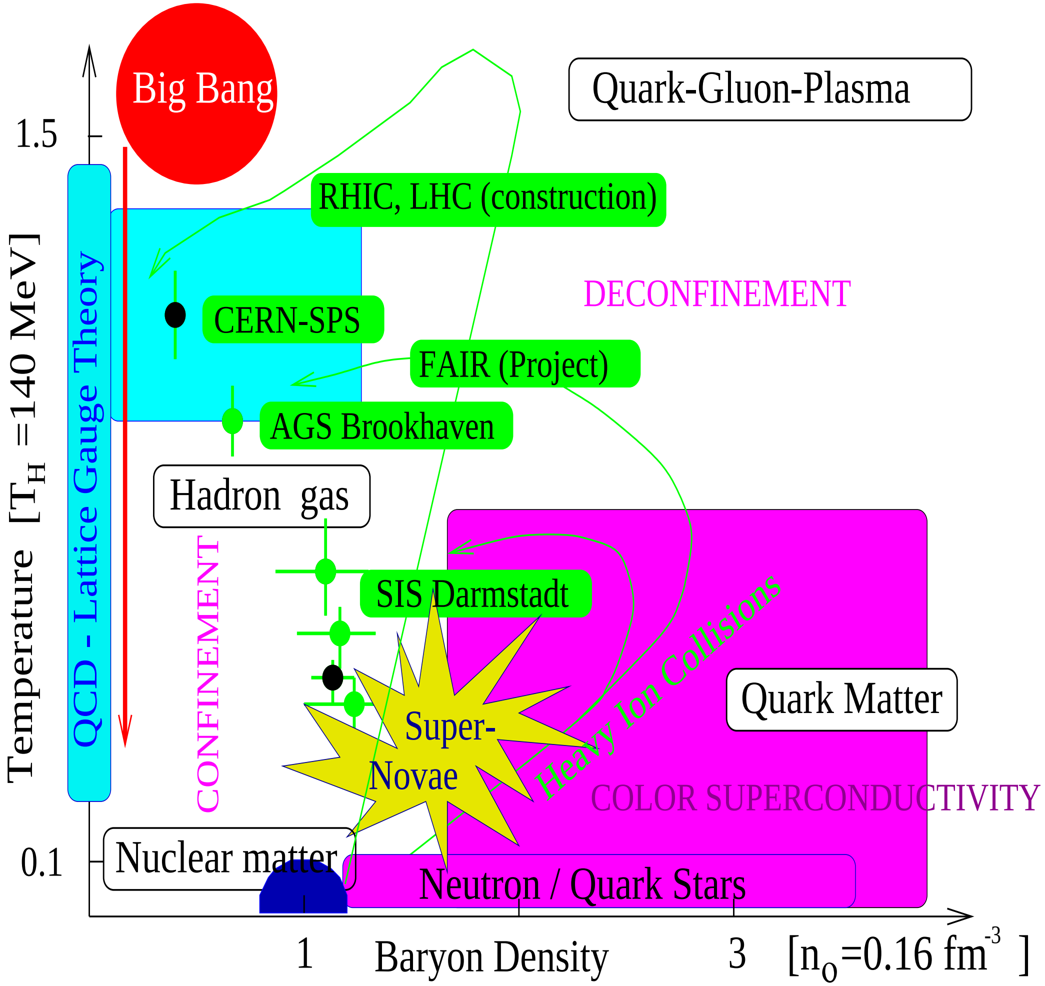
PHASE TRANSITIONS IN DENSE NUCLEAR MATTER



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Contents:

- Walecka Model
- NJL-type Model
- Phenomenology

Related Lectures:

H. Grigorian, A. Sedrakian,
V. Toneev, D. Voskresensky, ...

Contributions:

J. Berdermann, T. Klähn, F. Sandin

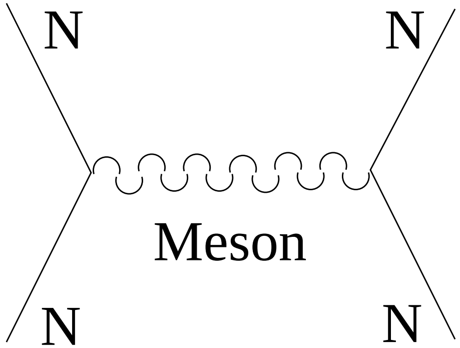
http://theory.gsi.de/Vir-Institute

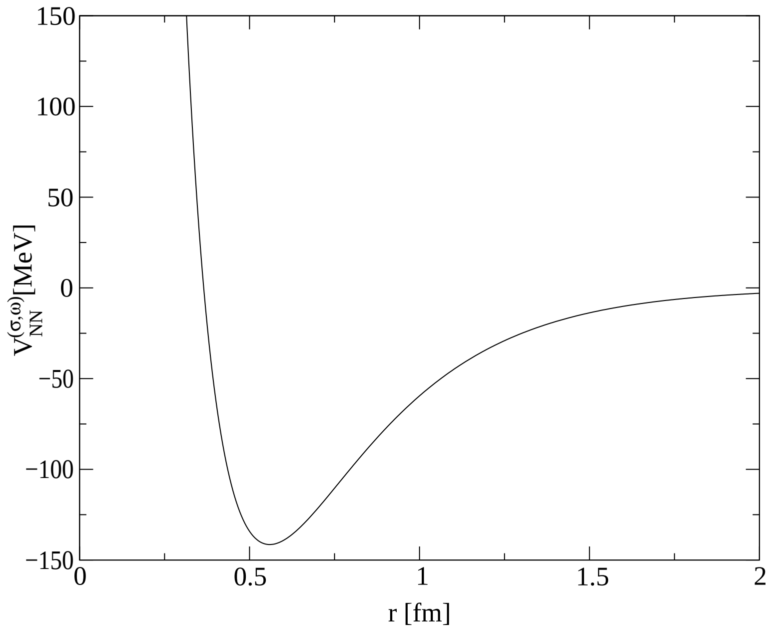
Walecka model for dense nuclear matter (I)

Meson exchange model

example: scalar (σ) meson

$$
\begin{aligned}
(-\triangle + m_\sigma^2)\sigma(\vec{r}) &= -g_\sigma \delta(\vec{r}) \\
\Rightarrow \quad \sigma(r) &= -\frac{g_\sigma}{4\pi}\frac{e^{-m_\sigma r}}{r} \\
V_{NN}^{(\sigma)}(r) &= g_\sigma \sigma(r) = -\frac{g_\sigma^2}{4\pi}\frac{e^{-m_\sigma r}}{r}
\end{aligned}
$$





