Indirect Methods in Nuclear Astrophysics

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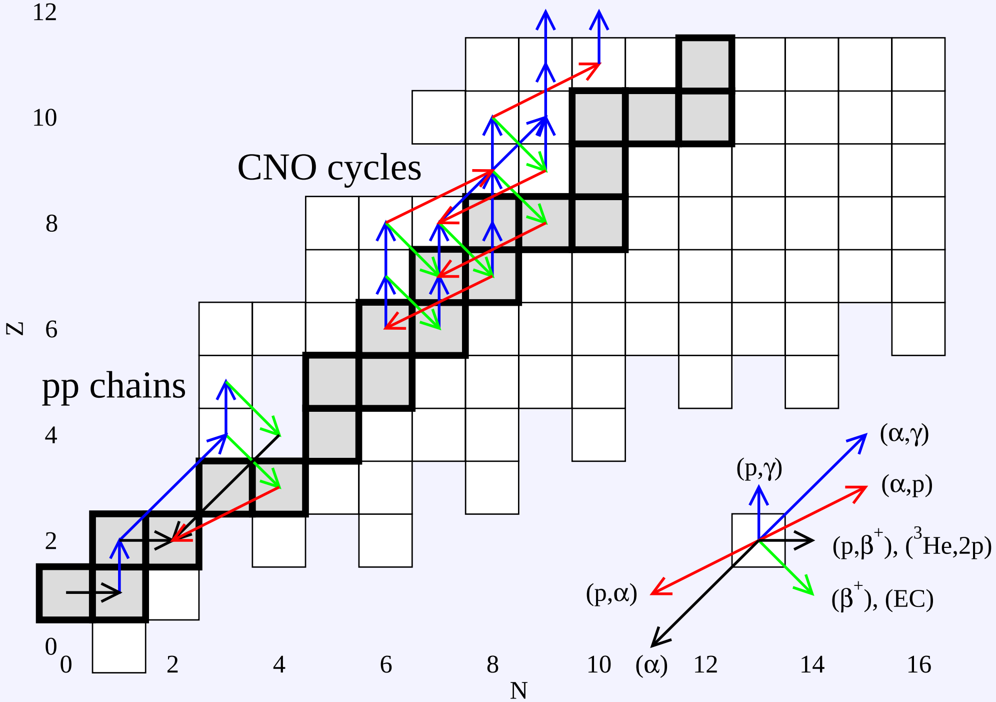
Outline

- **Motivation**
  nuclear reactions of astrophysical interest,
  astrophysical S factor, electron screening
- **Indirect Methods**
  - Coulomb Dissociation
    idea, theory, parameters, example,
    reduced transition probabilities and ANC
  - ANC method
    idea, theory of transfer reactions, application
  - Trojan-horse method
    idea, theory, approximations, application, examples
- **Summary**

Nuclear Reactions of Astrophysical Interest



nuclear astrophysics

nuclear reaction rates at small energies needed in many astrophysical models (primordial nucleosynthesis, stellar evolution, novae, supernovae, . . .)

for various processes (pp chains, CNO cycles, s, r, p, rp, . . .)

direct measurements

preferable, but difficult:

- small cross sections
- often unstable nuclei

} ⇒ low yields

- direct nuclear reactions: (p, α), (α, p), . . .
- radiative capture/dissociation reactions with charged particles: (p, γ), (α, γ), . . .
- weak interaction reactions: β^+, β^-, EC

alternative: indirect methods

depending on type of reaction,

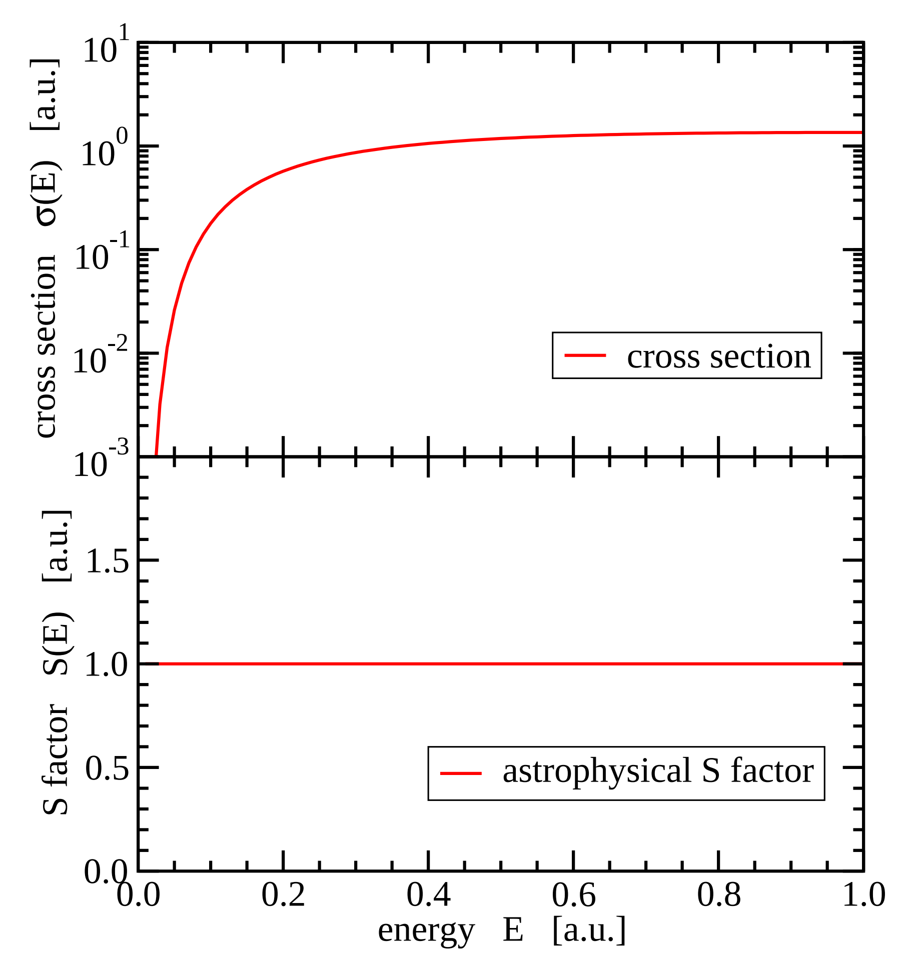
here: non-resonant charged-particle reactions

Cross Section and Astrophysical S Factor

nuclear reaction $b + c \rightarrow a + \ldots$
with charged particles b, c

$\Rightarrow$ Coulomb barrier

- strong energy dependence of (non-resonant) cross section $\sigma(E)$
- introduce astrophysical S factor
  $$S(E) = \sigma(E) E \exp(2\pi\eta)$$
  with Sommerfeld parameter
  $$\eta = Z_b Z_c e^2/(\hbar v)$$
  and relative velocity $v = \sqrt{2E/\mu}$
- extrapolation of measured cross sections to low energies



Reaction Rate

astrophysical environment: nuclei in hot plasma

⇒ temperature-dependent distribution of velocities

- Maxwellian-averaged reaction rate

$$r_{bc} = \frac{\varrho_b \varrho_c}{1+\delta_{bc}} \langle \sigma v \rangle$$

with densities ϱ_b, ϱ_c and

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu}} \int_0^\infty \sigma(E)\, E\, e^{-\frac{E}{k_B T}} \frac{dE}{(k_B T)^{3/2}}$$

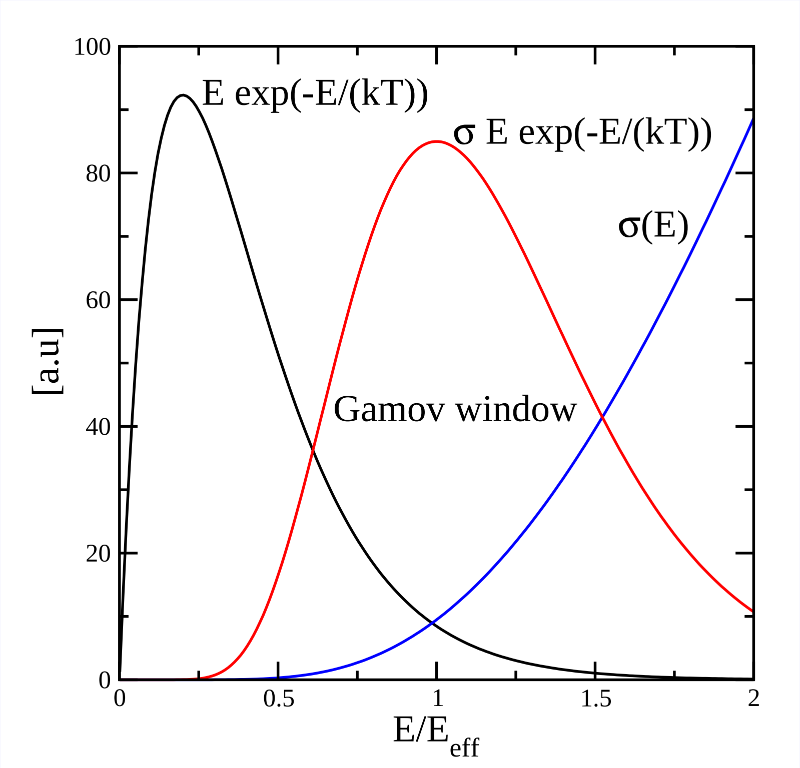
⇒ cross sections needed in Gamov window with effective energy

$$E_{\text{eff}} = 0.1220\, \mu^{1/3} (Z_b Z_c T_9)^{2/3} \text{ MeV}$$

and width

$$\Delta E = 0.2368\, \mu^{1/6} (Z_b Z_c)^{1/3} T_9^{5/6} \text{ MeV}$$

with temperature T_9 in 10^9 K
and reduced mass μ in amu



reaction	E_{eff} [keV]	$\sigma(E_{\text{eff}})$ [pb]
^{3}He(^{3}He,2p)^{4}He	22.0	1.5
^{7}Be(p,γ)^{8}B	18.4	1.5×10^{-3}
^{3}He(α,γ)^{7}Be	23.0	3.0×10^{-5}
^{14}N(p,γ)^{15}O	27.2	2.2×10^{-7}

for $T = 15.5 \times 10^6$ K (center of the sun)

