Relativistic Mean-Field Models with Effective Hadron Masses and Coupling Constants

(Direct-Urca reactions and limiting Neutron Star Mass)

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- Neutron star cooling data
- Neutron star cooling scenario. Direct Urca process
- Neutron star masses
- Equation of state in RMF models
- Generlized RMF model. Dropping hadron masses
- Tuning RMF model
- Rho-meson condensation

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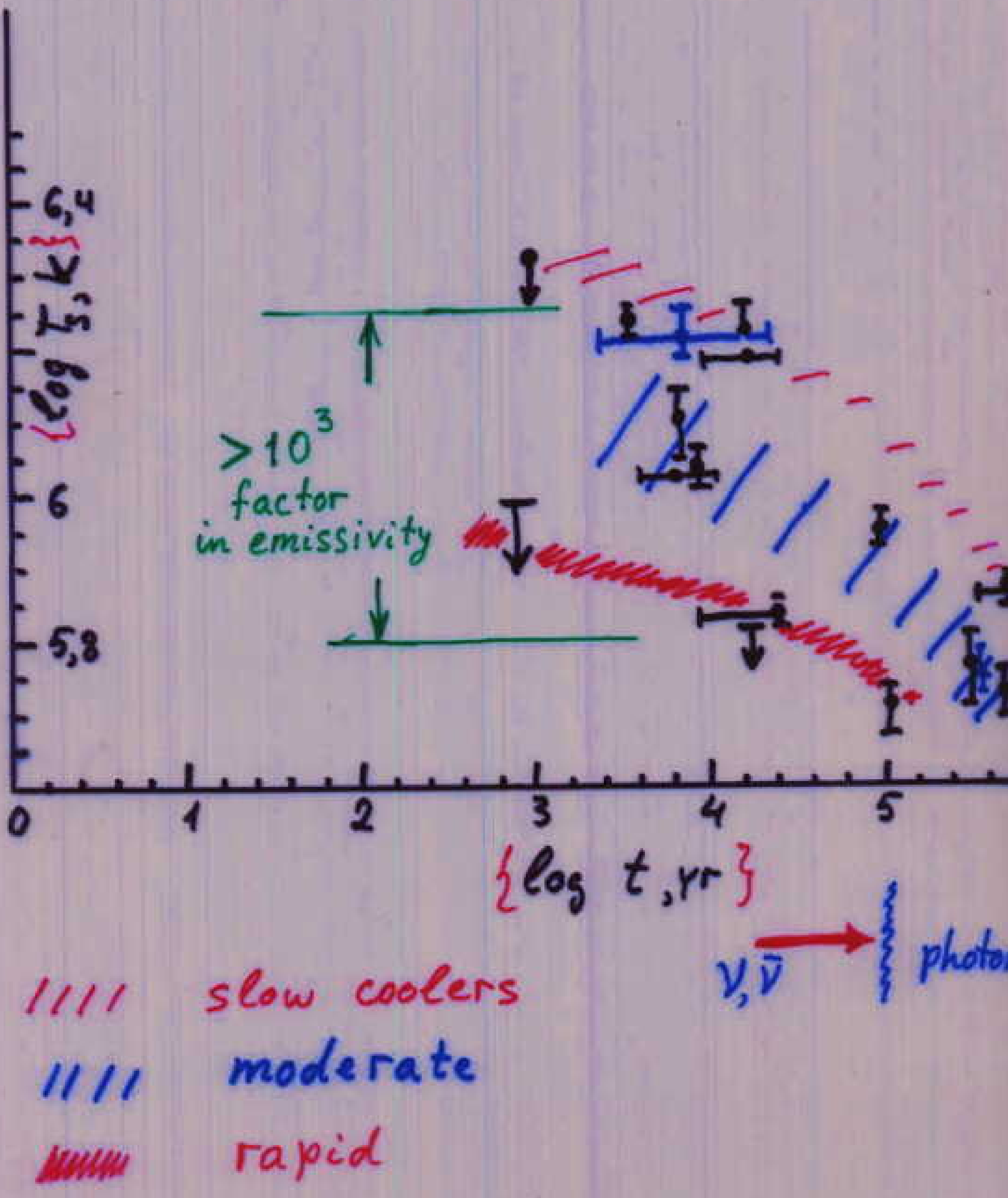
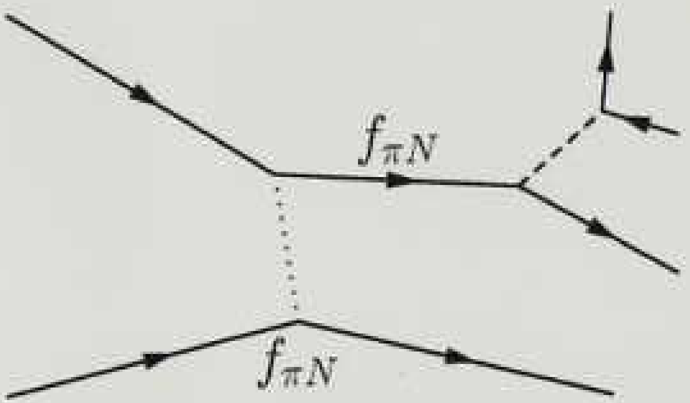
cooling of NS



Standard scenario

Friman & Maxwell AJ (1979). The only diagram in FOPE model which contributes to the MU and NB:



Dots symbolize FOPE: $D_\pi^{-1} = \omega^2 - m_\pi^2 - k^2$,

$$\epsilon_\nu \sim 10^{21}\, T_9^8 (n/n_0)^{1/3}, \quad \frac{\text{erg}}{\text{cm}^3\ \text{sec}}$$

$$T_9 = T/10^9 K, \quad n_0 \simeq 0.17\ fm^{-3}$$

Two nucleon process. One nucleon DU process is assumed forbidden up to high density. Sometimes one uses quasiparticle Green functions ($m_N \to m_N^*$) but vacuum vertices.

- ➡ Inconsistent picture!
- Explains only slow coollers!

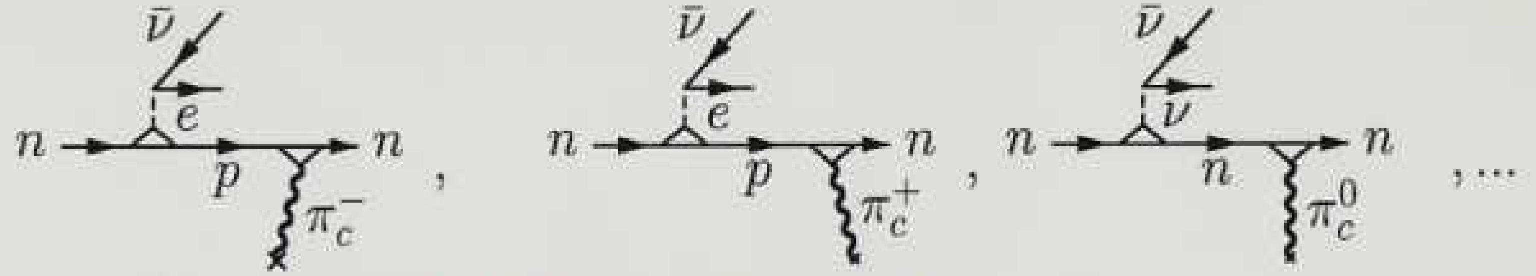
Standard scenario + exotics

For $n > n_c^{\mathrm{DU}}$ $(M > M_c^{\mathrm{DU}})$ DU processes (**Vacuum vertices**):

$$\epsilon_\nu \sim 10^{27}\, T_9^6 (n/n_0)^{2/3}, \ \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{sec}}$$

Extra factor $10^4 \div 10^6$ for typical temperatures compared to MU.

For $n > n_c^{\mathrm{PU}}$ $(M > M_c^{\mathrm{PU}})$ PU processes:



with free vertices: $\epsilon_\nu \sim 10^{26}\, T_9^6 (n/n_0)^{1/3}, \ \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{sec}}$.
Kaon condensate processes yield a smaller contribution.

PFB processes as they are inserted into standard scenario

Permitted only for $T < T_c$ (Flowers et al., AJ, **205** (1976)).
For neutrons correct asymptotic expression for $T \ll \Delta_{nn}$ (V. & Senatorov Sov. J. Nucl. Phys., **45** (1987); Senatorov & V., Phys. Lett., **B184** (1987); see V. astro-ph/0101514:

$$\epsilon_\nu \sim 10^{29} \left[\frac{\Delta_{nn}}{\mathrm{MeV}}\right]^7 \left[\frac{T}{\Delta_{nn}}\right]^{1/2} (n/n_0)^{1/3}\, \xi^2_{nn}, \ \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{sec}},$$

Δ_{nn} is neutron gap, $\xi_{nn} = \exp[-\Delta_{nn}/T]$ is superfluid suppression factor. (not $\epsilon_\nu \sim 10^{20} T_9^7\, \xi^2_{nn}$ as in Flowers et al. (1976):) For $T_9 = 0.1$ this $\epsilon_\nu^{\mathrm{PFB}} \sim \epsilon_\nu^{\mathrm{MU}}$, whereas with correct asymptotic for $\Delta = 0.5$ MeV, $T_9 = 1$ one gets **extra** 10^7.
With free vertices (incorrect!), the emissivity of the process on proton is significantly less than that for neutron.

- With PFB included, depending on values of gaps, one may explain intermediate and slow coolers, even if in-medium effects were artificially suppressed, cf. Schaab et al. AA, **321** (1997)

