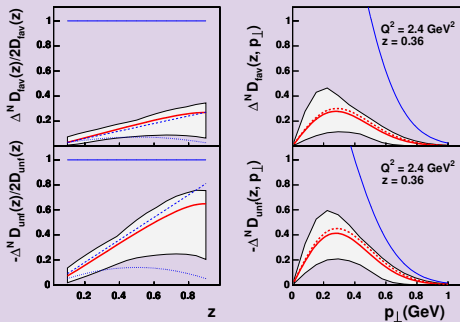
Description of COMPASS data $A_{UT}^{\sin(\phi_h+\phi_S+\pi)}$ COMPASS $A_{UT}^{\sin(\phi_h+\phi_S+\pi)}$ $\mu D \rightarrow \mu h X, p_{lab} = 160 \text{ GeV.}$ 

Why $A_{UT}^{\sin(\phi_h+\phi_S+\pi)} \sim 0$? One of the reasons is that $\langle x \rangle \sim 0.03$
 ($\langle x \rangle_{HERMES} \sim 0.1$) is very small and $\Delta_T q(x) \rightarrow 0$.

Description of COMPASS data $A_{UT}^{\sin(\phi_h+\phi_S+\pi)}$ COMPASS $A_{UT}^{\sin(\phi_h+\phi_S+\pi)}$ $\mu D \rightarrow \mu h X, p_{lab} = 160 \text{ GeV.}$ 

But deuteron target allows us to fit $\Delta_T d(x)$ as combination of $\Delta_T u(x) + \Delta_T d(x)$ enters into the asymmetry.

Collins fragmentation function

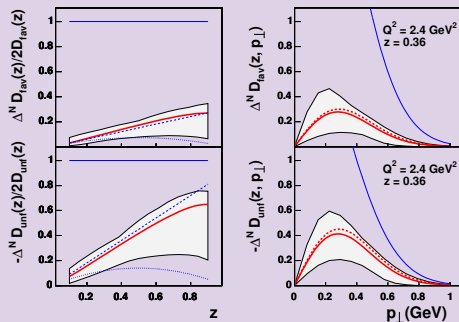


compared to Ref. [1] (dashed line) and Ref. [2] (dotted line)

[1] A. V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. **D73**, 094025 (2006).

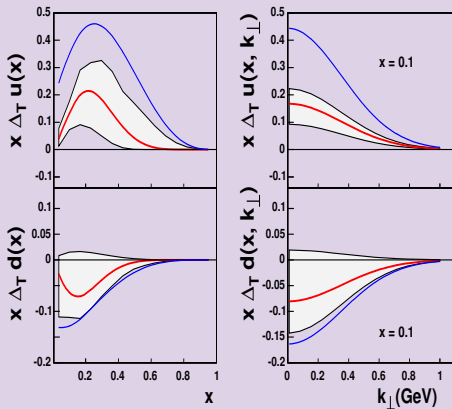
[2] W. Vogelsang and F. Yuan, Phys. Rev. **D72**, 054028 (2005).

Collins fragmentation function



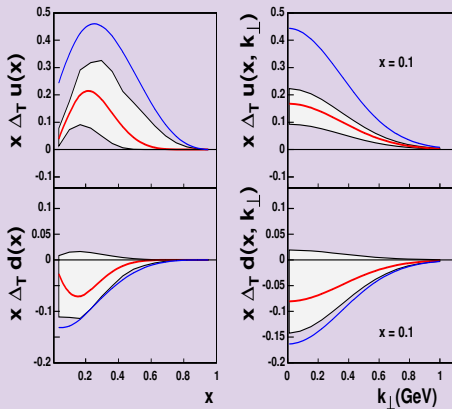
Right panel: solid line corresponds to **FIT II**, dashed line corresponds to **FIT I**