Electron Screening

electron screening in direct experiments

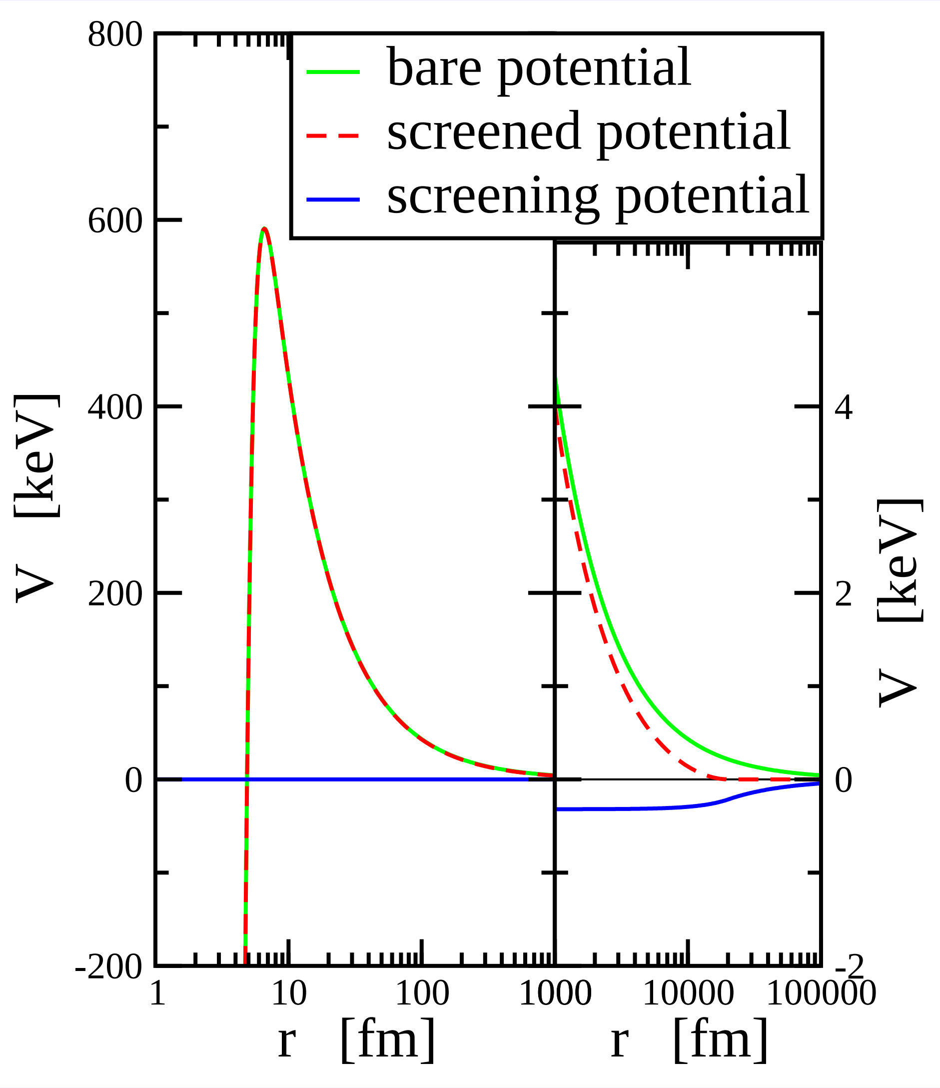
- reduction of Coulomb barrier by electron cloud of target nuclei
- enhanced cross section at low energies

$$\sigma_{\exp}(E) = \sigma_{\text{bare}}(E) f(E)$$

  with $f(E) = \exp(\pi\eta U_e/E)$ and

  electron screening potential energy U_e

- discrepancy between experimental observation and theoretical models, explanation?
- independent experimental information needed
- stellar conditions: electron screening in plasma



Indirect Methods

general characteristics:

- two-body reaction is replaced by three-body reaction at "high" energies
- relation of cross sections is found with the help of nuclear reaction theory

Coulomb dissociation

- study inverse of radiative capture reaction
  $b(c,\gamma)a \;\Leftrightarrow\; a(\gamma,c)b$
- use Coulomb field of target nucleus X as source of photons
  $a(\gamma,c)b \;\Leftrightarrow\; X(a,bc)X$

$\Downarrow$

absolute S factors as a function of energy

ANC method

- extract asymptotic normalization coefficient of ground state wave function of nucleus c from transfer reactions
- calculate matrix elements for radiative capture reaction $b(c,\gamma)a$

$\Downarrow$

S factor at zero energy

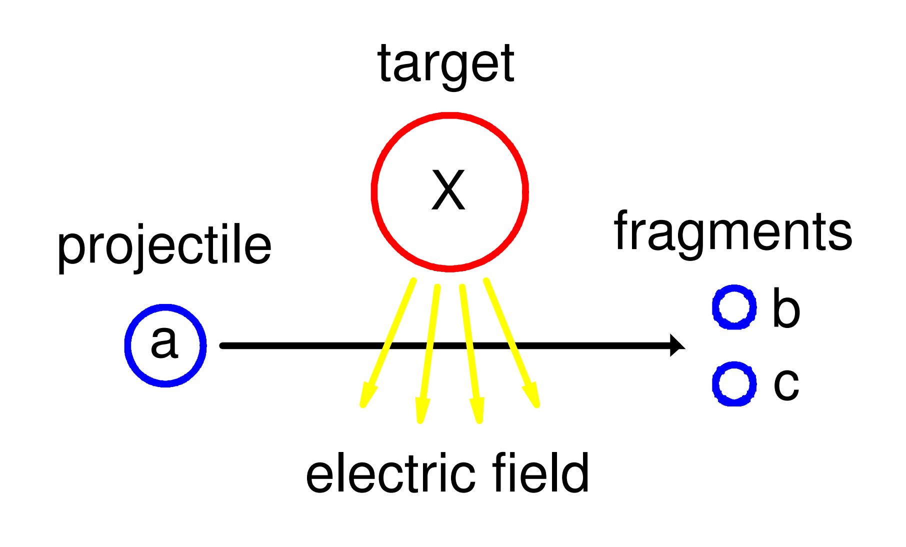
Trojan-horse method

- study three-body reaction
  $A + a \to C + c + b$
  with Trojan horse
  $a = b + x$
  and spectator b
- extract cross section of two-body reaction
  $A + x \to C + c$

$\Downarrow$

energy dependence of S factor

Idea of the Coulomb Dissociation Method



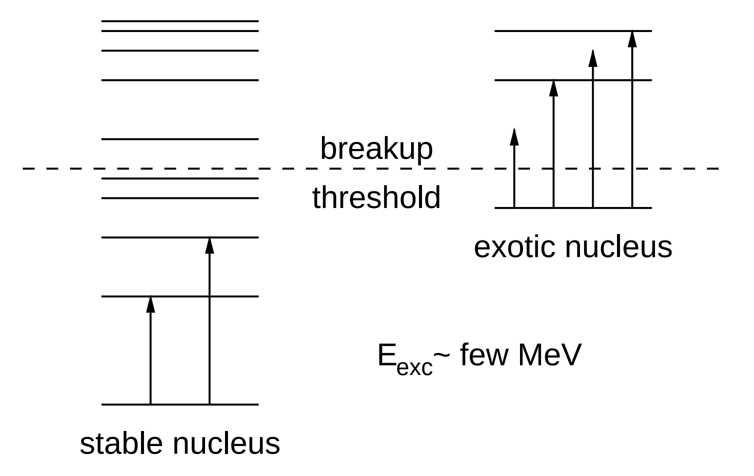
radiative capture	$c(b,\gamma)a$
detailed balance	$\Updownarrow$
photo dissociation	$a(\gamma,b)c$
equivalent photons in Coulomb field of target X	$\Updownarrow$
Coulomb dissociation	$X(a,bc)X$

(G. Baur, H. Rebel, C. Bertulani, Nucl. Phys. A 458 (1986) 188)

correspondence

(Fermi 1924, Weizsäcker-Williams 1932)

time-dependent electromagnetic field
of highly-charged nucleus X
during scattering of projectile a
$\Updownarrow$
spectrum of (virtual, equivalent) photons



only ground state transitions !

Theory of Coulomb Dissociation I

Coulomb dissociation reaction: $a + X \to b + c + X$

with three-body final state in the continuum

$\Rightarrow$ only approximate theoretical treatment

• **semiclassical methods**

– classical description of projectile-target relative motion
(valid for heavy targets if $\eta_{aX} = Z_a Z_X e^2/(\hbar v) \gg 1$ with beam velocity v)

– time-dependent perturbation $V(t)$ of projectile system

– time-dependent perturbation theory

$\Rightarrow$ excitation amplitude a_{fi}

• **quantal methods**

– valid for all projectile/target combinations and all beam energies

– time-independent scattering theory

$\Rightarrow$ T-matrix element T_{fi}

Theory of Coulomb Dissociation II

- **first-order theory** well known
  K. Alder et al., Rev. Mod. Phys. 28 (1956) 432

- **relativistic corrections** can be considered
  A. Winther et al., Nucl. Phys. A 319 (1979) 518

- **higher-order effects** $\Leftrightarrow$ multi-photon exchange:
  change of fragment momenta in Coulomb field of target
  after breakup ("post-acceleration")
  – higher-order perturbation theory
  – sudden approximation
  – dynamical calculations
    (solving the time-dependent Schrödinger equation)

- **nuclear contribution to breakup** can be considered
  (small contribution for forward-angle scattering/large impact parameters)
  $\Rightarrow$ selection of kinematical conditions in experiments