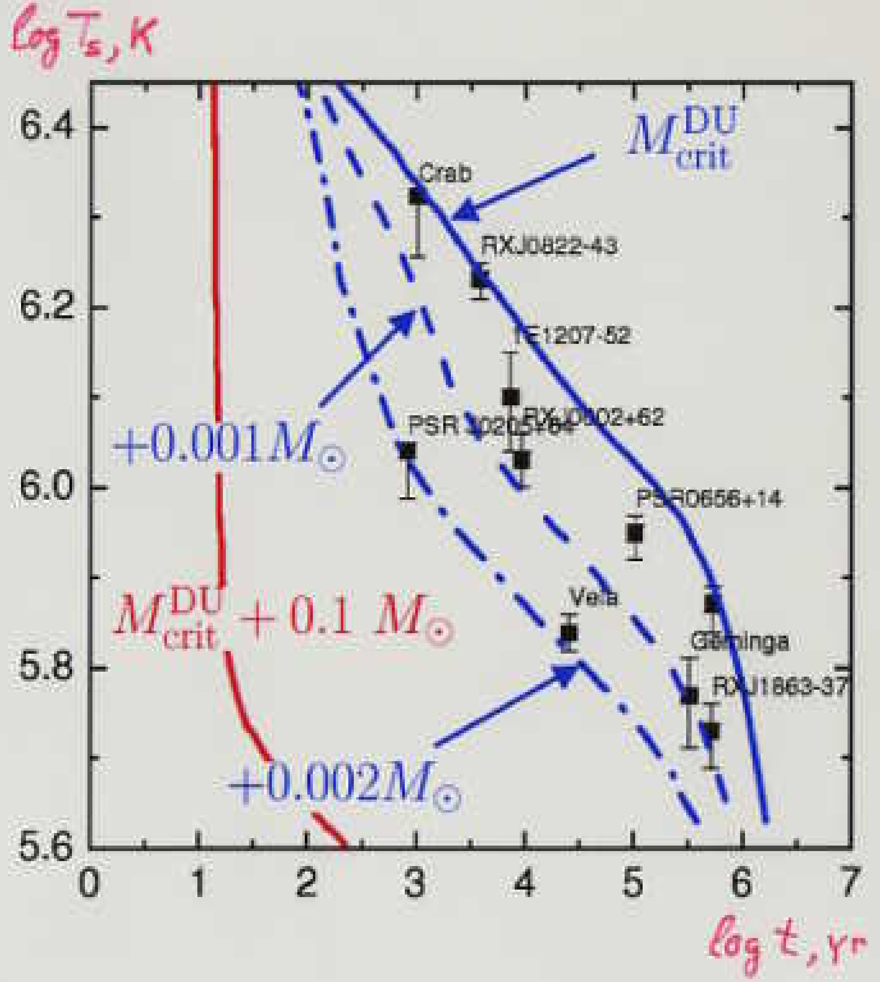
Neutron Star Cooling Scenario

standard scenario (MU+pairing)
only "slow" cooling can be described

Direct-Urca scenario
NS masses close to $M_{\rm crit}^{\rm DU}$

Neutron stars with $M > M_{\rm crit}^{\rm DU}$
will be too cold

<u>Are masses of all NS so close to each others ?</u>



Neutron Star Masses

1995

Thorsett-Chakrabarty ApJ 512

NS mass ≈ $1.35\ M_\odot \pm 0.04$

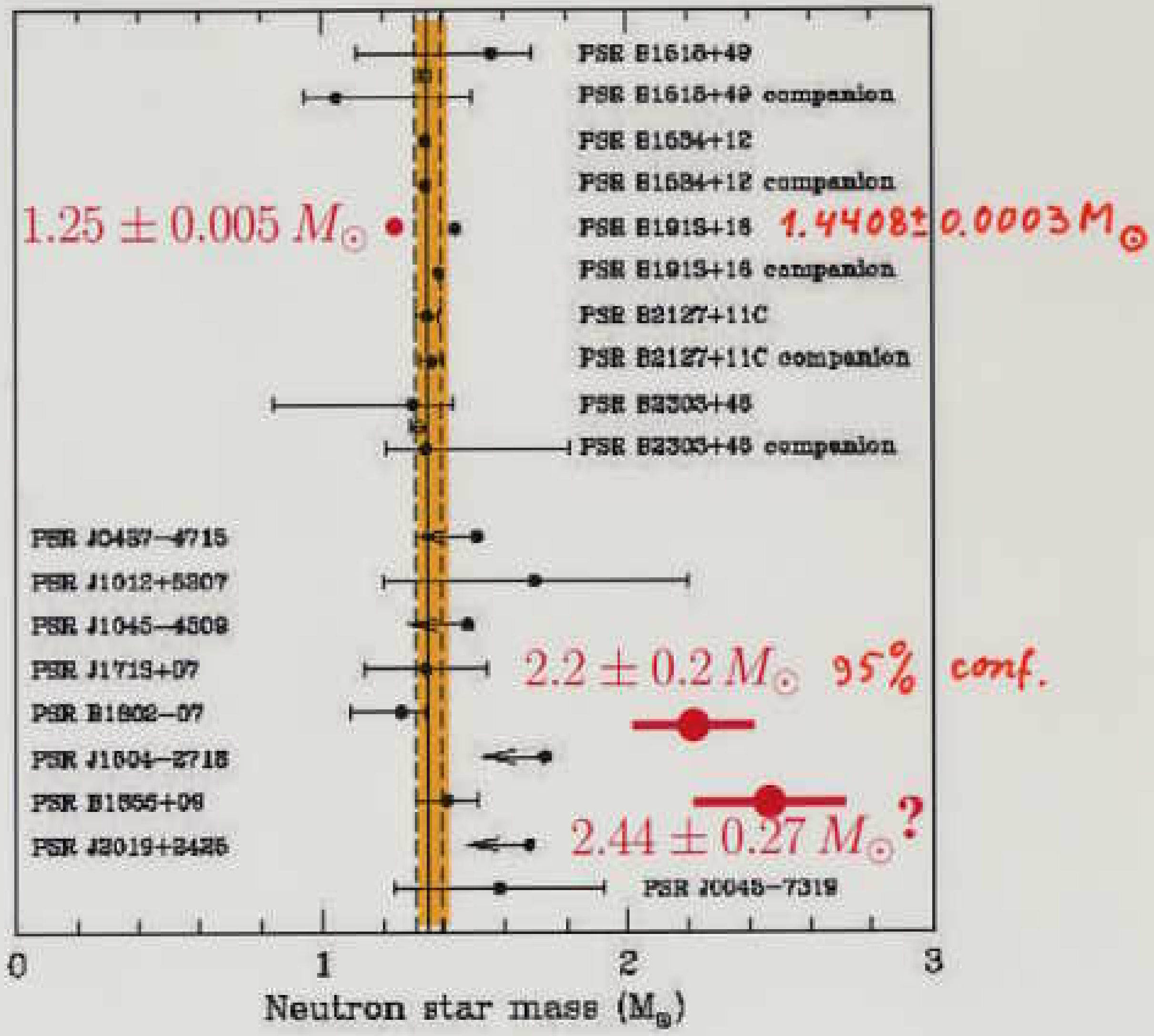
Direct Urca scenario can be realized if EoS is adjusted so that

$M^{\rm DU}_{\rm crit} \simeq 1.35\ M_\odot$

Bethe-Brown mechanism
of BH formation
based on "kaon condensation"
(limiting NS mass $\simeq 1.5\ M_\odot$)

2003-2004

too light & too heavy NS



A critics of Standard + exotics scenario

One may explain "intermediate cooling" data by varying density dependence of gaps artificially.

✔ How density dependence of gaps is able to know about necessity to fit cooling curves? They should be microscopically calculated!

✔ One may explain data with a sharp transition from "slow cooling" to "rapid cooling" by switching on the DU at $M \simeq \bar{M} \simeq 1.35 \ \ M_{\odot}$. Very narrow NS mass interval!

How DU reaction threshold is able to know about necessity to have $M_{crit}^{\rm DU} \simeq \bar{M} \simeq 1.35 \ \ M_{\odot}$, i.e. how small proton fraction is able to govern EoS?
It is based on assumption that all NS masses are in a narrow range near $\bar{M}$.

How about new data on NS masses?

One may still play on uncertainties: errobars for cooling data, different envelopes for slow and rapid coolers (why?), etc.

Nuclear medium cooiling scenario

- Processes following V. & Senatorov (1984), (1986), (1987), Migdal et al. (1990) :

→

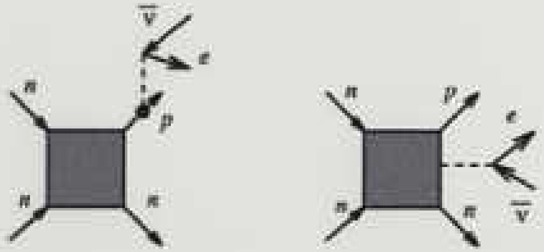
Medium effects show strong density (NS mass) dependence.

Conjectured that NS masses are different: →

cooling data can be well explained.

- Included in code by Schaab et al. (1997), Blaschke et al. (2001), (2004), Popov et al. (2004), Grigorian & Voskresensky (2005).

Neutrino Emission Reactions



modified Urca

pair formation breaking for $T < T_{\text{crit}}$

direct Urca for $n > n_{\text{crit}}^{\text{DU}}$

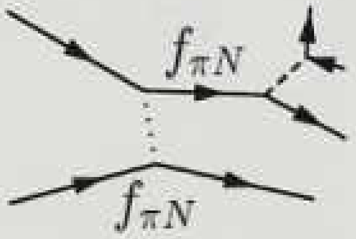
processes on meson condensates

for $n > n_{\text{crit}}^{\text{cond}}$

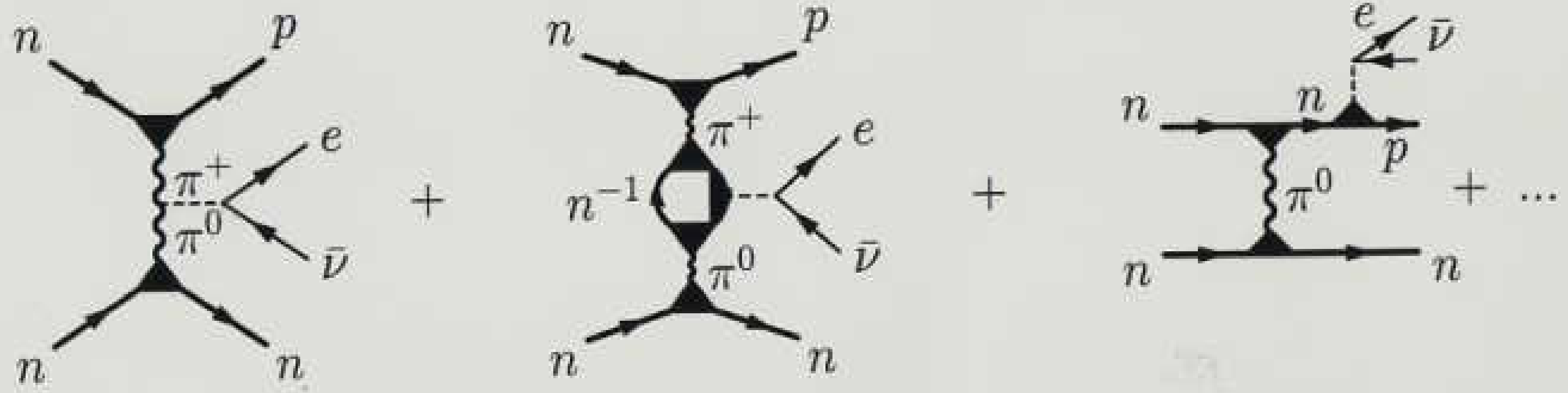
Medium effects in two-nucleon processes, MMU

(cf. V & Senatorov JETP (1986); Migdal et al. Phys. Rep. (1990))

MU(FOPE) :



➡ MMU(MOPE) :



First diagram yields main contribution, second diagram – a less contribution, third diagram (generalizes the MU(FOPE)) yields much less term for $n \gtrsim n_0$.

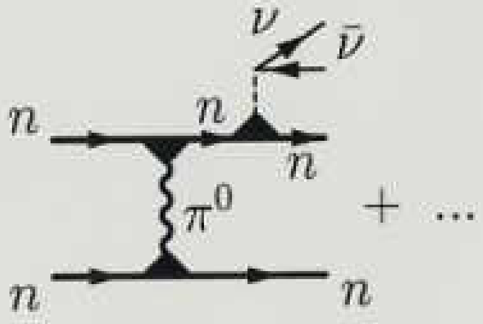
pion softening ➡ enhancement of the rate towards the n_c^{PU}.

$$\frac{\epsilon_\nu[\mathrm{MMU}]}{\varepsilon_\nu[\mathrm{MU}]} \sim 10^3\ (n/n_0)^{10/3} \frac{\Gamma^6(n)}{[\omega^*(n)/m_\pi]^8}.$$

Medium effects in two-nucleon processes, MNB

NB(FOPE) :

➡ MNB(MOPE) :



pion softening ➡ enhancement of the rate towards the n_c^{PU}.

$$\frac{\varepsilon_\nu[\mathrm{MNB}]}{\varepsilon_\nu[\mathrm{NB}]} \sim 10^3 \ (n/n_0)^{4/3} \frac{\Gamma^6(n)}{[\omega^*(n)/m_\pi]^3}.$$

A different enhancement factor for the MNB processes compared to MMU.

