Experiments with fractional magnetic flux quanta

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Arrays of Quantum Dots and Josephson Junctions
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Where is Tübingen?

Tübingen:

- small university town on Neckar
- population ~80000
- 30 km south from Stuttgart
  -- a capital of Baden-Württemberg

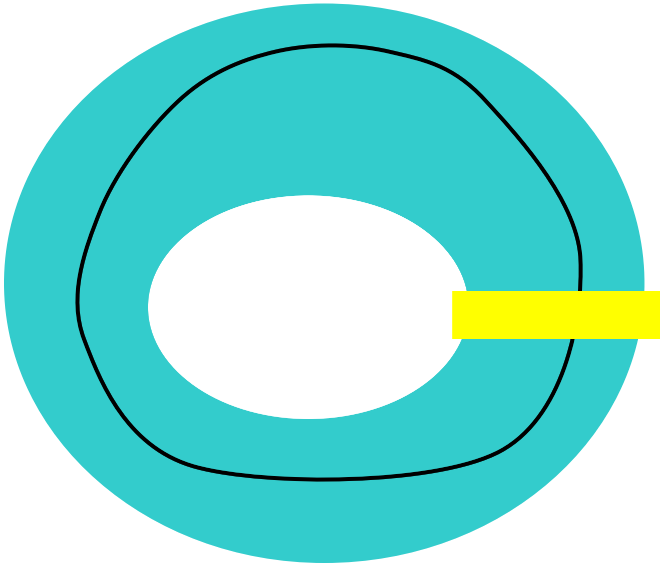
University:

- students ~30000
- university is 530 years old !!!
- two faculties of theology ;-)
- strong medicine
- phys., chem., math are small

Our Group (~30 people):

- 2 Profs: R. Kleiner, D. Koelle
- 1 Assistant Prof.
- 4 Post Docs
- ~12 PhD students
- ~10 Diploma students

Fluxoid quantization



$$\frac{\Phi_0}{2\pi}\oint_C \nabla\theta\, d\mathbf{l} = \oint_C \mathbf{A}\, d\mathbf{l}.$$

$$\oint_C \mathbf{A}\, d\mathbf{l} = \oint_S \mathrm{rot}\,\mathbf{A}\, d\mathbf{S} = \oint_S \mathbf{B}\, d\mathbf{S} = \Phi,$$

$$\Phi = \frac{\Phi_0}{2\pi}\oint_C \nabla\theta\, d\mathbf{l}.$$

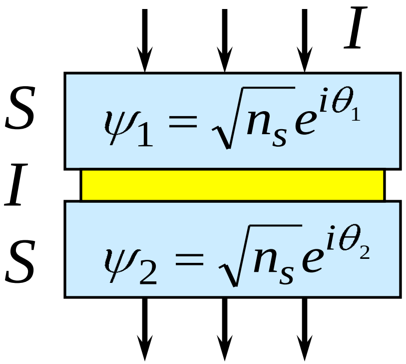
$$\oint_C \nabla\theta\, d\mathbf{l} = 2\pi n.$$

$$\boxed{\Phi = n\Phi_0}$$

$$\mathbf{j}_s = \frac{1}{\Lambda}\left(\frac{\Phi_0}{2\pi}\nabla\theta - \mathbf{A}\right)$$

$$\Phi_0 = \frac{h}{2e} \approx 2.07 \cdot 10^{-15}\,\mathrm{Wb}$$

Josephson junction

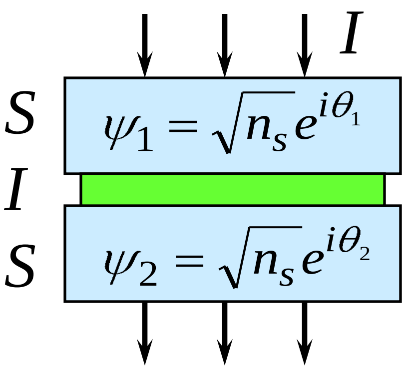


Conventional JJ. (0-junction)

Josephson phase: $\phi = \theta_2 - \theta_1$

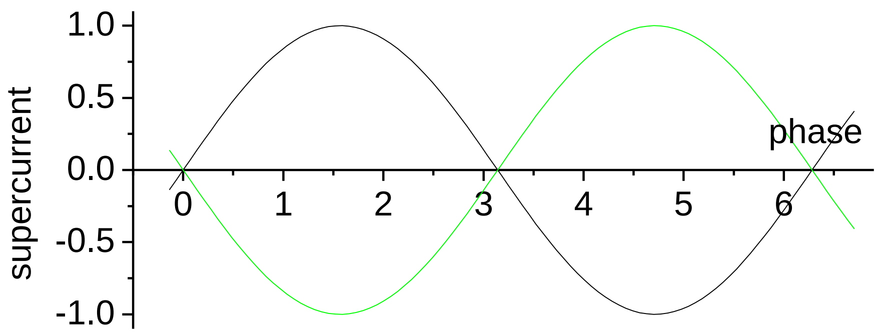
$$I = I_c \sin(\phi)$$

+other terms



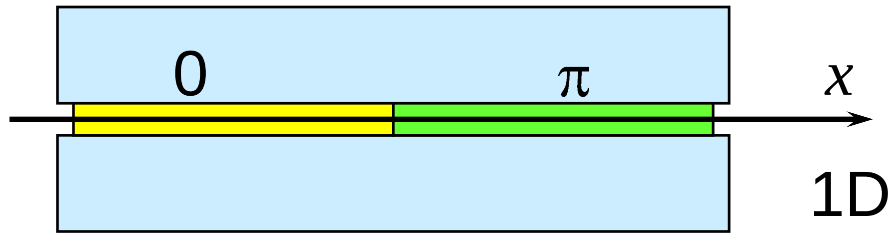
Unconventional JJ (π-junction)

$$I = -I_c \sin(\phi) = I_c \sin(\phi + \pi)$$



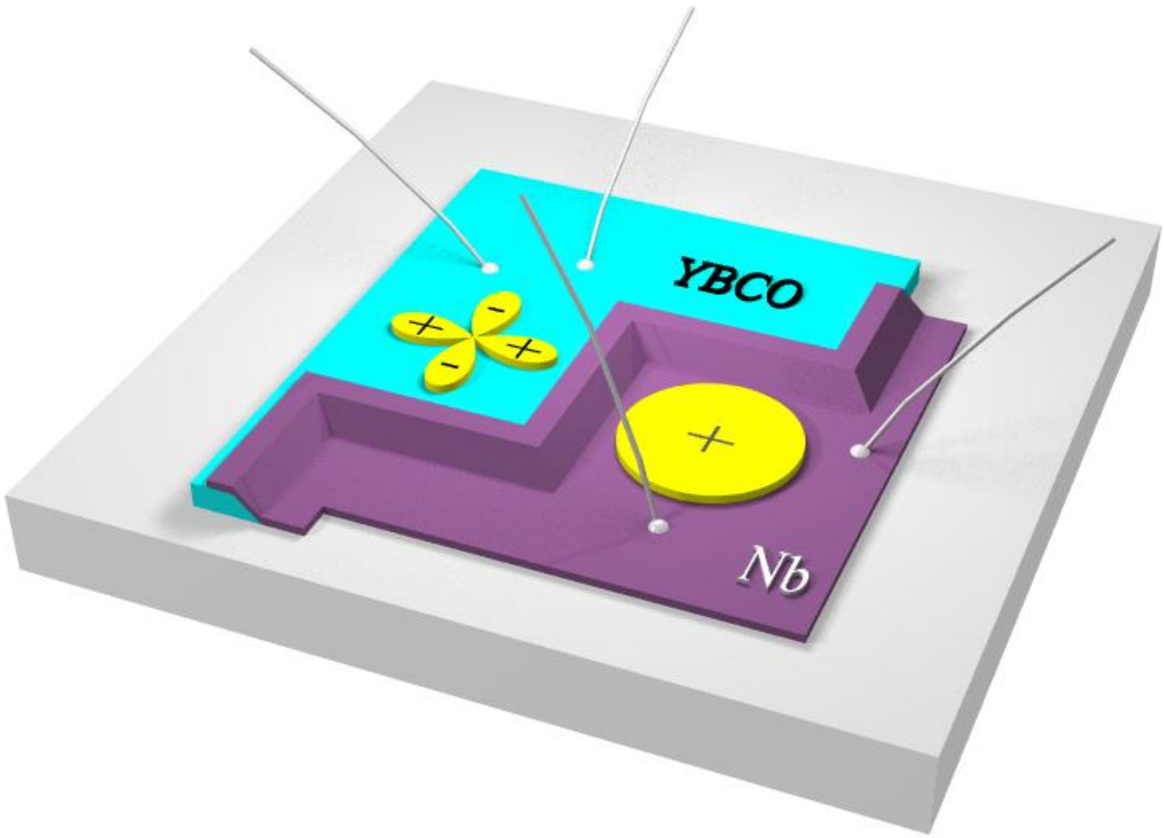
j_c=1–5000 A/cm^2, (Nb-AlO$_x$-Nb)

Long Josephson 0-π junction

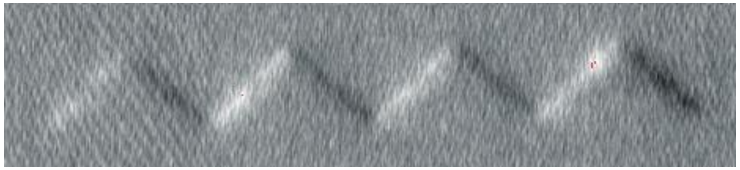


1D model

YBCO-Nb ramp zigzags



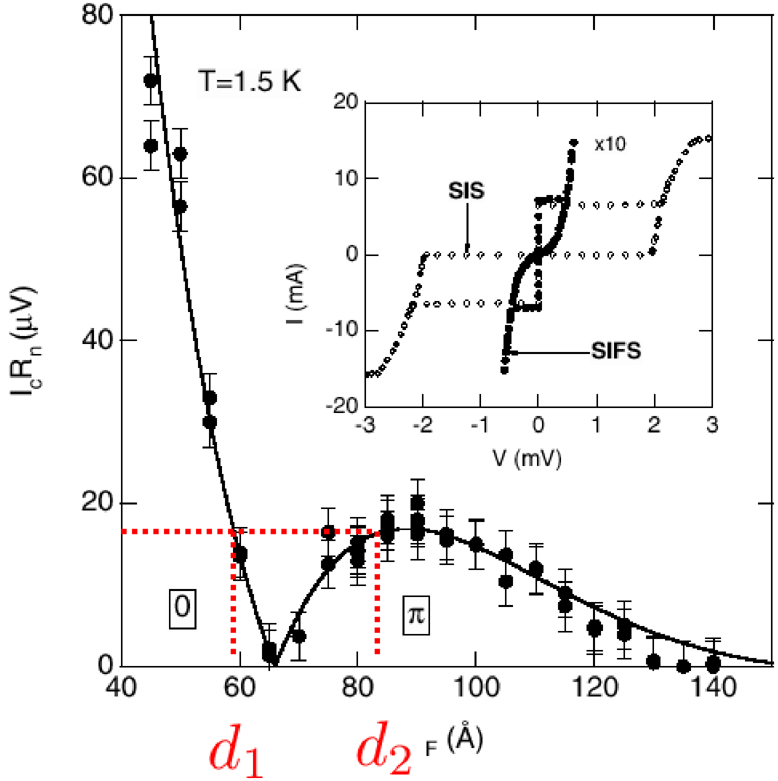
E. Goldobin, R. Straub,
D. Dönitz,
D. Koelle, R. Kleiner,
H. Hilgenkamp (2003).

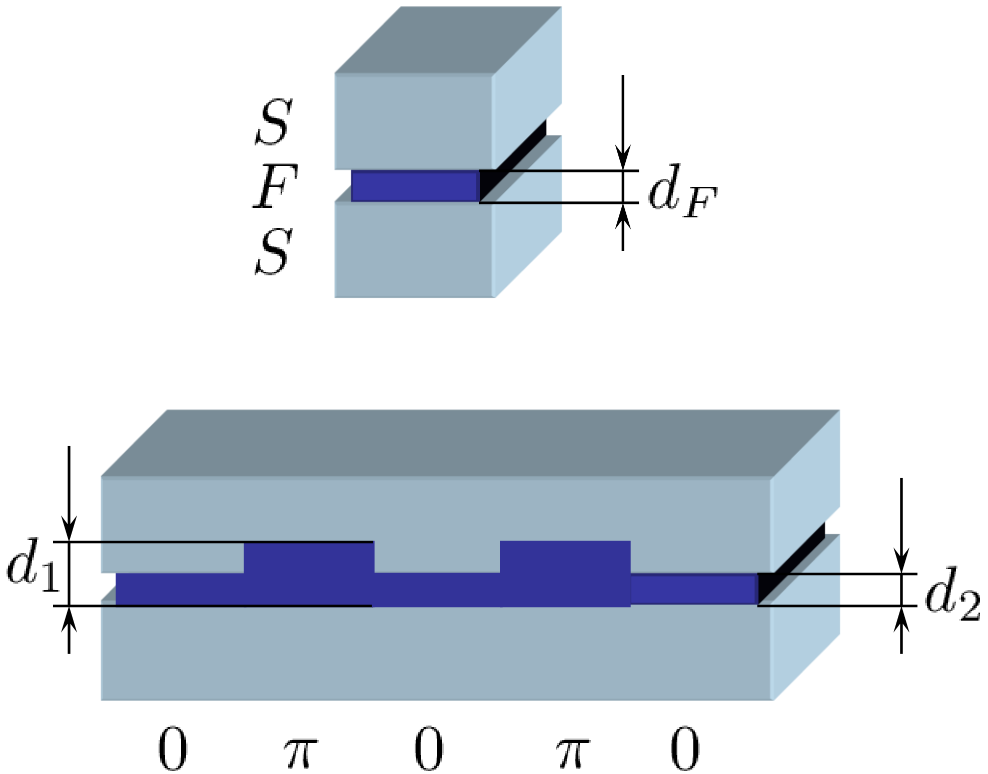


LTSEM image of supercurrent

📖 H.-J. Smilde at al. PRL **88**, 57004 (2002)

SFS/SIFS junctions

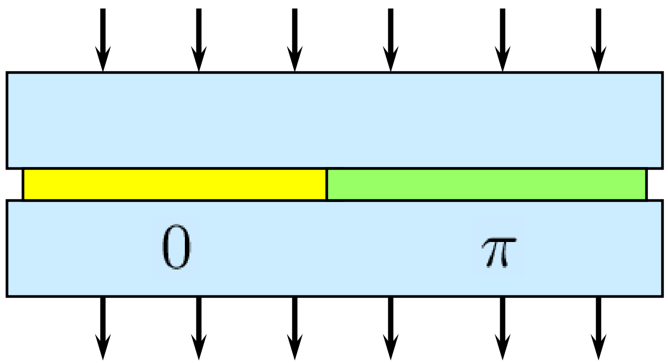


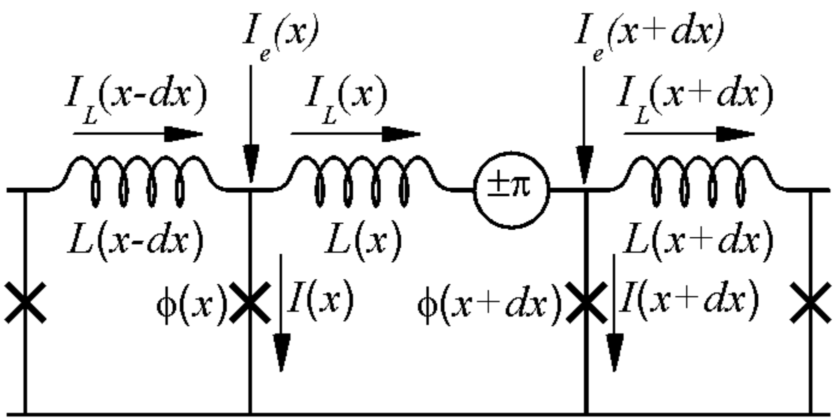


📖 V. Ryazanov et al. PRL **86**, 2427 (2001)

📖 T. Kontos et al. PRL **89**, 137007 (2002)

Deriving sine-Gordon equation





$$\phi(x+dx)-\phi(x)=\frac{2\pi}{\Phi_0}[\Phi_e-I_L(x)L(x)]+\Pi(x),$$

$$I_L(x)+I_e(x)=I_L(x+dx)+I(x),$$

$$\Pi(x)=\theta(x+dx)-\theta(x)$$, where

$$\theta(x)=\pi\sum_{k=1}^{N_c}\sigma_k\mathcal{H}(x-x_k),$$

$$I=j(x)w\,dx,\quad I_e=j_e(x)w\,dx,\quad L=\frac{\mu_0 d'}{w}dx,$$

$$\Phi_e=\mu_0(\mathbf{H}\cdot\mathbf{n})\Lambda dx=\mu_0 H(x)\Lambda\,dx$$

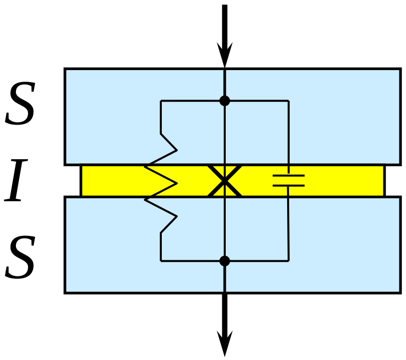
$$\phi_x=\frac{2\pi}{\Phi_0}\left[H\Lambda-\frac{I_L}{\mu_0 d'}\right]+\theta_x(x),$$

$$\frac{dI_L}{dx}=(j_e-j)w.$$

Exclude I_L...

