

*"Off-shell effects in the associated top quark pair and Higgs boson production."*

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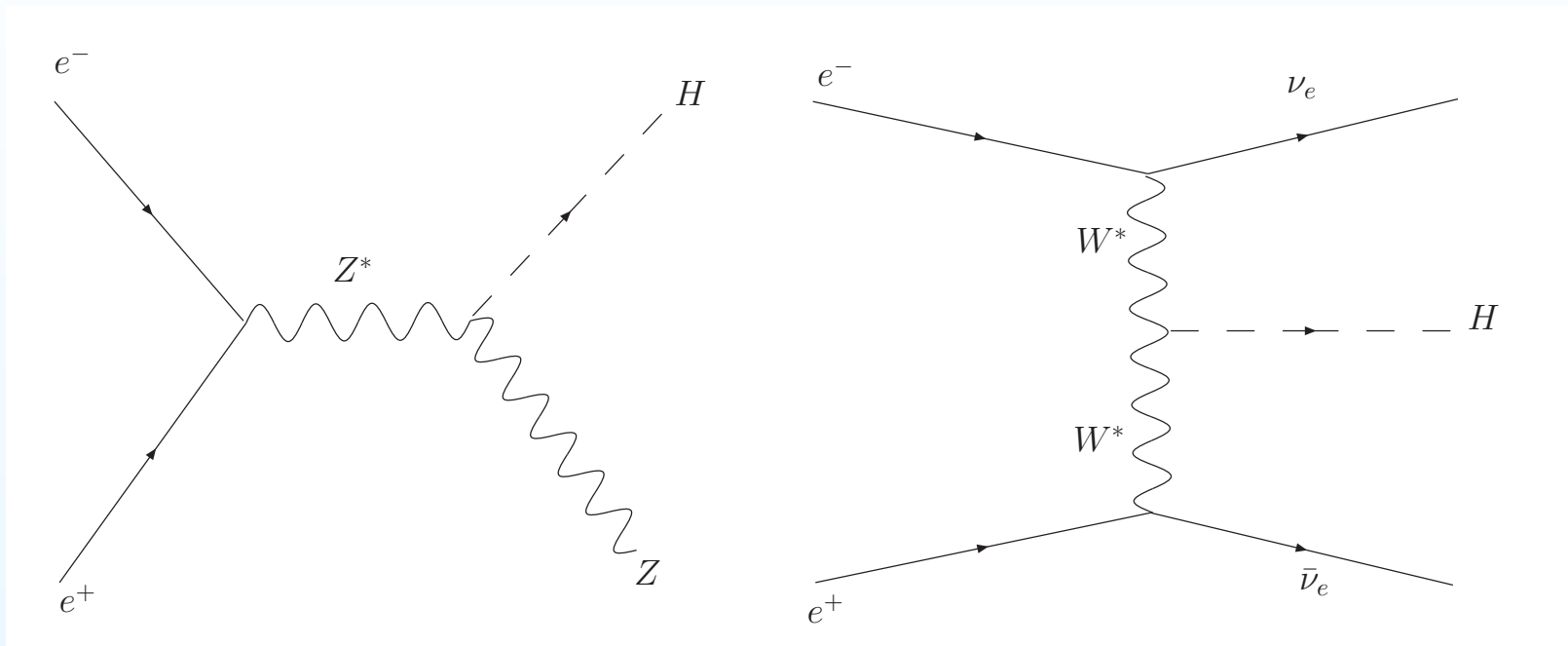
- the Higgsstrahlung process (dominates at low energies):

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- and the  $WW$  fusion process (dominates at high energies):

$$e^+e^- \rightarrow W^*W^* \rightarrow \bar{\nu}_e\nu_e H.$$

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Main production mechanisms of the SM Higgs boson at linear collider.