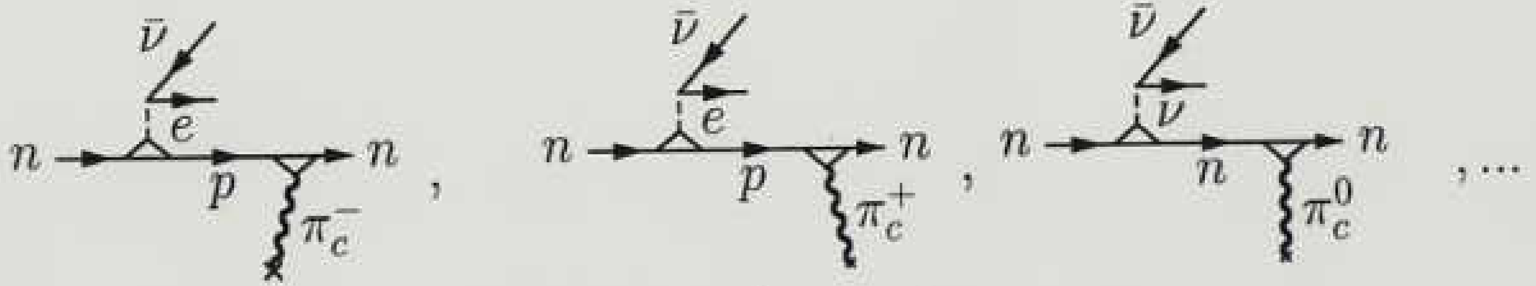
Proper DU processes

They are forbidden up to the density n_c^{DU} when triangle inequality $p_{\mathrm{Fn}} < p_{\mathrm{Fp}} + p_{\mathrm{Fe}}$ begins to fulfill. For traditional EoS like $V18 + \delta v + UIX^*$ DU processes are permitted only for $n > 5\,n_0$.

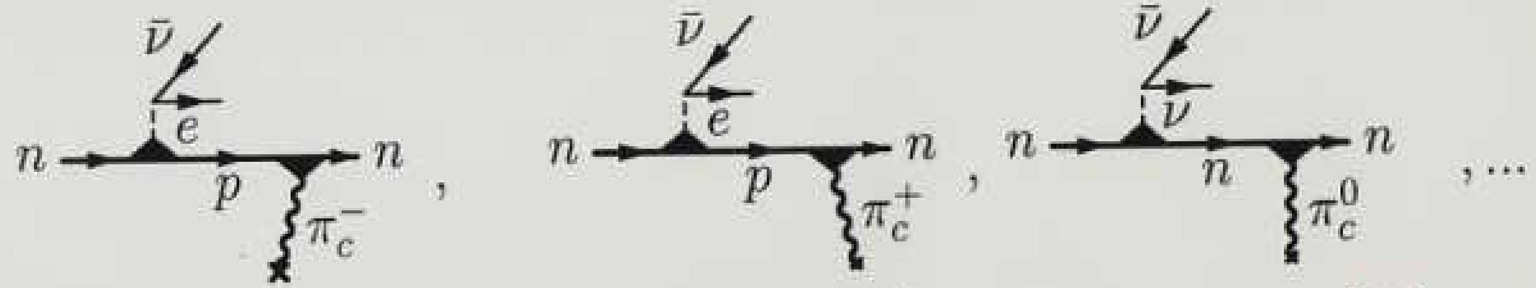
Due to full vertices a factor $\Gamma^2_{\mathrm{w-s}}$ in emissivity. (Numerically it is a minor modification).

DU-like processes (on condensates)

For $n > n_c^{\rm PU}$ ($M > M_c^{\rm PU}$) PU processes:



with free vertices: $\epsilon_\nu \sim 10^{26}\, T_9^6 (n/n_0)^{1/3}$, $\frac{\rm erg}{\rm cm^3\ sec}$. ➡

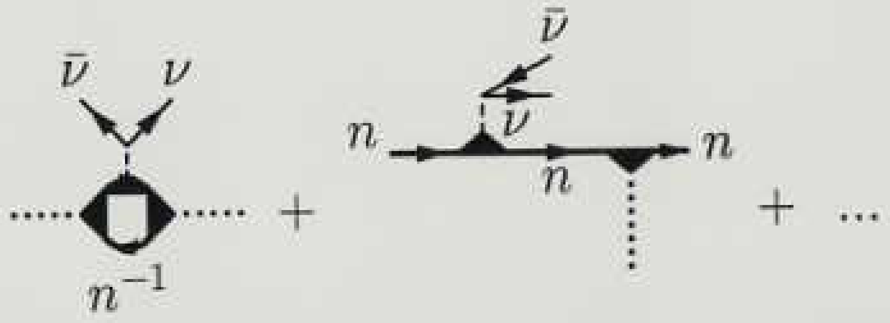


with full vertices: $\epsilon_\nu \sim 10^{26} \Gamma_s^2 \Gamma_{\rm w-s}^2\, T_9^6 (n/n_0)^{1/3}$, $\frac{\rm erg}{\rm cm^3\ sec}$. ➡ $\Gamma_s^2 \Gamma_{\rm w-s}^2 \sim \mathbf{0.1 \div 0.01}$.
(Tatsumi, Prog. Theor. Phys., **69** (1983); V. & Senatorov, JETP Lett., **40** (1984))
Kaon condensate, charged ρ condensate (Kolomeitsev & V. nucl-th/0410063) processes yield a smaller contribution.

Other resonance processes

There are many other in-medium reaction channels, e.g., with zero sound excitations.

- The most essential contribution comes from the neutral current processes



The dotted line is zero sound quantum of appropriate symmetry. These are resonance processes (second, of DU-type) similar to processes going on condensates with the only difference that rates of reactions with zero sounds are proportional to thermal occupations of the corresponding spectrum branches. Contribution of the resonance reactions is rather small due to a small phase space volume ($q \sim T$) associated with zero sounds, **cf. phonon processes.**

- Bubble rearrangement of the Fermi sea in a narrow vicinity of the π^0 condensation critical point (V. et al. AJ, **533** (2000)) ➡ most efficient process $\epsilon_\nu \sim 10^{27} T_9^5$. (This possibility is not included in cooling scenarios).

DU-like processes. MNPBF processes

Permitted only for $T < T_c$.

Renormalization of the proton vertex (vector part of $V_{pp}^N + V_{pp}^\gamma$) is governed by processes

forbidden in vacuum. ➡ 10^2 enhancement! cf. VS (1987), incorporated in cooling code by Schaab et al. AA, **321** (1997), Blaschke et al., AA, **368**, (2001); **424** (2004).

Both for neutrons and protons:

$$\epsilon_\nu \sim 10^{29} \left[\frac{\Delta_{nn}}{\text{MeV}}\right]^7 \left[\frac{T}{\Delta_{nn}}\right]^{1/2} (n/n_0)^{1/3}\, \xi_{ii}^2, \quad \frac{\text{erg}}{\text{cm}^3\ \text{sec}},$$

Δ_{ii} is NN gap, $i = n, p$, $\xi_{ii} = \exp[-\Delta_{ii}/T]$ is superfluid suppression factor.

Urbana-Argonne based EoS

$A18 + \delta v + UIX^*$:

$M_{crit}^{\mathrm{DU}} \simeq 2\ M_\odot$, $M_{max} \simeq 2.2\ M_\odot$ ($n_{cent} \simeq 7\ n_0$).

a-causal for $n > 4n_0$.

Improvement: Heiselberg, Hjorth-Jensen (HHJ) causal interpolation EoS:

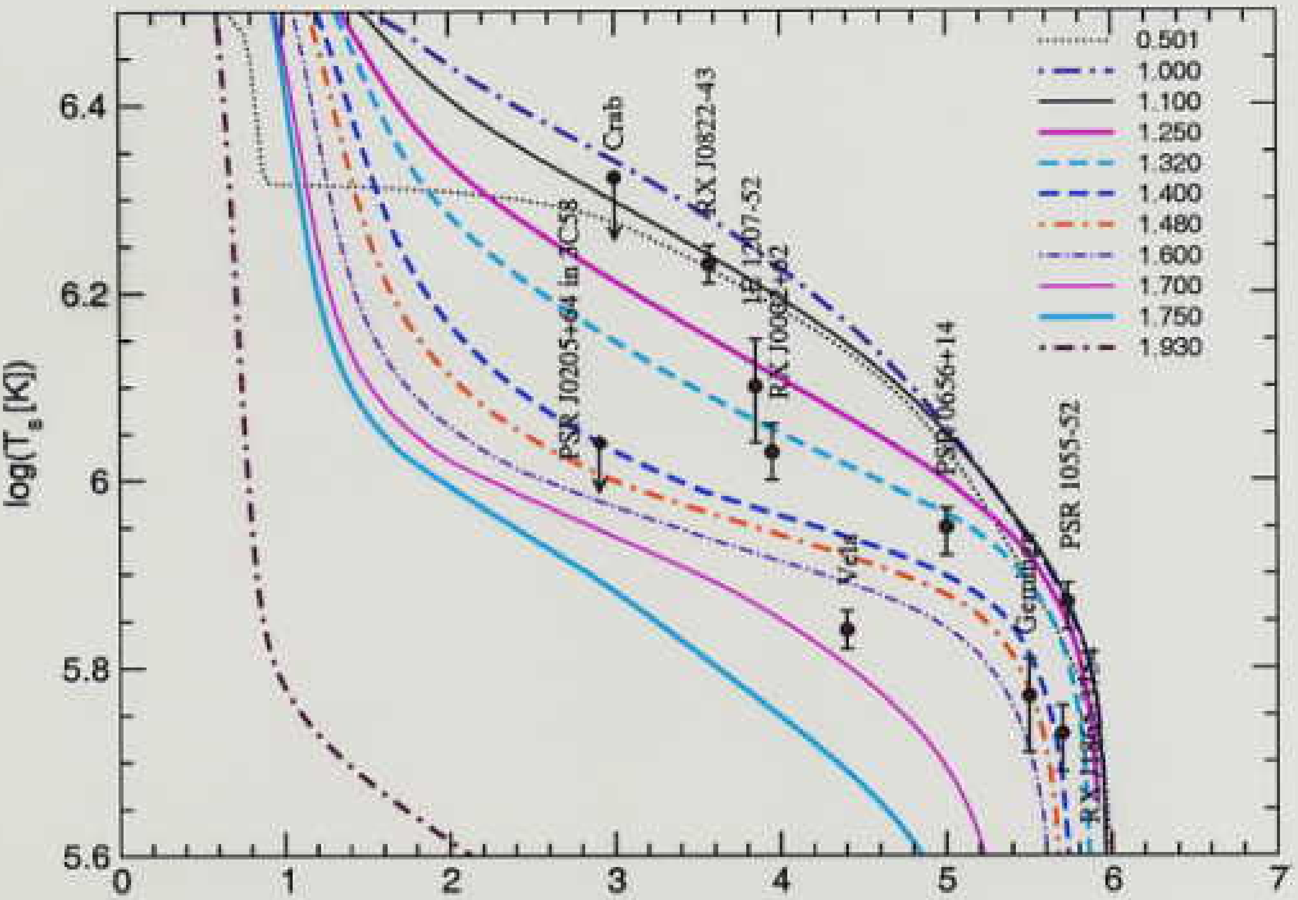
$M_{crit}^{\mathrm{DU}} \simeq 1.839\ M_\odot$, $M_{max} \simeq 1.96\ M_\odot$ ($n_{cent} \simeq 7\ n_0$).