📖 E. Goldobin et al. PRB **66**, 100508 (2002)

Deriving sine-Gordon equation



$$(j_e - j) = \frac{1}{\mu_0 d'}\left\{\mu_0 \Lambda H_x(x) - \frac{\Phi_0}{2\pi}[\phi_{xx} - \theta_{xx}(x)]\right\}$$

$$j(x) = j_c \sin(\phi) + \frac{\Phi_0}{2\pi\rho}\phi_t + C'\frac{\Phi_0}{2\pi}\phi_{tt}$$

$$\lambda_J^2\phi_{xx} - \omega_p^{-2}\phi_{tt} - \sin(\phi) = \omega_c^{-1}\phi_t - \gamma(x) + QH_x(x) + \lambda_J^2\theta_{xx}(x),$$

$\lambda_J = \sqrt{\Phi_0/(2\pi\mu_0 j_c d')}$ → Josephson penetration depth ~0.3–100μm

$\omega_p = \sqrt{2\pi j_c/(\Phi_0 C')}$ → Josephson plasma frequency ~10–10^3GHz

$\omega_c = 2\pi j_c \rho/\Phi_0$ → Josephson critical frequency ~1–10^5 GHz

$\gamma(x) = j_e(x)/j_c$ → normalized bias current density

$Q = 2\pi\mu_0\Lambda\lambda_J^2/\Phi_0$

New normalized units:coordinate x to λ_J, time t to ω_p^{-1}

E. Goldobin et al. PRB **66**, 100508 (2002)

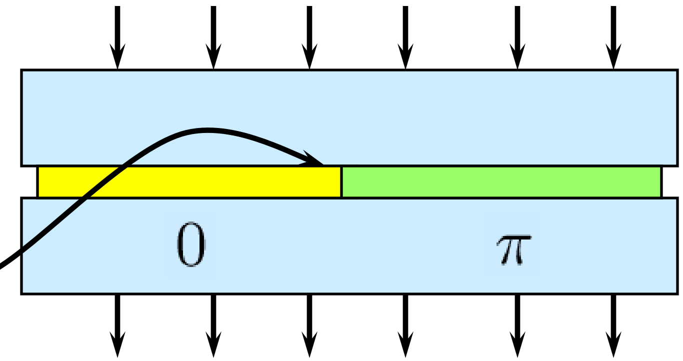
sine-Gordon equation for 0-π LJJ

$$\phi_{xx} - \phi_{tt} - \sin(\phi) = \alpha\phi_t - \gamma(x) + h_x(x) + \theta_{xx}(x)$$

$\alpha = \omega_p/\omega_c \equiv 1/\sqrt{\beta_c}$ — dimensionless damping

$h(x) = 2H(x)/H_{c1}$ — dimensionless field

$H_{c1} = \Phi_0/(\pi\mu_0\Lambda\lambda_J)$ — the first critical field



Phase discontinuity points!

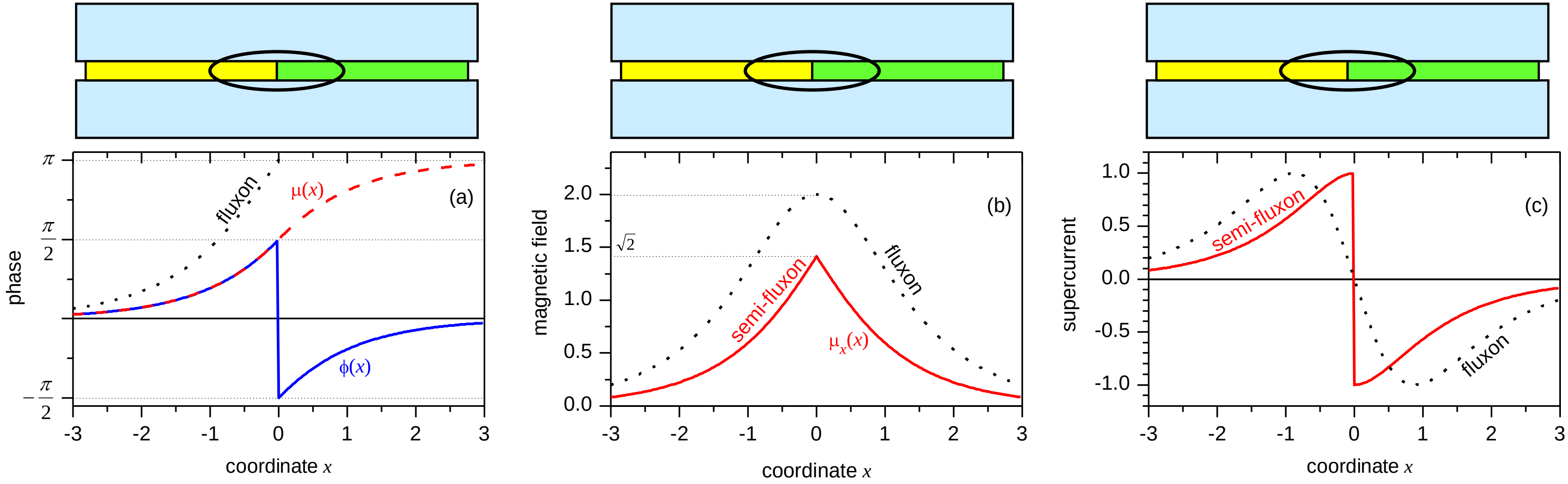
$$\phi(x,t) = \mu(x,t) + \theta(x).$$

$\mu(x,t)$ — magnetic component of the phase

$$\mu_{xx} - \mu_{tt} - \sin(\mu)\underbrace{\cos(\theta)}_{\pm 1} = \alpha\mu_t - \gamma(x) + h_x(x).$$

Semifluxon=vortex carrying $\Phi_0/2$

$$\phi_{xx} - \sin(\phi) = \theta_{xx}(x)$$



$$\phi(x) = -4\,\mathrm{sign}(x)\arctan\left(\mathcal{G}e^{-|x|}\right),$$

$$\mu_x(x) = \frac{2}{\cosh(|x| - \ln \mathcal{G})}$$

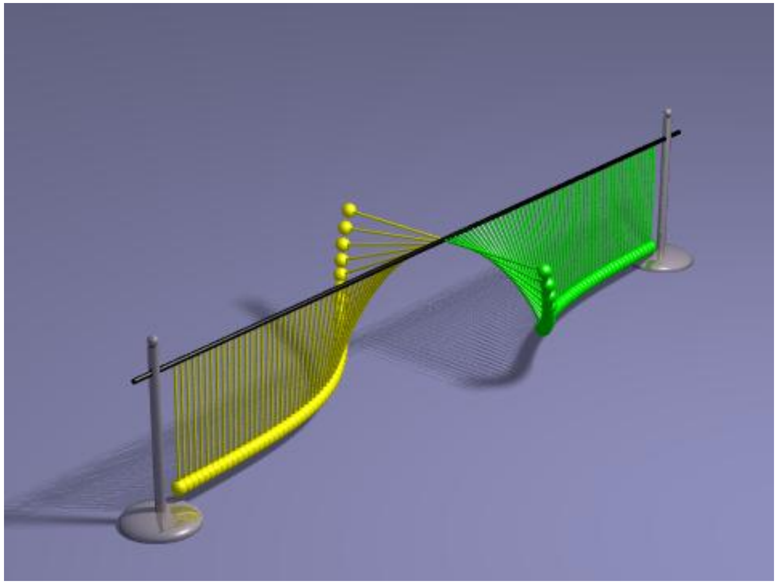
$$\mathcal{G} = \tan(\pi/8) = \sqrt{2} - 1 \approx 0.4$$

Pinned, two degenerate states ↑ and ↓

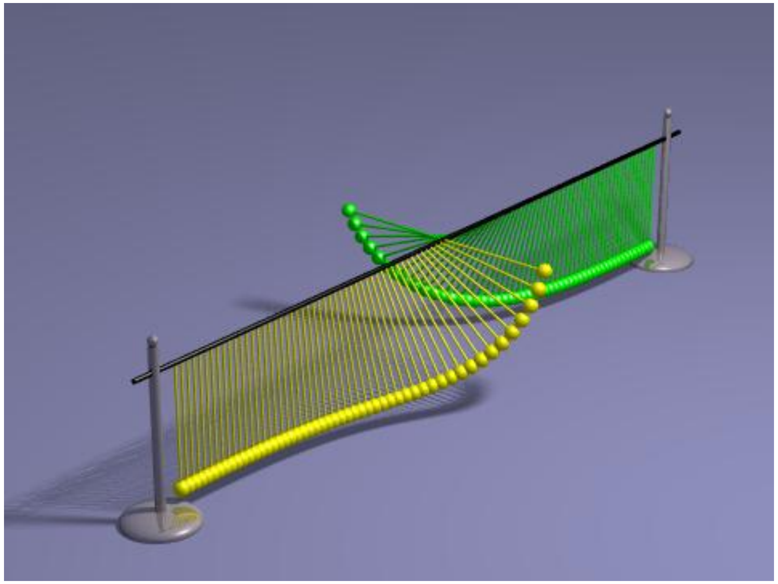
📖 Xu et al., PRB **51**, 11958 (1995)

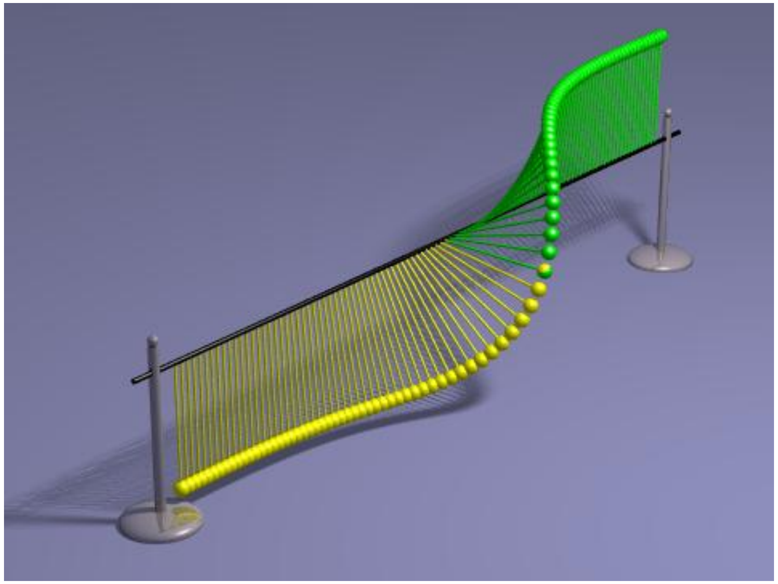
📖 Goldobin et al., PRB **66**, 100508 (2002)

Mechanical analog:pendula chain

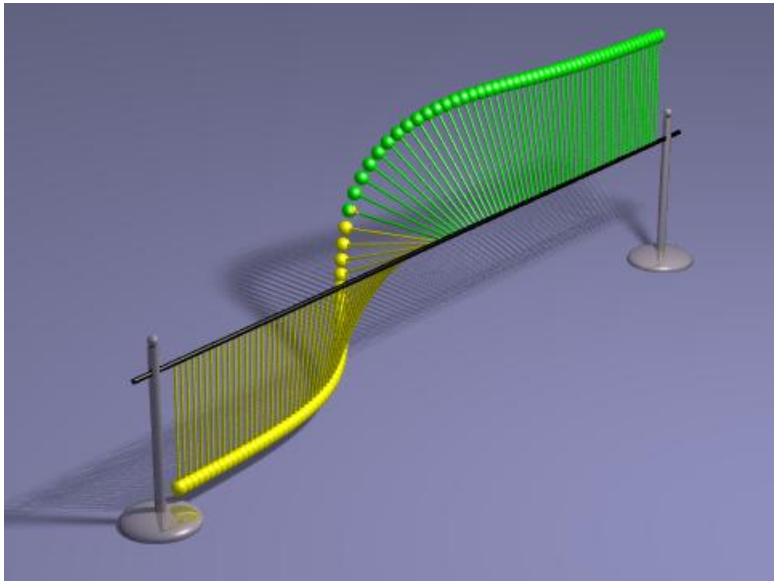


$\phi(x)$



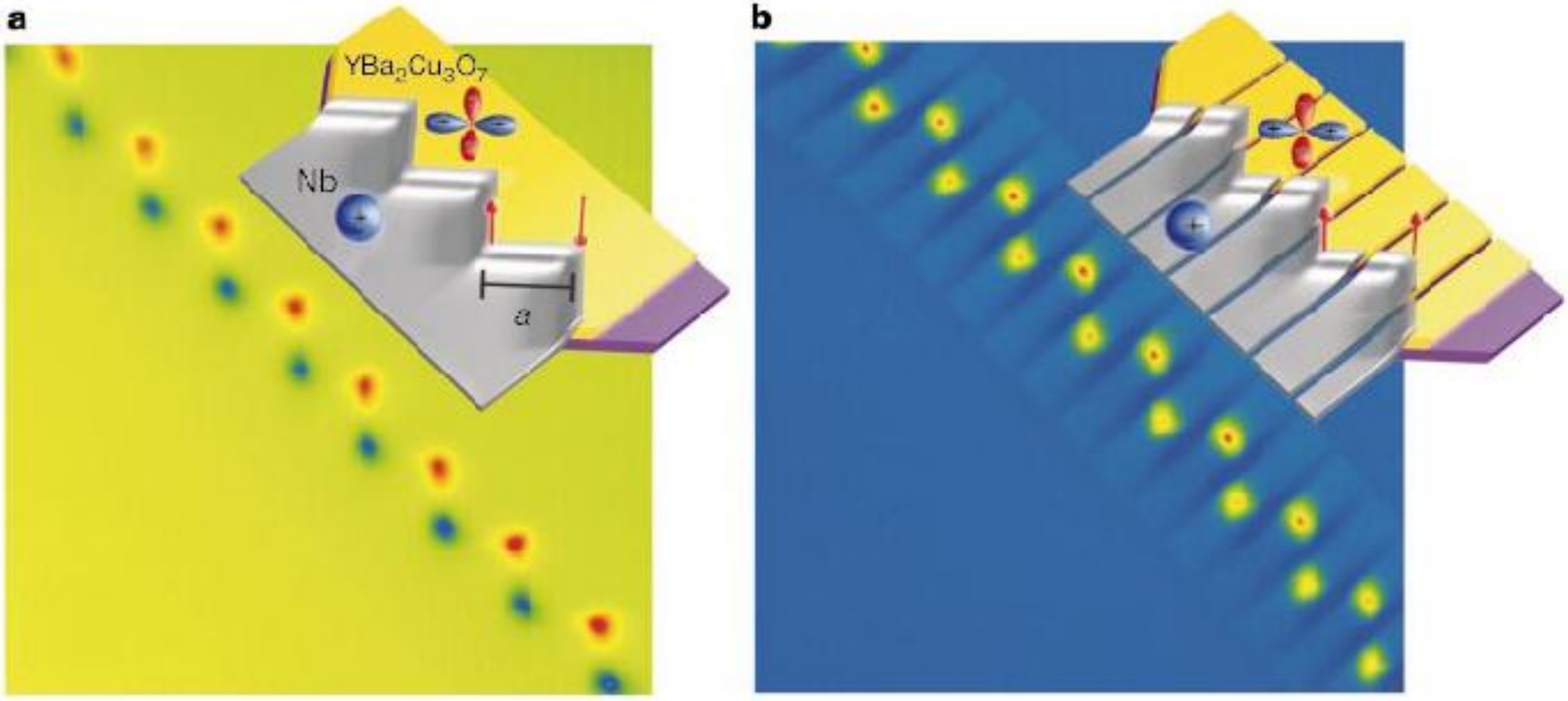


$\mu(x)$



Semifluxons observation

SQUID microscopy on YBCO-Nb ramp zigzag LJJs



📖 H. Hilgenkamp et al. Nature **422**, 50 (2003).

Ground states

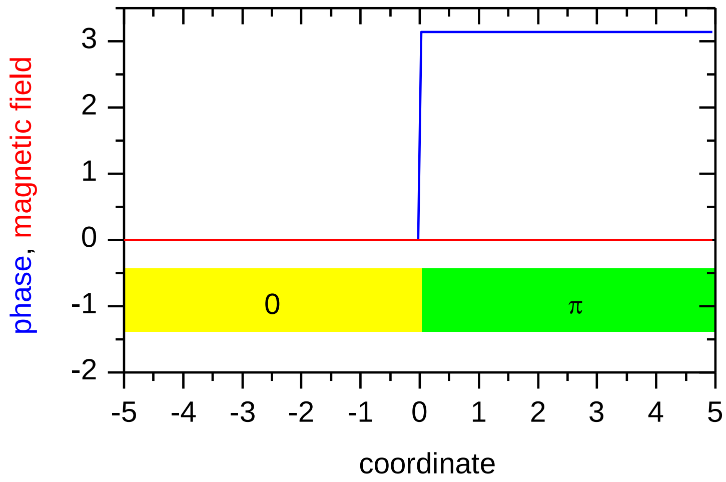
When do the semifluxons appear?

When the semifluxon solution is stable?

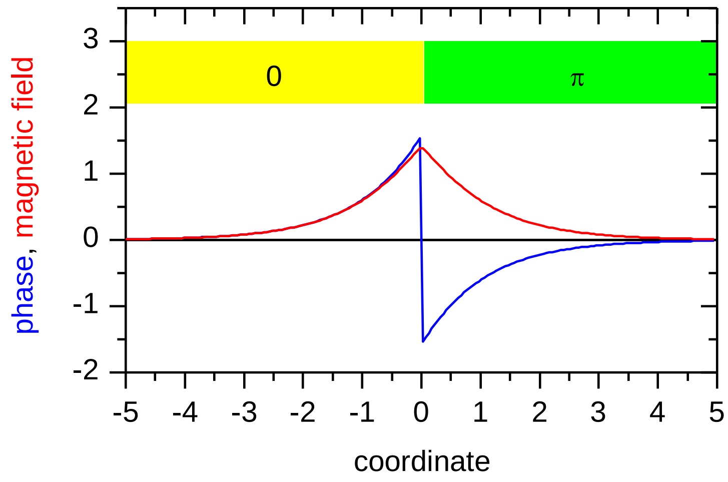
0-π boundary: semifluxon vs. μ=0

$$\phi_{xx} - \sin(\phi) = \theta_{xx}(x)$$

flat phase state



semifluxon



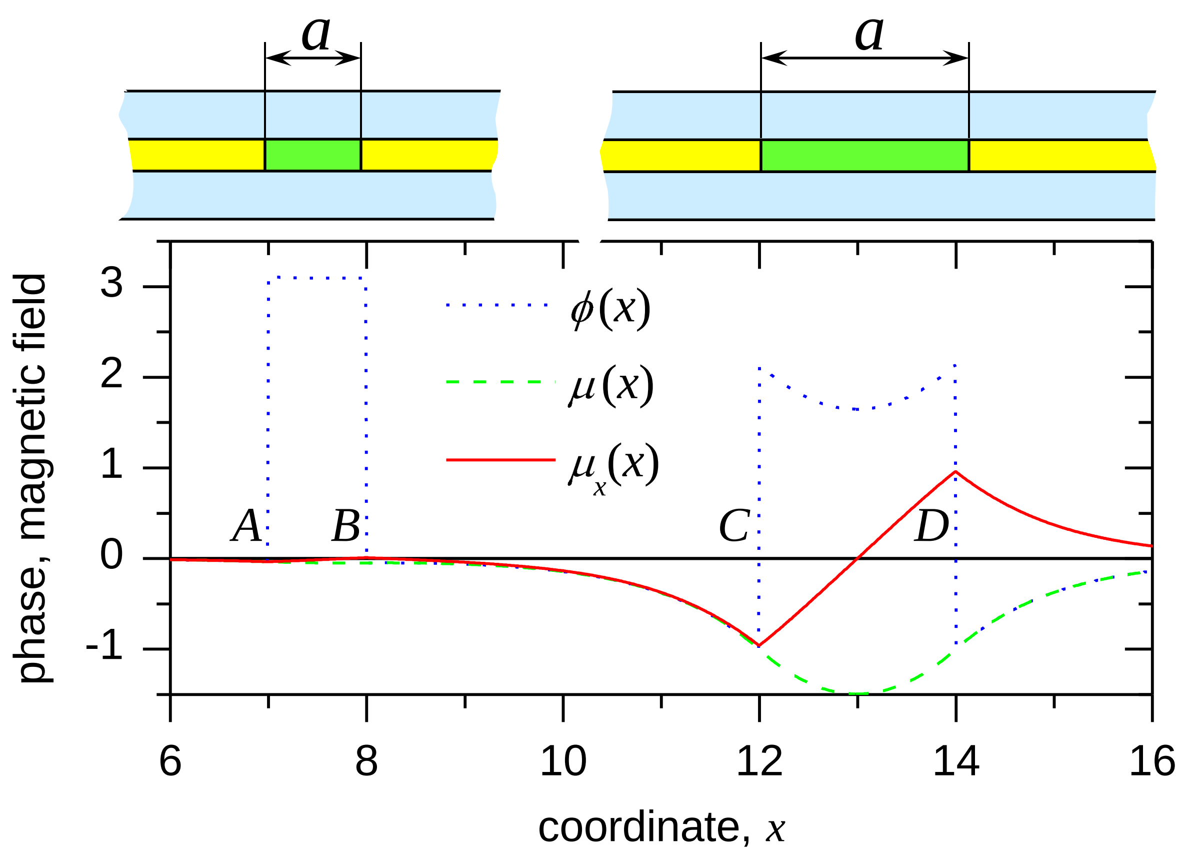
The energy of flat phase is 2 per unit of length, i.e. diverges at large L.

$$U = 16\frac{\mathcal{G}^2}{1+\mathcal{G}^2} = 8 - 4\sqrt{2} \approx 2.343$$

- One 0-π-boundary:
  - always semifluxon
  - flux-less flat phase solution (0-π) is unstable (has infinite energy)!