Theory of Coulomb Dissociation III

first-order semiclassical approximation for reaction $X(a,bc)X$

- classical description of projectile-target relative motion $\Rightarrow$ $\vec{R}_X(t)$

  valid for heavy targets if $\eta_{aX} = Z_a Z_X e^2/(\hbar v_{aX}) \gg 1$ with beam velocity v

- time-dependent perturbation of projectile system (magnetic interaction neglected)

$$V(t) = \frac{Z_b Z_X e^2}{|\vec{r}_b - \vec{R}_X(t)|} + \frac{Z_c Z_X e^2}{|\vec{r}_c - \vec{R}_X(t)|} - \frac{Z_a Z_X e^2}{|\vec{r}_a - \vec{R}_X(t)|}$$

- excitation amplitude in first-order time-dependent perturbation theory

$$a_{fi} = \frac{1}{i\hbar} \int dt \ \exp(i\omega t) \langle f | V(t) | i \rangle \qquad \begin{array}{l} |i\rangle = |J_a M_a\rangle \\ |f\rangle = |\vec{k}_{bc} J_b M_b J_c M_c\rangle \end{array}$$

  with excitation energy $\hbar\omega = E_f - E_i = E_\gamma = E_{bc} + S_{bc} = \frac{\hbar^2 k_{bc}^2}{2\mu_{bc}} + S_{bc}$

Theory of Coulomb Dissociation IV

- excitation probability $$P_{fi} = \frac{1}{2J_a+1} \sum_{M_a} \sum_{M_b M_c} |a_{fi}|^2 \frac{\mu_{bc} k_{bc}}{(2\pi)^3 \hbar^2}$$

- Coulomb breakup cross section $$\frac{d^3\sigma}{dE_{bc} d\Omega_{bc} d\Omega_{aX}} = \frac{d\sigma_R}{d\Omega_{aX}} P_{fi}$$

  with Rutherford cross section $d\sigma_R/d\Omega_{aX}$ for elastic aX scattering

- angular integration over relative momentum between fragments, multipole expansion ($\pi = E, M$, $\lambda = 1, 2, \ldots$)

$$\Rightarrow \quad \frac{d^2\sigma}{dE_{bc} d\Omega_{aX}} = \frac{1}{E_\gamma} \sum_{\pi\lambda} \sigma_{\pi\lambda}(a+\gamma \to b+c) \frac{dn_{\pi\lambda}}{d\Omega_{aX}}$$

with photo absorption cross section $\sigma_{\pi\lambda}(a+\gamma \to b+c)$
and virtual photon number $\frac{dn_{\pi\lambda}}{d\Omega_{aX}}$ (depending on kinematics)

E2 enhancement $\frac{dn_{E2}}{d\Omega_{aX}} \Big/ \frac{dn_{E1}}{d\Omega_{aX}} \approx \frac{4\hbar^2 c^2}{E_\gamma^2 b^2}$ M1 suppression $\frac{dn_{M1}}{d\Omega_{aX}} \Big/ \frac{dn_{E1}}{d\Omega_{aX}} \approx \frac{v^2}{c^2}$

Relation of Cross Sections

- Coulomb breakup cross section

$$\frac{d^2\sigma}{dE_{bc}d\Omega_{aX}}(a+X\to b+c+X)=\frac{1}{E_\gamma}\sum_{\pi\lambda}\sigma_{\pi\lambda}(a+\gamma\to b+c)\frac{dn_{\pi\lambda}}{d\Omega_{aX}}$$

with photo absorption cross section $\sigma_{\pi\lambda}(a+\gamma\to b+c)$

- theorem of detailed balance

$$\sigma_{\pi\lambda}(b+c\to a+\gamma)=\frac{2(2J_a+1)}{(2J_b+1)(2J_c+1)}\frac{k_\gamma^2}{k_{bc}^2}\sigma_{\pi\lambda}(a+\gamma\to b+c)$$

with radiative capture cross section $\sigma_{\pi\lambda}(b+c\to a+\gamma)$

- phase space factor $\dfrac{k_\gamma^2}{k_{bc}^2}=\dfrac{(E_{bc}+Q)^2}{2\mu_{bc}c^2E_{bc}}\ll 1$ for (not too) small E_{bc}

$\Rightarrow$ cross section for photo absorption $\gg$ cross section for radiative capture
$\Rightarrow$ large Coulomb dissociation cross section

Characteristic Parameters

- **adiabaticity parameter**

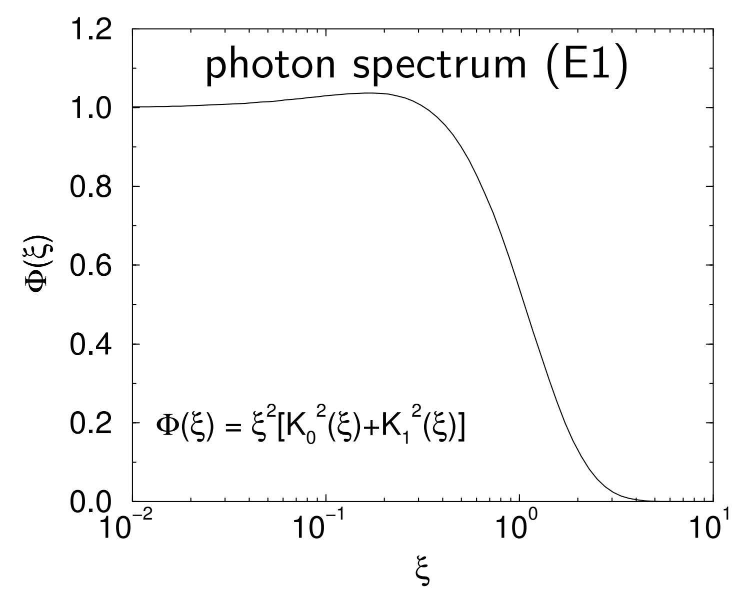
$$\xi = \frac{\omega b}{\gamma v}$$

$\hbar\omega$	excitation energy
b	impact parameter
v	projectile velocity

$\xi = 0$: sudden excitation

$\xi \gg 1$: adiabatic excitation

$\xi \approx 1 \Rightarrow E_{\mathrm{exc}}^{\mathrm{max}} \approx \gamma v \hbar / b$



- **strength parameter**

$$\chi = \frac{Z_X e \langle f || \mathcal{M}(\pi\lambda) || i \rangle}{\hbar v b^{\lambda}}$$

with target charge number Z_X

and multipole operator $\mathcal{M}(\pi\lambda)$

χ small $\Rightarrow$ first order perturbation theory sufficient

χ large $\Rightarrow$ higher order effects

- **structure** of nucleus a:

nucleon $(b = n, p)$ + core (c)

$$\Rightarrow \langle f || \mathcal{M}(E\lambda) || i \rangle \propto Z_{\mathrm{eff}}^{(\lambda)} e$$

with effective charge number

$$Z_{\mathrm{eff}}^{(\lambda)} = Z_b \left(\frac{m_c}{m_b + m_c} \right)^{\lambda} + Z_c \left(- \frac{m_b}{m_b + m_c} \right)^{\lambda}$$

$\Rightarrow$ p+core: E1-E2 interference

$\Rightarrow$ n+core: E2 suppression $\propto A^{-1}$

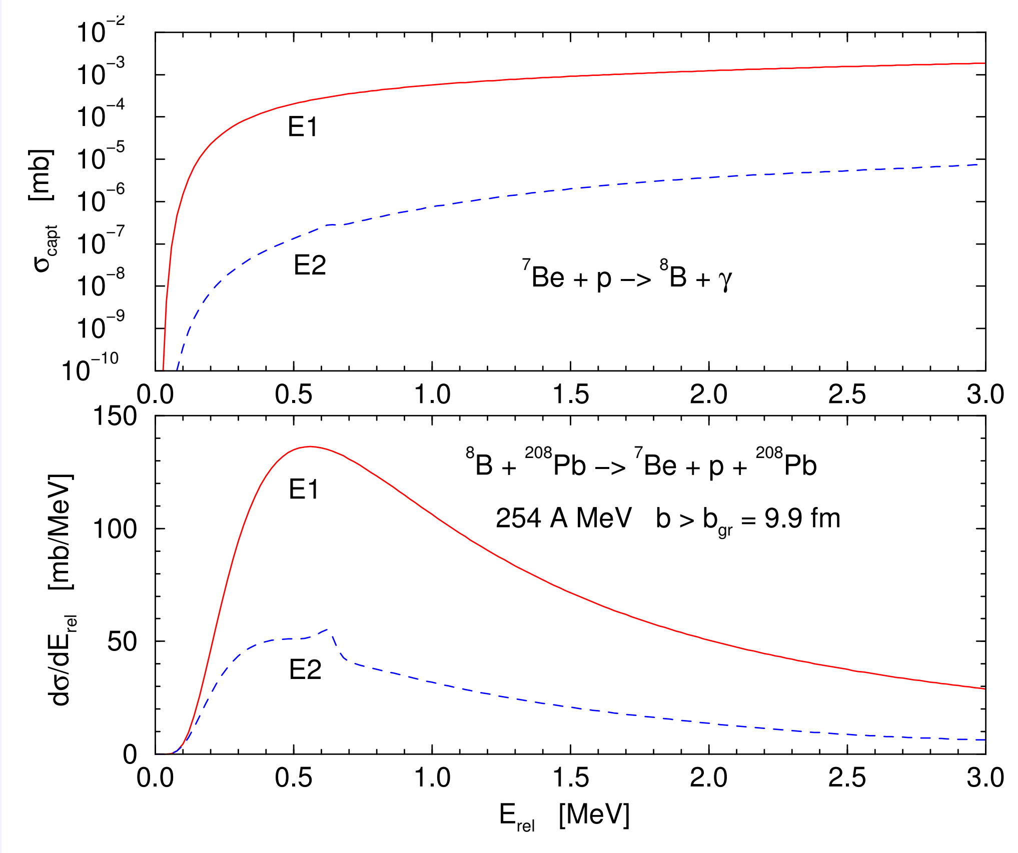
Example: ^{7}Be(p,γ)^{8}B

nuclear astrophysics

- small branch of pp chain:
  $\ldots\ ^7\text{Be}(p,\gamma)^8\text{B}(e^+\nu_e)^8\text{Be} \rightarrow 2^4\text{He}$
  $\Rightarrow$ source of high-energy neutrinos
- $E_{\text{eff}} \approx 20$ keV for $^7\text{Be}(p,\gamma)^8\text{B}$ in sun
- capture cross section at low energies dominated by non-resonant E1 transition to p-wave ground state with 137 keV binding energy

model calculation

- single-particle model ($p+^7$Be)
- compare radiative capture cross section with Coulomb dissociation cross section for conditions of GSI experiment