Deficiency (?) A decreasing of M_{max}.

Nuclear Medium Cooling Scenario

Blaschke, Grigorian, D.V., A& A **424** (2004)

- Urbana-Argonne $A18 + \delta v + UIX^*$ based EoS.
- Medium effects included in calcul. of all processes. $3P_2$ gaps from Schwenk & Friman model.



- This result has passed $\log N - \log S$ (population synthesis) controle (S.Popov et al., astro-ph/0411618)

Intermediate Conclusion

- EoS is required with a large maximum NS mass

(one may expect $M_{max} \gtrsim 2\ M_{\odot}$)

- A high value of the DU threshold density

(one may expect $M_{crit}^{DU} \gtrsim 1.6 \div 1.8\ M_{\odot}$)

➡ Urbana-Argonne based EoS is rather appropriate

Relativistic Mean-Field Model

Lagrangian

$$\mathcal{L} = \sum_N \bar{\mathrm{N}} \left[i\,(\hat{\partial} + i\, g_{\omega N} \hat{\omega} + i\, g_{\rho N} \boldsymbol{\tau}\, \hat{\boldsymbol{\rho}}) \right] - (m - g_{\sigma N}\, \sigma) \Big] \mathrm{N}$$

$$+ \underbrace{\frac{1}{2} (\partial_\mu \sigma\, \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma)}_{\text{scalar}}$$

$$- \underbrace{\frac{1}{4} \omega_{\mu\nu}\, \omega^{\mu\nu} + \frac{1}{2} m_\omega^2\, \omega_\mu\, \omega^\mu}_{\text{vector}} - \underbrace{\frac{1}{4} \boldsymbol{\rho}_{\mu\nu}\, \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2} \boldsymbol{\rho}_\mu\, \boldsymbol{\rho}^\mu}_{\text{iso-vector}} m_\rho^2$$

equations of motion

$$\left[i\,(\hat{\partial} + i\, g_{\omega N} \hat{\omega} + i\, g_{\rho N} \boldsymbol{\tau}\, \hat{\boldsymbol{\rho}}) \right] - (m - g_{\sigma N}\, \sigma) \Big] \mathrm{N} = 0$$

$\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$,

$\vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$,

$U(\sigma) = \frac{g}{3} \sigma^3 + \frac{c}{4} \sigma^4$.

$$(\partial^2 + m_\sigma^2)\, \sigma + \frac{dU}{d\sigma} = g_{\sigma N} \sum_N \bar{N} N$$

$$(\partial^2 + m_\omega^2)\, \omega_\mu = g_{\omega N} \sum_N \bar{N} \gamma_\mu N$$

$$(\partial^2 + m_\rho^2)\, \boldsymbol{\rho}_\mu = g_{\rho N} \sum_N \bar{N} \boldsymbol{\tau} \gamma_\mu N$$

Relativistic Mean-Field Model

$$\varepsilon_N(p) = \sqrt{m_N^{*2} + p^2} + g_{\omega N}\,\omega_0 + g_{\rho N}\,I_N\,\rho_{03} \quad m_N^* = m_N - g_{\sigma N}\,\sigma$$

field Ansatz

$\sigma(r,t) = \sigma$

$\omega_\mu(r,t) = \delta_{\mu,0}\,\omega_0$

$\rho_\mu^a(r,t) = \delta^{a,3}\,\delta_{\mu,0}\,\rho_0^{(3)}$

constant fields

$$\begin{aligned} g_{\sigma N}\,\sigma + \frac{\mathrm{d}U}{\mathrm{d}\sigma} &= \frac{g_{\sigma N}^2}{m_\sigma^2}\,\rho^{\text{scalar}} = \underline{C_\sigma^2}\,\frac{\rho_p^{\text{scalar}} + \rho_p^{\text{scalar}}}{m_N^2} \\ g_{\omega N}\,\omega_0 &= \frac{g_{\omega N}^2}{m_\omega^2}\,\rho_{\text{baryon}} = \underline{C_\omega^2}\,\frac{\rho_p + \rho_n}{m_N^2} \\ g_{\rho N}\,\rho_{03} &= \frac{g_{\rho N}^2}{m_\rho^2}\,\rho_{\text{isospin}} = \underline{C_\rho^2}\,\frac{\rho_p - \rho_n}{m_N^2} \end{aligned}$$

$$E = \frac{m_\sigma^2\,\sigma^2}{2} + U(\sigma) - \frac{m_\omega^2\,\omega_0^2}{2} - \frac{m_\rho^2\,\rho_0^{(3)2}}{2} + \sum_N \int_0^{p_{F,N}} \frac{dp\,p^2}{\pi^2}\,\varepsilon_N(p)$$

Paremeters $\left\{ C_i^2 = \dfrac{g_{iN}^2\,m_N^2}{m_i^2} \right\}$ are adjusted to properties of nuclear matter at saturation

6, c>0;

$$\begin{aligned} n_0 &\simeq 0.16 \pm 0.015\ \mathrm{fm}^{-3} \\ E_{\text{bind}} &\simeq -15.6 \pm 0.6\ \mathrm{MeV} \\ m_N^*(\rho_0) &\simeq (0.75 \pm 0.1)\,m_N \\ K &\simeq 240 \pm 40\ \mathrm{MeV} \\ a_{\text{sym}} &\simeq 32 \pm 4\ \mathrm{MeV} \end{aligned}$$

Neutron Star Composition

NS matter $(e+p+n)$:

- β-equilibrium $e+p \leftrightarrow n$: $\mu_e = \mu_n - \mu_p$
- electroneutrality: $n_e = n_p$

$\Longrightarrow$ EoS for NS matter $P = P(\epsilon)$, ϵ– total energy density

- Equilibrium condition for non-rotating NS (Tolman-Oppenheimer-Volkoff equation) $[G = 1]$

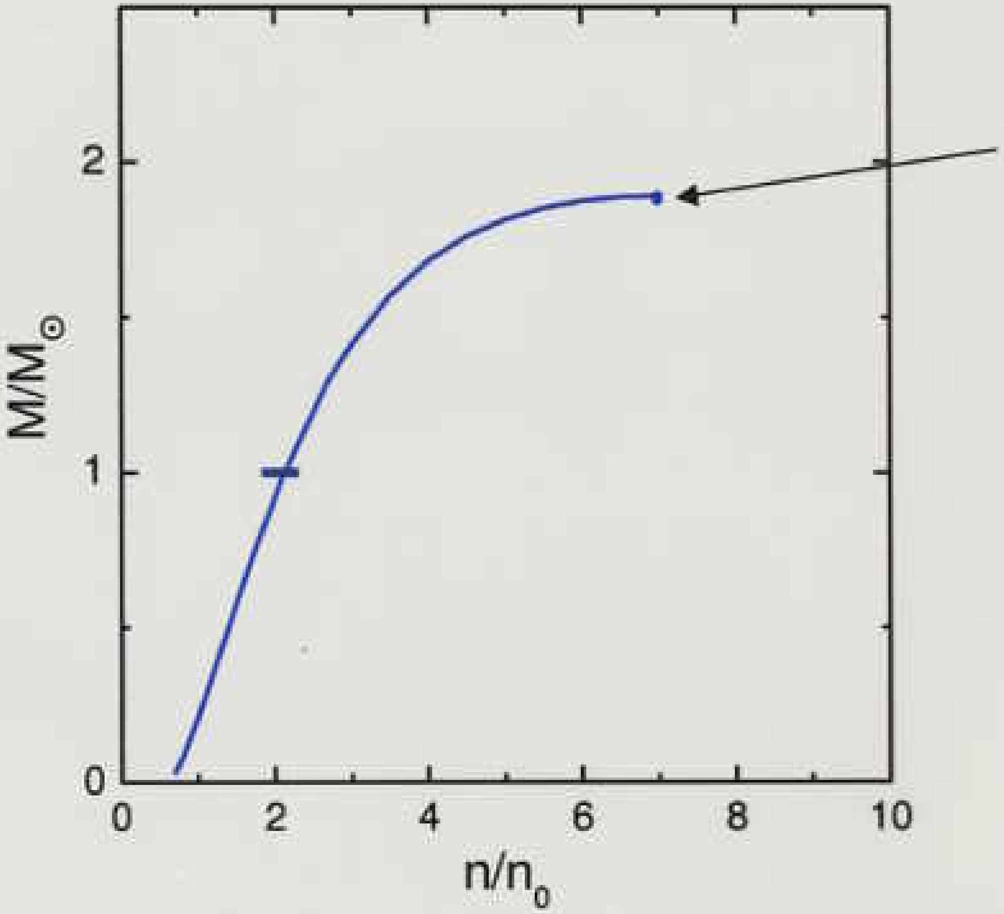
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{(P+\epsilon)\,(M + 4\,\pi\,r^3\,P)}{r\,(1-2M/r)}, \qquad P(0) = P(\epsilon_c)$$

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\,\pi^2\,r^2\,\epsilon(r)\,, \qquad M(0) = 0$$

$\epsilon_c = \epsilon(n_c)$ where n_c–central baryon density

- NS radius R: $P(R) = 0$; NS mass: $M = M(R)$

input parameters: $n_0 = 0.16\ \mathrm{fm}^{-3}, \quad e_\mathrm{B} = -16\ \mathrm{MeV}, \quad a_\mathrm{sym}(n_0) = 32\ \mathrm{MeV},$
$K = 270\ \mathrm{MeV}, \quad m_N^*(n_0)/m_N = 0.8$