📖 E. Goldobin et al. Phys. Rev. B **67**, 224515 (2003)

π-facet of length a: ↑↓ vs. μ=0

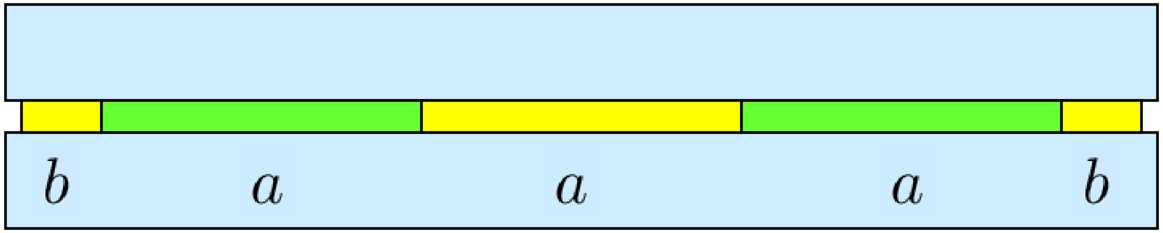


- ➔ Two 0-π-boundaries at a distance a:
  - ➔ semifluxons in ↑↓ state are formed for $a > a_c$
  - ➔ flux-less flat phase solution (0–π–0) for $a < a_c$.
- ➔ Important e.g. for SQUID microscopy!

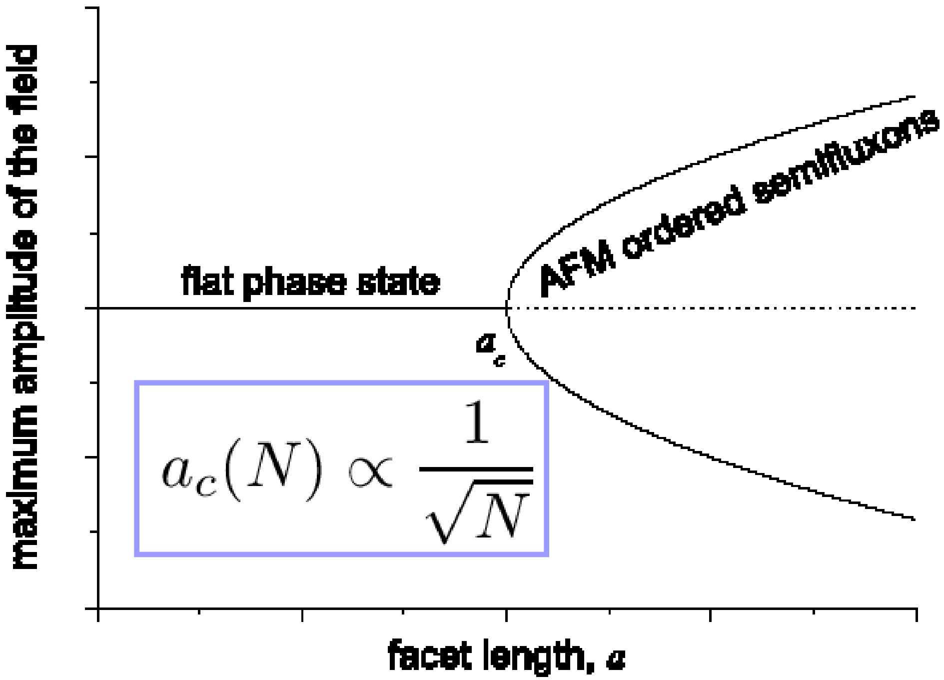
$a_c \sim \pi/2\ \lambda_J$

📖 Goldobin et al., Phys. Rev. B **67**, 224515 (2003)

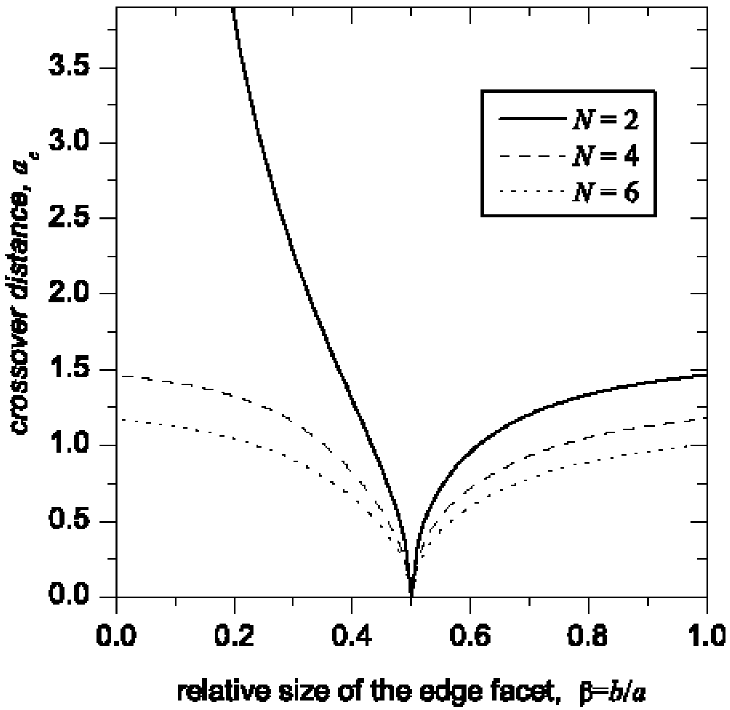
Behavior of $a_c^{(N)}(b)$



For odd N $a_c^{(N)} = 0$.



$$a_c(N) \propto \frac{1}{\sqrt{N}}$$

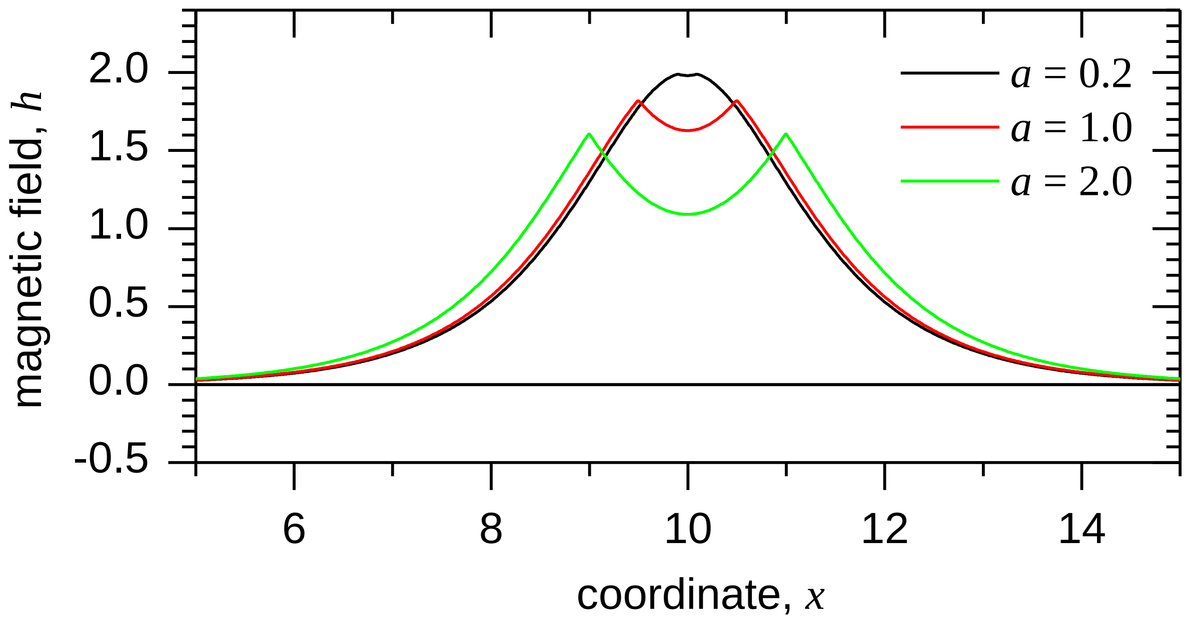


$a_c^{(2)} \approx 1.55 \pm 0.05$ $a_c^{(4)} = 1.35 \pm 0.05$ $a_c^{(6)} = 1.15 \pm 0.05$

📖 A. Zenchuk, E. Goldobin, PRB **69**, 024515 (2004).

Unipolar (FM) states

- semifluxon+semifluxon = fluxon!



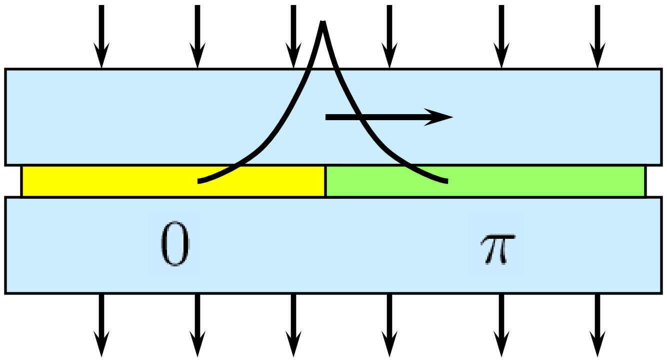
$$U_{\uparrow\uparrow}(\infty) = 2U_{SF} \approx 4.6, \quad U_{\uparrow\uparrow}(0) = U_F = 8$$

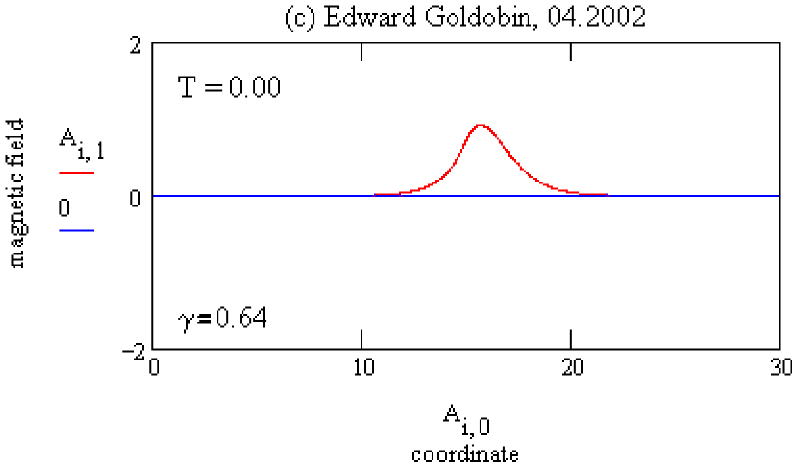
$$U_{\uparrow\downarrow}(\infty) = 2U_{SF} \approx 4.6, \quad U_{\uparrow\downarrow}(0) = 0$$

$$U_{\uparrow\uparrow}(a) \geq U_{\uparrow\downarrow}(a)$$

Switching on the current...

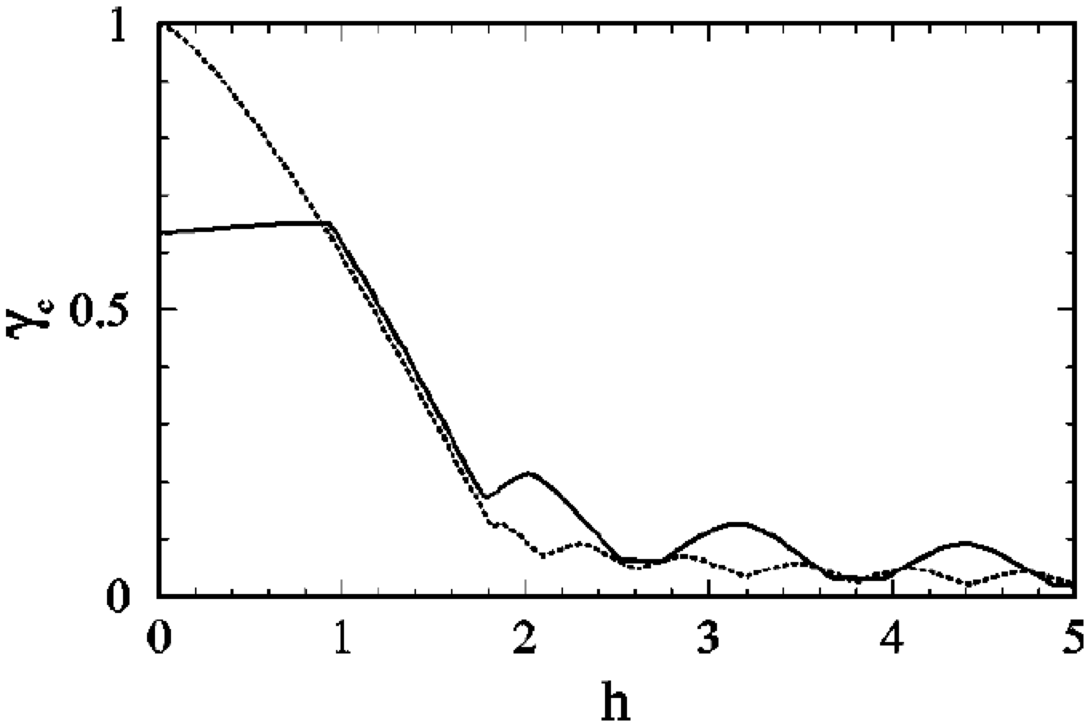
- at zero bias we have a static pinned semifluxon.
- bias current pulls semifluxons, just like fluxons.
- but semifluxons are pinned --> deformation
- at bias=$2/\pi\ I_c \approx 0.64\ I_c$ switching to the R-state





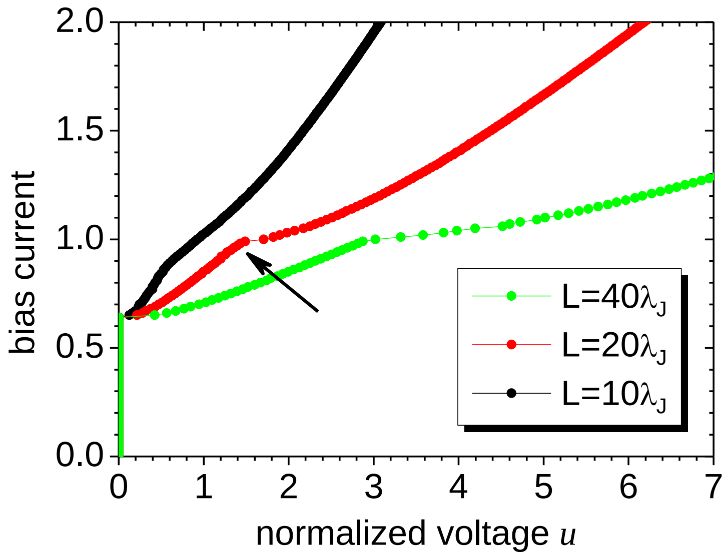
📖 Goldobin et al., PRB **67**, 224515 (2003).

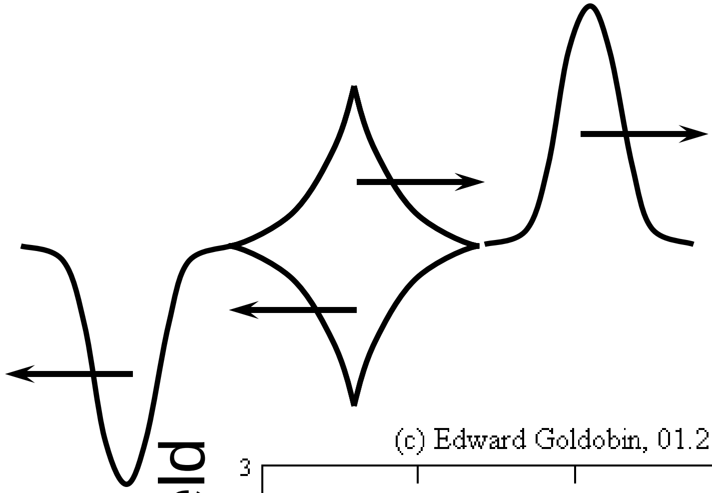
$I_c(H)$ of long 0-π JJ

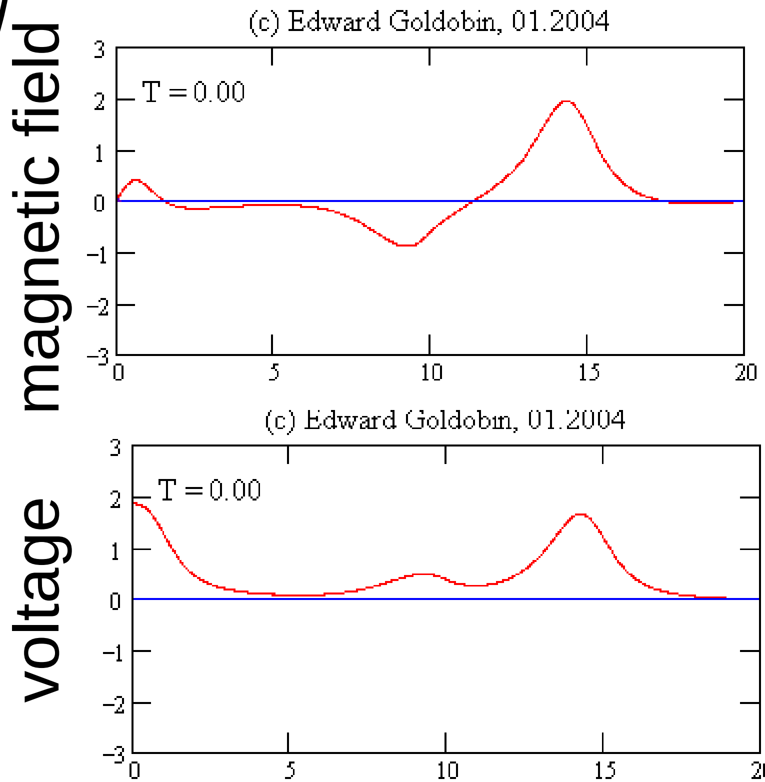


📖 N. Lazarides, PRB **69**, 212501 (2004).