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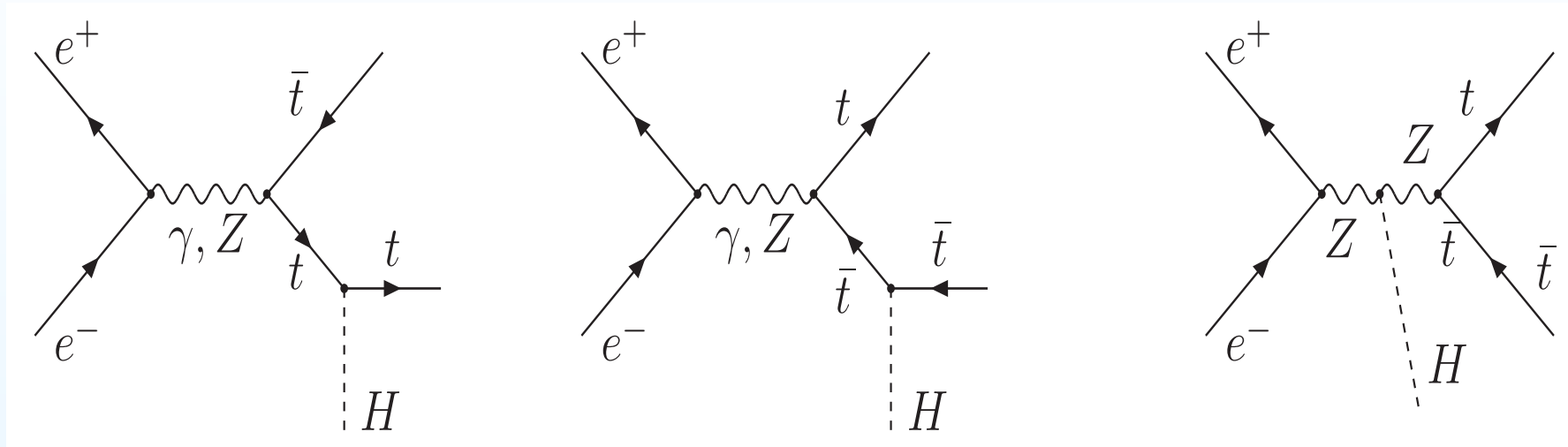
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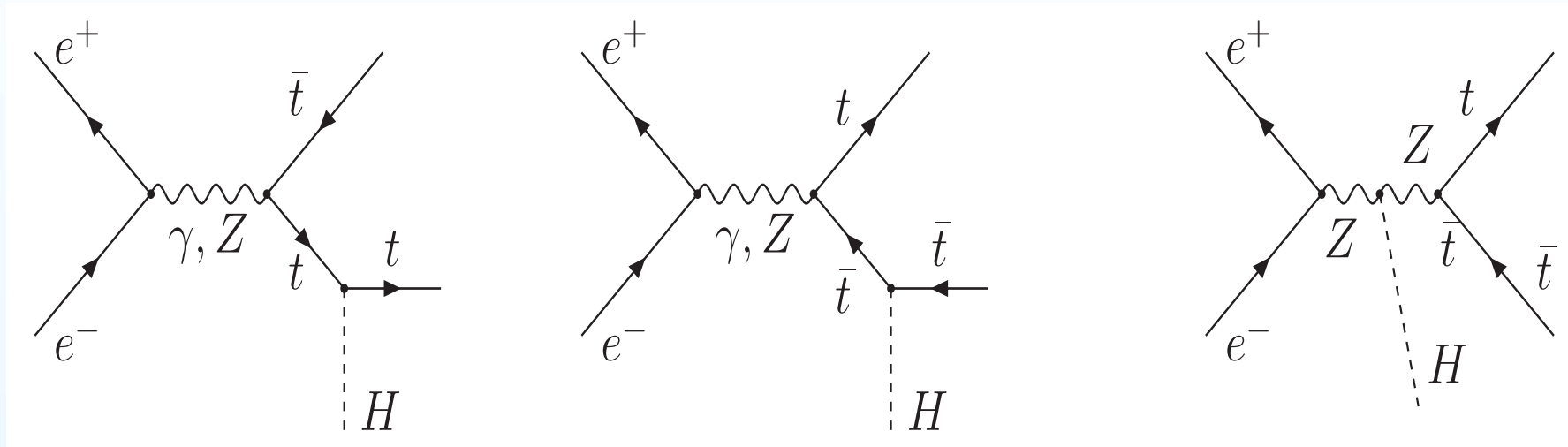
If  $M_H > 2m_t$ , the Higgs top Yukawa coupling can be measured from the  $H \rightarrow t\bar{t}$  branching ratio.