(M1 contribution of sharp resonance at 632 keV not shown)

Example: ^{7}Be(p,γ)^{8}B

recent direct experiments (^{7}Be target)

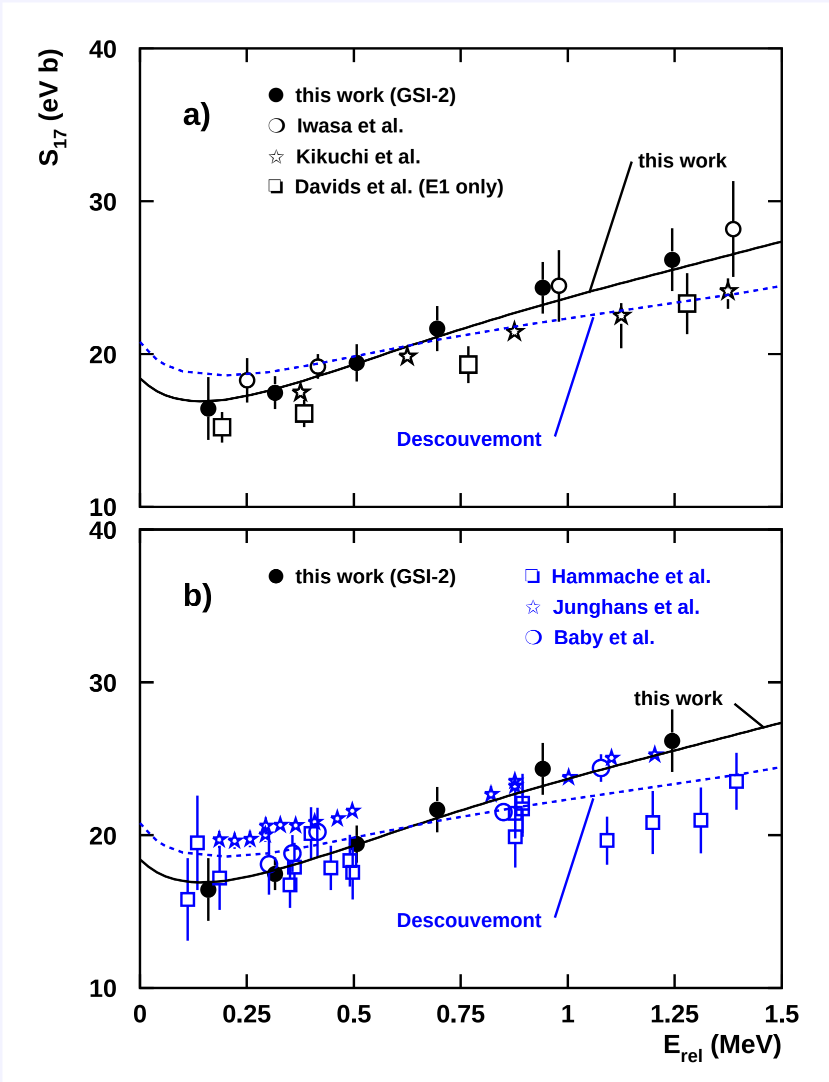
- Orsay: F. Hammache et al.,
  Phys. Rev. Lett. 80 (1998) 928; 86 (2001) 3985
- University of Washington, Seattle: A.R. Junghans et al.,
  Phys. Rev. Lett. 88 (2002) 041101; Phys. Rev. C 68 (2003) 065803
- Weizmann Institute, Rehovot: L.T. Baby et al.,
  Phys. Rev. C 67 (2003) 065805

Coulomb breakup experiments (Pb target)

- RIKEN: 46.5 A MeV/51.2 A MeV
  T. Motobayashi et al., Phys. Rev. Lett. 73 (1994) 2680
  T. Kikuchi et al., Eur. Phys. J. A3 (1998) 213
- GSI: 254 A MeV
  N. Iwasa et al., Phys. Rev. Lett. 83 (1999) 2910
  F. Schümann et al., Phys. Rev. Lett. 90 (2003) 232501
- MSU: 83 A MeV
  B. Davids et al., Phys. Rev. C 63 (2001) 065806

theoretical models (extrapolation to E=0 MeV)

- P. Descouvement, D. Baye, Nucl. Phys. A 567 (1994) 341
- F. Schümann et al., Phys. Rev. Lett. 90 (2003) 232501

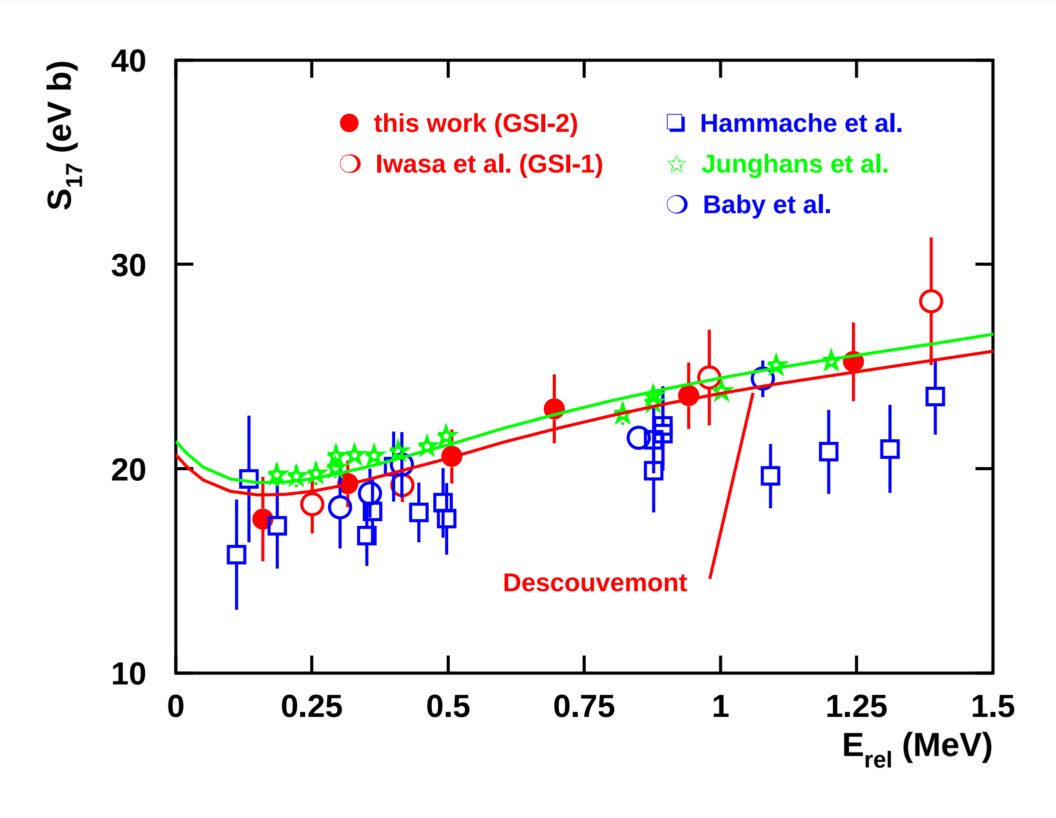


(from F. Schümann et al., PRL 90 (2003) 232501)

Example: ^{7}Be(p,γ)^{8}B

new analysis of GSI-2 experiment

- improved efficiency
- only E1 contribution in first-order calculation
- no indication of E2 or higher-order effects from angular distributions



theoretical model

(for extrapolation to E=0 MeV)

- new calculation in cluster model with Minnesota potential
  (P. Descouvement, PRC 70 (2004) 065802)

$\Rightarrow S_{17}(0) = (20.6 \pm 0.8 \pm 1.2)$ eV b
consistent with direct experiments

Reduced Transition Probability

- **photo dissociation cross section**

$$\sigma_{\pi\lambda}(a+\gamma \to b+c) = \frac{\lambda+1}{\lambda}\frac{(2\pi)^3}{[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(\pi\lambda)}{dE}$$

with photon energy $E_\gamma = S_{bc} + E$

- **reduced transition probability**

$$\frac{dB}{dE}(\pi\lambda) = \frac{2J_f+1}{2J_i+1}\sum_{j_f l_f}\left|\sum_{j_i l_i j_c}\langle k J_f j_f l_f j_c || \mathcal{M}(\pi\lambda) || J_i j_i l_i j_c\rangle\right|^2 \frac{\mu k}{(2\pi)^3\hbar^2}$$

electric multipole operator $\mathcal{M}(E\lambda\mu) = Z_{\text{eff}}^{(\lambda)} e r^\lambda Y_{\lambda\mu}(\hat{r})$ in long-wavelength limit

with effective charge $Z_{\text{eff}}^{(\lambda)} = Z_b\left(\frac{m_c}{m_b+m_c}\right)^\lambda + Z_c\left(-\frac{m_b}{m_b+m_c}\right)^\lambda$

for nucleus a with nucleon b + core c structure

- Eλ transitions at low relative energies
  $\Rightarrow$ matrix elements determined by asymptotic of wave functions ($r > R$)