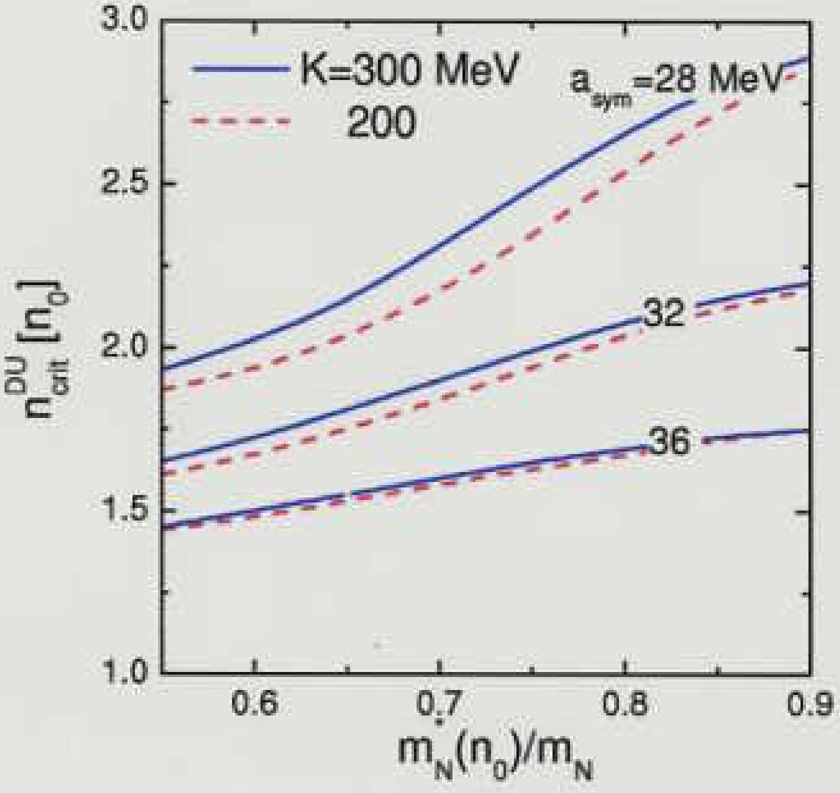


limiting NS mass

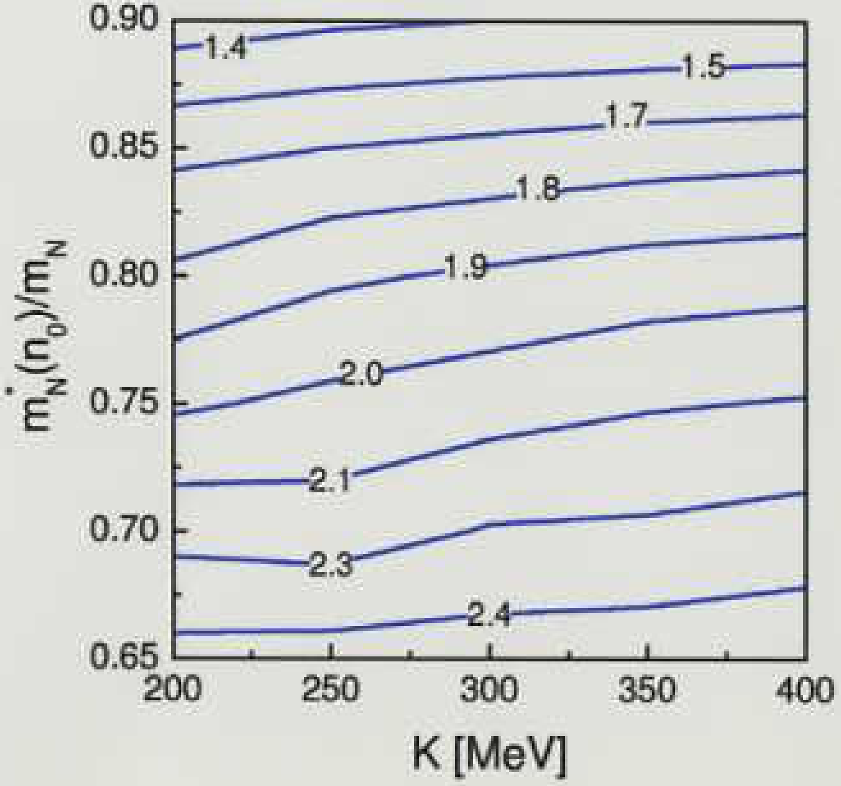
$n_\mathrm{crit}^\mathrm{DU} = 2.07\, n_0$

$M_\mathrm{crit}^\mathrm{DU} = 0.99\, M_\odot$

$n_0 = 0.16\ \mathrm{fm}^{-3}, \quad e_\mathrm{B} = -16\ \mathrm{MeV}$



threshold of DU process



limiting NS mass

Chiral Symmetry Restoration with Density

motivates dropping of effective meson and nucleon masses

- in standard RMF model m_σ, m_ω, and m_ρ do not change

  Can the in-medium modification (decrease) of meson masses
  be included in an RMF model??

- Song, Brown, Min, Rho (1997) $m^*_\sigma/m_\sigma \approx m^*_\omega/m_\omega \approx m^*_\rho/m_\rho = \Phi(n)$
- decreasing functions of σ: $m^*_\omega(\sigma)$, $m^*_\rho(\sigma) \longleftarrow$ self-consistent σ field
  results in *increase* of ρ an ω masses
- σ field dependent masses and couplings constant
  too many functions to tune

Generalized RMF Model

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_l\,,$$

$$\mathcal{L}_N = a_N\,\bar\Psi_N\left(i\,D\cdot\gamma\right)\Psi_N - m_N\,\phi_N\,\bar\Psi_N\,\Psi_N\,,$$

$$D_\mu = \partial_\mu + i\,g_\omega\,\widetilde\chi_\omega\omega_\mu + \frac{i}{2}g_\rho\,\widetilde\chi_\rho\,\boldsymbol{\rho}_\mu\,\boldsymbol{\tau}\,,$$

$$\mathcal{L}_M = a_\sigma\frac{\partial^\mu\sigma\partial_\mu\sigma}{2} - \phi_\sigma^2\frac{m_\sigma^2\sigma^2}{2} - \widetilde U(\sigma)$$

$$-a_\omega\,\frac{\omega_{\mu\nu}\,\omega^{\mu\nu}}{4} + \phi_\omega^2\,\frac{m_\omega^2\,\omega_\mu\omega^\mu}{2} - a_\rho\,\frac{\boldsymbol{\rho}_{\mu\nu}\,\boldsymbol{\rho}^{\mu\nu}}{4} + \phi_\rho^2\,\frac{m_\rho^2\,\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu}{2}\,,$$

$$\omega_{\mu\nu} = \partial_\nu\omega_\mu - \partial_\mu\omega_\nu\,,\quad \boldsymbol{\rho}_{\mu\nu} = \partial_\nu\boldsymbol{\rho}_\mu - \partial_\mu\boldsymbol{\rho}_\nu + g_\rho^{'}\,\widetilde\chi_\rho^{'}\,[\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu]\,,$$

$$\mathcal{L}_l = \sum_l \bar\Psi_l[i(\gamma\cdot\partial) - m_l]\Psi_l\,.$$

non-Abelian gauge boson

$\{g_\rho^{'} = g_\rho\}$

All scaling functions a_i, $\widetilde\chi_i$, ϕ_i depend on $g_\sigma\,\widetilde\chi_\sigma\,\sigma$

$$\Psi_N \to \Psi_N/\sqrt{a_N}\,,\quad \sigma\to\sigma/\sqrt{a_\sigma}\,,\quad \omega_\mu\to\omega_\mu/\sqrt{a_\omega}\,,\quad \boldsymbol{\rho}_\mu\to\boldsymbol{\rho}_\mu/\sqrt{a_\rho}$$

Generalized RMF Model

$$
\begin{aligned}
\mathcal{L}_N &= \bar{\Psi}_N \left(i\, D \cdot \gamma \right) \Psi_N - m_N \, \Phi_N \, \bar{\Psi}_N \, \Psi_N \,, \\
& \qquad D_\mu = \partial_\mu + i\, g_\omega \, \chi_\omega \omega_\mu + \frac{i}{2} g_\rho \, \chi_\rho \, \boldsymbol{\rho}_\mu \, \boldsymbol{\tau} \,, \\
\mathcal{L}_M &= \frac{\partial^\mu \sigma \partial_\mu \sigma}{2} - \Phi_\sigma^2 \frac{m_\sigma^2 \sigma^2}{2} - U(\sigma) \\
& \quad - \frac{\omega_{\mu\nu} \, \omega^{\mu\nu}}{4} + \Phi_\omega^2 \, \frac{m_\omega^2 \, \omega_\mu \omega^\mu}{2} - \frac{\boldsymbol{\rho}_{\mu\nu} \, \boldsymbol{\rho}^{\mu\nu}}{4} + \Phi_\rho^2 \frac{m_\rho^2 \, \boldsymbol{\rho}_\mu \boldsymbol{\rho}^\mu}{2} \,, \\
& \qquad \omega_{\mu\nu} = \partial_\nu \omega_\mu - \partial_\mu \omega_\nu \,, \quad \boldsymbol{\rho}_{\mu\nu} = \partial_\nu \boldsymbol{\rho}_\mu - \partial_\mu \boldsymbol{\rho}_\nu + g_\rho^{'} \, \chi_\rho^{\prime} \, [\boldsymbol{\rho}_\mu \times \boldsymbol{\rho}_\nu] \,,
\end{aligned}
$$

$$
m_i^*/m_i = \phi_i(\chi_\sigma \sigma)/\sqrt{a_i(\chi_\sigma \sigma)} = \Phi_i(\chi_\sigma \sigma) \quad \chi_i = \widetilde{\chi}_i(\chi_\sigma \sigma)/\sqrt{a_i(\chi_\sigma \sigma)}
$$

Energy-Density Functional

minimized with respect to ω and ρ mean fields

$$E[n_n, n_p, n_l; f] = E_N[n_n, n_p; f] + E_l[n_e, n_\mu]$$

$$E_N[n_n, n_p; f] = \frac{m_N^4\, f^2}{2\, C_\sigma^2}\, \eta_\sigma(f) + U(f) + \frac{C_\omega^2\, (n_n + n_p)^2}{2\, m_N^2\, \eta_\omega(f)} + \frac{C_\rho^2 (n_n - n_p)^2}{8\, m_N^2\, \eta_\rho(f)}$$
$$+ \left(\int\limits_0^{p_{\mathrm{F},n}} + \int\limits_0^{p_{\mathrm{F},p}} \right) \frac{\mathrm{d}p p^2}{\pi^2} \sqrt{m_N^2\, \Phi_N^2(f) + p^2}\,,$$

$$E_l[n_e, n_\mu] = \sum_{i=e\,,\mu} \int\limits_0^{p_{\mathrm{F},i}} \frac{\mathrm{d}p p^2}{\pi^2} \sqrt{m_i^2 + p^2}$$

scalar field $f = g_\sigma\, \underline{\chi_\sigma}\, \underline{\sigma}$

$$\eta_i(f) = \frac{\Phi_i^2(f)}{\chi_i^2(f)}\,, \quad i = \sigma\,, \omega\,, \rho$$

Equivalence of RMF models

12 scaling functions

$a_{N,\sigma,\omega,\rho}$, $\widetilde{\chi}_{\rho,\omega,\sigma}$, $\phi_{N,\sigma\,\omega,\rho}$ and $\widetilde{U}(\sigma)$

4 scaling functions

$\eta_{\sigma,\rho,\omega}(f)$ and $U(f)$

3 independent scaling functions

$\eta_{\omega,\rho}(f)$ and $U(f)$

$$U \to U + \frac{m_N^4 f^2}{2\,C_\sigma^2}\,(1 - \eta_\sigma(f))$$

Can we put some constraints
on scaling functions ??

RMF Model and Brown-Rho scaling

assume couplings are constant

$$\Phi_N = \Phi_\sigma = \Phi_\omega = \Phi_\rho = \Phi(f) = 1 - f \quad \Longrightarrow \quad \eta_i = \Phi^2(f) \longleftarrow \text{decreasing!}$$

- Effective hadron masses do not decrease monotonously with the density increase. Decreasing at small densities, they start to increase at higher densities.

$$\frac{\partial f}{\partial n} = \left(\frac{C_\omega^2 n}{m_N^2 \eta_V^2} \frac{\partial \eta_\omega}{\partial f} - \frac{\partial \Phi_N}{\partial f} \frac{m_N^2 \Phi_N(f)}{\sqrt{m_N^2 \Phi_N^2(f) + p_F^2}} \right) \Bigg|_{f(n)} \Bigg/ \frac{\partial^2 E_N[n;f]}{\partial f^2} \Bigg|_{f(n)}$$

- Discontinuity of the density dependence of the scalar field f. The equation of motion for f may have several solutions

- decreasing η_σ leads to overbinding of the nuclear matter at saturation DBHF calculations [Rapp,Machleidt,Durso,Brown]

- if $\eta_\omega \neq 1 \quad \longrightarrow$ problems with flow in HIC [Ko,Li,Brown]
  for n near n_0

- DBHF calculations [Li,Kuo,Lee,Brown]: $a_{\rm sym} \propto n \longrightarrow \eta_\rho \sim 1$
  Urbana-Argonne EoS : $a_{\rm sym} \propto n^{0.6} \longrightarrow \eta_\rho \simeq (n/n_0)^{0.4}$

Universal scaling

Assume $\eta_\sigma = \eta_\omega = \eta_\rho = 1$, $U(f) = \frac{b}{3}f^3 + \frac{c}{4}f^4$, $c > 0$:

➡ RMF models with the universal scaling are equivalent to RMF models without any scaling.