Semifluxon based oscillator







Frequency depends on:

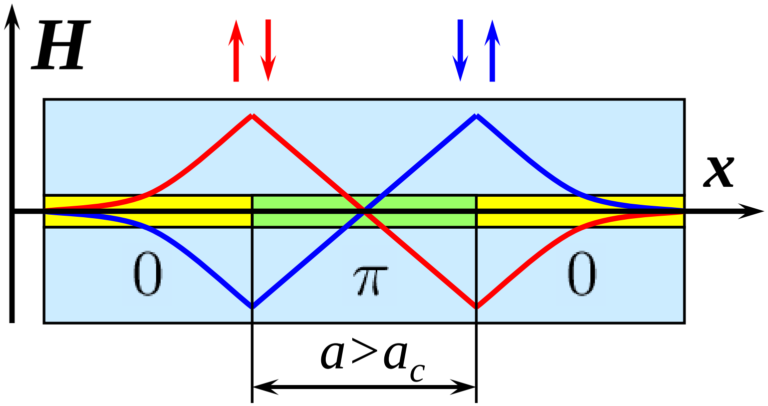
- bias current, damping, length
- two outputs shifted by 180°
- more stable than flux-flow due to semifluxon pinning

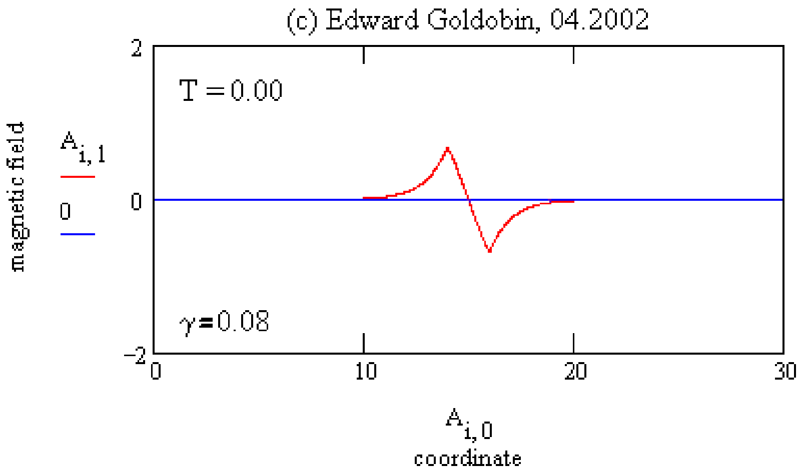
N. Lazarides, PRB **69**, 212501 (2004).

Two biased semifluxons

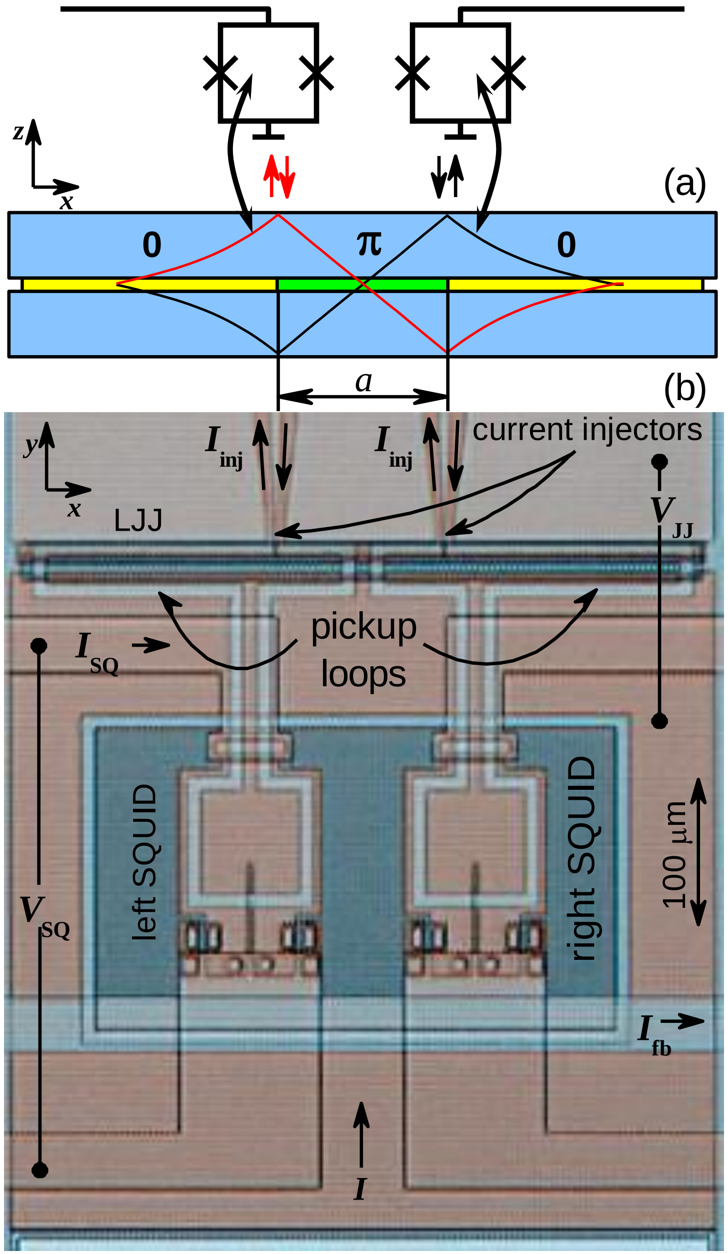
- The distance between 0-π boundaries $a > a_c$
- ↑↓ state at zero bias
- current pushes semifluxons to each other => swap

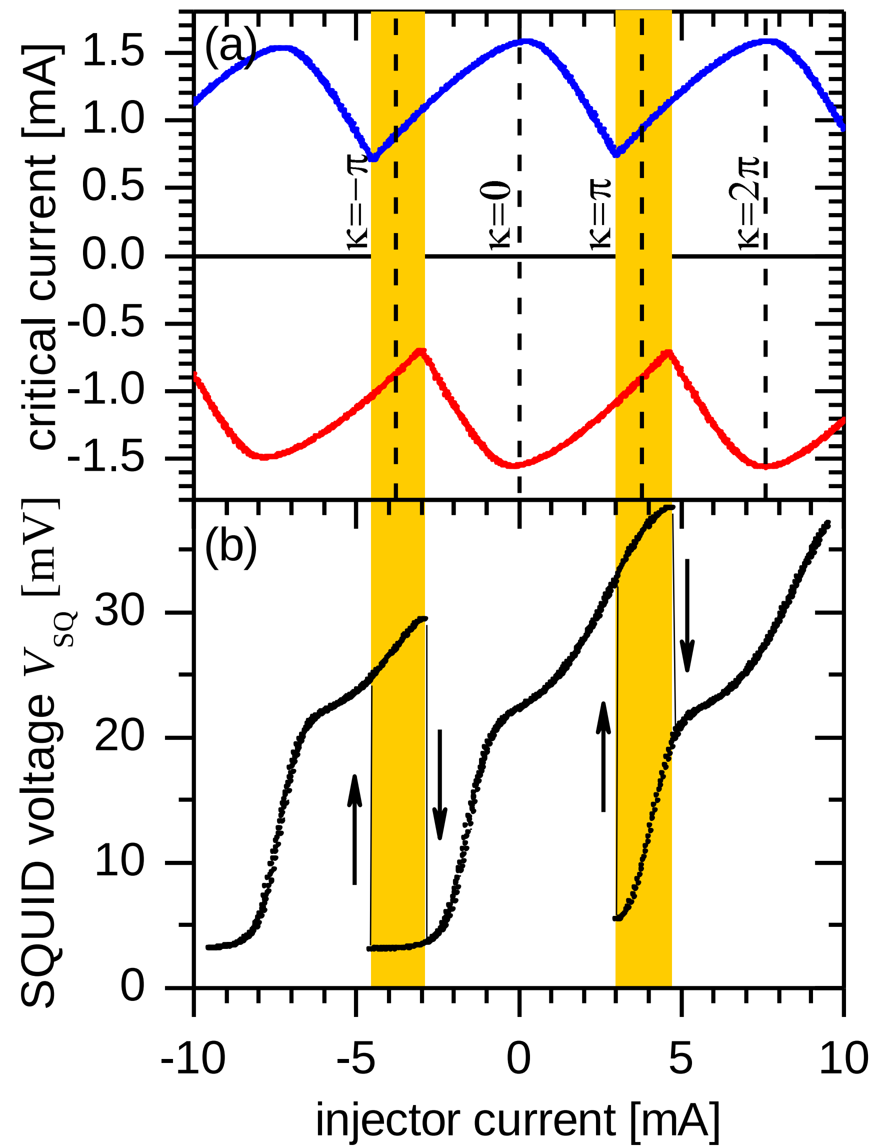
$$\uparrow\downarrow \xrightarrow{\gamma=0.08} \downarrow\uparrow$$





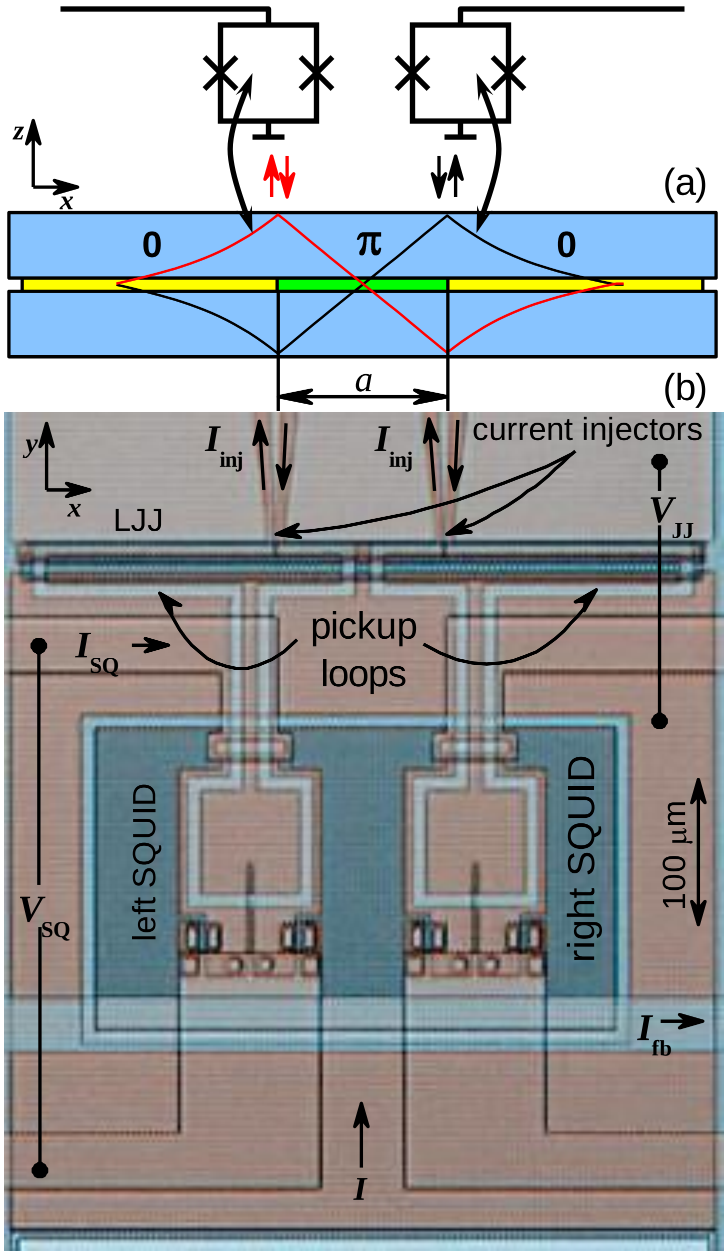
Sample & injectors calibration

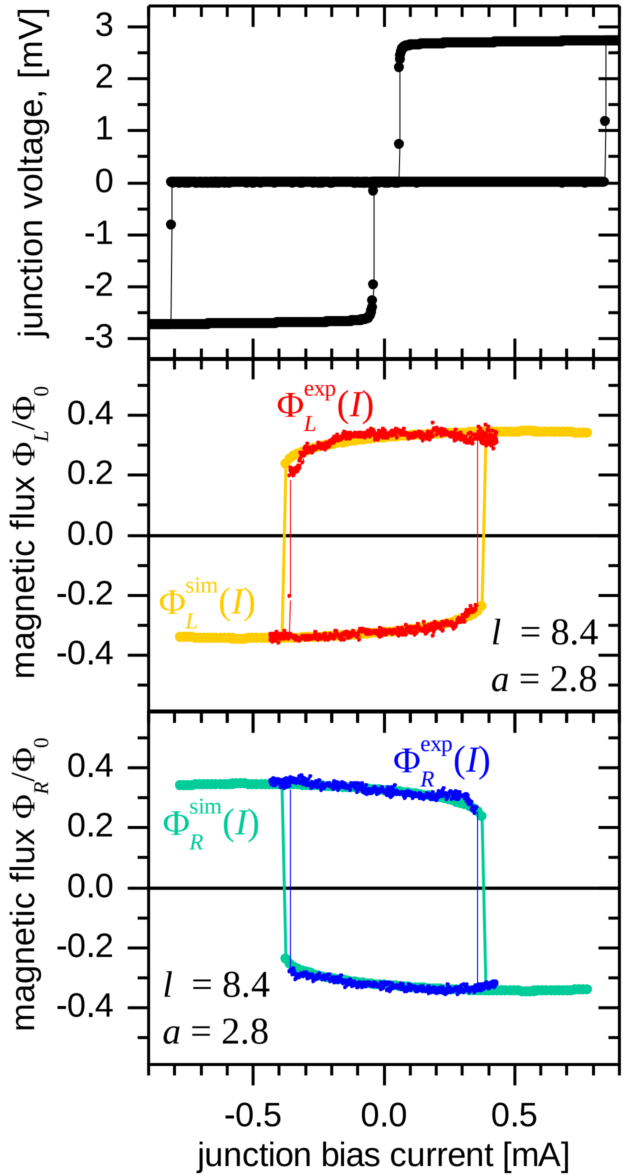




A. Dewes et al., PRL **101**, 247001, (2008)

Observed rearrangement: ↑↓↔↓↑



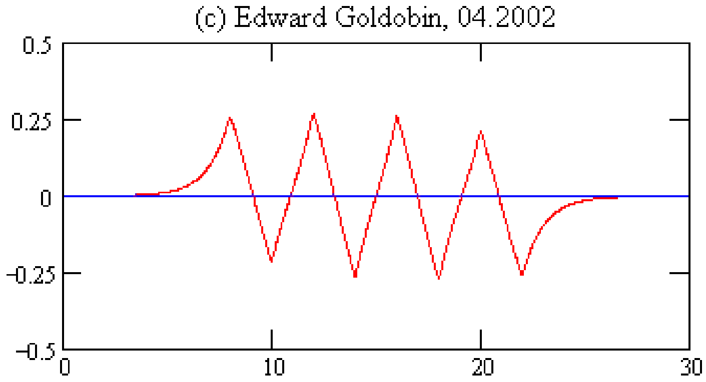


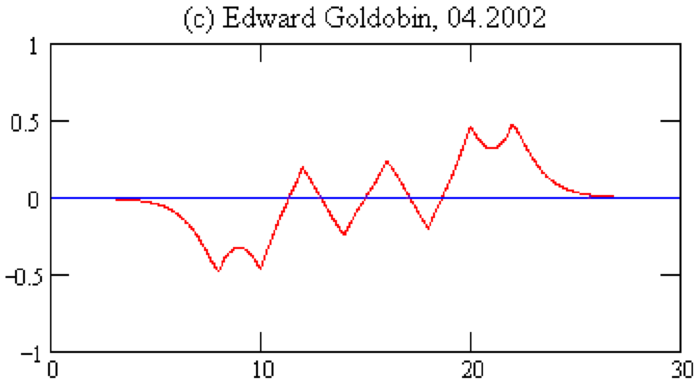
📖 A. Dewes et al., PRL **101**, 247001, (2008)

8 biased semifluxons

- The distance between corners $a > a_c$
- AFM ordered state at zero bias

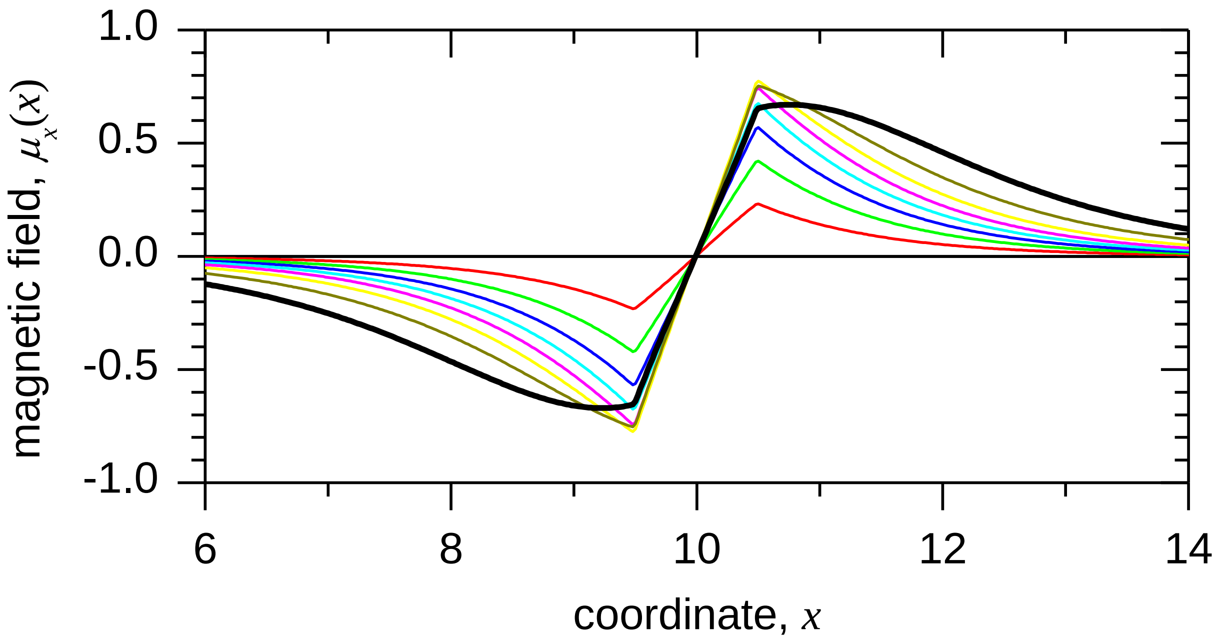
$$\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow \xrightarrow{\gamma=+0.13} \downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow \xrightarrow{\gamma=+0.23} \downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow$$