Transversity



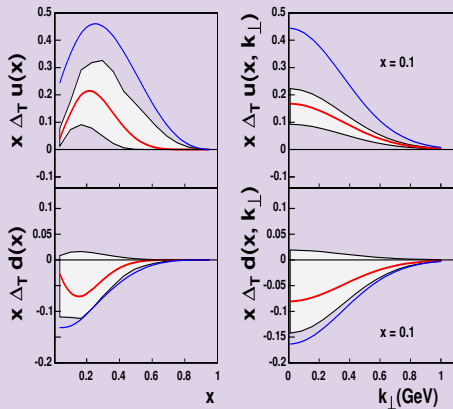
- This is the first extraction of **transversity** from experimental data.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Both $\Delta_T u(x)$ and $\Delta_T d(x)$ do not saturate Soffer bound.
- HERMES data alone fixes well $\Delta_T u(x)$ while HERMES+COMPASS allows us to extract $\Delta_T d(x)$.

Transversity



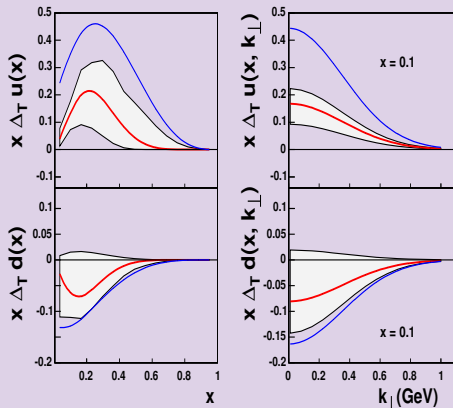
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Transversity



- This is the first extraction of **transversity** from experimental data.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
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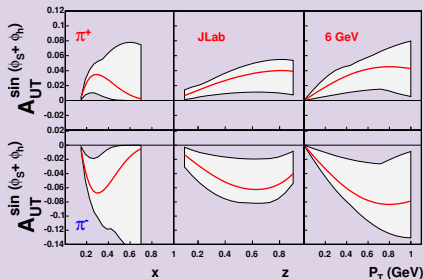
Transversity



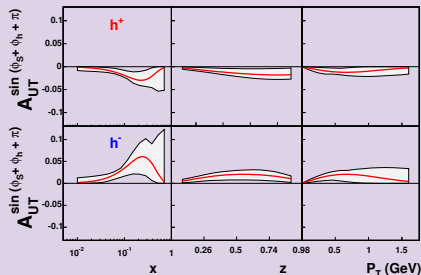
- This is the first extraction of **transversity** from experimental data.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Both $\Delta_T u(x)$ and $\Delta_T d(x)$ do not saturate Soffer bound.
- **HERMES** data alone fixes well $\Delta_T u(x)$ while **HERMES+COMPASS** allows us to extract $\Delta_T d(x)$.

PREDICTIONS

JLab

 $ep \rightarrow e\pi X$, $p_{lab} = 6$ GeV.


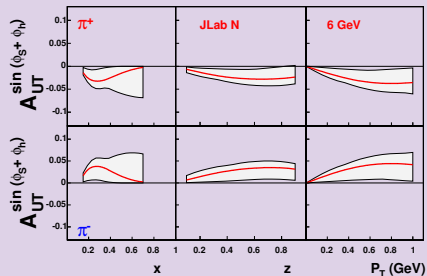
COMPASS

 $\mu p \rightarrow \mu h X$, $p_{lab} = 160$ GeV.


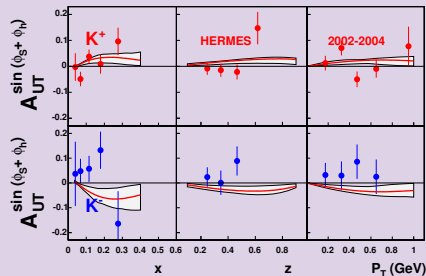
JLab can improve our knowledge of transversity in high x region.
 COMPASS operating on proton target is expected to measure 5% asymmetry at $x \sim 0.2$

PREDICTIONS

JLab

 $eN \rightarrow e\pi X$, $p_{lab} = 6$ GeV.

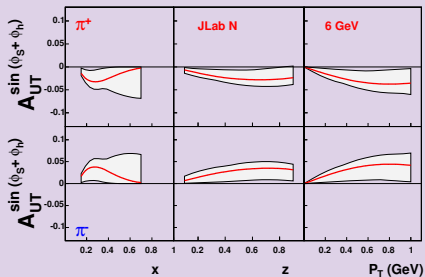
HERMES

 $ep \rightarrow eKX$, $p_{lab} = 27.57$ GeV.