Meson	I^π	T	S	M[MeV]
$\pi^0, \pi^\pm$	0^-	1	0	140
σ	0^+	0	0	≈ 500
$K^0, K^\pm$	0^-	1/2	± 1	495
η	0^-	0	0	550
$\rho^0, \rho^\pm$	1^-	1	0	770
ω	1^-	0	0	780
δ	0^+	1	0	900

Walecka model for dense nuclear matter (II)

Field theoretical formulation: Lagrangian and Path Integral for Partition Function

$$\mathcal{Z}_{gk}(T,V,\{\mu_i\}) = \int [d\overline{\Psi}][d\Psi] \exp\left\{ \int_0^{\beta=1/T} d\tau \int_V d^3\vec{x} \, \left(\mathcal{L}_0 + \mathcal{L}_I + \mu_p \Psi_p^+ \Psi_p + \mu_n \Psi_n^+ \Psi_n\right) \right\}$$

$$\mathcal{L}_0(\tau,\vec{x}) \;=\; \overline{\Psi}(\tau,\vec{x})\,(i\gamma_\mu \partial_\mu - m_N)\,\Psi(\tau,\vec{x})\ , \qquad \mathcal{L}_I(\tau,\vec{x}) = j_{\omega_\mu}(\tau,\vec{x}) \frac{G_\omega}{2} j_{\omega_\mu}(\tau,\vec{x}) - j_{\sigma_\mu}(\tau,\vec{x}) \frac{G_\sigma}{2} j_{\sigma_\mu}(\tau,\vec{x})$$

$$j_\sigma(\tau,\vec{x}) \;=\; \overline{\Psi}(\tau,\vec{x})\Psi(\tau,\vec{x})$$
$$j_{\omega_\mu}(\tau,\vec{x}) \;=\; \overline{\Psi}(\tau,\vec{x})\gamma_\mu\Psi(\tau,\vec{x})$$

$$\Psi = \begin{pmatrix} \psi_n \\ \psi_p \end{pmatrix}; \quad \psi_n = \begin{pmatrix} u_{n,\uparrow} \\ u_{n,\downarrow} \\ v_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ b \end{matrix}}\right\} \text{Neutron} \\ \left.\vphantom{\begin{matrix} a \\ b \end{matrix}}\right\} \text{Antineutron} \end{matrix}$$

$\mu_n = \mu_p$	$\longrightarrow$	symmetric nuclear matter
$\mu_n \neq 0;\ \mu_p = 0$	$\longrightarrow$	pure neutron matter
$\mu_n = \mu_p + \mu_{e^-}$	$\longrightarrow$	neutron star matter (β-equilibrium)

WALECKA MODEL FOR DENSE NUCLEAR MATTER (III)

Evaluation of the Path Integral: Hubbard-Stratonovich trick

$$\exp\left(-\left(\overline{\Psi}\Psi\right)\frac{G_\sigma}{2}\left(\overline{\Psi}\Psi\right)\right) = \left(\det G_\sigma^{-1}\right)^{\frac{1}{2}}\int[d\sigma]\exp\left(\frac{\sigma^2}{2G_\sigma}+\sigma\overline{\Psi}\Psi\right) \quad (1)$$

Effective action quadratic $\Longrightarrow$ Gaussian Path Integral

$$\mathcal{S} \equiv \int_0^\beta d\tau\int d^3\vec{x}\,\overline{\Psi}(\vec{x},\tau)\left\{\left(-\gamma_0\frac{\partial}{\partial\tau}+i\vec{\gamma}\vec{\nabla}-m_N+\gamma_0\mu+\sigma-\gamma_\mu\omega_\mu\right)\Psi(\vec{x},\tau)+\frac{\sigma^2}{2G_\sigma}-\frac{\omega_\mu^2}{2G_{\omega_\mu}}\right\}$$

Fourier representation: $\Psi(\vec{x},\tau) = \sqrt{\frac{T}{V}}\sum_n\sum_{\vec{p}}e^{i(\vec{p}\vec{x}+\omega_n\tau)}\Psi_n(\vec{p})$, with $\omega_n \equiv \pi T(2n+1)$

$$\begin{aligned}
&\int_0^\beta d\tau\int d^3\vec{x}\,\overline{\Psi}(\vec{x},\tau)\left(-\gamma_0\frac{\partial}{\partial\tau}+i\vec{\gamma}\vec{\nabla}-m_N+\gamma_0\mu+\sigma-\gamma_0\omega_0\right)\Psi(\vec{x},\tau)\\
= &\ \frac{1}{\beta V}\int_0^\beta d\tau\int d^3\vec{x}\sum_{n,n'}\sum_{\vec{p},\vec{p}'}\overline{\Psi}_{n'}(\vec{p}')\left(-i\gamma_0\omega_n-\vec{\gamma}\vec{p}-m_N^*+\gamma_0\mu^*\right)\Psi_n(\vec{p})\exp\left[i\left\{(\vec{p}-\vec{p}')\vec{x}+(\omega_n-\omega_{n'})\tau\right\}\right]\\
= &\ \beta\sum_n\sum_{\vec{p}}\overline{\Psi}_n(\vec{p})(-\gamma_\mu p_\mu-m_N^*)\Psi_n(\vec{p}) = \sum_n\sum_{\vec{p}}\overline{\Psi}_n(\vec{p})G^{-1}[\sigma,\omega_0]\Psi_n(\vec{p}) \quad (2)
\end{aligned}$$

Effective mass $m_N^* = m_N - \sigma$, chemical potential $\mu^* = \mu - \omega_0$ and quasiparticle propagator

$$G^{-1}[\sigma,\omega] = -\beta(\gamma_\mu p_\mu + m_N^*) \ , \ p_0 = i\omega_n - \mu^*$$

WALECKA MODEL FOR DENSE NUCLEAR MATTER (IV)