π facet of length $a<a_c$

- The distance between corners $a < a_c$
- flat phase state at zero bias
- increasing bias with step 0.1. $\gamma_c = 0.76$

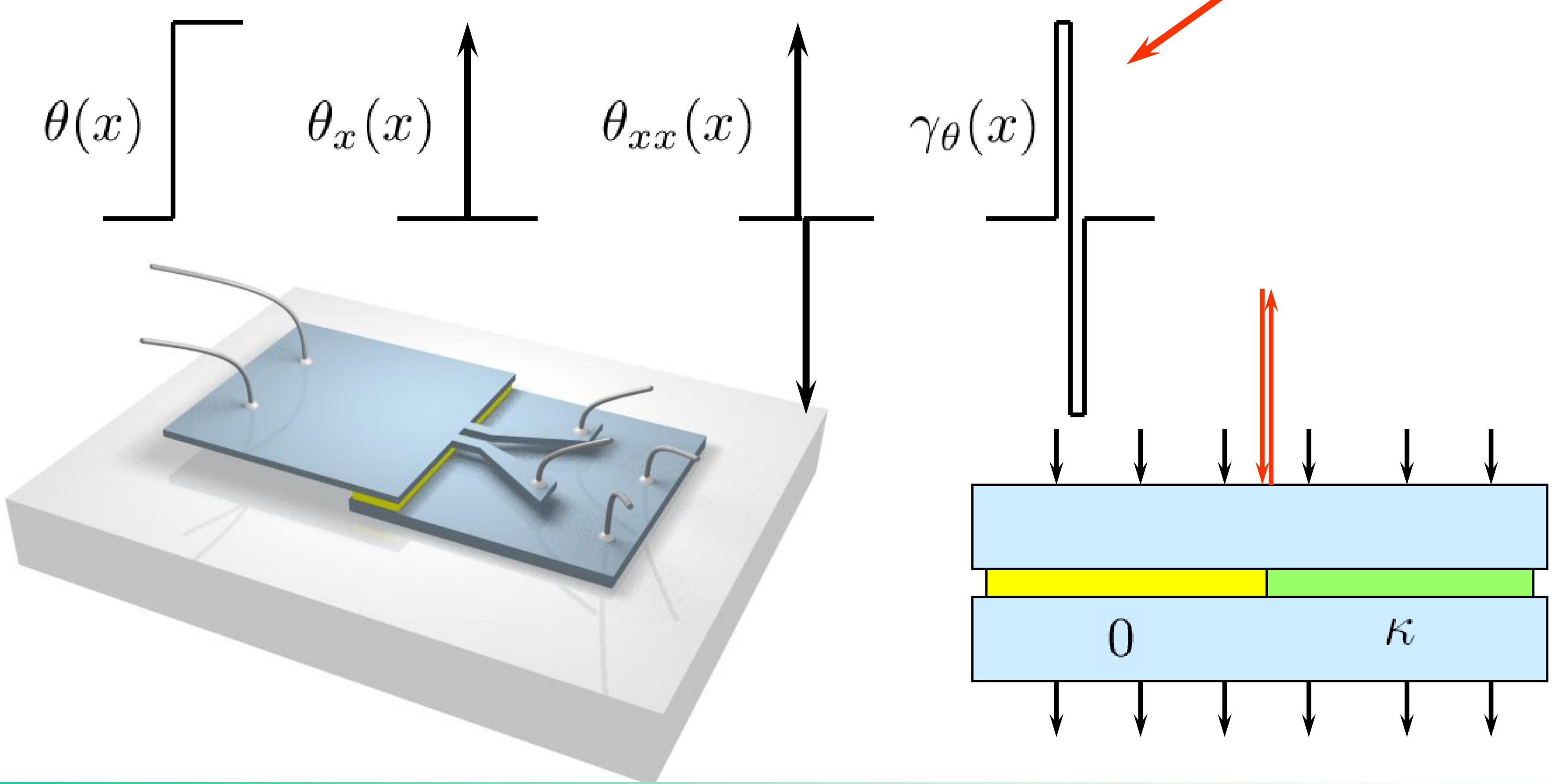


"semifluxons" emerge under the action of bias current

Goldobin et al., PRB **67**, 224515 (2003).

Artificial 0-κ junctions

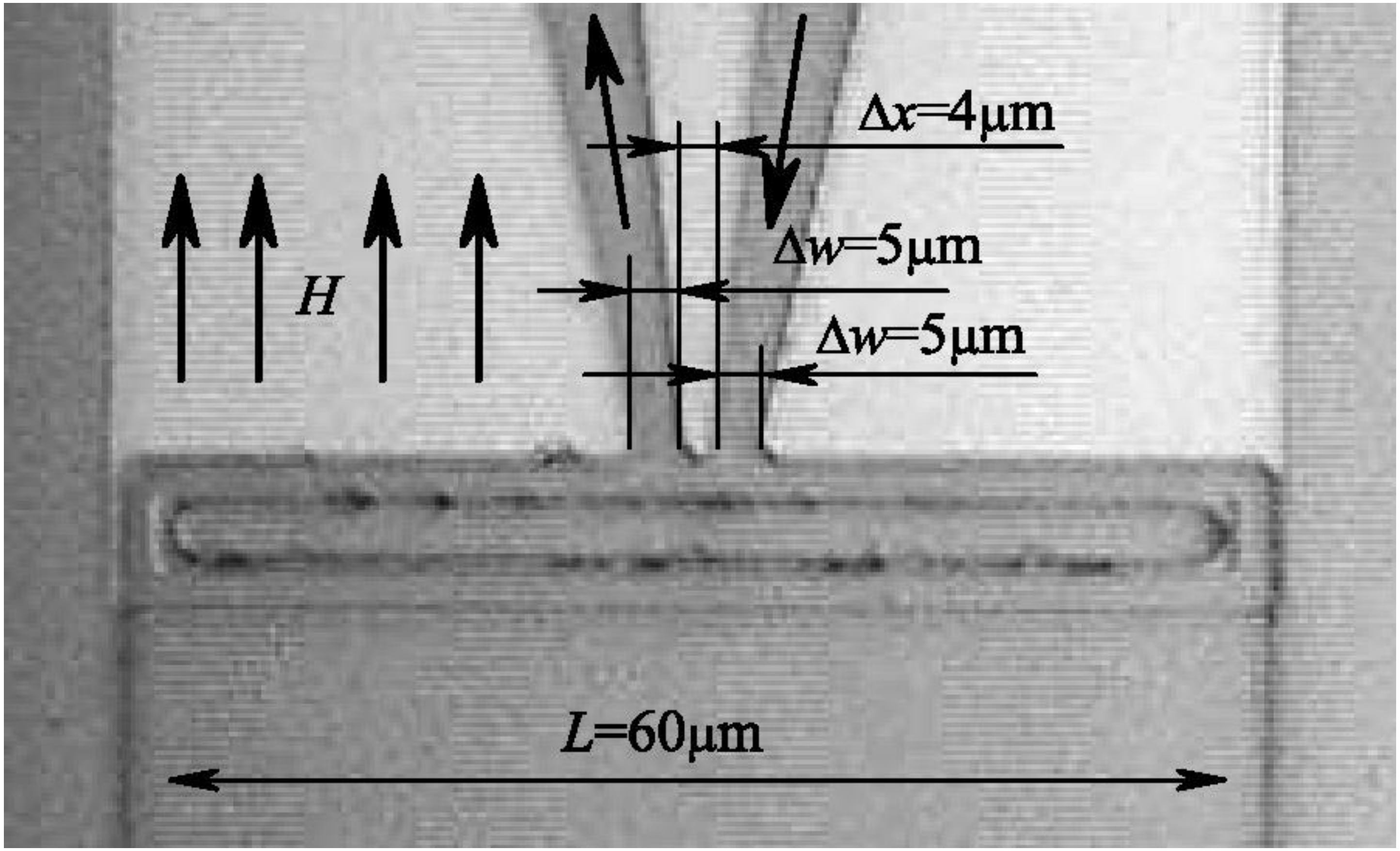
$$\phi_{xx} - \phi_{tt} - \sin(\phi) = \alpha\phi_t - \gamma(x) + \theta_{xx}(x)$$



A. Ustinov, Appl. Phys. Lett. **80**, 3153 (2000).

Goldobin et al., Phys. Rev. Lett. **92**, 057005 (2002)

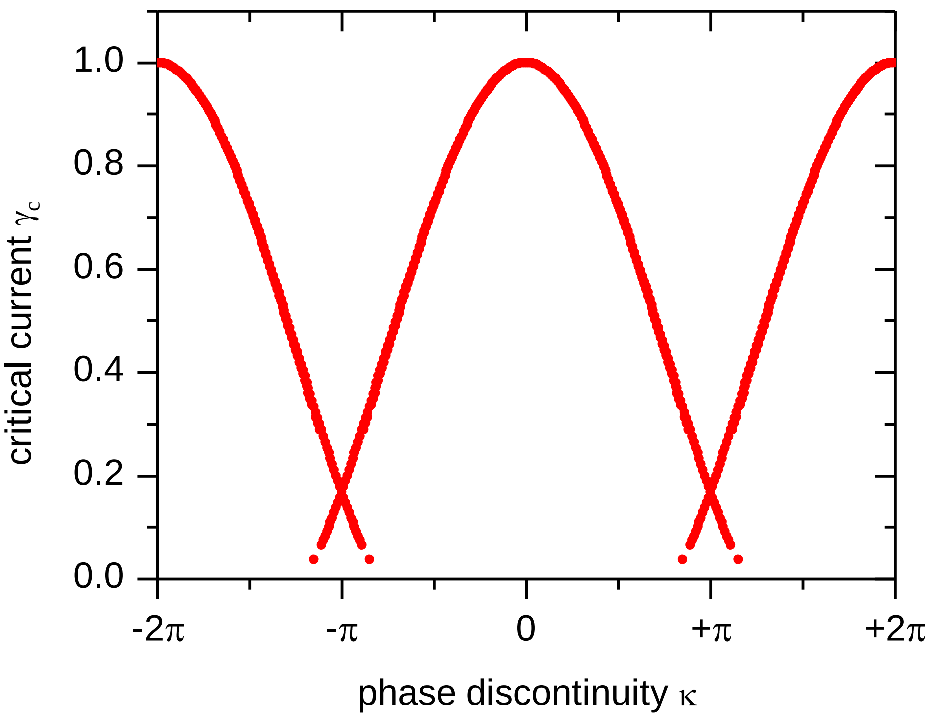
Nb LJJ with two injectors



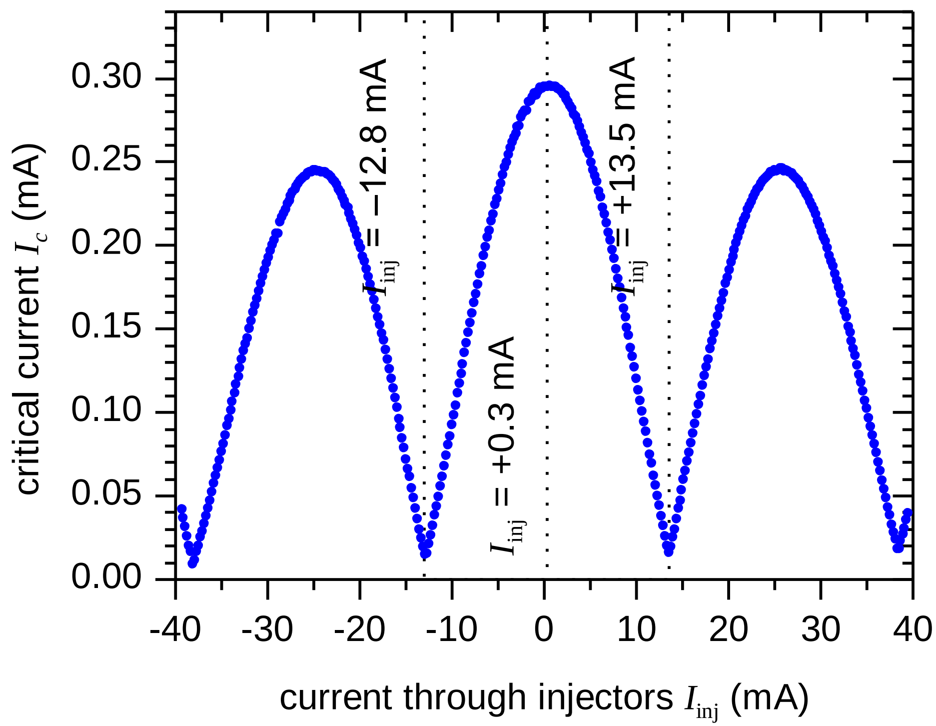
$\lambda_J \approx 30\ \mu\text{m}\ (j_c \approx 100\ \text{A/cm}^2)$

Calibration of injectors

Numerical

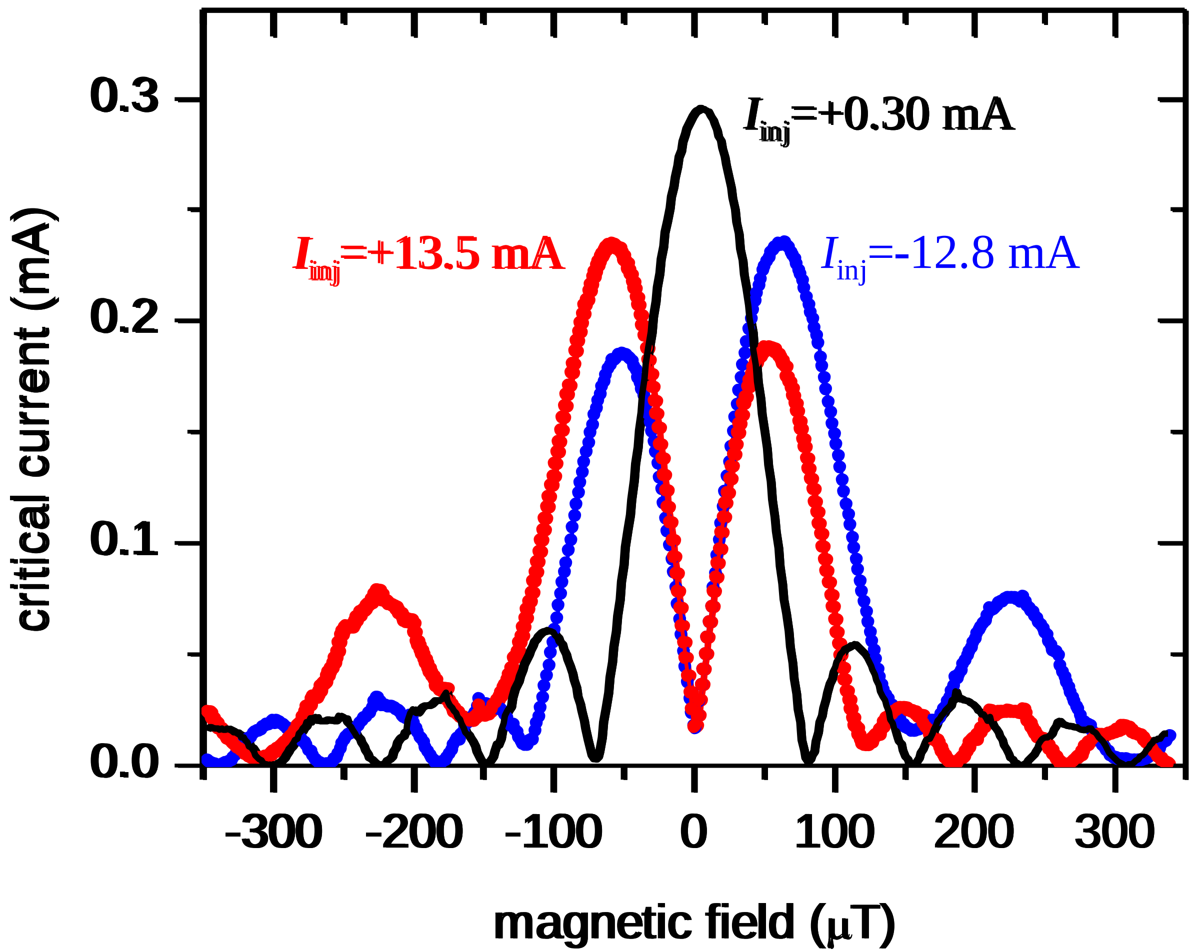


Experimental



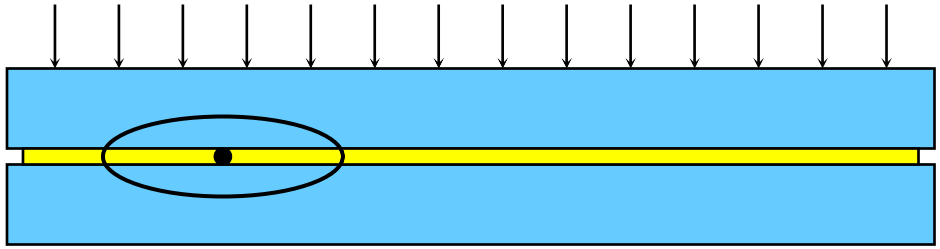
Goldobin et al., PRL **92**, 057005 (2004)