# Process $e^+e^- \rightarrow t\bar{t}H$



Diagrams contributing to the process  $e^+e^- \rightarrow t\bar{t}H$ .

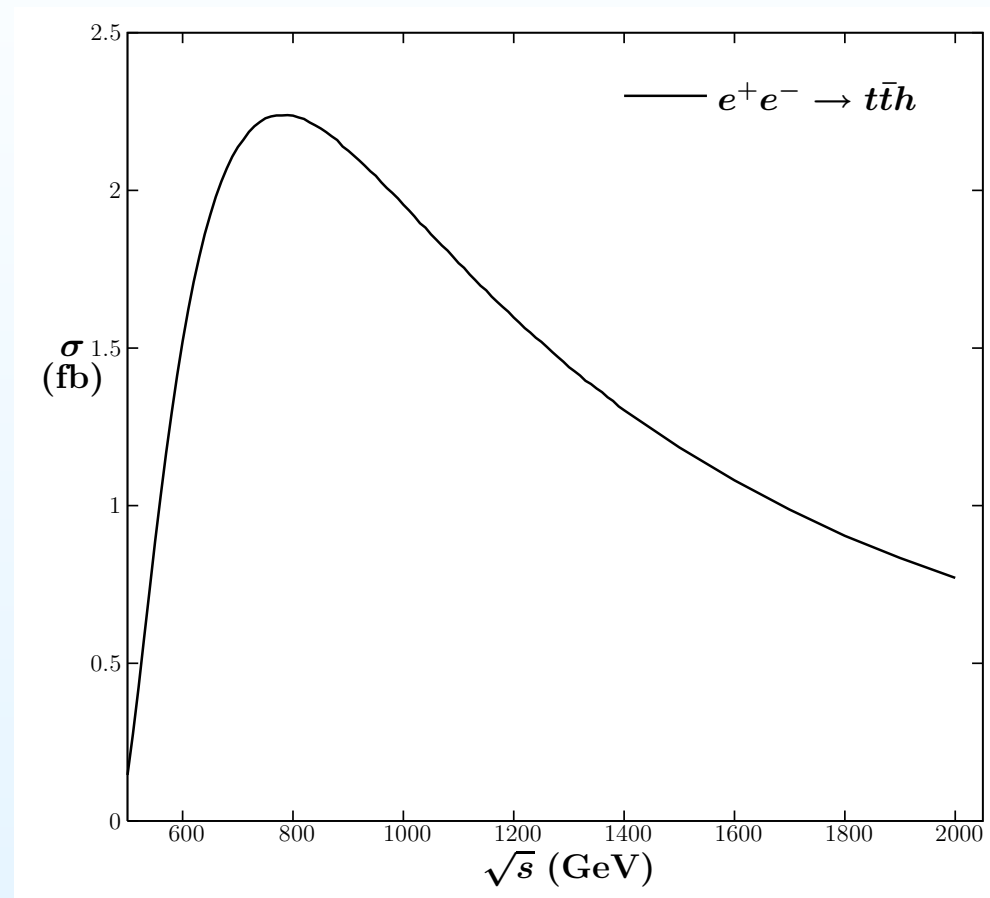
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Diagrams contributing to the process  $e^+e^- \rightarrow t\bar{t}H$ .