Evaluate fermionic Path Integral and mean field approximation:

$$\begin{aligned}
\mathcal{Z}_{gk}(T,V,\{\mu_i\}) &= \mathcal{N}\prod_{n,\vec{p}}\int[d\overline{\Psi}_n(\vec{p})][d\Psi_n(\vec{p})][d\sigma][d\omega_0]\exp\left\{\sum_{n,\vec{p}}\overline{\Psi}_n(\vec{p})G^{-1}[\sigma,\omega_0]\Psi_n(\vec{p})+\frac{\sigma^2}{2G_\sigma}-\frac{\omega_0^2}{2G_{\omega_0}}\right\}\\
&= \int[d\sigma][d\omega_0]\exp\left\{Tr\,\ln G^{-1}[\sigma,\omega_0]+\frac{\sigma^2}{2G_\sigma}-\frac{\omega_0^2}{2G_{\omega_0}}\right\}\\
&= \exp\left\{Tr\,\ln G^{-1}[\overline{\sigma},\overline{\omega}_0]+\frac{\overline{\sigma}^2}{2G_\sigma}-\frac{\overline{\omega}_0^2}{2G_{\omega_0}}\right\}
\end{aligned}$$

Stationarity condition: $\partial\ln Z_{gk}/\partial\overline{\sigma}=\partial\ln Z_{gk}/\partial\overline{\omega}_0=0$ corresponds to

$$\overline{\sigma} = -G_\sigma Tr\,G[\overline{\sigma},\overline{\omega}_0]=G_\sigma n_s\ ,\quad \overline{\omega}_0=-G_\omega Tr\,\gamma_0 G[\overline{\sigma},\overline{\omega}_0]=G_\omega n\ .$$

Thermodynamics: $\Omega(T,V,\mu)=-T\ln\mathcal{Z}_{gk}=-pV$

$$p(\mu,T)=\frac{1}{2}G_\omega n^2-\frac{1}{2}G_\sigma n_s^2+4T\int\frac{d^3\vec{p}}{(2\pi)^3}\left[\ln\left(1+e^{-\beta(E^*-\mu^*)}\right)+\ln\left(1+e^{-\beta(E^*+\mu^*)}\right)\right]$$

$$n=4\int\frac{d^3\vec{p}}{(2\pi)^3}[f_-(E^*)-f_-(E^*)]\ ,\ n_s=4\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{m_N^*}{E^*}[f_-(E^*)-f_+(E^*)]\ ,\ f_\pm(E^*)=\frac{1}{e^{\beta(E^*\mp\mu^*)}+1}$$

Quasiparticle properties $E^*=\sqrt{\vec{p}^2+{m_N^*}^2}$, $m_N^*=m_n-G_\sigma n_s$, $\mu^*=\mu-G_\omega n$.

WALECKA MODEL FOR DENSE NUCLEAR MATTER (IV)

Evaluate Traces: $Tr \ln G^{-1} = tr_p tr_D \ln G^{-1} = tr_p \ln \mathrm{det}_D G^{-1} = \sum_n \sum_{\vec{p}} \ln \mathrm{det}_D G^{-1}$

Scalar mean field

$$\begin{aligned}
\overline{\sigma} &= -G_{\overline{\sigma}} Tr\, G[\overline{\sigma}, \overline{\omega}_0] \\
&= -2G_\sigma T \sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} tr_D \left[\gamma_\mu p_\mu - (m - \overline{\sigma}) + i\gamma_0(\mu - \overline{\omega})\right]^{-1} \\
&= 2G_\sigma T \sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{m^*}{\vec{p}^2 + m^{*2} + (\omega_n + i\mu^*)^2} \right) \\
&= G_\sigma \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{m^*}{E^*} \left(\frac{1}{e^{\beta(E^* - \mu^*)} + 1} + \frac{1}{e^{\beta(E^* + \mu^*)} + 1} \right) \\
&\equiv G_\sigma n_s
\end{aligned}$$

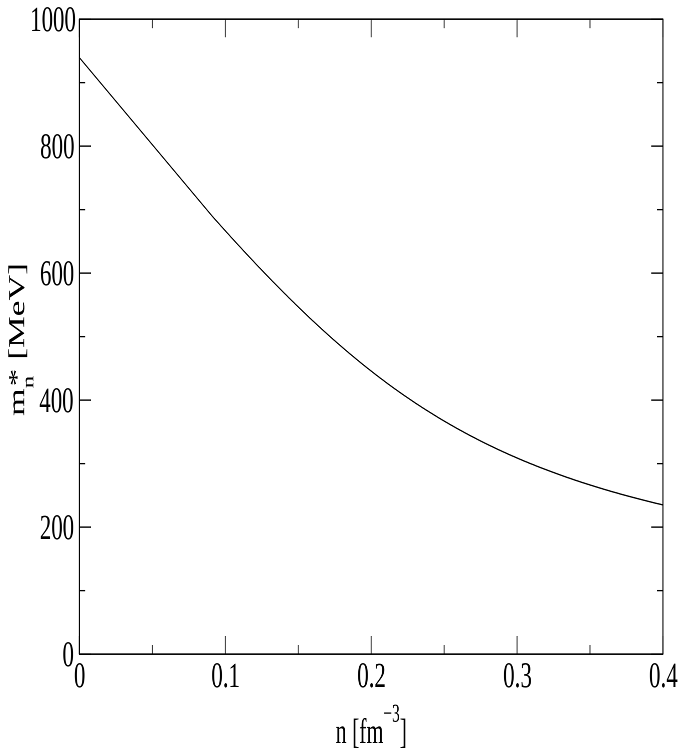
Vector mean field

$$\begin{aligned}
\overline{\omega}_0 &= -G_{\overline{\omega}_0} Tr\, \gamma_0 G[\overline{\sigma}, \overline{\omega}_0] \\
&= G_\omega \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{1}{e^{\beta(E^* - \mu^*)} + 1} - \frac{1}{e^{\beta(E^* + \mu^*)} + 1} \right) \\
&\equiv G_\omega n
\end{aligned}$$