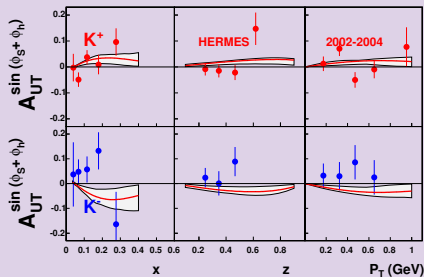
JLab can improve our knowledge of $\Delta_T d(x)$ transversity using neutron target. Prediction of the model are compatible with Kaon data from HERMES.

PREDICTIONS

JLab

 $eN \rightarrow e\pi X$, $p_{lab} = 6$ GeV.

HERMES

 $ep \rightarrow eKX$, $p_{lab} = 27.57$ GeV.

$$A_{UT}^{\sin(\phi_h+\phi_S)}|_{proton} \sim 4\Delta_T u(x)\Delta^N D_{h/u\uparrow}(z) + \Delta_T d(x)\Delta^N D_{h/d\uparrow}(z)$$

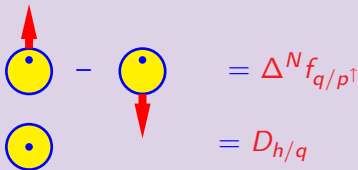
$$A_{UT}^{\sin(\phi_h+\phi_S)}|_{neutron} \sim 4\Delta_T d(x)\Delta^N D_{h/u\uparrow}(z) + \Delta_T u(x)\Delta^N D_{h/d\uparrow}(z)$$

Sivers effect

Sivers effect $A \propto \sin(\phi_h - \phi_S)$

The azimuthal asymmetry arises due to modulation in parton density, the so called Sivers function $\Delta^N f_{q/p\uparrow}$ is the difference of parton distributions in a polarized hadron.

$$A_N \sim \sin(\phi_h - \phi_S) \cdot \Delta^N f_{q/p\uparrow}(x, k_\perp) \otimes D_{h/q}(z)$$



D. Sivers, *Phys. Rev.* **D41**(1990) 83

Sivers effect

Sivers effect $A \propto \sin(\phi_h - \phi_S)$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, z) \sim \frac{\sum_q e_q^2 x z \Delta^N f_{q/p\uparrow}(x) D_{h/q}(z)}{\sum_q e_q^2 x f_q(x) D_{h/q}(z)},$$

Positivity constraints :

$$|\Delta^N f_{q/p\uparrow}(x, \mathbf{k}_\perp)| \leq 2f_q(x, \mathbf{k}_\perp)$$

Two different notations:

$$\begin{aligned} f_{q/p\uparrow}(x, \mathbf{k}_\perp) &= f_{q/p}(x, \mathbf{k}_\perp) + \frac{1}{2} \Delta^N f_{q/p\uparrow}(x, \mathbf{k}_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp) \\ &= f_{q/p}(x, \mathbf{k}_\perp) - f_{1T}^{\perp q}(x, \mathbf{k}_\perp) \frac{\mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \mathbf{k}_\perp)}{m_p}, \end{aligned}$$

Sivers effect

Sivers effect $A \propto \sin(\phi_h - \phi_S)$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, z) \sim \frac{\sum_q e_q^2 x z \Delta^N f_{q/p^\uparrow}(x) D_{h/q}(z)}{\sum_q e_q^2 x f_q(x) D_{h/q}(z)},$$

Positivity constraints :

$$|\Delta^N f_{q/p^\uparrow}(x, k_\perp)| \leq 2f_q(x, k_\perp)$$

Two different notations:

Relation

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = -\frac{2|k_\perp|}{m_p} f_{1T}^{\perp q}(x, k_\perp).$$

Trento conventions: A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Phys. Rev. **D70**, 117504 (2004).