Wave Functions

- **bound state** wave function $\Phi_i(\vec{r})$

$$\langle \vec{r} | J_i M_i j_i l_i j_c \rangle = \frac{1}{r} \sum_{m_i m_c} (j_i \; m_i \; j_c \; m_c | J_i \; M_i) f^{j_c}_{J_i j_i l_i}(r) \mathcal{Y}^{l_i}_{j_i m_i}(\hat{r}) \phi_{j_c m_c}$$

$$f^{j_c}_{J_i j_i l_i}(r) \to C^{j_c}_{J_i j_i l_i} W_{-\eta_i, l_i+1/2}(2qr) \quad \text{for} \quad r \to \infty$$

with asymptotic normalization coefficient (ANC) $C^{j_c}_{J_i j_i l_i}$,

Whittaker function $W_{-\eta_i, l_i+1/2}$ for

nucleon separation energy $S_{bc} = \frac{\hbar^2 q^2}{2\mu}$, Sommerfeld parameter $\eta_i = \frac{Z_b Z_c e^2 \mu}{\hbar^2 q}$

- **continuum** wave function $\Phi_f(\vec{r})$ for relative energy $E = \frac{\hbar^2 k^2}{2\mu}$

$$\langle \vec{r} | \vec{k} J_f M_f j_f l_f j_c \rangle = \frac{4\pi}{kr} \sum_{m_f m_c} (j_f \; m_f \; j_c \; m_c | J_f \; M_f) g^{j_c}_{J_f j_f l_f}(r) i^{l_f} Y^*_{l_f m_f}(\hat{k}) \mathcal{Y}^{l_f}_{j_f m_f}(\hat{r}) \phi_{j_c m_c}$$

$$g^{j_c}_{J_f j_f l_f}(r) \to \exp\left[i(\sigma_{l_f} + \delta^{j_c}_{J_f j_f l_f}) \right] \left[\cos(\delta^{j_c}_{J_f j_f l_f}) F_{l_f}(\eta_f; kr) + \sin(\delta^{j_c}_{J_f j_f l_f}) G_{l_f}(\eta_f; kr) \right]$$

with nuclear phase shift $\delta^{j_c}_{J_f j_f l_f}$, Coulomb phase shift σ_{l_f},

Coulomb wave functions F_{l_f}, G_{l_f}, Sommerfeld parameter $\eta_f = \eta_i q / k$

Reduced Transition Probability and ANC

- reduced radial integrals

$$\mathcal{I}_{l_i}^{l_f}(\lambda) = q^{\lambda+1} \int_R^\infty dr\, r^\lambda \left[\cos(\delta_{l_f}) F_{l_f}(kr) + \sin(\delta_{l_f}) G_{l_f}(kr)\right] W_{-\eta_i, l_i+1/2}(2qr)$$

- shape function $\mathcal{S}_{l_i}^{l_f}(\lambda) = \frac{1}{x}\left|\mathcal{I}_{l_i}^{l_f}(\lambda)\right|^2$ depends only on phase shift δ_{l_f} and

  dimensionless parameters $\gamma = qR$, $x = k/q = \sqrt{E/S_{bc}}$, $\eta_i = \sqrt{E_G/S_{bc}}$

  with Gamov energy $E_G = (Z_b Z_c e^2)^2 \mu_{bc}/(2\hbar^2)$

- reduced transition probability for $E\lambda$ transition $l_i \to l_f$:

$$\Rightarrow \frac{dB(E\lambda)}{dE} = \left[Z_{\text{eff}}^{(\lambda)} e\right]^2 \frac{2\mu_{bc}}{\pi\hbar^2} D_s \frac{|C_{l_i}|^2}{q^{2\lambda+3}} \mathcal{S}_{l_i}^{l_f}(\lambda)$$

  with spin factor D_s

- at low energies: effective-range expansion for phase shift

  $\Rightarrow$ scattering length a_{l_f} and scaling laws

(S. Typel and G. Baur, preprint nucl-th/0411069, accepted for publication in Nucl. Phys. A)

Coulomb Dissociation of ^{11}Be

- $E1$ transition from s-wave halo ground state ($S_n = 504$ keV, $\gamma = 0.41$, $R = 2.78$ fm) to p-wave continuum states with $j = 3/2, 1/2$
- effective-range expansion for phase shifts

$$\tan \delta_l^j = -(c_l^j x \gamma)^{2l+1}$$

with reduced scattering length c_l^j

- expansion of shape function for small γ

$$\mathcal{S}_0^1(1) = \frac{4x^3}{(1+x^2)^4}\left[1 - c_1^3(1+3x^2)\gamma^3 + \ldots\right]$$

- fit to experimental data from Coulomb breakup of ^{11}Be at 520 A·MeV on Pb

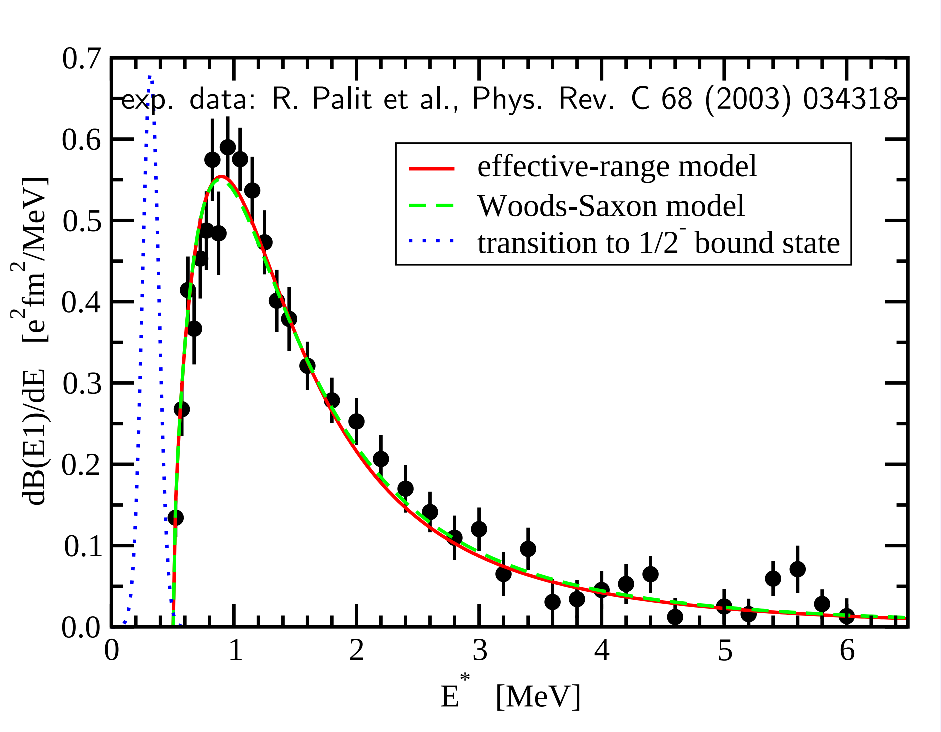
  $\Rightarrow$ ANC $C_0 = 0.724(8)$ fm$^{-1/2}$

  $\Rightarrow$ spectroscopic factor $C^2S = 0.704(15)$

  $\Rightarrow$ reduced scattering lengths

$$c_1^{3/2} = -0.41(86, -20)$$
$$c_1^{1/2} = 2.77(13, -14)$$

(S. Typel and G. Baur, Phys. Rev. Lett. 93 (2004) 142502)



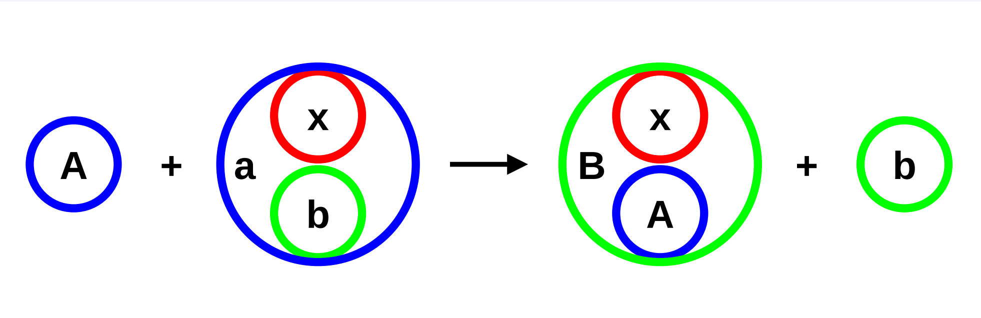
- $c_1^{1/2}$ unnaturally large

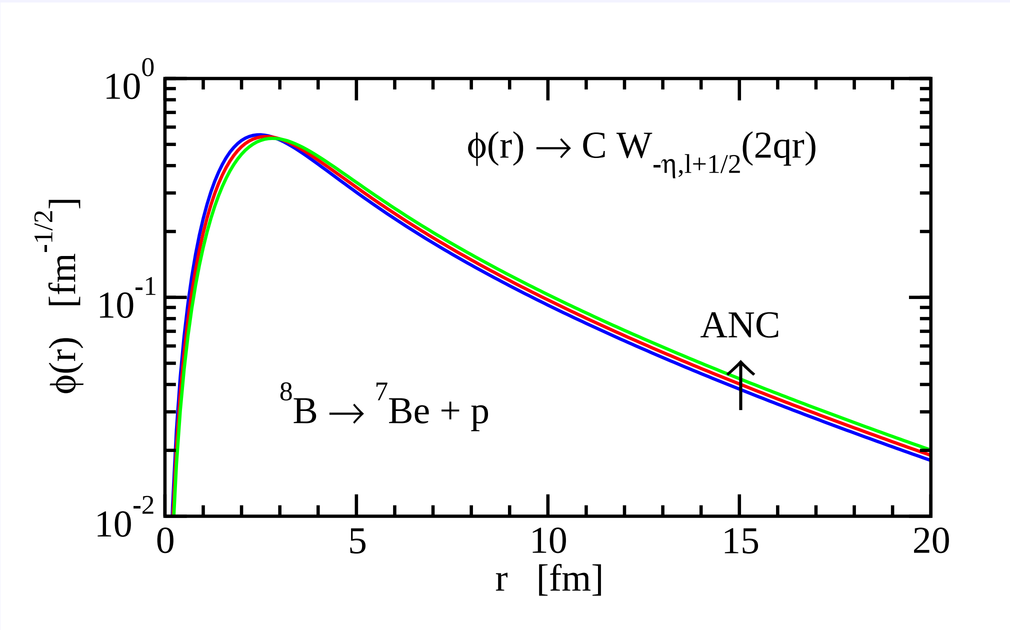
  $\Leftrightarrow$ existence of bound $1/2^-$ state 320 keV above ground state