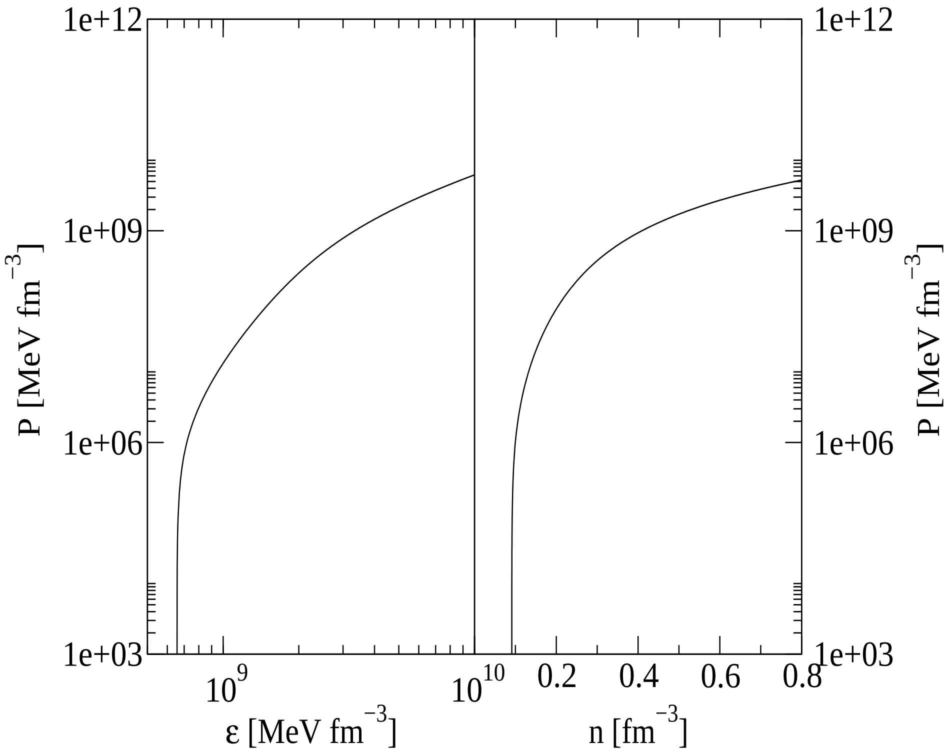
Matsubara sums $\longrightarrow$ Seminar!!

WALECKA MODEL (VI) - RESULTS

Effective mass



Pressure vs. energy density / density



See, e.g., Kapusta: 'Finite temperature field theory': liquid - gas phase transition

NJL model for neutral 3-flavor quark matter

$$\Omega(T,\mu) = \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - T\sum_n \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\mathrm{Tr}\ln\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right) + \Omega_e - \Omega_0.$$

Inverse propagator of Nambu-Gorkov spinors

$$S^{-1}(i\omega_n,\vec{p}) = \begin{bmatrix} \not{p} - M + \mu\gamma^0 & \widehat{\Delta} \\ \widehat{\Delta}^\dagger & \not{p} - M - \mu\gamma^0 \end{bmatrix},$$

with diquark gaps ($\Delta_{ur} = \Delta_{ds}, \ldots$)

$$\Delta_{k\gamma} = 2G_D\langle \bar{q}_{i\alpha} i\gamma_5 \epsilon_{\alpha\beta\gamma}\epsilon_{ijk} q^C_{j\beta}\rangle.$$

as elements of the gap matrix

$$\widehat{\Delta} = i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\Delta_{k\gamma}.$$

Fermion determinant (Tr ln D = ln det D)

$$\mathrm{ln det}\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right) = 2\sum_{a=1}^{18}\ln\left(\frac{\omega_n^2+\lambda_a(\vec{p})^2}{T^2}\right).$$

Result for thermodynamic potential

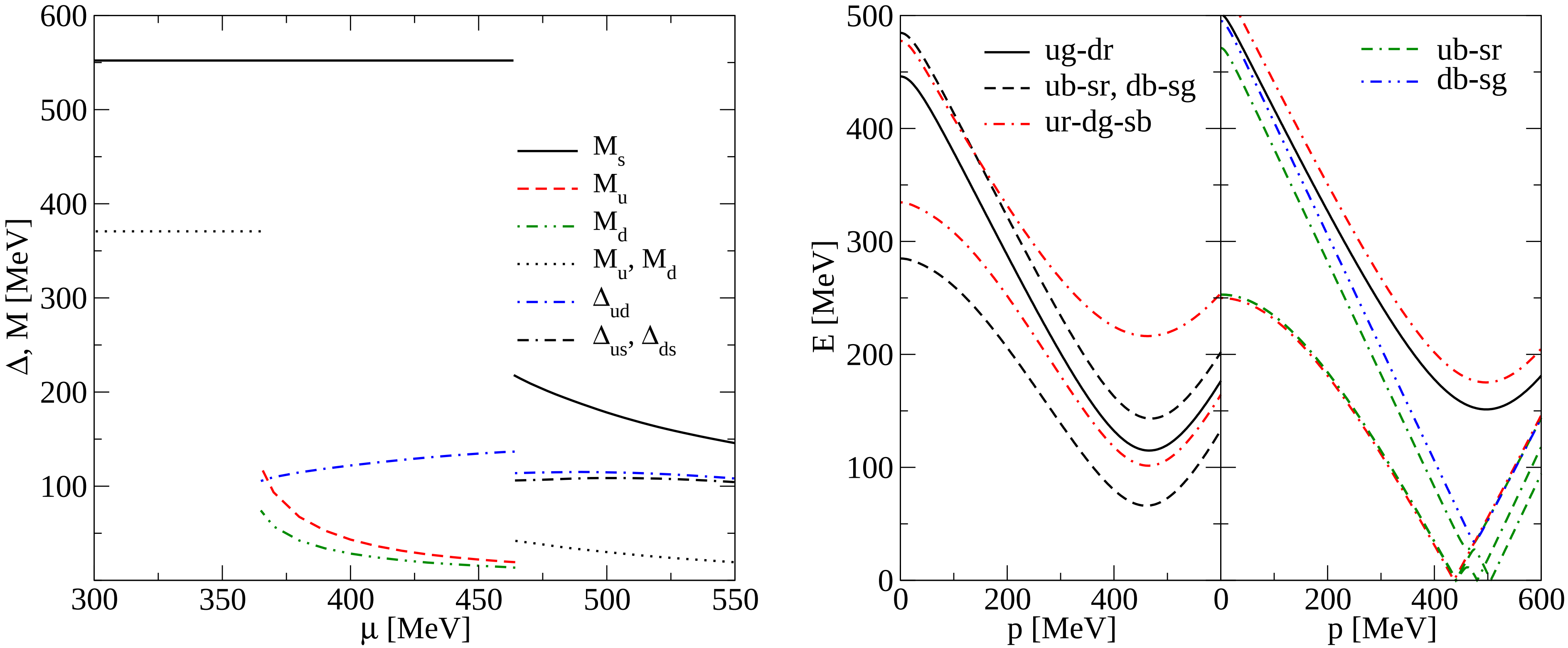
$$\Omega(T,\mu) = \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - \int\frac{d^3p}{(2\pi)^3}\sum_{a=1}^{18}\left(\lambda_a + 2T\ln\left(1+e^{-\lambda_a/T}\right)\right) + \Omega_e - \Omega_0.$$