Sivers function

Model for Siverson function

$\Delta^N f_{q/p\uparrow}(x, k_\perp) \implies$ we use factorization of x and k_\perp
and Gaussian dependence on k_\perp

$$\Delta^N f_{q/p\uparrow}(x, k_\perp) = 2 \mathcal{N}_q(x) f_q(x) h(k_\perp) \frac{e^{-p_\perp^2 / \langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle},$$

with

$$\mathcal{N}_q(x) = N_q x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}$$

$$h(k_\perp) = \sqrt{2} e \frac{k_\perp}{M'} e^{-k_\perp^2 / M'^2},$$

where N_q , a_q , b_q , and M' are parameters.

Sivers function

Model for Sivers function

$\Delta^N f_{q/p\uparrow}(x, k_\perp) \implies$ we use factorization of x and k_\perp
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$$\Delta^N f_{q/p\uparrow}(x, k_\perp) = 2 \mathcal{N}_q(x) f_q(x) h(k_\perp) \frac{e^{-p_\perp^2 / \langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle},$$

with

$$\mathcal{N}_q(x) \leq 1$$

$$h(k_\perp) \leq 1$$

positivity constraint $|\Delta^N f_{q/p\uparrow}(x, k_\perp)| \leq 2f_q(x, k_\perp)$ is fulfilled.

Description of $A_{UT}^{\sin(\phi_h - \phi_S)}$

We use [HERMES](#) and [COMPASS](#) data sets on $A_{UT}^{\sin(\phi_h - \phi_S)}$ in the fitting procedure.

u , d and *sea* Sivers functions are fitted.

For sea Sivers functions we use

$$\Delta^N f_{\bar{u}/p^\uparrow}(x, \mathbf{k}_\perp), \Delta^N f_{\bar{d}/p^\uparrow}(x, \mathbf{k}_\perp), \Delta^N f_{s/p^\uparrow}(x, \mathbf{k}_\perp), \Delta^N f_{\bar{s}/p^\uparrow}(x, \mathbf{k}_\perp).$$

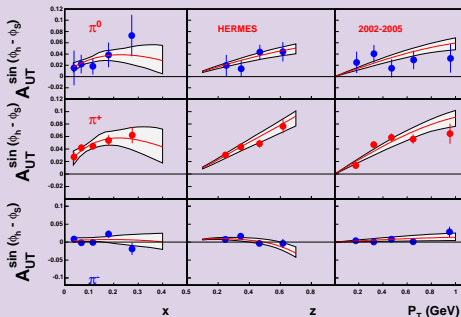
HERMES Collaboration, Diefenthaler M., HERMES measurements of Collins and Sivers asymmetries from a transversely polarised hydrogen target, arXiv:0706.2242

COMPASS Collaboration, Martin A. COMPASS results on transverse single-spin asymmetries Czech. J. Phys. **B56**, F33-F52 (2006).

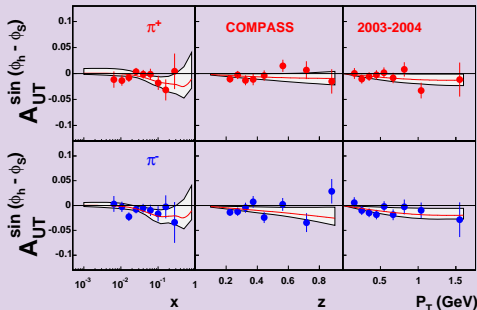
Description of the data

Table: Best values of the free parameters for the u , d and sea Siverson functions.