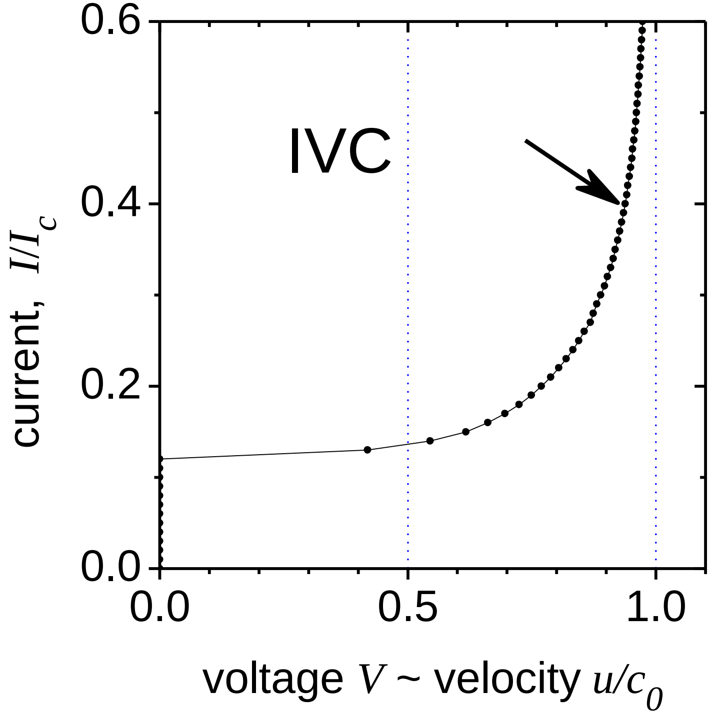
$I_c(H)$ in 0-0 and in 0-π states

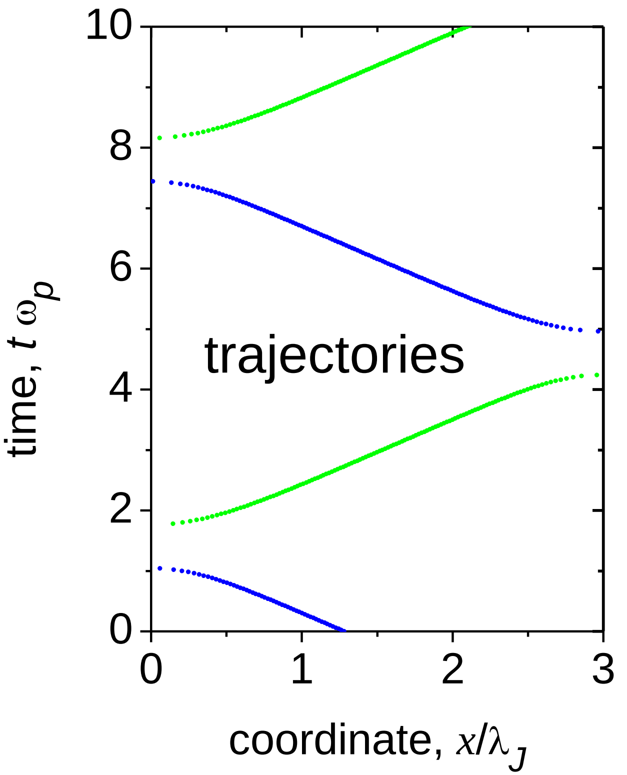


Goldobin et al., PRL **92**, 057005 (2004)

Classical Zero Field Step (ZFS)

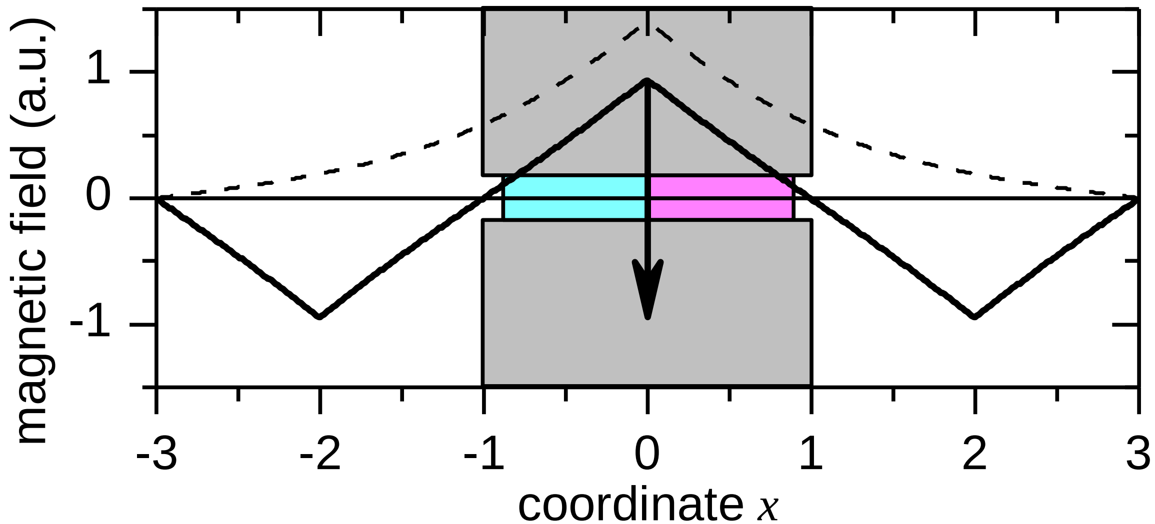






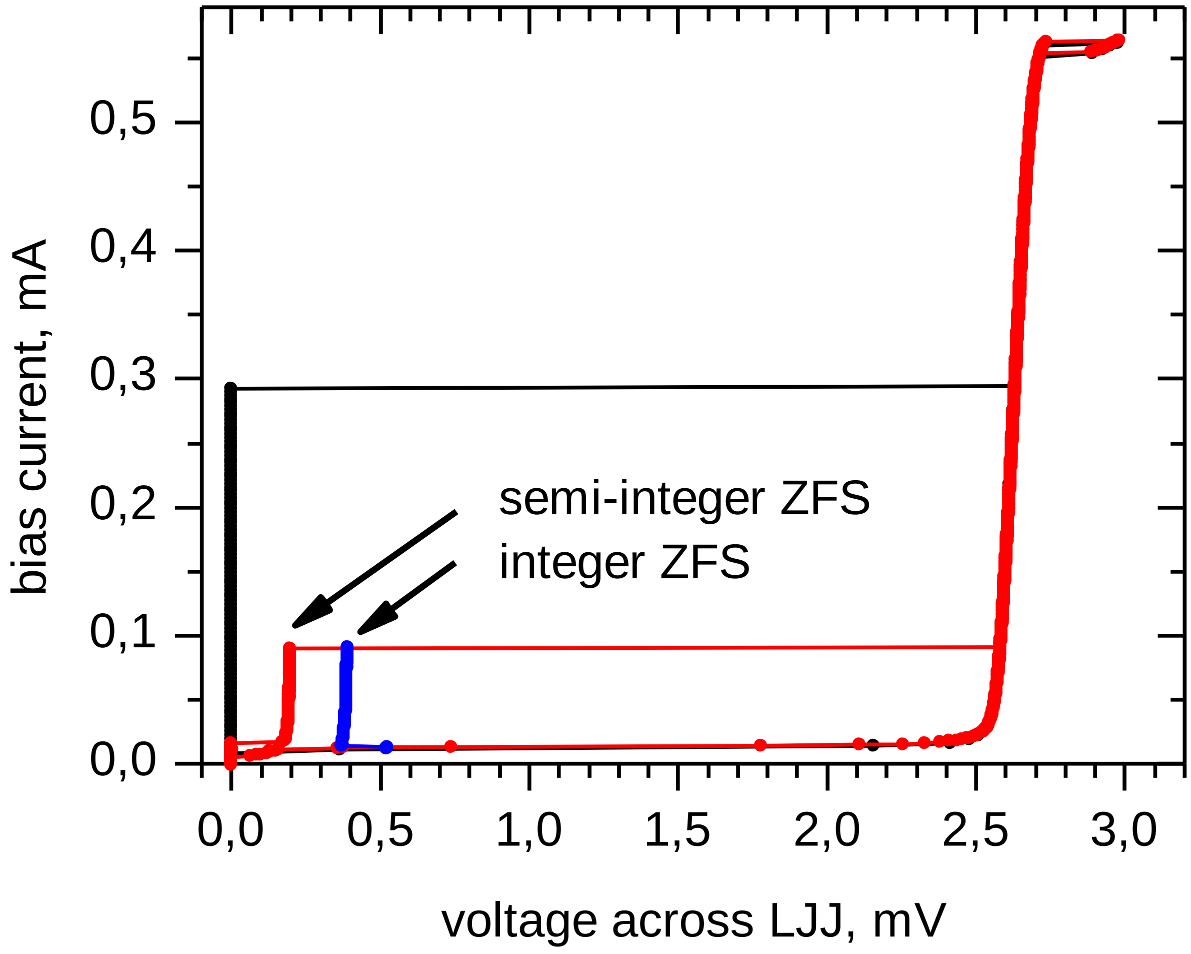
Semifluxon -- half-integer ZFS

- Finite length --> image technique:
  - 1 real semifluxon + 2 anti-semifluxons (images)
- Bias current → Force → SF hopping.



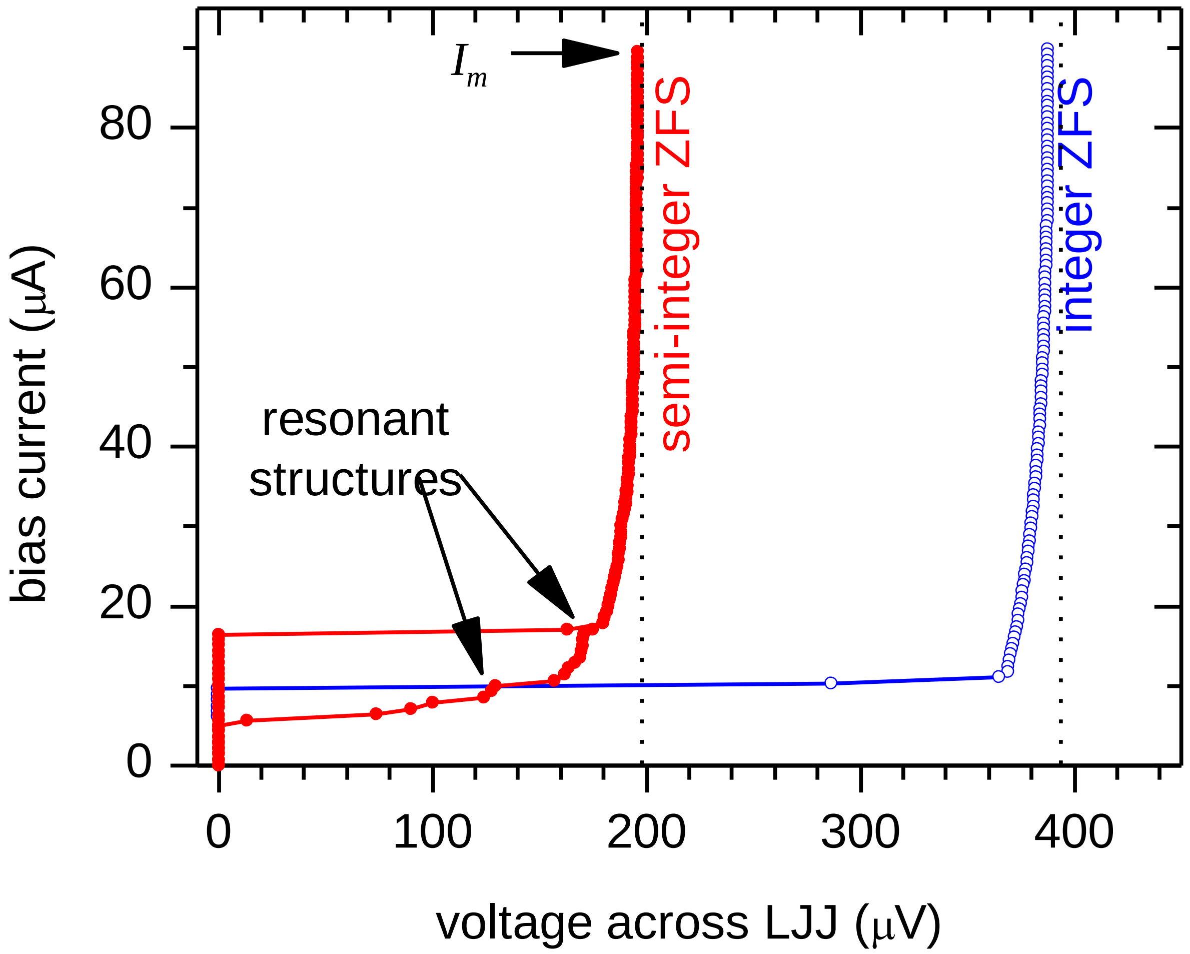
Goldobin et al., Phys. Rev. B **67**, 224515 (2003).

Half integer ZFS (full IVC)



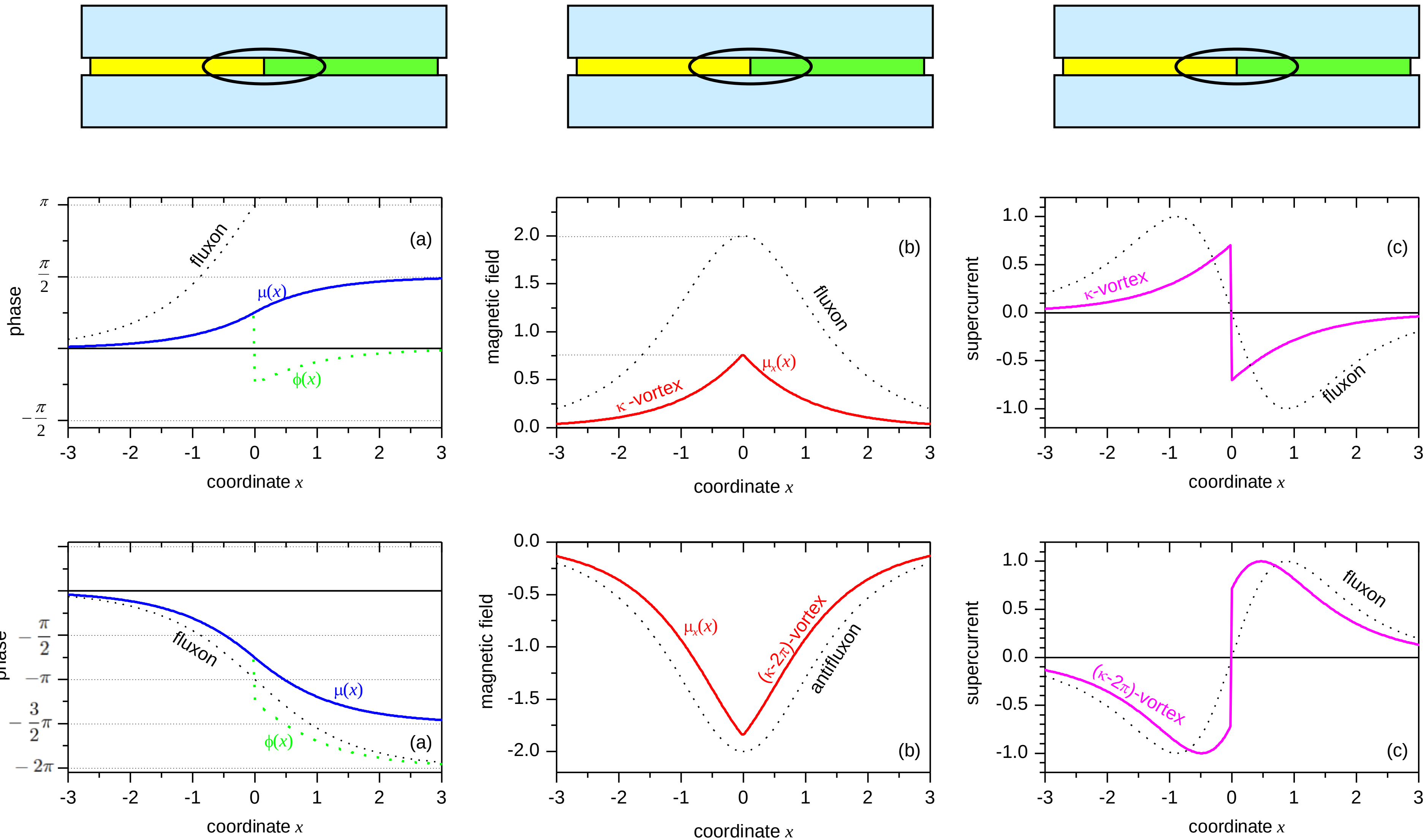
Goldobin et al., PRL **92**, 057005 (2004)

Half integer ZFS



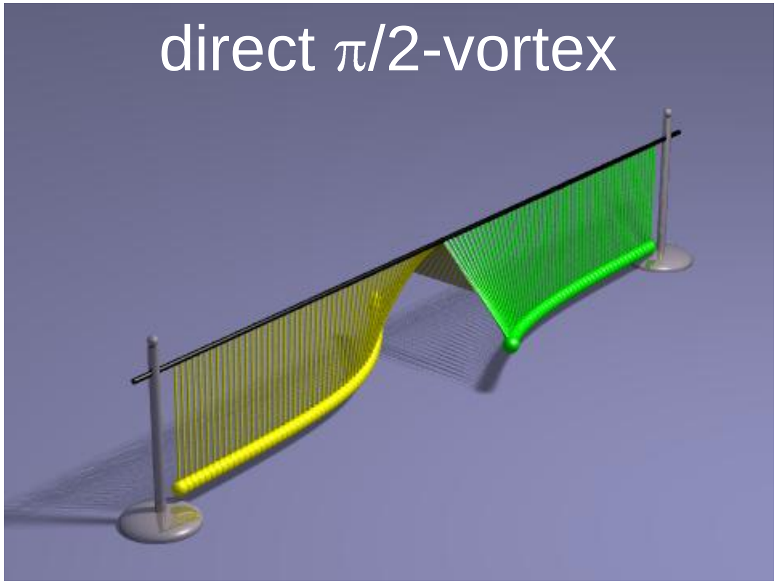
Goldobin et al., PRL **92**, 057005 (2004)