Neutrality conditions: $n_Q = n_8 = n_3 = 0$,

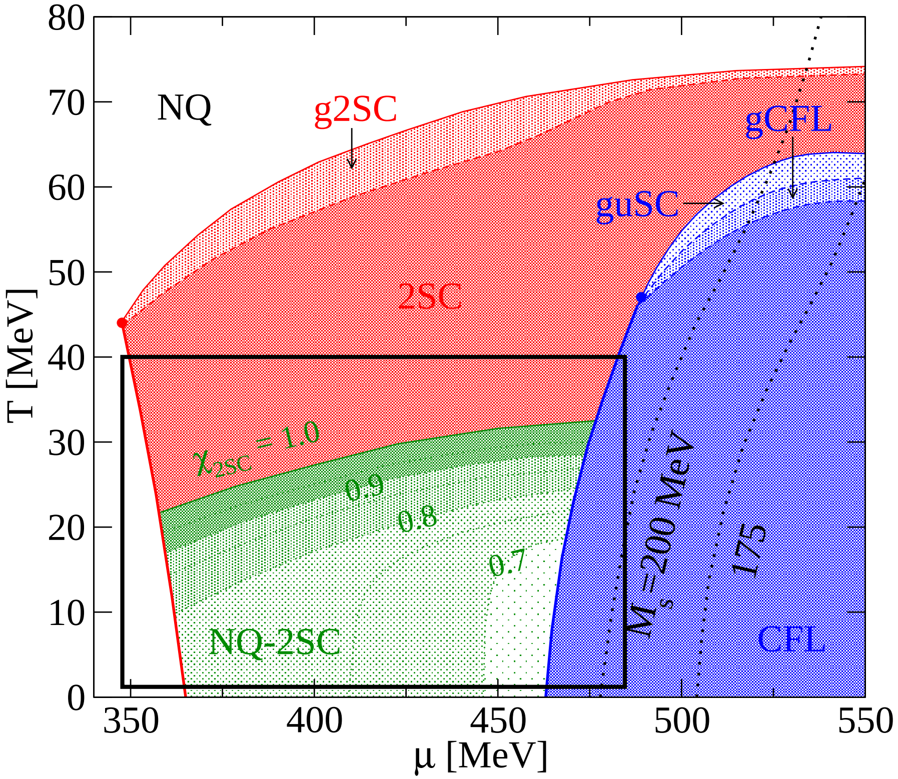
$$n_i = -\frac{\partial\Omega}{\partial\mu_i} = 0,$$

Equation of state: $P = -\Omega$, etc.

MASSES, DIQUARK GAPS AND GAPLESS MODES



Quark matter in compact stars



Blaschke et al: hep-ph/0503194
Rüster et al: hep-ph/0503184
Color neutrality problem: hep-ph/0507271

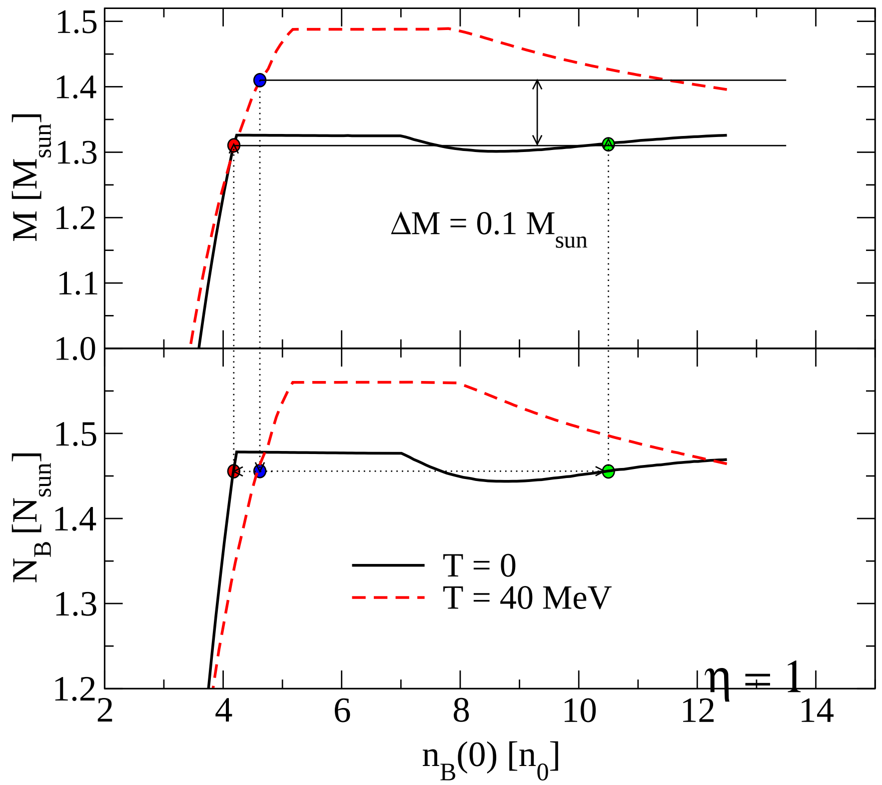
The phases are:

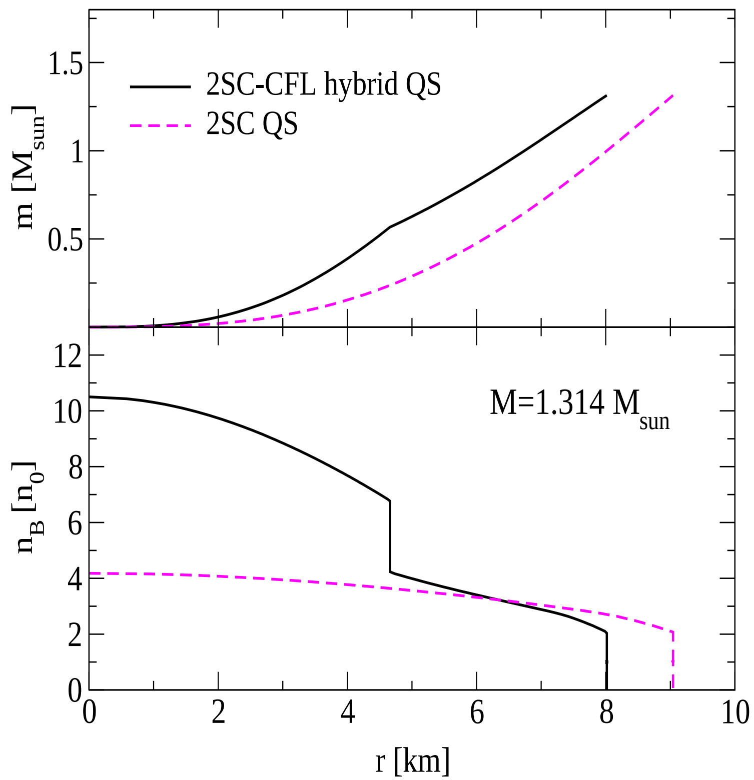
- NQ: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$;
- NQ-2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$, $0 \leq \chi_{2SC} \leq 1$;
- 2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$;
- uSC: $\Delta_{ud} \neq 0$, $\Delta_{us} \neq 0$, $\Delta_{ds} = 0$;
- CFL: $\Delta_{ud} \neq 0$, $\Delta_{ds} \neq 0$, $\Delta_{us} \neq 0$;