We want:

$\eta_\omega < 1$ to demonstrate possibility to increase M_{max};

$\eta_\rho > 1$ to increase n_{crit}^{DU};

$\eta_\omega \simeq 1$, $\eta_\rho \simeq 1$ for $n \simeq n_0$ not to spoil out fit at nuclear saturation

➡ A non-universal scaling.

RMF model with non-universal scaling

Modified Walecka (MW) model:
$\Phi_N(f) = 1 - f$ and $U(f) = \frac{b}{3} f^3 + \frac{c}{4} f^4$

non-universal scaling:
$\eta_\sigma = 1; \quad \eta_\omega < 1; \quad \eta_\rho > 1$

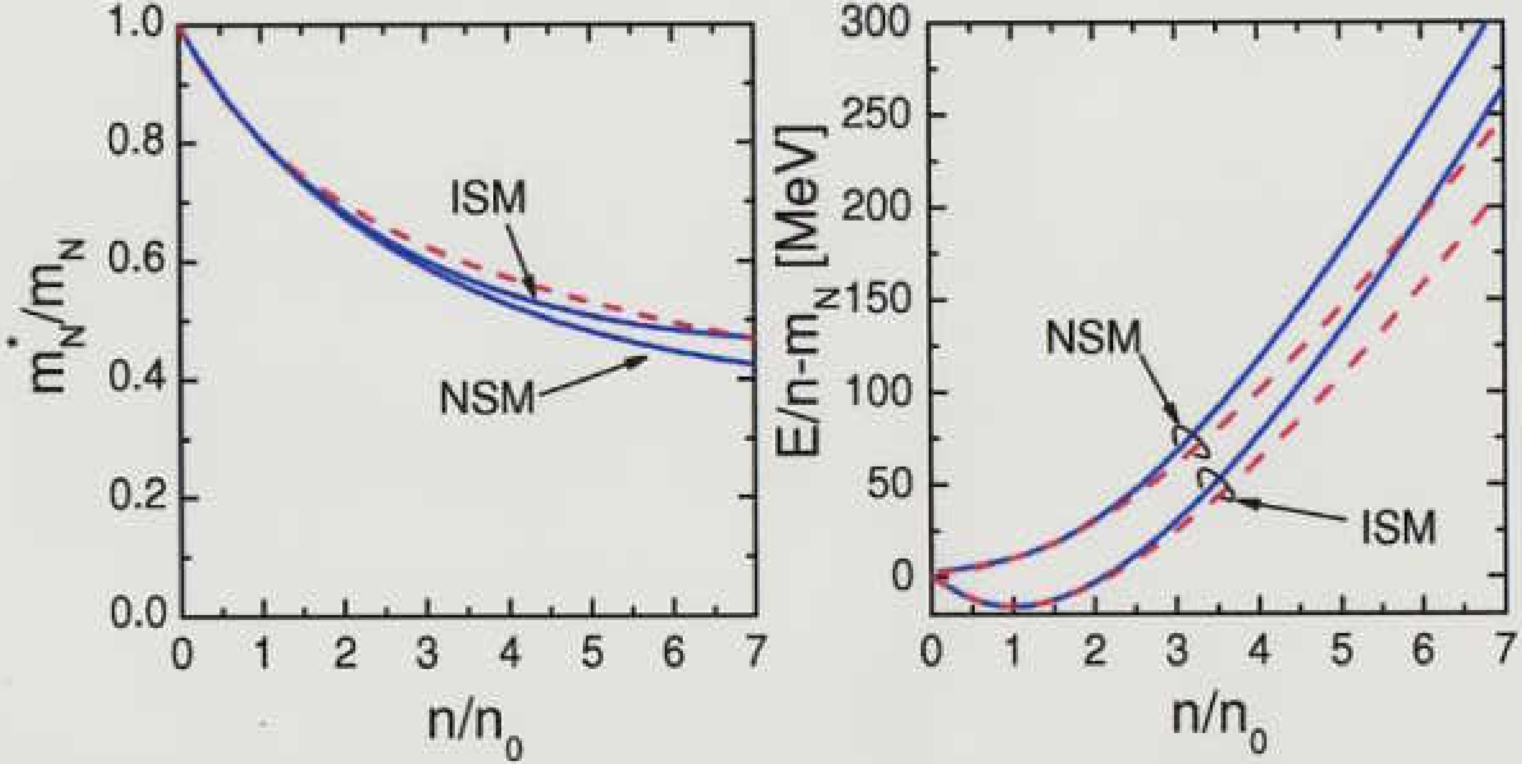
$$\eta_\sigma = 1, \quad \eta_\omega(f) = \frac{1 + z\, f_0}{1 + z\, f}, \quad \eta_\rho(f) = \frac{\eta_\omega(f)}{\eta_\omega(f) + 4\,\frac{C_\omega^2}{C_\rho^2}\,(\eta_\omega(f) - 1)},$$

$$f_0 = 1 - m_N^*(n_0)/m_N \quad \longrightarrow \{\, \eta_\omega(f_0) = \eta_\rho(f_0) = 1 \,\}$$

input parameters:
$n_0 = 0.16\ \mathrm{fm}^{-3}, \quad e_{\mathrm{B}} = -16\ \mathrm{MeV}, \quad a_{\mathrm{sym}}(n_0) = 32\ \mathrm{MeV},$
$K = 275\ \mathrm{MeV}, \quad m_N^*(n_0)/m_N = 0.805$

$z = 0.65$

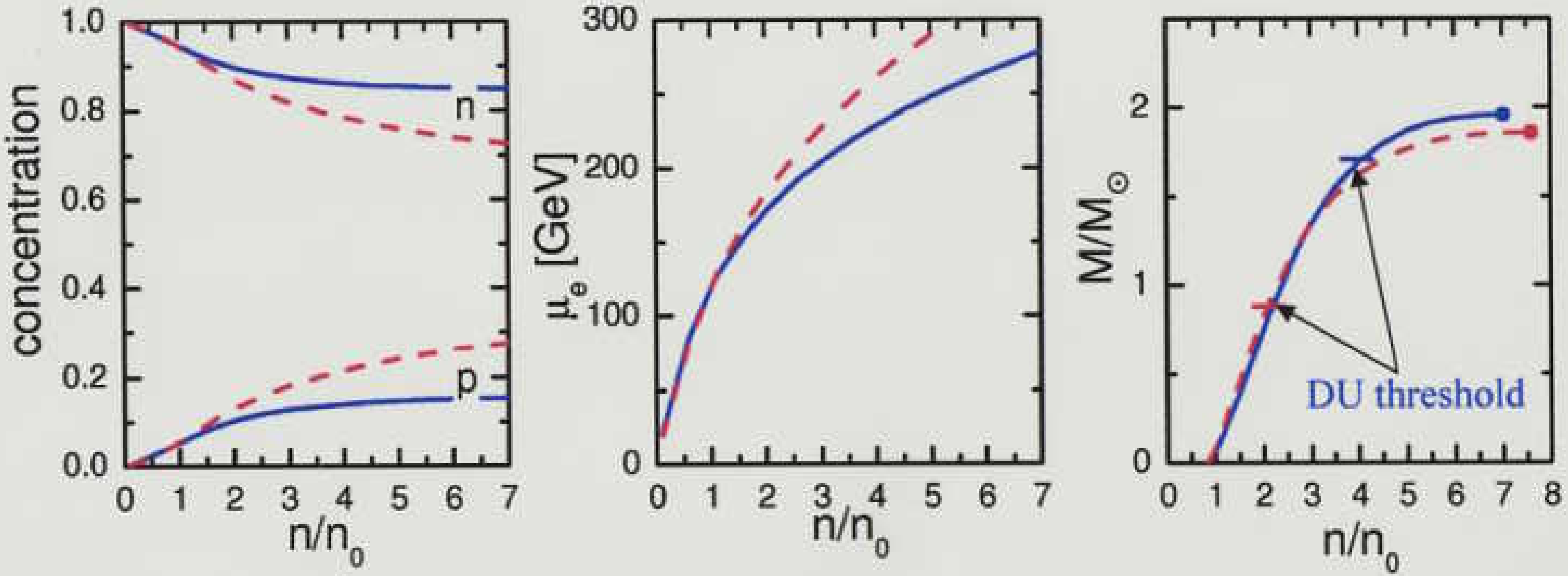
RMF model with non-universal scaling



isospin symmetrical matter (ISM)
neutron star matter (NSM)

solid lines: MW(nu)
dashed lines: MW(u)=MW

RMF model with non-universal scaling



solid lines: MW(nu)
dashes lins: MW(u)=MW

RMF model for Urbana-Argonne EoS

Heiselberg & Hjorth-Jensen fit to the A18+δv+UIX* EoS of Urbana-Argonne group for $n < 4\,n_0$ and causal for higher densities

$$E[n_p, n_n] = (n_p + n_n)\ \left[e_{\rm B}\, u\,\frac{2.2 - u}{1 + 0.2\,u} + a_{\rm sym}\, u^{0.6}\,\frac{(n_p - n_n)^2}{(n_p + n_n)^2}\right]\,, \quad \boxed{\text{HHJ EoS}}$$

$$u = (n_p + n_n)/n_0\,,\ \ e_B = -15.8\ \text{MeV}\ \ a_{\rm sym} = 32\ \text{MeV}\ .$$

$$M_{\rm lim} = 2.0\,M_\odot \qquad n_{\rm crit}^{\rm DU} = 5.2\,n_0 \qquad M_{\rm crit}^{\rm DU} = 1.84\,M_\odot$$

- HHJ EoS can be fitted by MW model with:

$$e_{\rm B} = -15.8\ \text{MeV}\,,\ \ K = 250\ \text{MeV}\,,\ \ m_N^*(n_0) = 0.8\,m_N\,,\ \ a_{\rm sym} = 28\ \text{MeV}$$