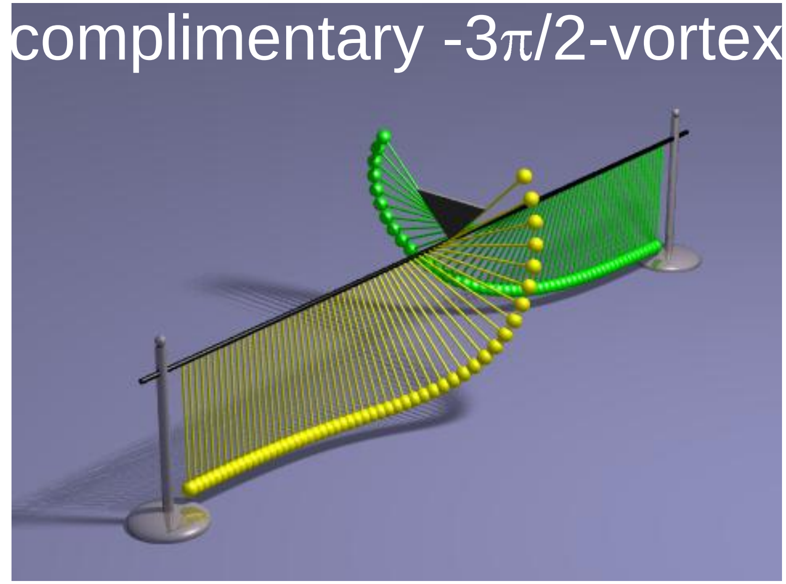
κ-vortex: broken symmetry



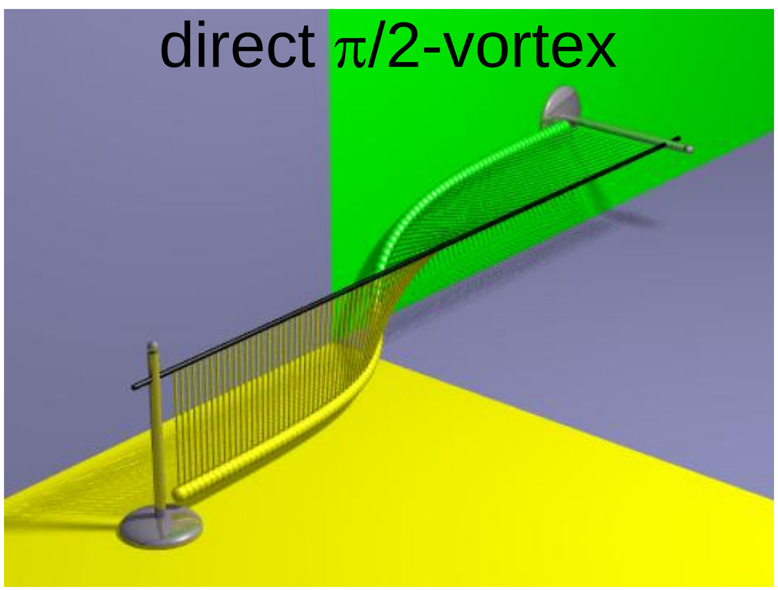
Goldobin et al., PRB **70**, 174519 (2004)

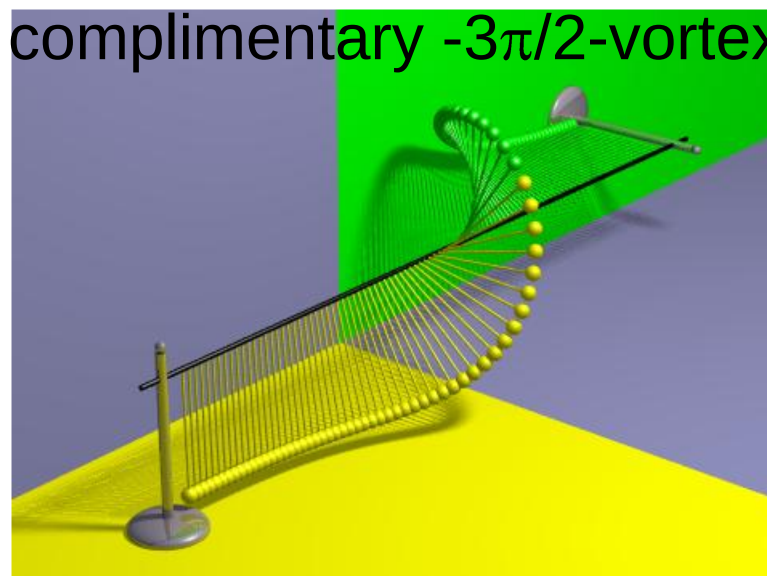
Mechanical analog:pendula chain





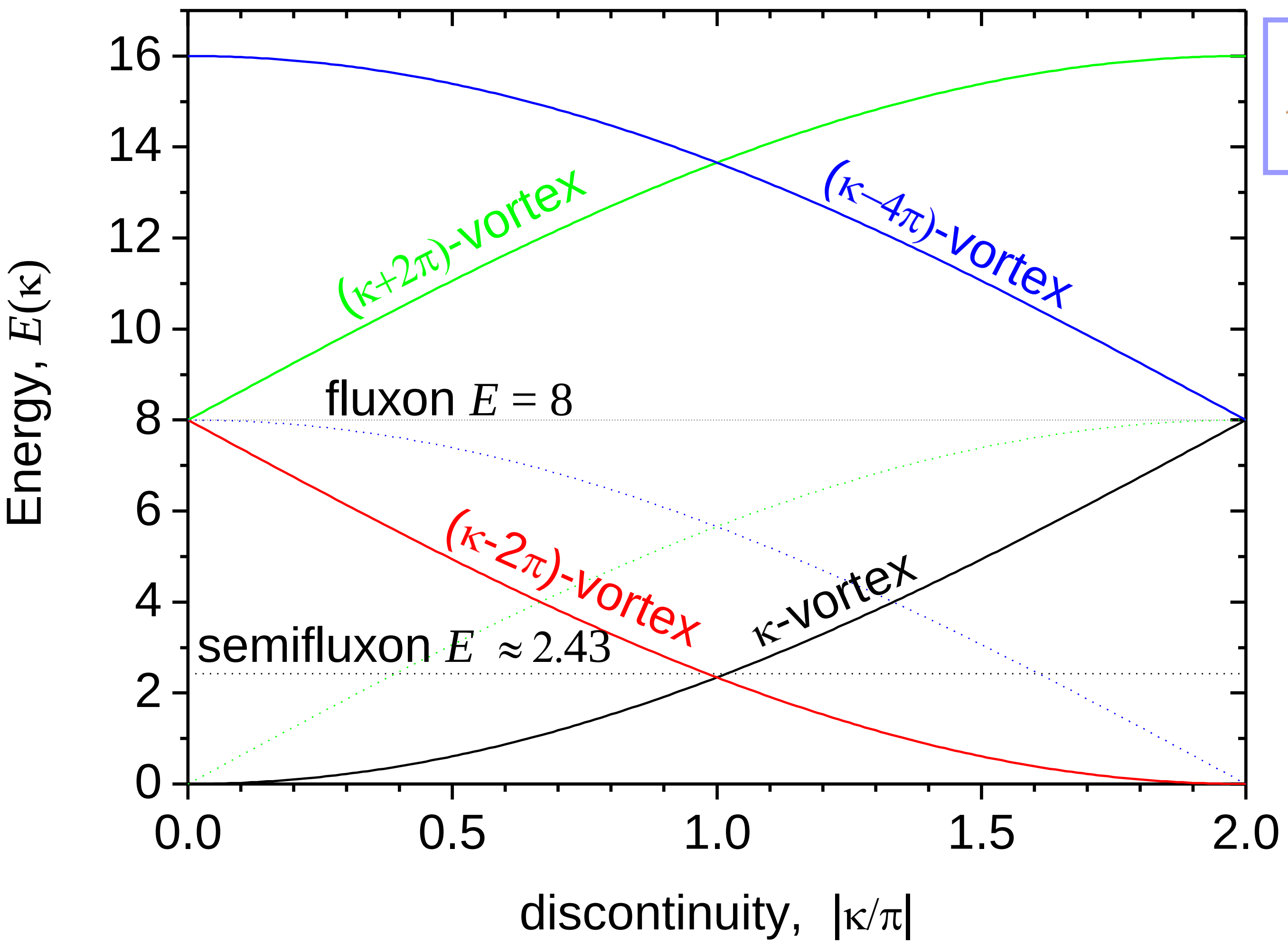
$\phi(x)$





$\mu(x)$

Energy of a single vortex

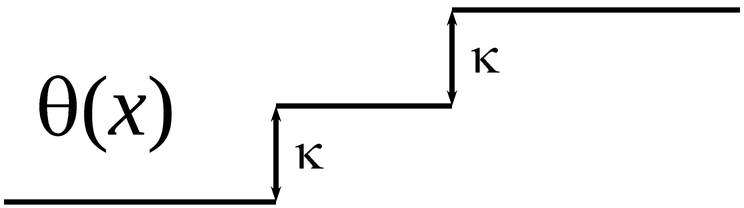


$$E(\kappa) = 16\sin^2\left(\frac{\kappa}{8}\right)$$

Kogan et al., PRB **61**, 9122 (2000)

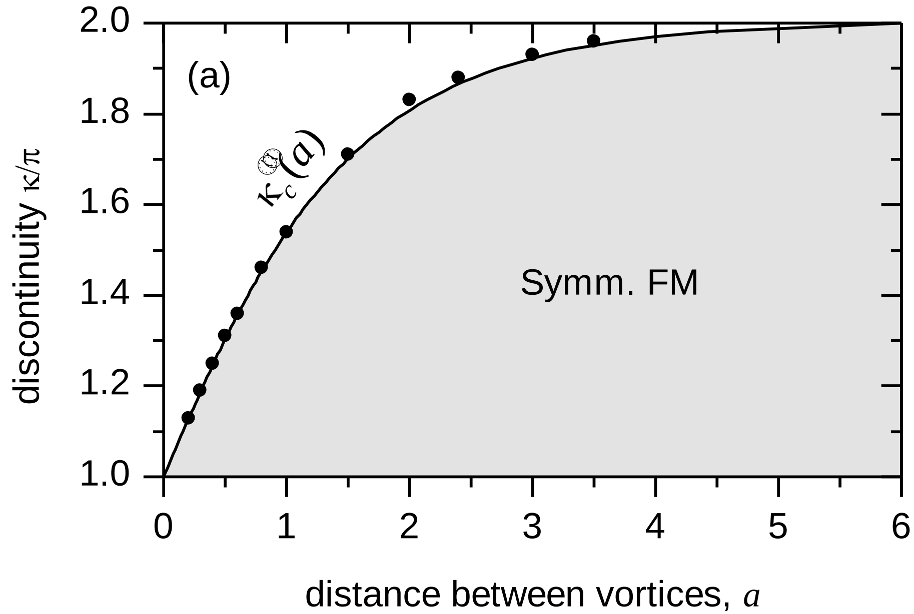
Goldobin et al., Phys. Rev. B **70**, 174519 (2004)

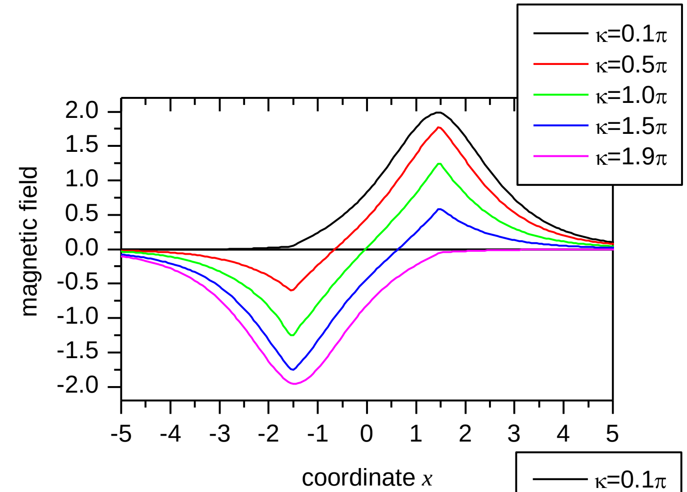
Ground states @ (+κ, +κ) discont.

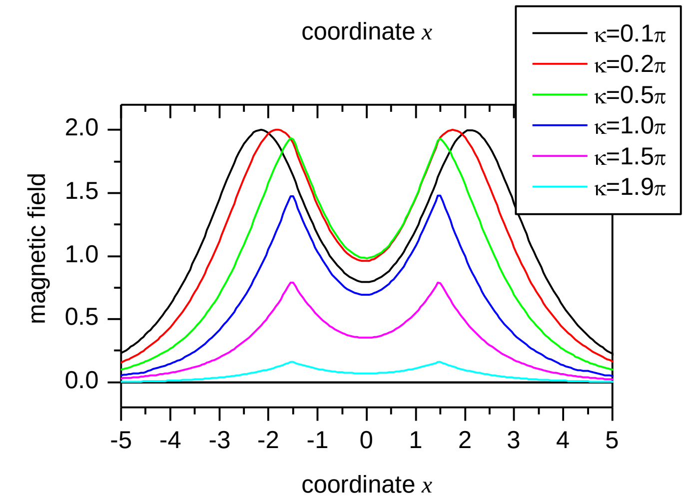


↓ ⇑ asymm. AFM

↓ ↓ symm. FM (⇑⇑)







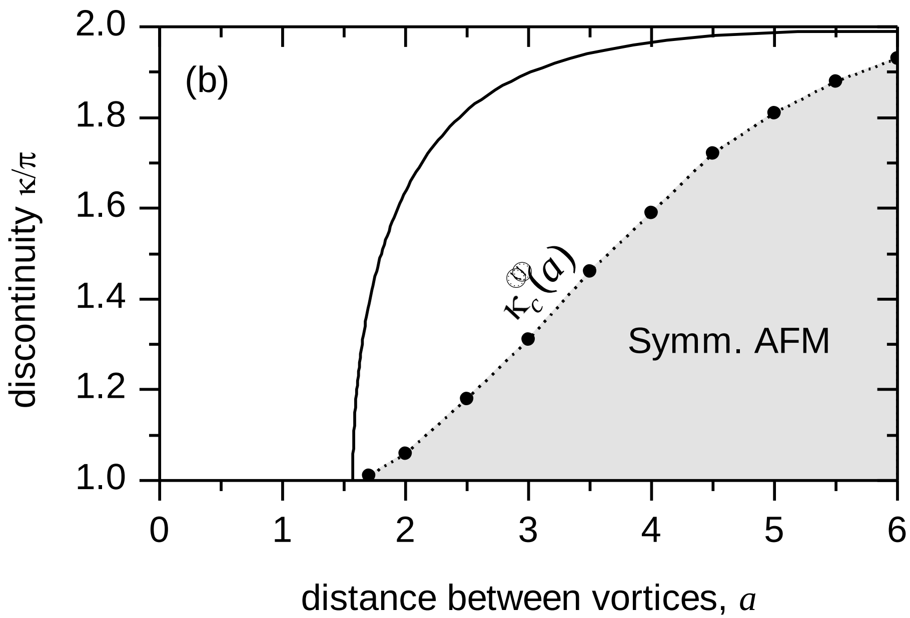
Crossover @ $\kappa_c = 0.07\pi$

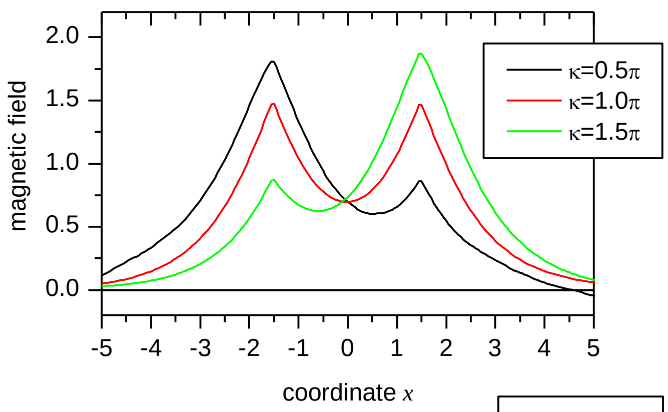
Goldobin et al., PRB **70**, 174519 (2004)

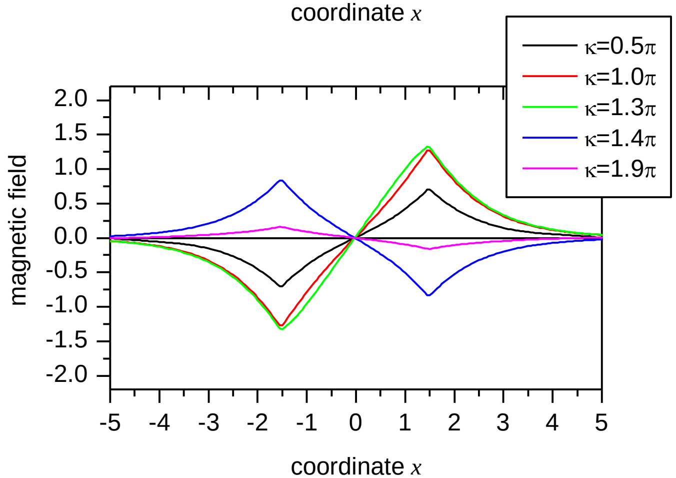
Ground states @ (+κ,−κ) discont.

↓ ⇓ (asymm. FM)

↓ ↑ (symm. AFM)



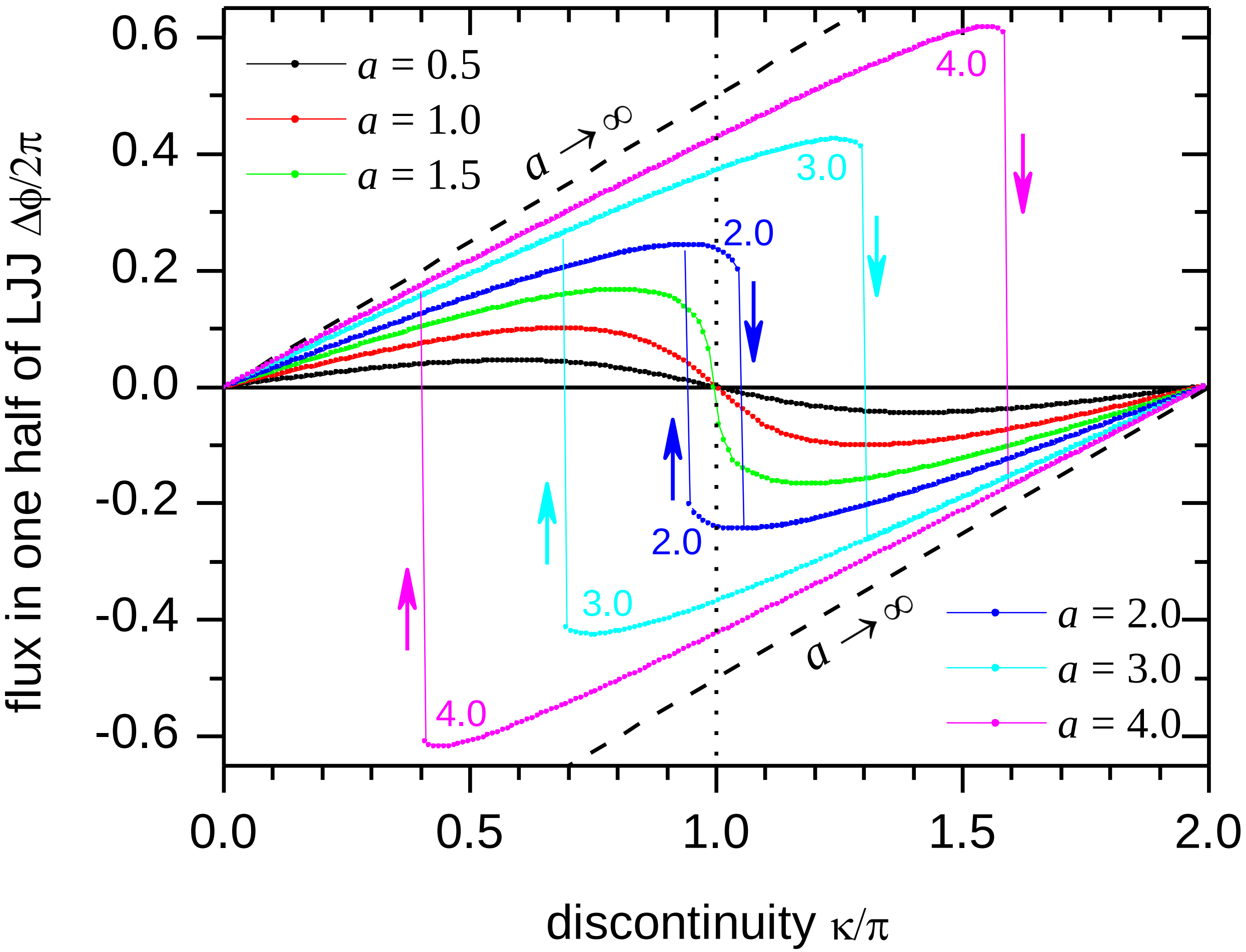




Bias may change crossover point!