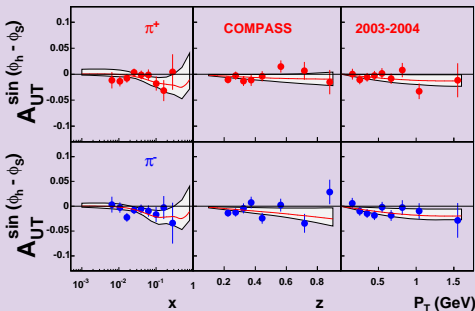
		$\chi^2/\text{d.o.f.} = 1.$			
u	N_u	$=$	$0.33^{+0.062}_{-0.067}$		
	Siverson function	a_u	$=$	$0.58^{+0.86}_{-0.46}$	$b_u = 2.6^{+4.4}_{-2.3}$
d	N_d	$=$	$-1.00^{+0.004}_{-0.000}$		
	Siverson function	a_d	$=$	$0.75^{+0.65}_{-0.36}$	$b_d = 1.1^{+2.5}_{-0.92}$
sea	$N_{\bar{u}}$	$=$	$0.005^{+0.24}_{-0.15}$	$N_{\bar{d}} = -0.36^{+0.39}_{-0.51}$	
		N_s	$=$	$-0.19^{+0.61}_{-0.74}$	$N_{\bar{s}} = 1.00^{+0}_{-0.00059}$
	Siverson function	a_{sea}	$=$	$1.5^{+2.6}_{-1.2}$	$b_{sea} = 11^{+31}_{-11}$
		$\langle k_{\perp}^2 \rangle$	$=$	0.25 GeV^2	$M'^2 = 0.41^{+0.41}_{-0.18} \text{ GeV}^2$

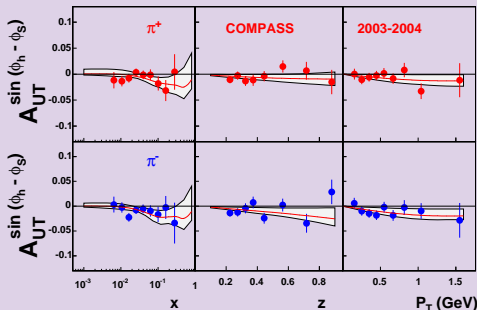
Description of HERMES data $A_{UT}^{\sin(\phi_h - \phi_S)}$ HERMES $A_{UT}^{\sin(\phi_h - \phi_S)}$ $ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.

HERMES Collaboration, Diefenthaler M., HERMES measurements of Collins and Sivers asymmetries from a transversely polarised hydrogen target, arXiv:0706.2242

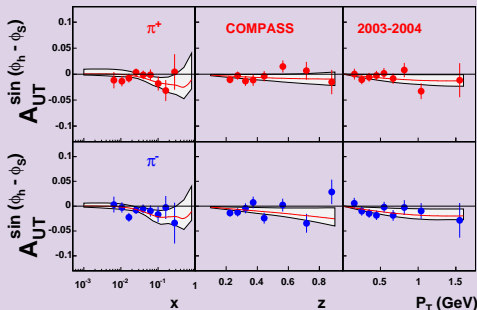
Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$ COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$ $\mu D \rightarrow \mu h X, p_{lab} = 160 \text{ GeV.}$ 

COMPASS Collaboration, Martin A. COMPASS results on transverse single-spin asymmetries Czech. J. Phys. **B56**, F33-F52 (2006).

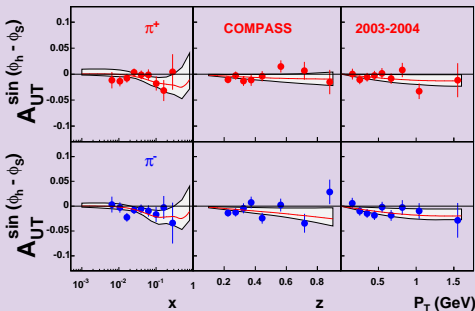
Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$ COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$ $\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.Why $A_{UT}^{\sin(\phi_h - \phi_S)} \sim 0$?

Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$ COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$ $\mu D \rightarrow \mu h X, p_{lab} = 160 \text{ GeV.}$ 

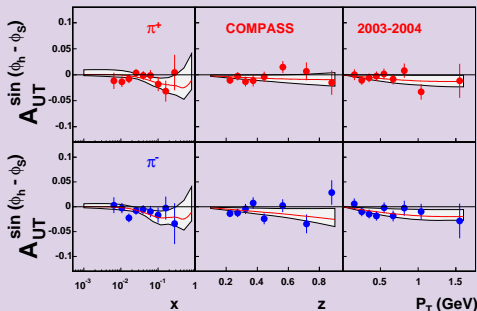
$$\left(A_{UT}^{\sin(\phi_h - \phi_S)} \right)_{\text{hydrogen}} \sim 4 \Delta N_{f_{u/p\uparrow}} D_u^h + \Delta N_{f_{d/p\uparrow}} D_d^h$$

Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$ COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$ $\mu D \rightarrow \mu h X, p_{lab} = 160 \text{ GeV.}$ 

$$\left(A_{UT}^{\sin(\phi_h - \phi_S)} \right)_{\text{deuterium}} \sim \left(\Delta N_{f_{u/p\uparrow}} + \Delta N_{f_{d/p\uparrow}} \right) \left(4 D_u^h + D_d^h \right)$$

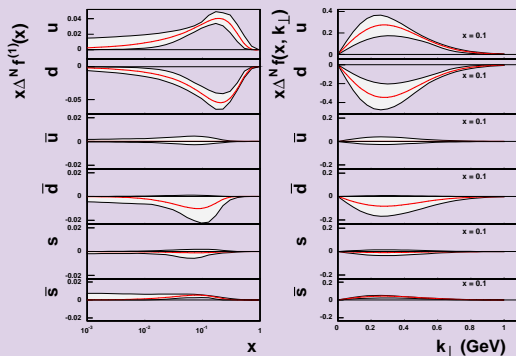
Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$ COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$ $\mu D \rightarrow \mu h X, p_{lab} = 160 \text{ GeV.}$ 

$$\left(A_{UT}^{\sin(\phi_h - \phi_S)} \right)_{\text{deuterium}} \sim \left(\Delta N_{f_{u/p\uparrow}} + \Delta N_{f_{d/p\uparrow}} \right) \sim 0$$

Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$ COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$ $\mu D \rightarrow \mu h X, p_{lab} = 160 \text{ GeV.}$ 

But **deuteron** target allows us to fit better $\Delta^{Nf}_{d/p^\uparrow}$ as combination of $\Delta^{Nf}_{u/p^\uparrow} + \Delta^{Nf}_{d/p^\uparrow}$ enters into the asymmetry.

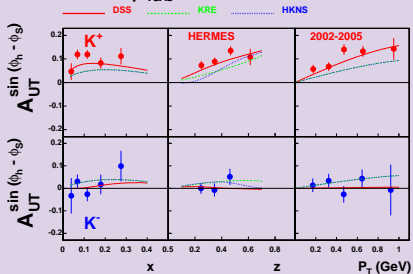
Sivers function



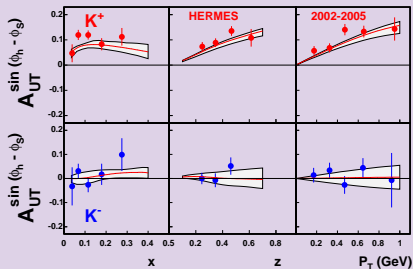
$$\Delta^{Nf_q^{(1)}}(x) \equiv \int d^2 \mathbf{k}_\perp \frac{k_\perp}{4m_p} \Delta^{Nf_{q/p^\uparrow}}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x).$$

KAON HERMES AND COMPASS DATA

HERMES

 $ep \rightarrow eKX$, $p_{lab} = 27.57$ GeV.

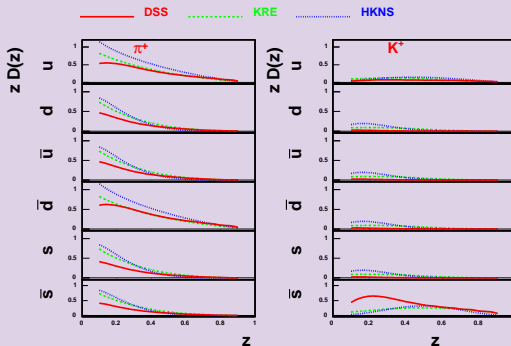
HERMES

 $ep \rightarrow eKX$, $p_{lab} = 27.57$ GeV.

Kaon FF as given by De Florian *et al.* in Ref.

de Florian D., Sassot R., and Stratmann M. Phys. Rev. **D75** 114010 (2007)
 (right panel) are compared the Kretzer (dotted lines) and HKNS
 set (dashed lines) of fragmentation functions (left panel).