  $\Rightarrow$ reduced $E1$ strength in continuum
- non-energy-weighted sum rule

$$B(E1, l_i) = \left[Z_{\text{eff}}^{(1)} e\right]^2 \frac{3}{4\pi}\langle r^2\rangle_{l_i}$$

Idea of the ANC Method





extract asymptotic normalization coefficient (**ANC**)

for breakup of nucleus B into $A + x$
or nucleus a into $b + x$

from cross section of transfer reaction

$$A + a \rightarrow B + b$$

with $a = b + x$ and $B = A + x$

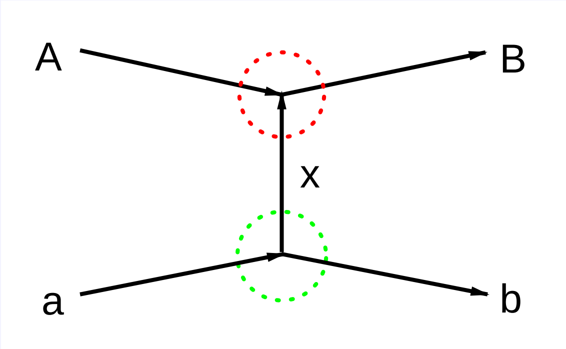
$$\Downarrow$$

calculate astrophysical S factor $S(E)$

in the limit $E \rightarrow 0$

(H.M. Xu et al., Phys. Rev. Lett. 73 (1994) 2027)

Theory of Transfer Reactions I



- transfer reaction $A + a \to B + b$

  with $a = b + x$ and $B = A + x$
- cross section

$$d\sigma = \frac{2\pi}{\hbar} \frac{\mu_{Aa}}{p_{Aa}} \frac{d^3 k_{Bb}}{(2\pi)^3} \left|T_{fi}\right|^2 \delta(E_B + E_b - E_A - E_a - Q)$$

  with general T-matrix element in post formulation

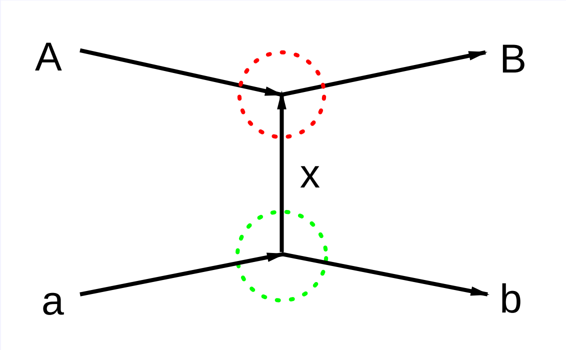
$$T_{fi} = \langle \exp(i\vec{k}_{Bb} \cdot \vec{r}_{Bb}) \phi_B \phi_b | V_{Bb} | \Psi^{(+)}_{Aa} \rangle$$

  and exact initial state wave function $\Psi^{(+)}_{Aa}$
- introduce optical potentials U_{ij} and distorted waves $\chi^{(\pm)}_{ij}$ $(ij = Aa, Bb)$

  with $(T_{ij} + U_{ij})\chi^{(\pm)}_{ij} = E_{ij}\chi^{(\pm)}_{ij}$
- apply Gell-Mann–Goldberger relation (Phys. Rev. 91 (1953) 398)

$$\Rightarrow T_{fi} = \langle \chi^{(-)}_{Bb} \phi_B \phi_b | V_{Bb} - U_{Bb} | \Psi^{(+)}_{Aa} \rangle$$

Theory of Transfer Reactions II



- distorted-wave Born approximation (DWBA)

$$\Psi^{(+)}_{Aa} \approx \chi^{(+)}_{Aa} \phi_A \phi_a$$

- approximation for potential

$$V_{Bb} - U_{Bb} = V_{Ab} + V_{xb} - U_{Bb} \approx V_{xb}$$

- T-matrix element in post-form DWBA for transfer reaction $A + a \to B + b$

$$T_{fi} = \langle \chi^{(-)}_{Bb}(r\vec{_{Bb}}) \phi_B(\vec{r}_{Ax}) \phi_b | V_{xb}(\vec{r}_{xb}) | \chi^{(+)}_{Aa}(\vec{r}_{Aa}) \phi_A \phi_a(\vec{r}_{xb}) \rangle$$

  with relative coordinates $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ (internal coordinates suppressed)

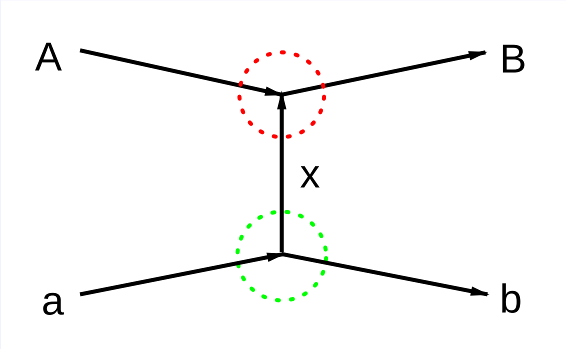
- define overlap functions ("wave functions of transferred particle")

$$\Phi^B_{Ax}(\vec{r}_{Ax}) = \langle \phi_B(\vec{r}_{Ax}) | \phi_A \rangle^*, \Phi^a_{bx}(\vec{r}_{xb}) = \langle \phi_b | \phi_a(\vec{r}_{xb}) \rangle$$

  ($\Rightarrow$ spectroscopic factors $\mathcal{S}^B_{Ax} = \langle \Phi^B_{Ax} | \Phi^B_{Ax} \rangle$, $\mathcal{S}^a_{bx} = \langle \Phi^a_{bx} | \Phi^a_{bx} \rangle$)

$$\Rightarrow \quad T_{fi} = \langle \chi^{(-)}_{Bb} \Phi^B_{Ax} | V_{xb} | \chi^{(+)}_{Aa} \Phi^a_{bx} \rangle$$

Theory of Transfer Reactions III



- T-matrix element in post-form DWBA

$$T_{fi} = \langle \chi^{(-)}_{Bb} \Phi^{B}_{Ax} | V_{xb} | \chi^{(+)}_{Aa} \Phi^{a}_{bx} \rangle$$

- asymptotics of Φ^{B}_{Ax} outside range of nuclear potential

$$\Phi^{B}_{Ax}(\vec{r}_{Ax}) \to \sum_{lm} \frac{C^{Ax}_{l}}{r_{Ax}} W_{-\eta_{Ax},l+1/2}(2q_{Ax}r_{Ax}) Y_{lm}(\hat{r}_{Ax}) \phi_{x}$$ (similar for Φ^{a}_{bx})

with asymptotic normalization coefficient (ANC) C^{Ax}_{l},
Whittaker function $W_{-\eta_{Ax},l+1/2}$,
and separation energy $S_{Ax} = \hbar^2 q^2_{Ax}/(2\mu_{Ax})$ of B into $A + x$

- strong absorption by optical potentials for small radii
$\Rightarrow$ main contribution to T_{fi} from radii outside optical potentials for small S_{Ax}, S_{bx}
$\Rightarrow$ cross section $d\sigma \propto |T_{fi}|^2 \propto \left| \sum_{ll'} C^{Ax}_{l} C^{bx}_{l'} \right|^2$ $\Rightarrow$ $\left| C^{bx}_{l'} \right|^2$

$\Rightarrow$ $\frac{dB}{dE}(E\lambda, a + \gamma \to b + x)$ $\Rightarrow$ ANC method

Application of the ANC method

$S_{17}(0)$ of radiative capture reaction ^{7}Be$(p,\gamma)^{8}$B

experiments (Texas A&M University)
extraction of ANC from

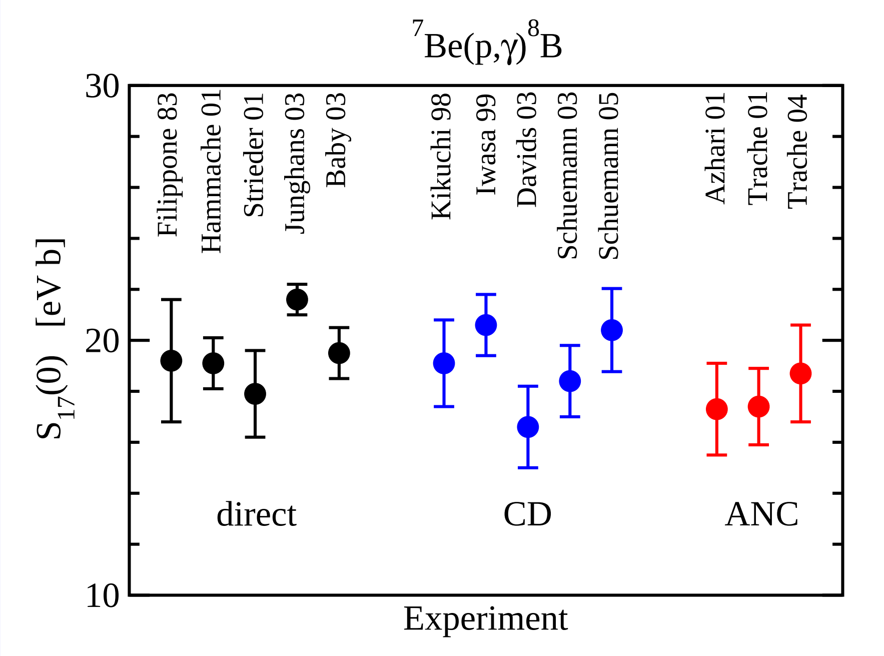
- proton transfer reactions
  ^{10}B(^{7}Be,^{8}B)^{9}Be, ^{14}N(^{7}Be,^{8}B)^{13}C
  with 85 MeV ^{7}Be beam