Contribution from the Higgs bremsstrahlung off the  $Z$  line is small. The cross section of  $e^+e^- \rightarrow t\bar{t}H$  depends mainly on  $g_{t\bar{t}H}$ .

# Process $e^+e^- \rightarrow t\bar{t}H$



The total cross section of  $e^+e^- \rightarrow t\bar{t}H$  as a function of CMS energy.  
( $M_H = 130$  GeV,  $m_t = 174.3$  GeV)

## Process $e^+e^- \rightarrow 8f$ .

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- hadronic channel: eight jets (38 %),
- semileptonic channel: lepton and six jets (37 %),
- leptonic channel: two leptons and four jets (25 %).

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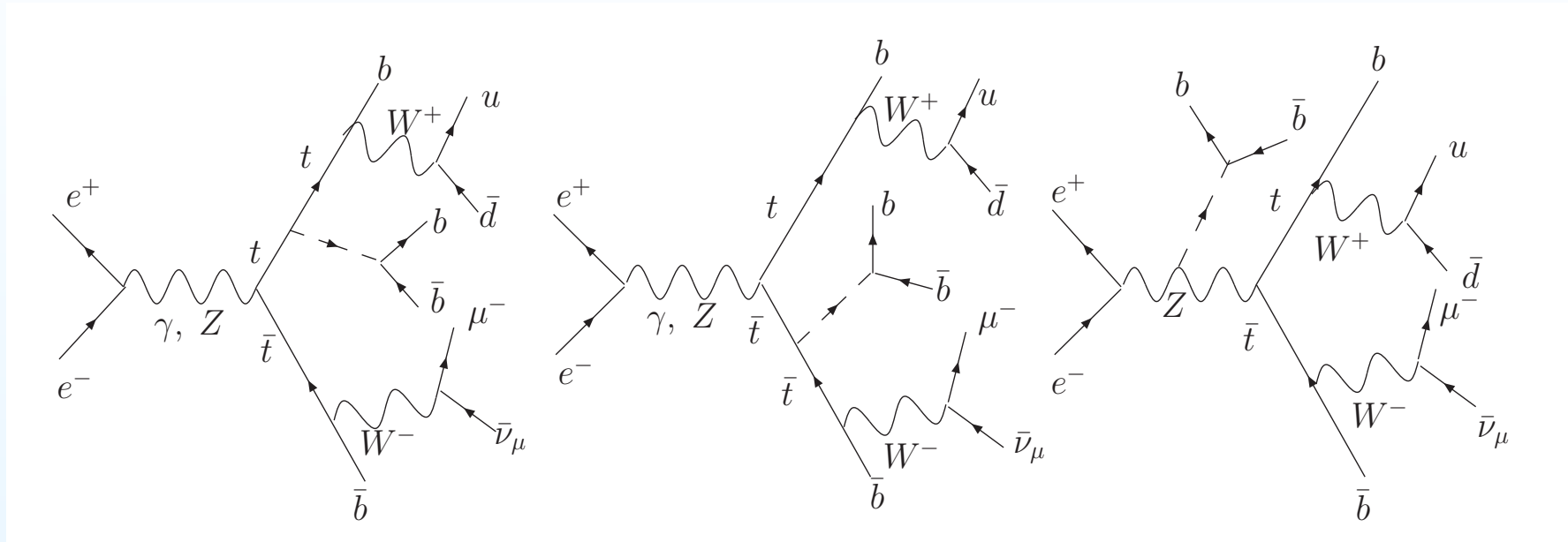
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- Total cross section with the signal diagrams can be compared with the total cross section of the process  $e^+e^- \rightarrow t\bar{t}H$  in the narrow width approximation.