Result:

- Gapless phases only at high T,
- CFL only at high chemical potential,
- At T $\leq$20-30 MeV: mixed NQ-2SC phase,
- Critical point (T_c, μ_c)=(44 MeV, 347 MeV),
- Strong coupling, $\eta = 1$, changes?.

2SC-CFL TWIN CONFIGURATIONS, ENERGY RELEASE

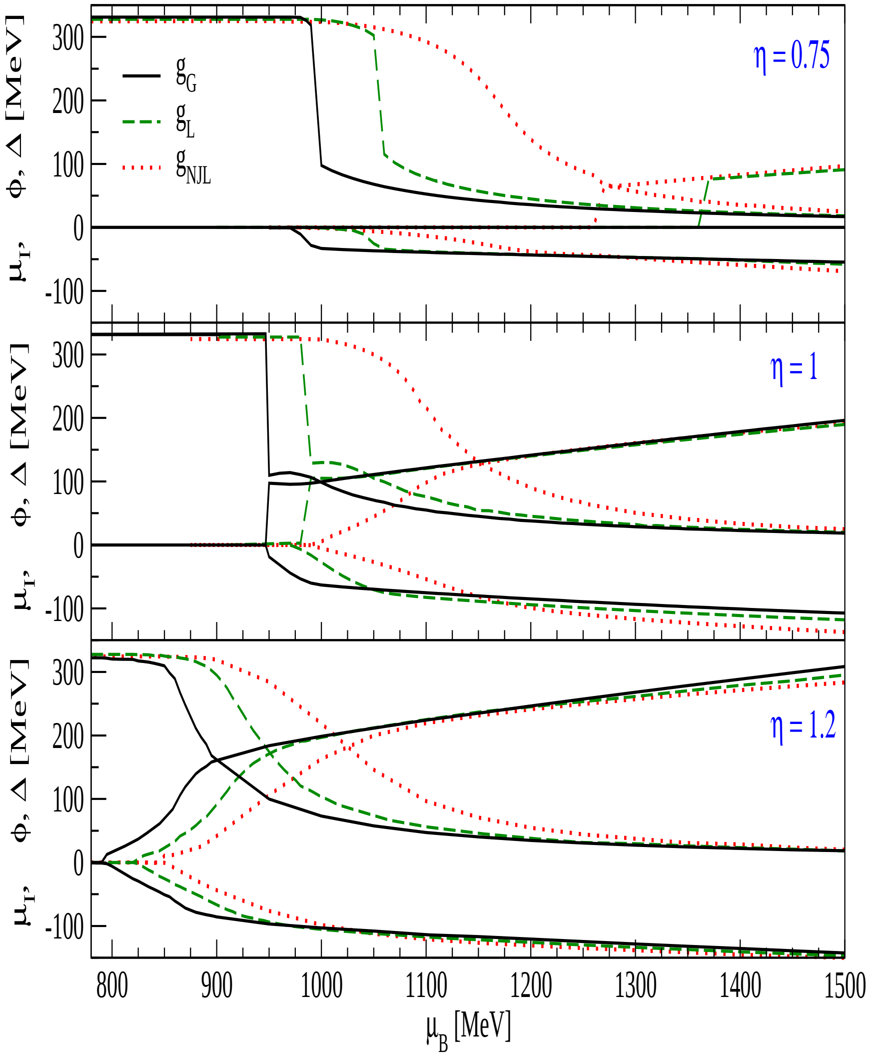




Energy release: $\Delta E \sim 0.1$ M$_\odot$c$^2 \sim 10^{52}$ erg.
See also: Aguilera et al: A&A 416, 991 (2004), DB et al: NPA 736, 203 (2004)

Caution: CFL core unstable against adding a hadronic shell!

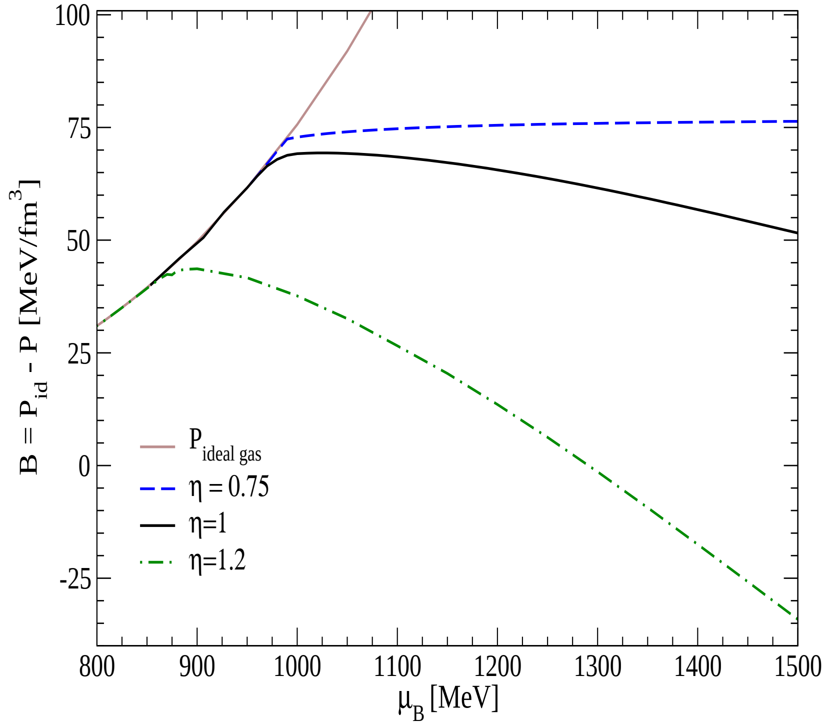
2SC QUARK MATTER IN COMPACT STARS?