  A. Azhari et al., Phys. Rev. C 63 (2001) 055803
- breakup ^{8}B $\longrightarrow$ ^{7}Be+p
  on C, Si, Sn, and Pb targets with
  beam energies from 30 to 1000 A MeV

  L. Trache et al., Phys. Rev. Lett. 87 (2001) 271102,
  nucl-th/0312101, Phys. Rev. C 69 (2004) 032802
- neutron transfer reaction
  ^{13}C(^{7}Li,^{8}Li)^{12}C with 63 MeV ^{7}Li beam
  and charge symmetry

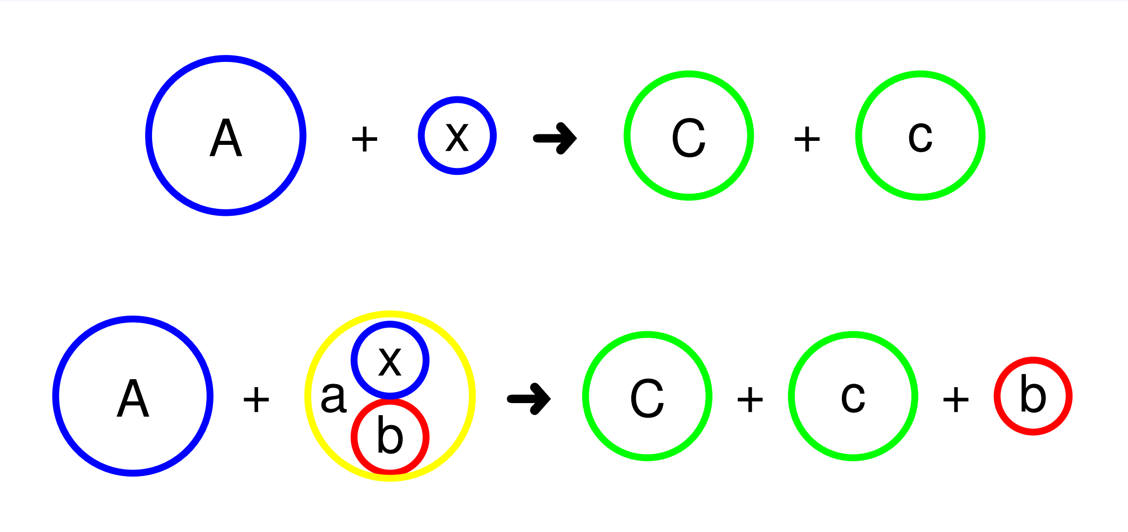
  L. Trache et al., Phys. Rev. C 67 (2003) 062801(R)

comparison to other methods



dependence of extrapolation to $E = 0$ MeV
on ^{7}Be-p nuclear potential

Idea of the Trojan-Horse Method



replace **two-body reaction**

$$A + x \to C + c$$

by **three-body reaction**

$$A + a \to C + c + b$$

with **Trojan horse** $a = b + x$

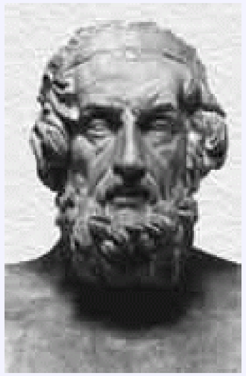
and **spectator** b

- small momentum transfer to spectator
  $\Rightarrow$ quasi-free scattering dominates
- large relative energy of system $A + a$
  $\Rightarrow$ no suppression of cross section
  $\Rightarrow$ no electron screening
- small relative energies of system $A + x$ accessible
  $\Rightarrow$ application to nuclear astrophysics

(G. Baur, Phys. Lett. B 178 (1986) 35)

$\ldots\ \kappa\epsilon\kappa\alpha\lambda\upsilon\mu\mu\epsilon\nu o\iota\ \iota\pi\pi o.$

Homer, Odyssey VIII, 503



Theory of the Trojan-Horse Method I

- find relation: three-body reaction ⇔ two-body reaction with Trojan horse $a = b + x$

$$A + a \to \underbrace{C + c}_{B} + b \qquad A + x \to \underbrace{C + c}_{B}$$

- triple differential cross section with T matrix element T_{fi}

$$\frac{d^3\sigma}{dE_{Cc}d\Omega_{Cc}d\Omega_{Bb}} = \frac{\mu_{Aa}\mu_{Bb}\mu_{Cc}}{(2\pi)^5\hbar^6}\frac{k_{Bb}k_{Cc}}{k_{Aa}}\frac{1}{2J_i+1}\sum_{M_i,M_f}|T_{fi}|^2$$

- post-form distorted wave Born approximation (DWBA, cf. transfer reactions)

$$T_{fi} \approx \langle \chi^{(-)}_{Bb}\phi_B\phi_b|V_{xb}|\chi^{(+)}_{Aa}\phi_A\phi_a\rangle$$

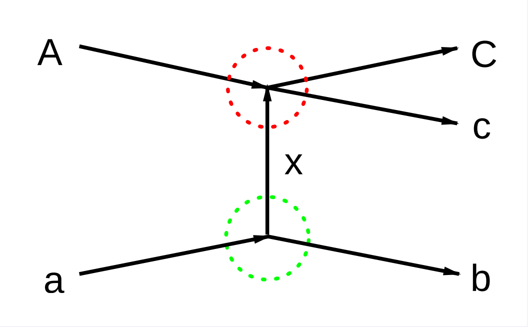
— distorted waves $\chi^{(+)}_{Aa}$, $\chi^{(-)}_{Bb}$

— ground state wave functions ϕ_A, ϕ_a, ϕ_b of nuclei A, a, b

— complete scattering state wave function $\phi_B = \Psi^{(-)}_{Cc}$

(contains information on two-body cross section)

— potential V_{xb} between x and b in Trojan horse a



(S. Typel, H.H. Wolter, Few-Body Systems 29 (2000) 75, S. Typel, G. Baur, Ann. Phys. 305 (2003) 228)

Theory of the Trojan-Horse Method II

- essential surface approximation: replace $\Psi_{Cc}^{(-)}$ by asymptotic form for $r > R$

  (strong absorption for $r < R$ due to optical potentials)
  $\Rightarrow$ THM approximation of T matrix element

$$T_{fi}^{TH} = \frac{1}{2ik_{Cc}} \sqrt{\frac{v_{Cc}}{v_{Ax}}} \sum_{l} (2l+1) \left[S_{AxCc}^{l} U_{l}^{(+)} - \delta_{(Ax)(Cc)} U_{l}^{(-)} \right]$$

  — S matrix elements S_{AxCc}^{l} of two-body reaction $C + c \rightarrow A + x \Rightarrow$ cross section

  — T_{fi}^{TH} has form of two-body scattering amplitude except factors $U_{l}^{(\pm)}(\vec{k}_{Bb}\vec{k}_{Cc}\vec{k}_{Aa})$: reduced DWBA matrix elements with particular momentum dependence

  $\Rightarrow$ suppression of cross section from S-matrix element $S_{AxCc}^{l} \propto \exp(-\pi\eta_{Ax})$ is cancelled!

- further approximation for simple physical interpretation:

  use plane waves instead of distorted waves $\chi_{Aa}^{(+)}, \chi_{Bb}^{(-)}$

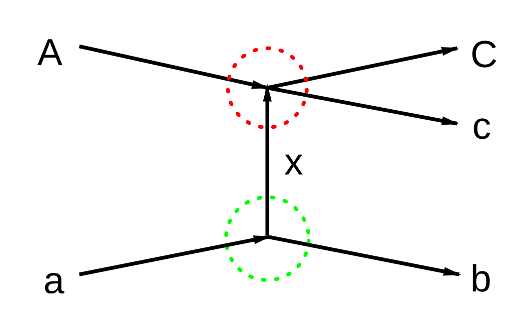
  $\Rightarrow$ factorization of three-body cross section

  $\Rightarrow$ cf. plane-wave impulse approximation (PWIA)

Modified Plane-Wave Approximation

- cross section of three-body reaction in modified plane-wave approximation

$$\frac{d^3\sigma}{dE_{Cc}d\Omega_{Cc}d\Omega_{Bb}} = KF \left| W(\vec{Q}_{Bb}) \right|^2 \frac{d\sigma^{TH}}{d\Omega}$$



∘ $KF \propto k_{Ax}^{-3}$ kinematic factor

∘ $W(\vec{Q}_{Bb})$ momentum amplitude

$=$ Fourier transform of $V_{xb}\phi_a$
with $\vec{Q}_{Bb} \hat{=}$ recoil momentum of spectator b

∘ $\frac{d\sigma^{TH}}{d\Omega} = P \frac{d\sigma}{d\Omega}$ TH cross section with cross section $\frac{d\sigma}{d\Omega}$ of two-body reaction $C + c \to A + x$ and penetrability factor $P \propto k_{Ax}^3 \exp(2\pi\eta_{Ax})$