# Process $e^+e^- \rightarrow 8f$



Signal diagrams for the process  $e^+e^- \rightarrow bud\bar{\mu}^- \bar{\nu}_\mu \bar{b}b\bar{b}$ .

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The narrow width approximation

$$\sigma(e^+e^- \rightarrow t\bar{t}H \rightarrow bud\bar{\mu}^- \bar{\nu}_\mu bb\bar{b}) = \sigma(e^+e^- \rightarrow t\bar{t}H) \times \frac{\Gamma_{W^+ \rightarrow u\bar{d}}}{\Gamma_W} \times \frac{\Gamma_{W^- \rightarrow \mu^- \bar{\nu}_\mu}}{\Gamma_W} \times \frac{\Gamma_{H \rightarrow b\bar{b}}}{\Gamma_H},$$

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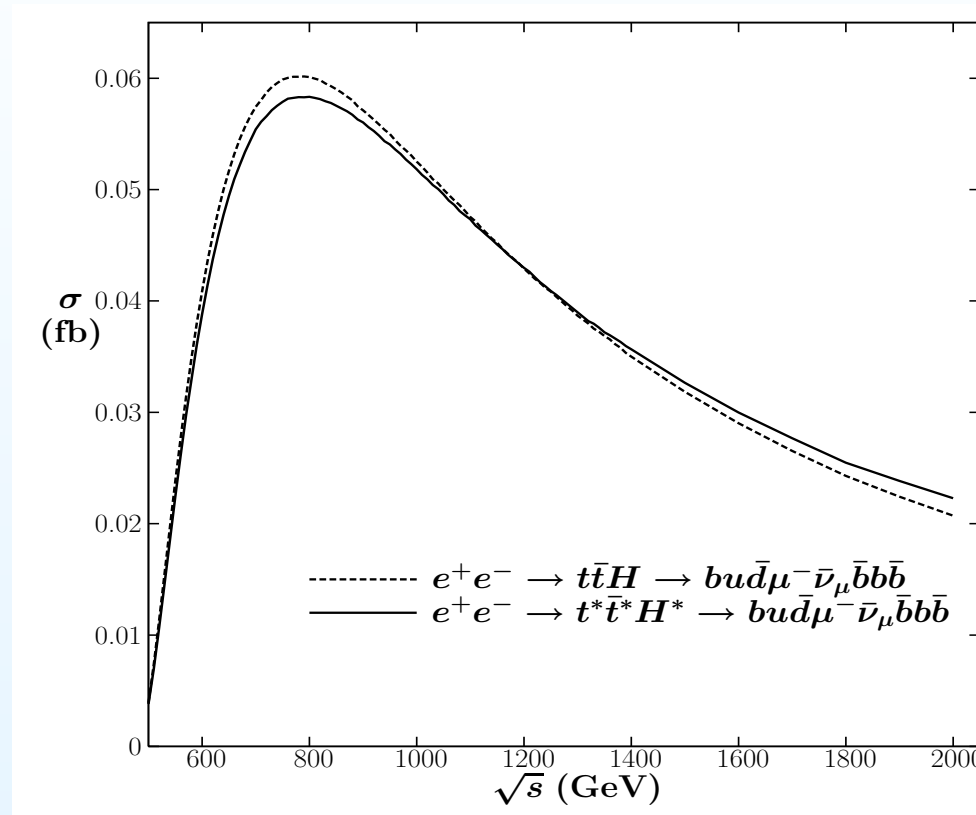
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where

$$\frac{\Gamma_{t \rightarrow Wb}}{\Gamma_t} = 1$$

# Process $e^+e^- \rightarrow 8f$



Total cross section of  $e^+e^- \rightarrow b\bar{u}d\bar{\mu}^-\bar{\nu}_\mu\bar{b}b\bar{b}$  and total cross section of  $e^+e^- \rightarrow t\bar{t}H$  in the narrow width approximation as a function of CMS energy.