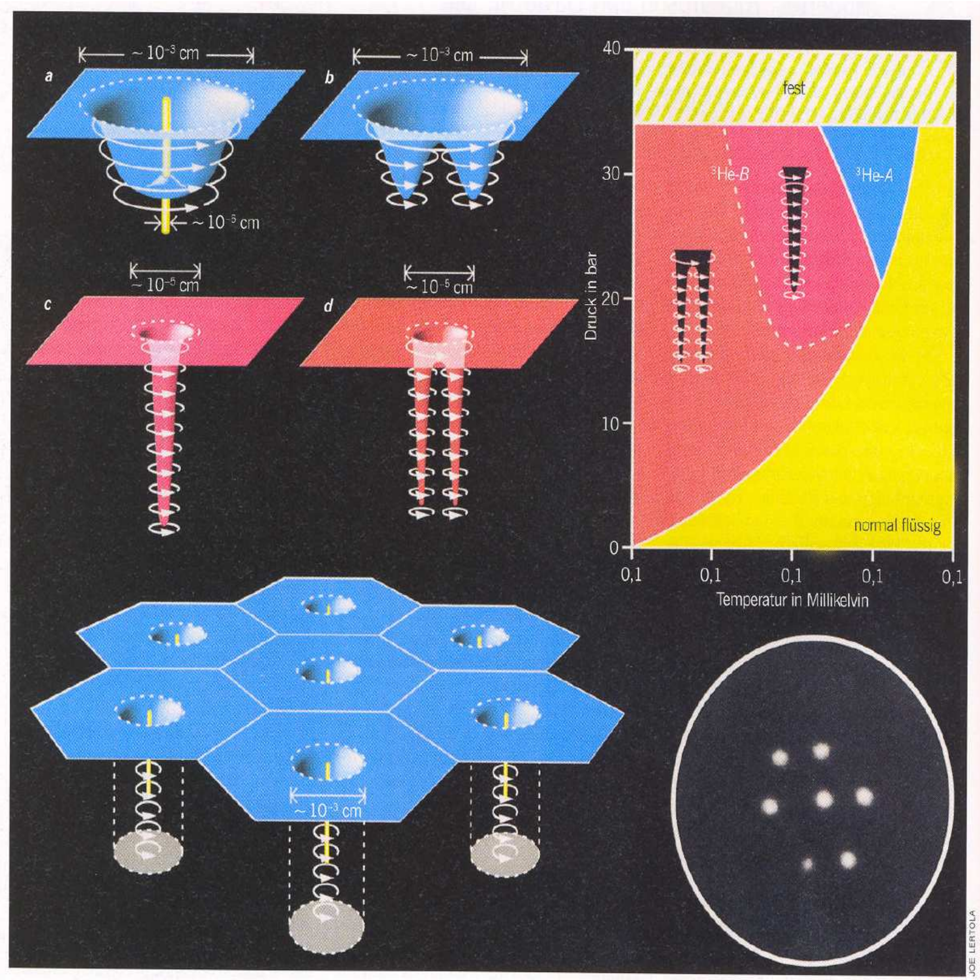


- Bag model Fit for Gaussian SM EoS

  Grigorian et al., PRC 69 (2004)