Goldobin et al., PRB **70**, 174519 (2004)

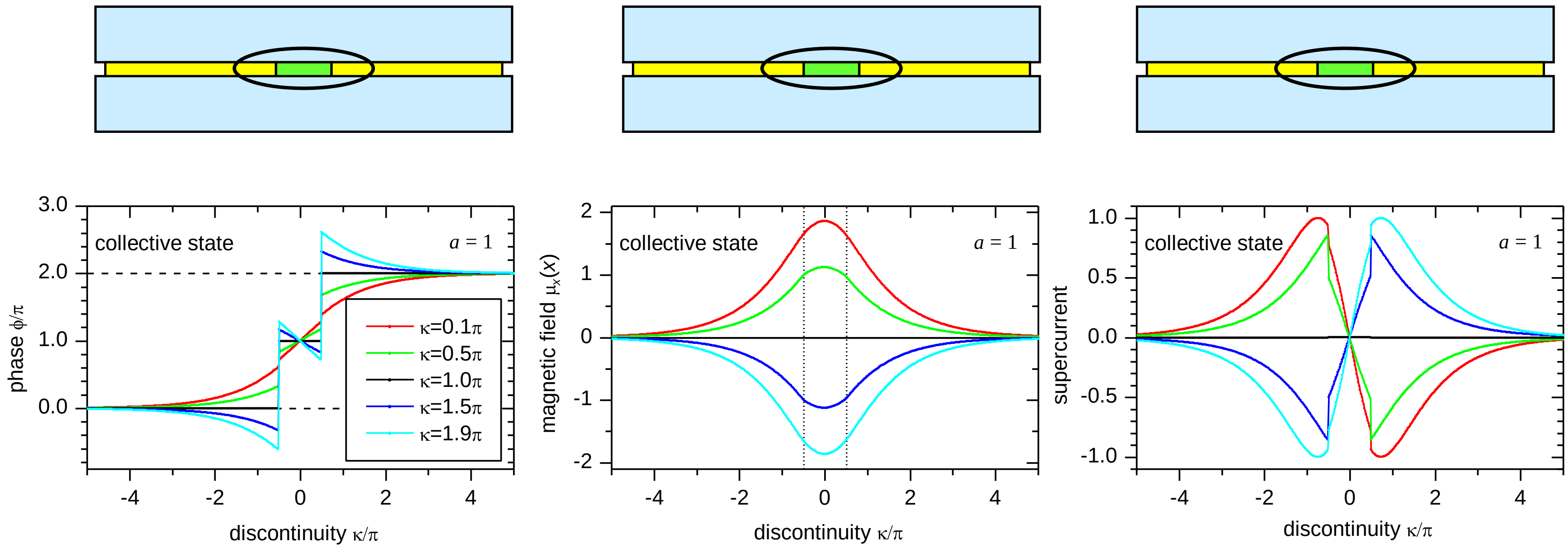
Crossover for symm. AFM state



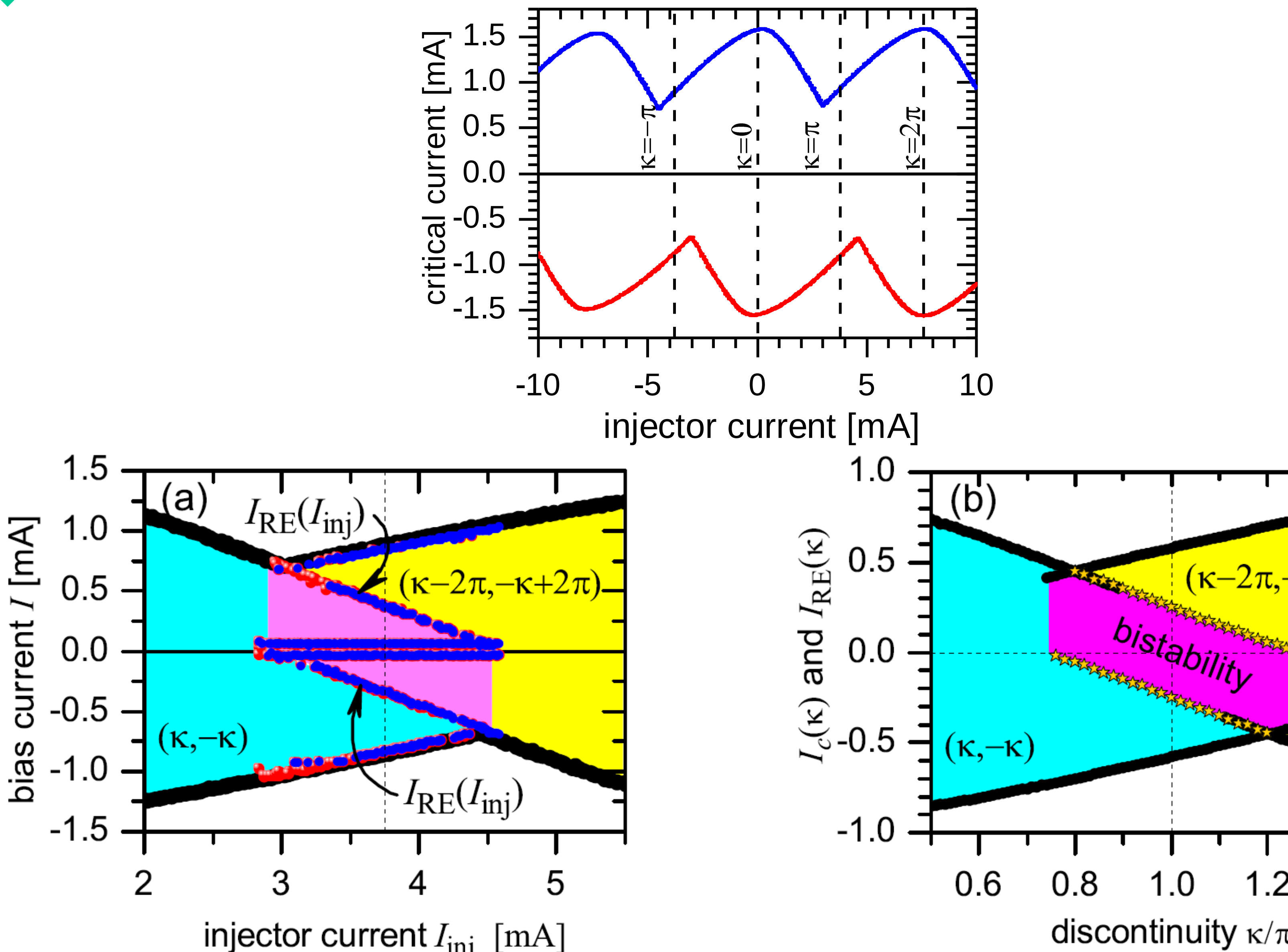
Goldobin et al., PRB **70**, 174519 (2004)

Crossover for asymm. AFM state

New: collective state!



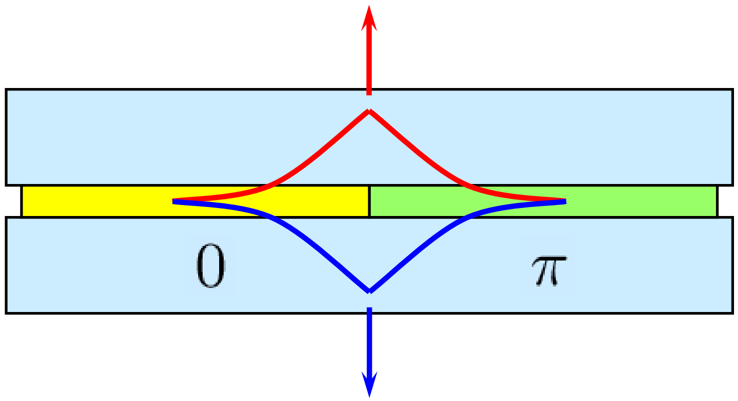
Bistability region (κ)



📖 A. Dewes et al., PRL **101**, 247001, (2008)

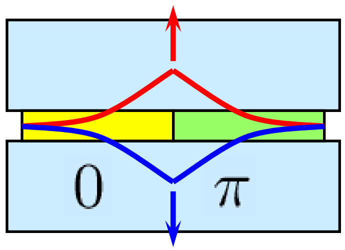
Quantum semifluxons?

MQC



long junction = NO

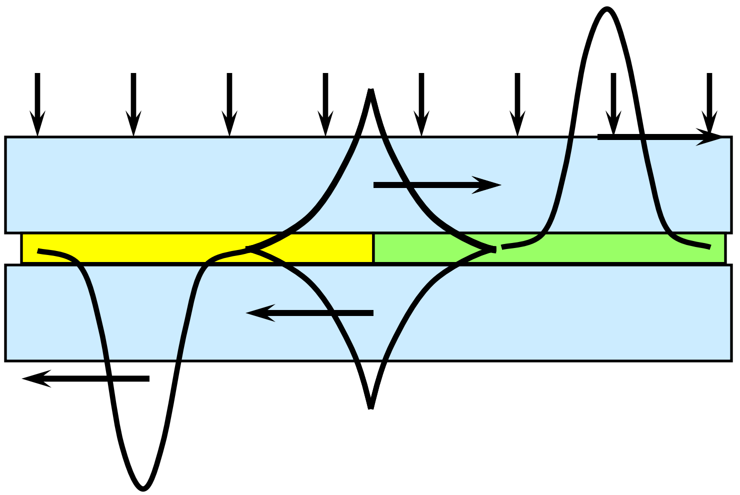
Goldobin et al., PRB **72**, 054527 (2005)

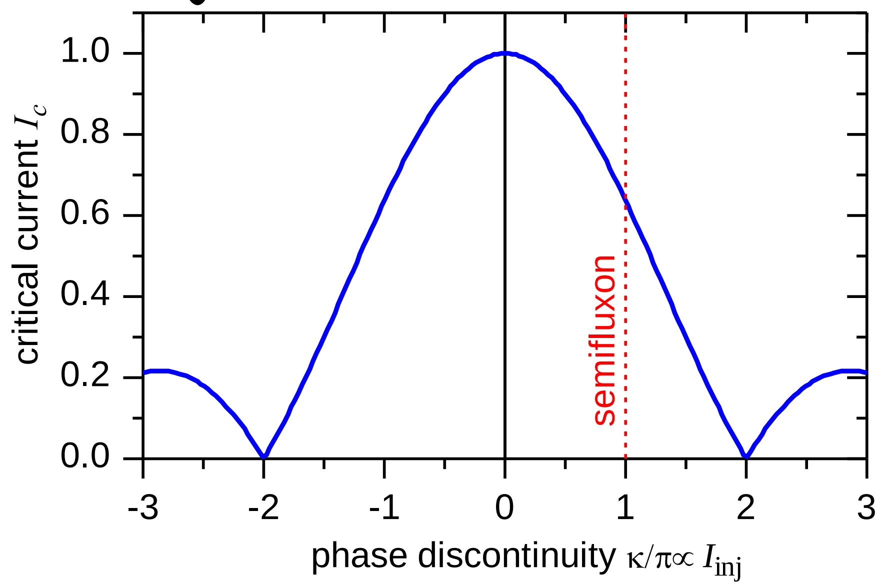


short junction = YES

K. Vogel et al., submitted to PRB.

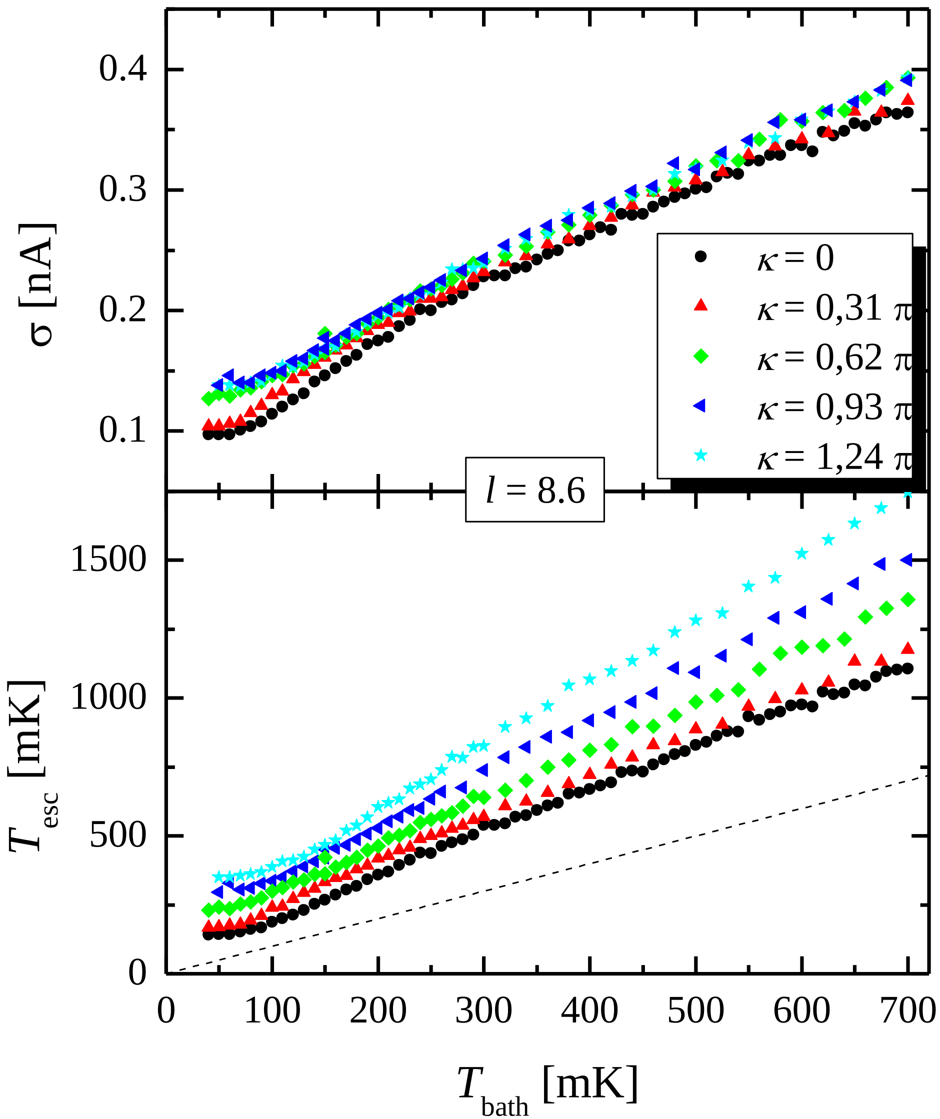
MQT

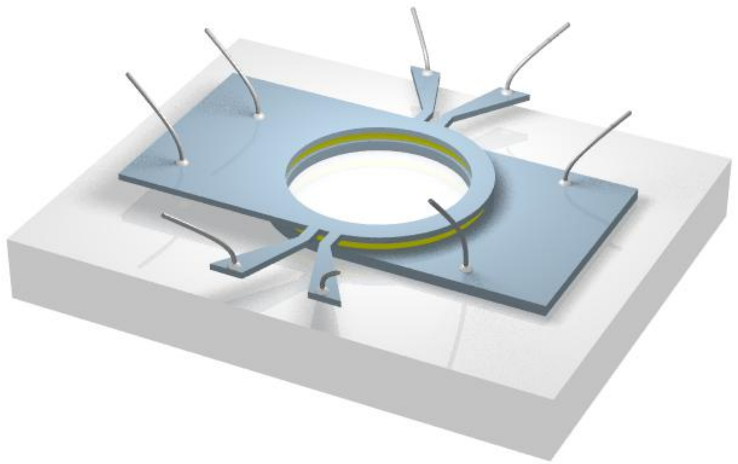


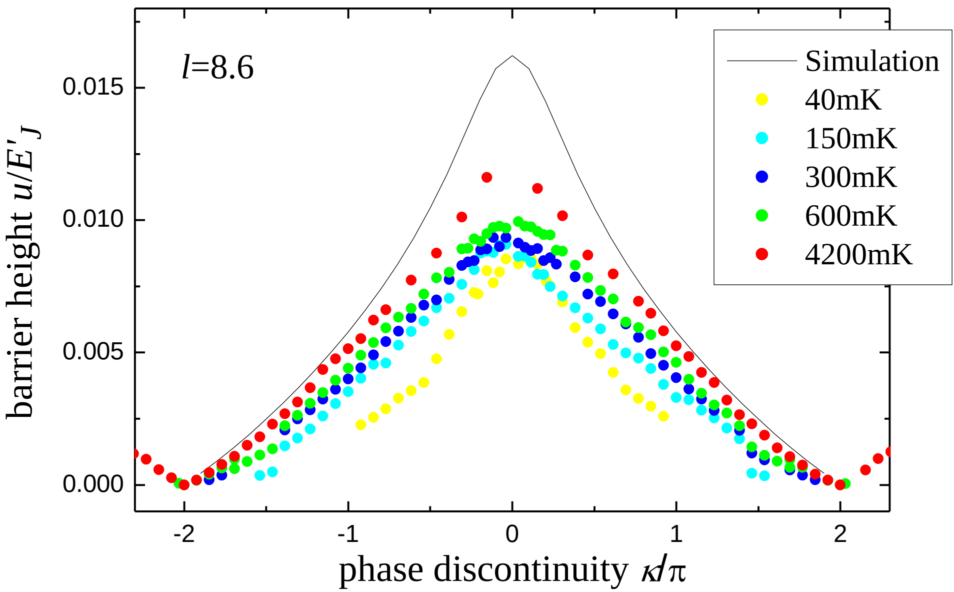


Malomed et al., PRB **69**, 64502 (2004)

Thermal escape vs. MQT







Summary

- Various types of 0-π LJJ: GB, d-wave/s-wave, SFS/SIFS
- sine-Gordon equation with discontinuities, semifluxon
- Ground state: zero phase (no flux) vs. semufluxon states
- Bias current:
  - rearrangement,
  - emerging,
  - half-integer ZFS,
  - oscillators
- Arbitrary fractional vortices
  - ground states
  - energy and stability
  - two vortex molecules
  - ...