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The numerical results for  $\sigma_{8f}$  and  $\sigma_{\text{NWA}}$  with the corresponding relative correction.

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$\sqrt{s}$ [GeV]	$\sigma_{8f}$ [at]	$\sigma_{\text{NWA}}$ [at]	$\delta$ [%]
500	3.80(1)	3.92(1)	3.0
800	58.33(6)	60.06(2)	2.9
1000	51.79(6)	52.56(3)	1.3
1200	42.98(6)	42.96(2)	0.1
2000	21.89(11)	20.76(2)	5.0

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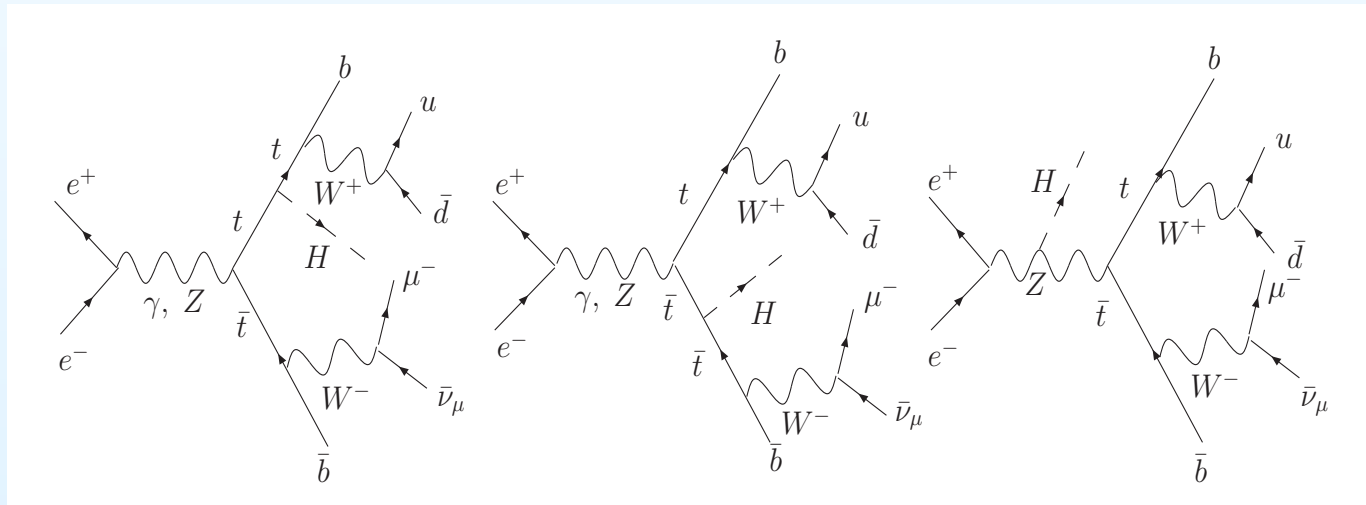
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Signal diagrams for the process  $e^+e^- \rightarrow bud\mu^-\bar{\nu}_\mu bH$ .

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$\sqrt{s}$ [GeV]	$\sigma_{all}$ [ab]	$\sigma_{sig}$ [ab]
500	4.21(1)	4.23(1)
800	64.45(8)	64.24(8)
2000	23.99(66)	22.23(3)

## Process $e^+e^- \rightarrow 6f + H$

$$|m_{W^+}^* - m_W| \leq 3 \Gamma_W$$

$$|m_{W^-}^* - m_W| \leq 3 \Gamma_W,$$

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$\sqrt{s}$ [GeV]	$\sigma_{all}$ [ab]	$\sigma_{sig}$ [ab]
500	3.47(1)	3.48(1)
800	53.17(8)	52.95(7)
2000	19.79(61)	18.34(3)



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- These effects are at the level of a few percent, they can be very important for the future experiments at linear collider.
- The full set of the Feynman diagrams for processes with eight fermions in the final state should be also considered.