  Aguilera et al., hep-ph/0412266

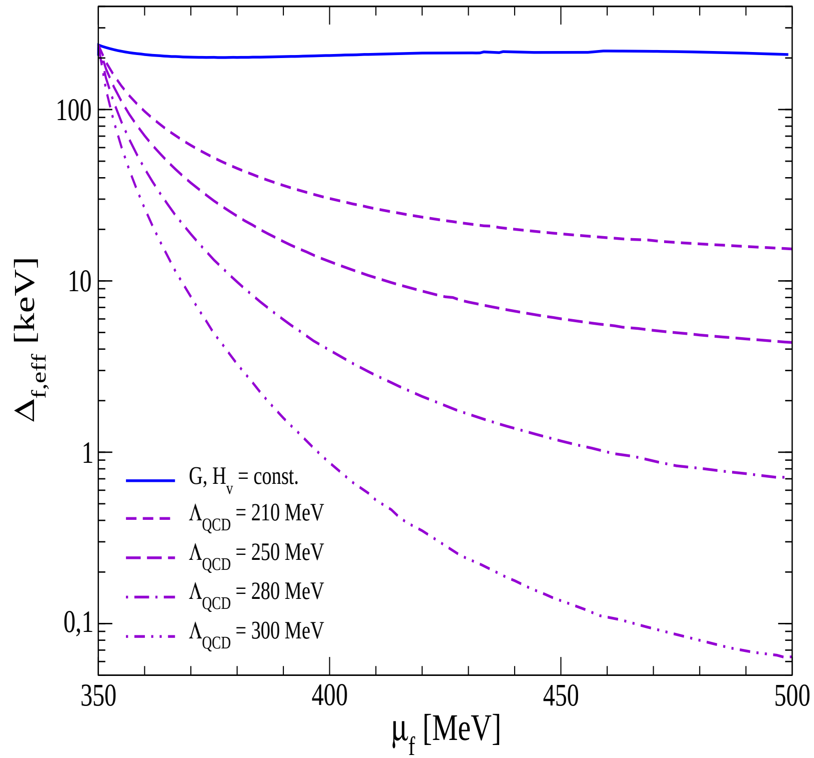




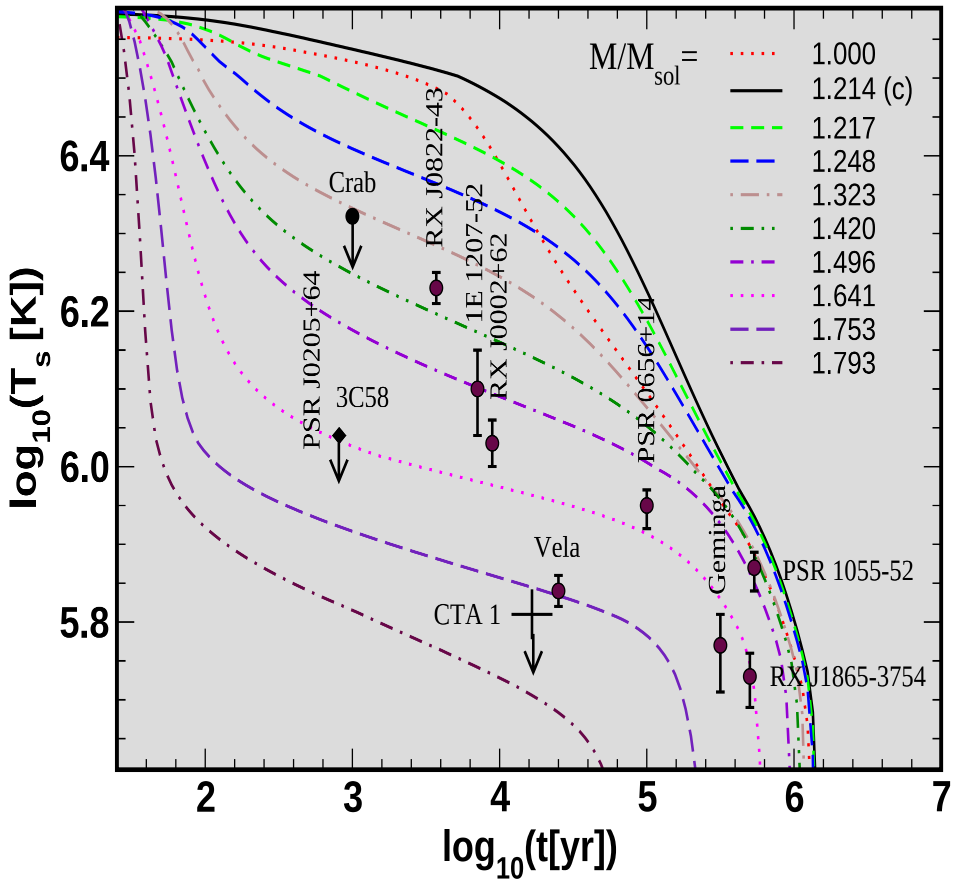
Phases of superfluid ^{3}He

- Color-spin-locking (CSL) condensate
  Aguilera et al., hep-ph/0503288

$$\langle q_f^T \ C\gamma^3\lambda_2 \ q_f\rangle = \langle q_f^T \ C\gamma^1\lambda_7 \ q_f\rangle = \langle q_f^T \ C\gamma^2\lambda_5 \ q_f\rangle \equiv \eta_f \ ,$$



Cooling of a Quark-Hadron Hybrid Star



Evolution of the surface temperature

$$\frac{dU}{dt} = \sum_i C_i \frac{dT}{dt} = -\varepsilon_\gamma - \sum_j \varepsilon_\nu^j$$

Data taken from:
Yakovlev et al., A & A **389** (2002) L24

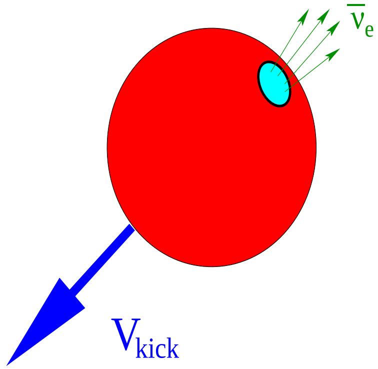
Calculation for neutron stars:
DB, Grigorian, Voskresensky,
astro-ph/0403170; A & A 424 (2004) 979

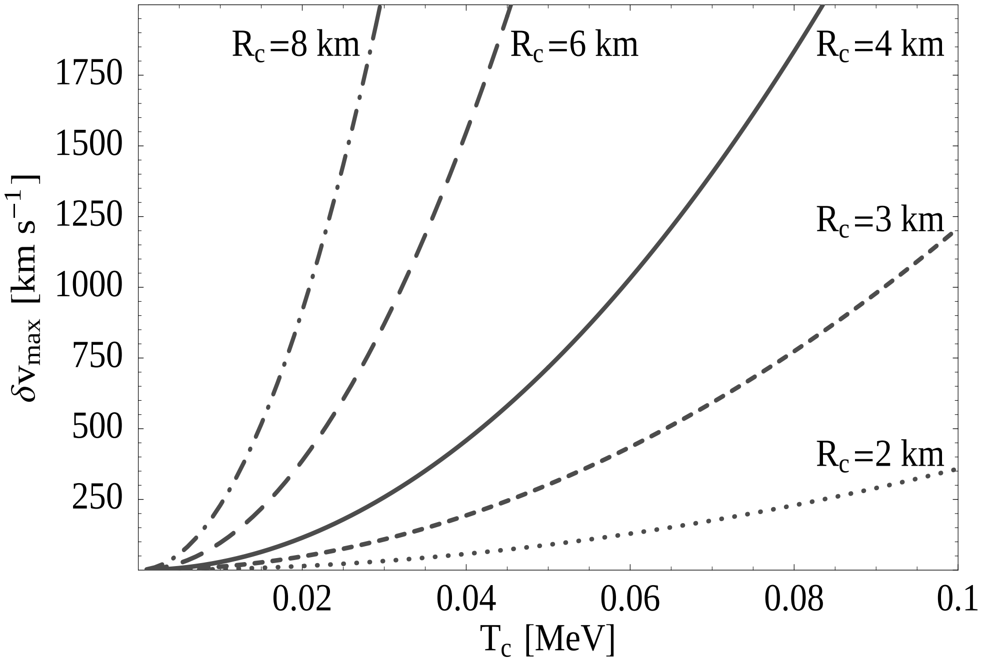
for hybrid stars: astro-ph/0411619;
PRC 71 (2005) 045801.

Small gaps: 2SC+X or CSL ?

PULSAR KICKS, GRAVITATIONAL WAVES ...

- Large kick velocities of pulsars at birth
  $v_{kick} = 500 ... 1000$ km s^{-1}
  Lyne, Lorimer: Nature **369**, 127 (1994)
  Arzoumanian et al: ApJ**568**, 289 (2002)
- Possible explanation: “Neutrino rocket”
  Schmitt et al: hep-ph/0502166





- Spin-1 condensates (CSL) can be anisotropic Schmitt, PRD 71 (2005)
- Anisotropic direct Urca ν emissivity leads to acceleration of the PNS, when $T \leq T_c$ for CSL pairing Acceleration stops when photons start dominating, resulting in:
  $\delta v_{\max} \approx 0.033\, \alpha_s G_F^2 \mu_e \mu_u \mu_d \frac{4\pi}{3} \frac{R_c^3}{1.4 M_\odot} T_0^4 T_c^2 t_0.$
- Gravitational waves: Pagliara et al. gr-qc/0405145