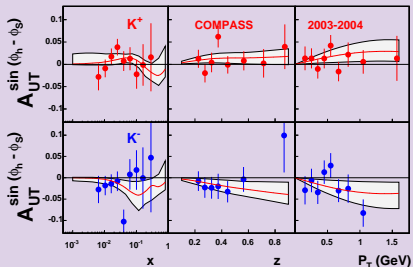
Fragmentation function



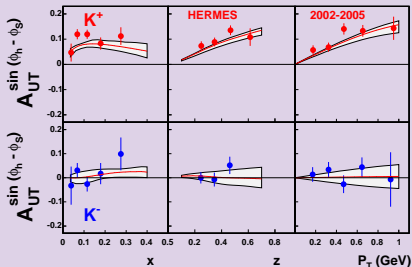
$K^+(u\bar{s}), \pi^+(u\bar{d})$ thus knowledge of $\bar{s} \rightarrow K^+$ FF is very important

KAON HERMES AND COMPASS DATA

COMPASS

 $\mu D \rightarrow \mu K X$, $p_{lab} = 160$ GeV.

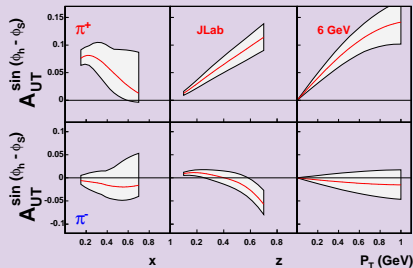
HERMES

 $ep \rightarrow eK X$, $p_{lab} = 27.57$ GeV.

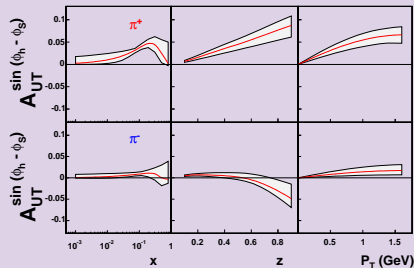
Model description of COMPASS and HERMES Kaon data.

PREDICTIONS

JLab

 $ep \rightarrow e\pi X, p_{lab} = 6 \text{ GeV.}$


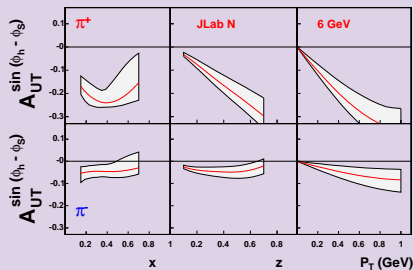
COMPASS

 $\mu p \rightarrow \mu\pi X, p_{lab} = 160 \text{ GeV.}$


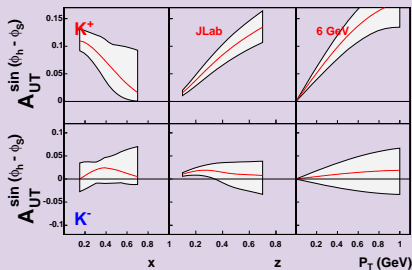
JLab can improve our knowledge of Sivers function in high x region. COMPASS operating on proton target is expected to measure 5% asymmetry for h^+ .

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JLab

 $ep \rightarrow eKX$, $p_{lab} = 6$ GeV.

JLab can improve our knowledge of $\Delta N_{f_{d/p\uparrow}}$ using neutron target.

CONCLUSIONS

- *First* extraction of transversity for u and d quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- Transversity $\Delta_T q(x)$ is found not to saturate Soffer bound $(q(x) + \Delta q(x))/2$.
 $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Estimates of the Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
 $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for u , d and *sea* quarks are extracted from HERMES and COMPASS data.
- Predictions for Collins and Sivers asymmetries at JLab and COMPASS (with the proton target) are presented and expected to be sizable.

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THANK YOU!

$$\Delta^N D_h^{tav}(z, |p_\perp|) > 0 \text{ and } \Delta^N D_h^{unt}(z, |p_\perp|) < 0$$

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