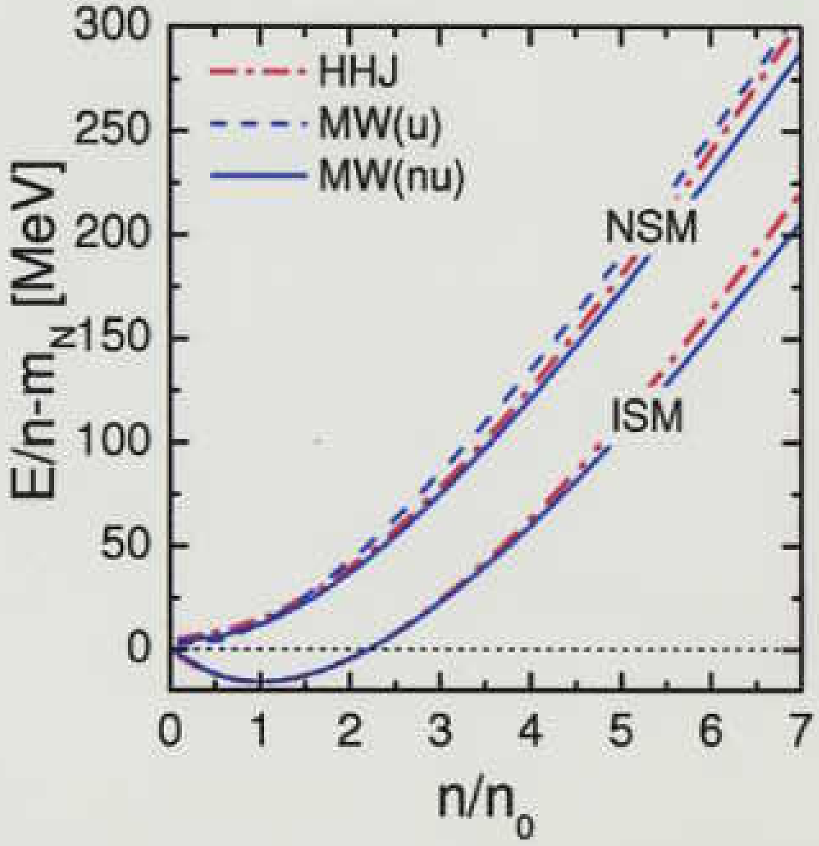
$$M_{\rm lim} = 1.98\,M_\odot \qquad n_{\rm crit}^{\rm DU} = 2.6\,n_0!!$$

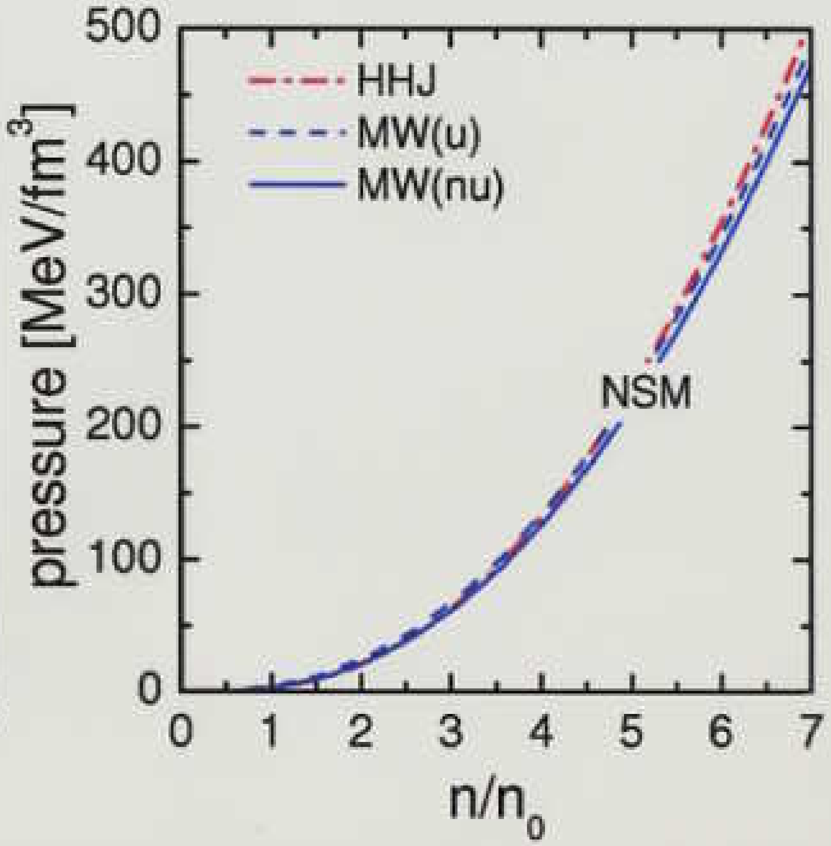
RMF model for Urbana-Argonne EoS

Modified Walecka (MW) model:
$\Phi_N(f) = 1 - f$ and $U(f) = \frac{b}{3} f^3 + \frac{c}{4} f^4$

non-universal scaling:
$\eta_\sigma = \eta_\omega = 1 \quad \eta_\rho > 1$

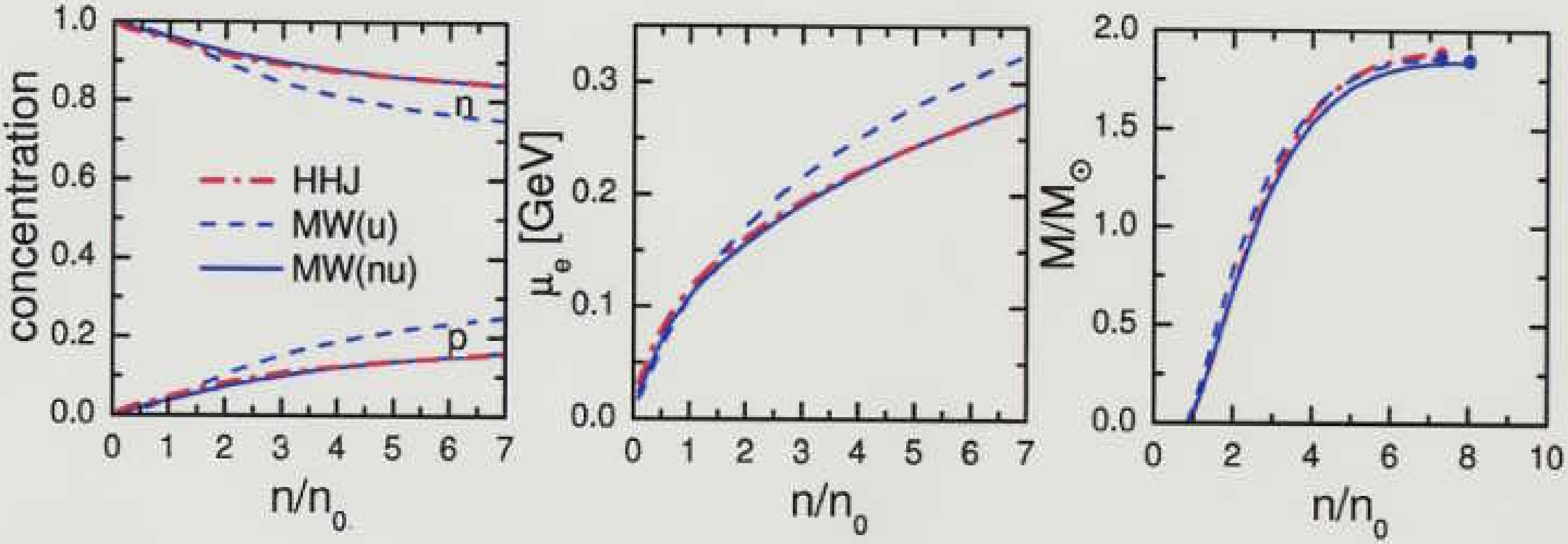
$$\eta_\sigma = \eta_\omega = 1 , \quad \eta_\rho(f) = \left(\frac{1 + z\, f}{1 + z\, f_0} \right)^2 , \quad z = 2.9 , \quad f_0 = 1 - m_N^*(n_0)/m_N$$





RMF model for Urbana-Argonne EoS

change of neutron star composition



less protons ⟶ $n_{\rm crit}^{\rm DU} = 5.19\, n_0$

Conclusions

- new data on neutron star masses
- Direct Urca reaction should be exotic

  too fast cooling otherwise
- EoS with high threshold of direct Urca reactions

  standard RMF models have too low threshold
- RMF models with a scaling of hadron masses and coupling constants
  - *increase DU threshold*
  - *increase limiting NS mass*

Rho meson fields

$$\mathcal{L}_{N\rho} = \bar{\Psi}_N \left(i\, D \cdot \gamma \right) \Psi_N - m_N \bar{\Psi}_N \Psi_N - \frac{1}{4} \boldsymbol{\rho}_{\mu\nu} \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \boldsymbol{\rho}^\mu$$

$$D_\mu = \partial_\mu + \frac{i}{2} g_\rho \boldsymbol{\rho}_\mu \boldsymbol{\tau} \qquad \boldsymbol{\rho}_{\mu\nu} = \partial_\nu \boldsymbol{\rho}_\mu - \partial_\mu \boldsymbol{\rho}_\nu + g'_\rho [\boldsymbol{\rho}_\mu \times \boldsymbol{\rho}_\nu]$$

ρ meson as a non-Abelian gauge boson: $g' = g$

gauge boson condensation:

gluons : A.B. Migdal, JETP Let. 28 (1978) 35
W-bosons : A.D. Linde, Phys. Lett 86 (1979) 39

boson mean fields:

$$\rho_0^{(3)} \neq 0\,, \qquad \rho_i^{\pm} = \frac{1}{\sqrt{2}} (\rho_i^{(1)} \pm i\, \rho_i^{(2)}) \neq 0\,, \quad i = 1, 2, 3$$

[D.N. Voskresensky Phys. Lett. B 392 (1997) 262]

energy-density functional:

$$E_{\rho N} = \frac{1}{2} g_\rho \,(n_p - n_n)\, \rho_0^{(3)} - \frac{1}{2}\left(\rho_0^{(3)}\right)^2 m_\rho^2 - \left[\left(g'_\rho \rho_0^{(3)}\right)^2 - m_\rho^2\right] |\rho_c|^2 + \frac{g_\rho'^2}{2}\left(\rho_i^\dagger \rho_j^- - \rho_i^- \rho_j^\dagger\right)^2$$

- $\left(\rho_i^+ \rho_j^- - \rho_i^- \rho_j^+\right) = 0 \quad \rho_i^- = a_i\, \rho_c \qquad \rho_i^+ = a_i\, \rho_c^\dagger$ $\boldsymbol{a}$ is the spatial unit vector

standard solution:

$$\rho_0^{(3)} = \frac{1}{2}\frac{g_\rho}{m_\rho^2}(n_p - n_n)\,, \quad \rho_c = 0\,, \qquad\qquad E_{\rho N} = \frac{g_\rho^2 (n_n - n_p)^2}{8\, m_\rho^2}$$

new solution:

$$\rho_0^{(3)} = -\frac{m_\rho}{g_\rho}\,\mathrm{sign}(n_n - n_p)\,, \quad |\rho_c|^2 = \frac{g_\rho}{g'_\rho}\frac{|n_p - n_n| - n^\rho}{4\, m_\rho}\,, \quad n^\rho = 2\,\frac{m_\rho^3}{g_\rho\, g'_\rho}$$

$$E_{\rho N} = -\frac{m_\rho^4}{2\, g_\rho'^2} + \frac{1}{2} m_\rho \frac{g_\rho}{g'_\rho}\,|n_n - n_p|$$

↑

10—20 n_0

scaling

$$\frac{m*_\rho}{m_\rho} \simeq \frac{g*_\rho}{g_\rho} = \Phi(f) \quad \frac{g_\rho^{'*}}{g_\rho^{'}} = \begin{cases} 1 & \text{, case 1} \\ \Phi(f) & \text{, case 2} \end{cases} \qquad n^{*\rho} = n^\rho \begin{cases} \Phi^2(f) & \text{, case 1} \\ \Phi(f) & \text{, case 2} \end{cases}$$