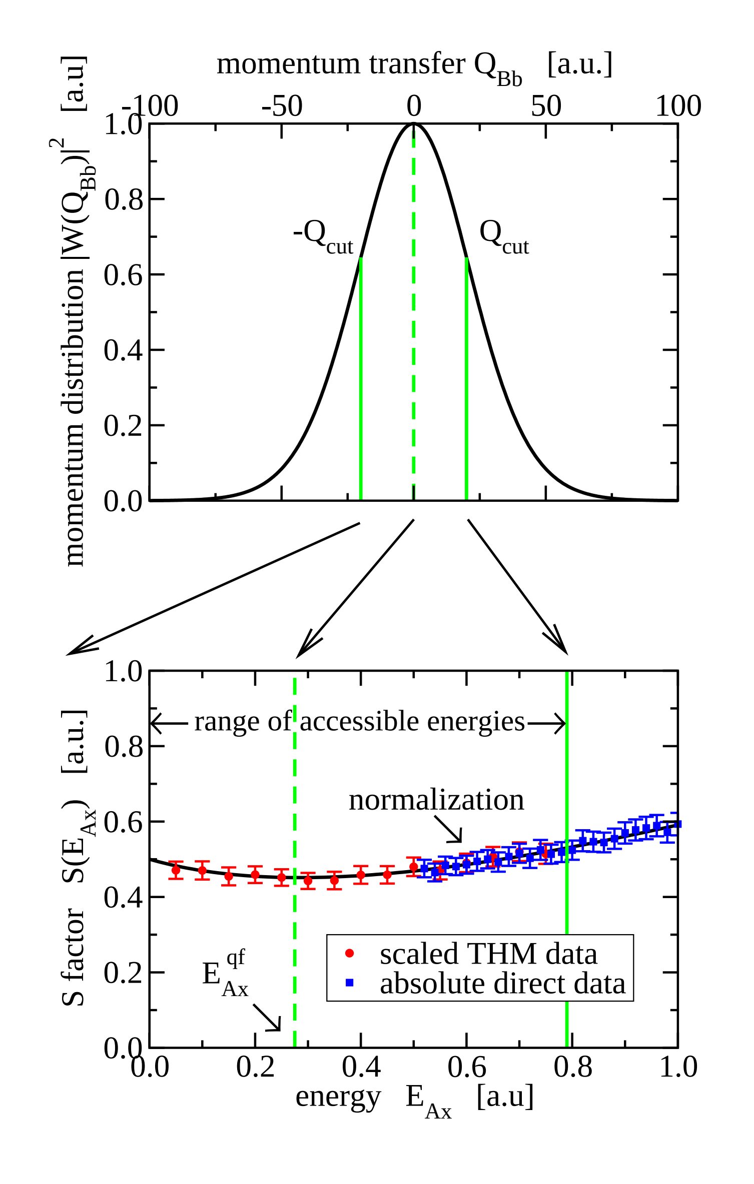
$\Rightarrow$ $KF \frac{d\sigma^{TH}}{d\Omega} \propto S(E_{Ax})$ astrophysical S factor for $E_{Ax} \to 0$

Application of the Trojan-Horse Method

- selection of Trojan horse $a = b + x$
  (e.g. ^{2}H $= n + p$, ^{6}Li $= \alpha + d$, . . .)
  with binding energy $\epsilon_a > 0$ and
  well known ground state wave function
  $\Rightarrow$ momentum amplitude $W(\vec{Q}_{Bb})$
- width of momentum amplitude W
  $\Leftrightarrow$ Fermi motion of x inside a
- condition $\vec{Q}_{Bb} = 0$ defines
  "quasi-free energy" in $A + x$ system

$$E_{Ax}^{qf} = E_{Aa} \left(1 - \frac{\mu_{Aa}}{\mu_{Bb}} \frac{\mu_{bx}^2}{m_x^2} \right) - \epsilon_a \ll E_{Aa}$$

- cutoff in $\vec{Q}_{Bb}$ determines range of
  accessible energies E_{Ax} around E_{Ax}^{qf}
- small momentum transfer
  $\Rightarrow$ dominance of quasi-free process
- normalization of cross section to direct data
  at higher E_{Ax}



D(^{6}Li,α)^{4}He

- **direct reaction**: D(^{6}Li,α)^{4}He
  - experiment with gas target

    (S. Engstler et al., Z. Phys. A 342 (1992) 471)
  - $S(0) = 17.4$ MeV b

    (corrected for electron screening)

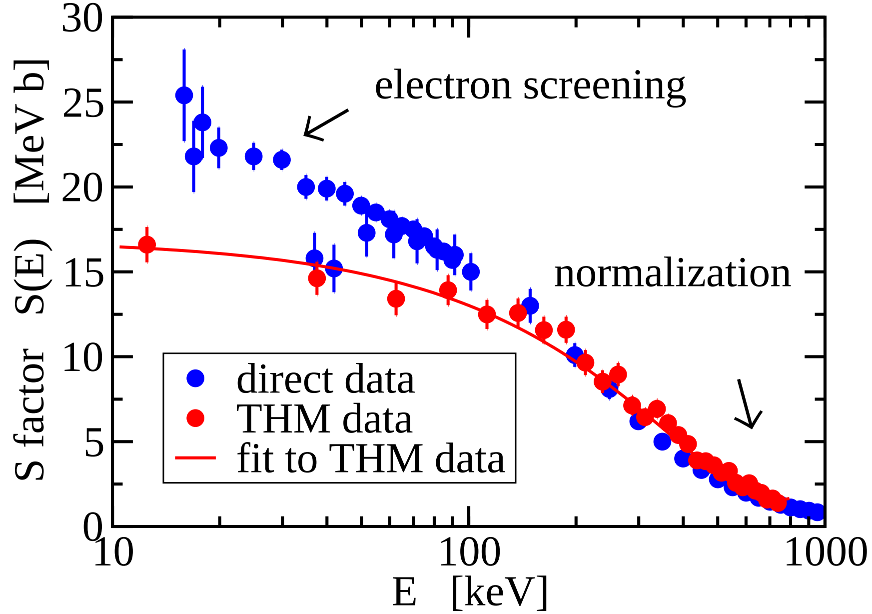
- **THM**: ^{6}Li(^{6}Li,$\alpha\alpha$)^{4}He
  - experiment with 6 MeV ^{6}Li beam

    (C. Spitaleri et al., Phys. Rev. C 63 (2001) 055801;

    A. Musumarra et al., Phys. Rev. C 64 (2001) 068801)
  - $E^{qf} = 25$ keV
  - target and projectile breakup
  - $l = 0$, $\hbar Q_{Bb} < 35$ MeV/c
  - normalization to direct data

    for $E > 600$ keV

    $\Rightarrow S(0) = (16.9 \pm 0.5)$ MeV b



- **electron screening potential**:

  U_e(direct) $= (330 \pm 120)$ eV

  U_e(THM) $= (320 \pm 50)$ eV

  U_e(theory) $= 186$ eV (adiabatic limit)

^{6}Li(p,α)^{3}He

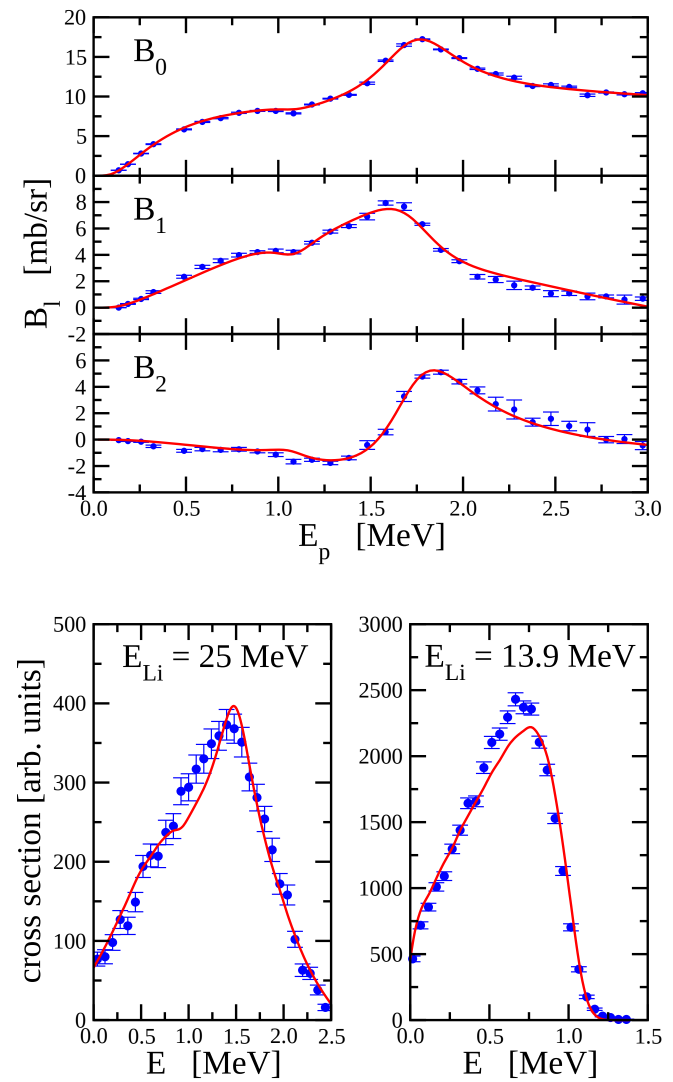
- **direct reaction**: ^{6}Li(p,α)^{3}He
  - ◦ experimental data

    (J. Elwyn et al., Phys. Rev. C 20 (1979) 1084)
  - ◦ differential cross section

    $d\sigma/d\Omega = \sum_l B_l P_l(\cos\theta)$
  - ◦ non-resonant s wave and

    resonant p wave contribution
  - ◦ S matrix from R-matrix fit

    $\Rightarrow$ simulation of THM experiment



- **THM**: ^{2}H(^{6}Li,α^{3}He)n
  - ◦ experiments with 13.9/25 MeV ^{6}Li beam

    (A. Tumino et al., Phys. Rev. C 67 (2003) 065803

    and preliminary results)
  - ◦ $E^{qf} = -0.24/1.35$ MeV
  - ◦ $\hbar Q_{Bb} < 30$ MeV/c
  - ◦ remaining discrepancies ?

Summary

- indirect methods give complementary information to direct measurements

- combination of nuclear reaction theory and experiments at "high" energies

- **Coulomb-dissociation method**

$\Rightarrow$ absolute S factors $S(E)$ of radiative capture reactions for ground state transitions via inverse photo dissociation reaction with equivalent photons

- **ANC method**

$\Rightarrow$ S factors $S(0)$ at energy zero of radiative capture reactions from asymptotic normalization coefficients determined in transfer/breakup reactions

- **Trojan-horse method**

$\Rightarrow$ energy dependence of S factors for direct nuclear reactions from related three-body reactions (transfer to continuum) under quasi-free scattering conditions, full theory not applied yet, method can be generalized to photo-nuclear reactions