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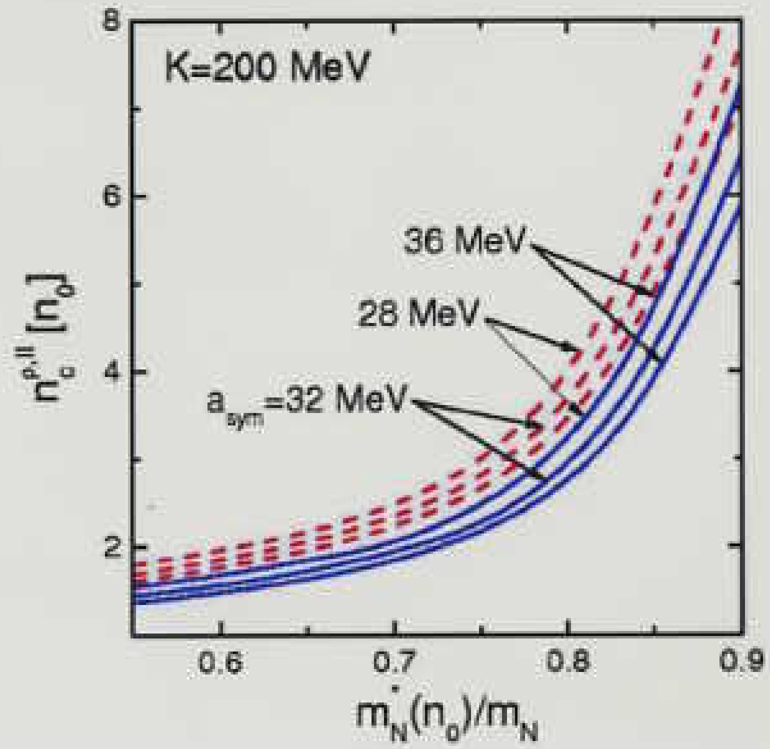
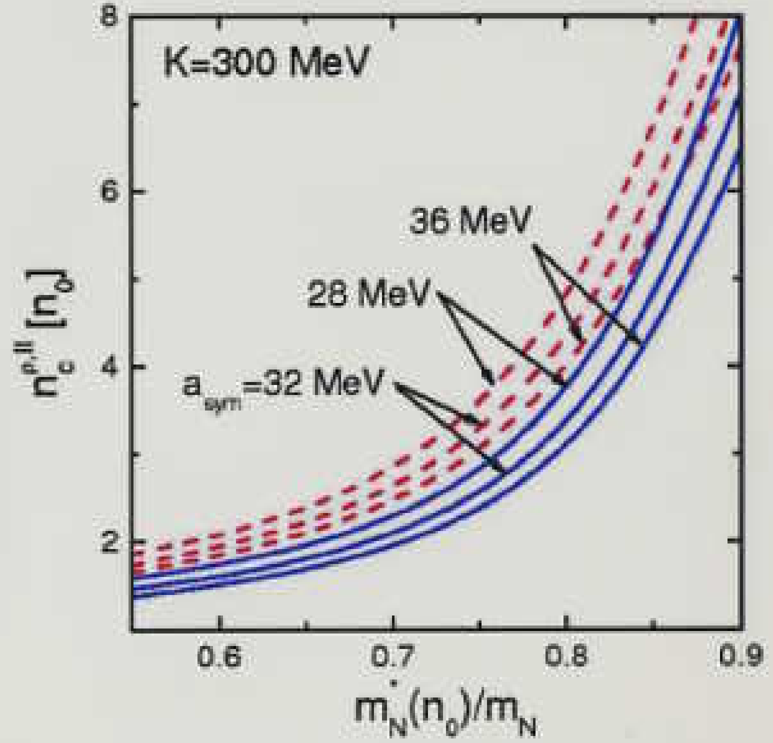
critical density second-order phase transition

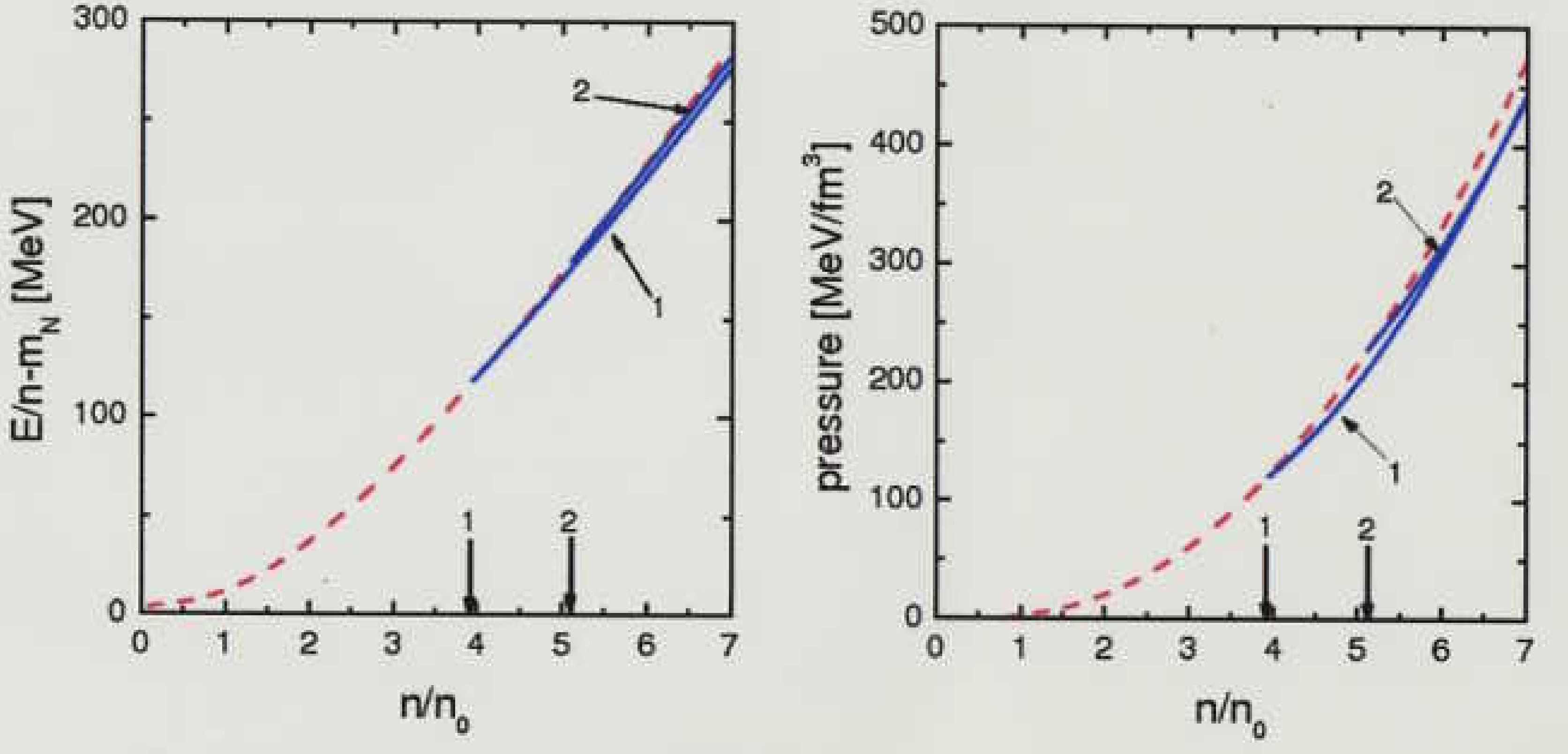




$n_0 = 0.16 \text{ fm}^{-3}, \quad e_B = -16 \text{ MeV}$

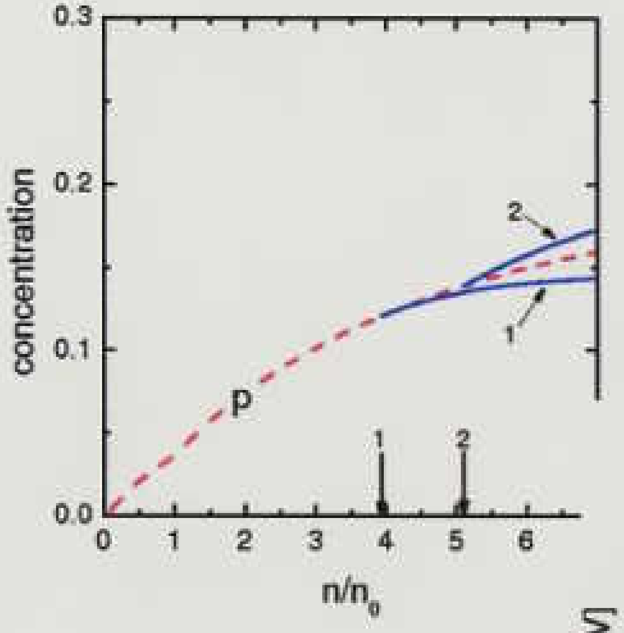
Neutron star with Rho-condensate

energy & pressure MW(nu) model for HHJ EoS (Urbana-Argonne EoS)

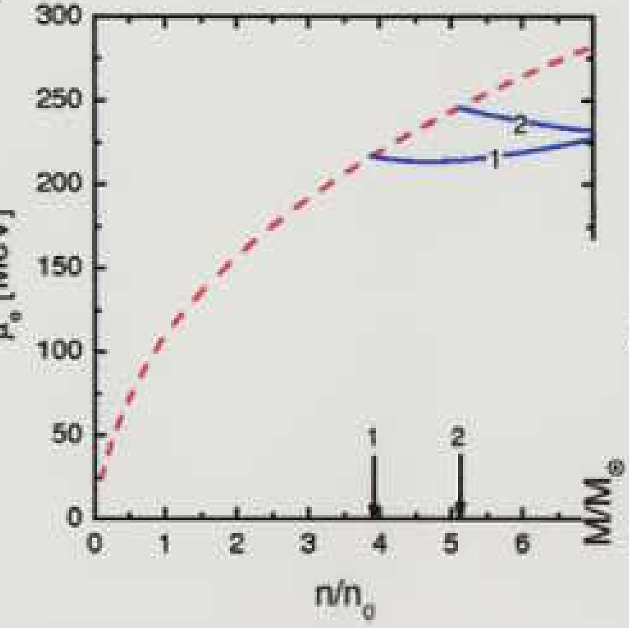


second order phase transition
weak softening of EoS

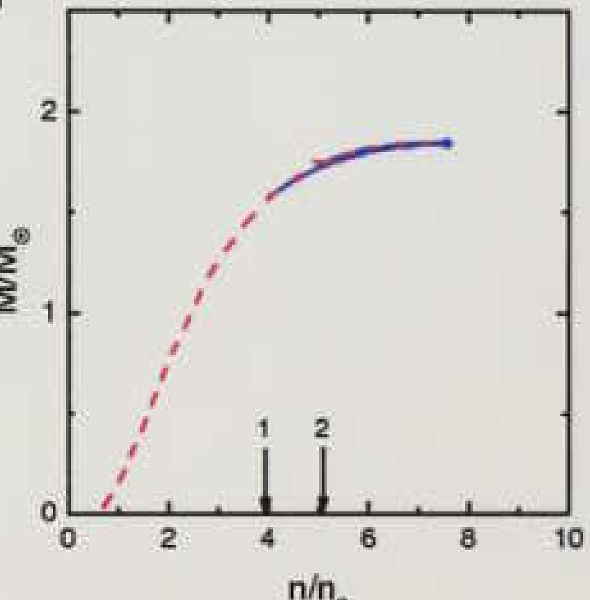
Neutron star with Rho-condensate



ρ^- condensate change proton concentration



replace electrons



no DU reactions in condensate phase!