Two-loop corrections to the pole masses of heavy quarks in the MSSM

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$$\frac{\Delta m_{t,b}}{m_{t,b}} \equiv \frac{M_{t,b}^{pole} - m_{t,b}^{\overline{\mathrm{DR}}}(\bar{\mu})}{m_{t,b}^{\overline{\mathrm{DR}}}(\bar{\mu})}$$

- Motivation
- Calculation method
- Results
- Conclusion

- Estimates of SUSY particles masses are necessary.
- One needs to know values of (running) parameters of MSSM Lagrangian at the "low-energy" (EW ≈ 100 GeV, or SUSY breaking ≈ 1000 GeV) scale.
- For large $\tan\beta$ and/or m_0 predicted values of masses are very sensitive to Yukawa couplings of heavy quarks. This dependence is particularly strong for Higgs boson masses.
- Thus, it is desirable to have determination of the heavy quarks Yukawa couplings as precise as possible.

In order to evaluate Yukawa couplings from

- M_Z^{pole}, M_W^{pole}
- M_t^{pole}
- $\alpha_s^{(5)}(M_Z)$, α_{em}
- $m_b(m_b)$, etc.

we need to calculate relation between the pole and running masses:

$$\frac{\Delta m_{t,b}}{m_{t,b}} \equiv \frac{M_{t,b}^{pole} - m_{t,b}^{\overline{\rm DR}}(\bar{\mu})}{m_{t,b}^{\overline{\rm DR}}(\bar{\mu})}$$

- Pierce, Matchev, Zhang, hep-ph/9606211:

$$\frac{\Delta m_{t,b}}{m_{t,b}} = \mathcal{O}(\alpha_s) + \mathcal{O}(y_t^2) + \mathcal{O}(y_b^2) + \mathcal{O}(g^2) + \mathcal{O}(g'^2)$$

  For the b quark $\mathcal{O}(\alpha_s)$ terms are reduced significantly by $\mathcal{O}(y_t^2 + y_b^2)$ ones.

- Bednyakov, Onishchenko, Velizhanin, Veretin, hep-ph/0210258:
  $\mathcal{O}(\alpha_s^2)$, obtained by asymptotic expansion in $m_{t,b}/M_{SUSY}$, leading term only. For the b quark this correction is comparable with one-loop one.

- $\mathcal{O}(\alpha_s y_t^2 + \alpha_s y_b^2 + y_t^4 + y_b^4)$ corrections for b-quark, since these may be of the same order of magnitude as $\mathcal{O}(\alpha_s^2)$ ones.
- second-order terms of asymptotic expansion for t-quark, since $m_t/M_{SUSY} \approx 0.2$.

full connected propagator of a quark:

$$i\left(\hat{p} - m - \Sigma(\hat{p}, m_i)\right)^{-1}$$

self-energy of the quark:

$$\Sigma(\hat{p}, m_i) = \hat{p}\Sigma_V(p^2, m_i^2) + \hat{p}\gamma_5\Sigma_A(p^2, m_i^2) + m\Sigma_S(p^2, m_i)$$

pole mass M_p:

$$\left(\left(1 + \Sigma_V(M_p^2, m_i^2)\right)^2 - \Sigma_A^2(M_p^2, m_i^2)\right) M_p^2 - m^2\left(1 - \Sigma_S(M_p^2, m_i)\right)^2 = 0$$

perturbative expansion of the pole mass:

$$\begin{aligned}
\frac{M_p - m}{m} &= \alpha M^{(1)} + \alpha^2 M^{(2)}, \\
M^{(1)} &= \Sigma_V^{(1)}(m^2, m_i^2) + \Sigma_S^{(1)}(m^2, m_i), \\
M^{(2)} &= \Sigma_V^{(2)}(m^2, m_i^2) + \Sigma_S^{(2)}(m^2, m_i) + \frac{1}{2}\Sigma_A^{(1)^2}(m^2, m_i^2) \\
&+ M^{(1)} \Big(\Sigma_V^{(1)}(m^2, m_i^2) \\
&\quad + 2m^2 \frac{\partial}{\partial p^2} \left(\Sigma_V^{(1)}(p^2, m_i^2) + \Sigma_S^{(1)}(p^2, m_i) \right) \Big)_{p^2 = m^2}
\end{aligned}$$

Asymptotic expansion (F.V. Tkachov):

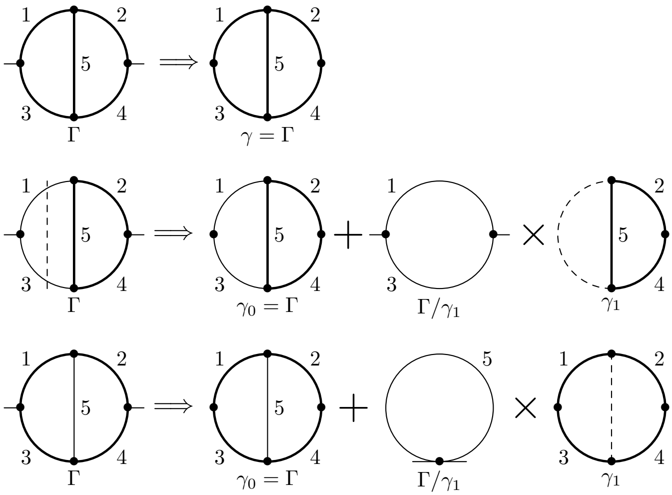
$$F_\Gamma(p,m,M) = \sum_\gamma F_{\Gamma/\gamma}(p,m)\mathcal{T}_\gamma(p_\gamma,m)F_\gamma(p_\gamma,m,M)$$

- γ: asymptotically irreducible subgraphs of Γ
- $\mathcal{T}_\gamma(p_\gamma,m)$: Taylor expansion in small masses and external (with respect to γ) momenta

about 160 two-loop propagator-type diagrams

$$m_t \ll m_{\tilde{q}}, m_{\tilde{g}}$$

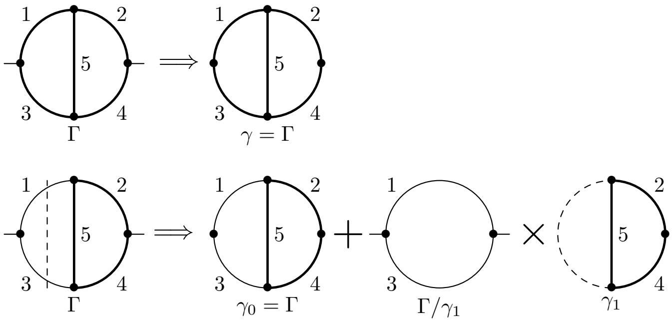
$$\frac{\Delta m_t}{m_t} \approx 1 + \alpha_s \sum_{n=-1}^{2} m_t^n \sigma_1^{(n)} + \alpha_s^2 \sum_{n=-1}^{2} m_t^n \sigma_2^{(n)}$$


Γ
γ = Γ
Γ
γ0 = Γ
Γ/γ1
γ1
Γ
γ0 = Γ
Γ/γ1
γ1

about 1200 two-loop propagator-type diagrams

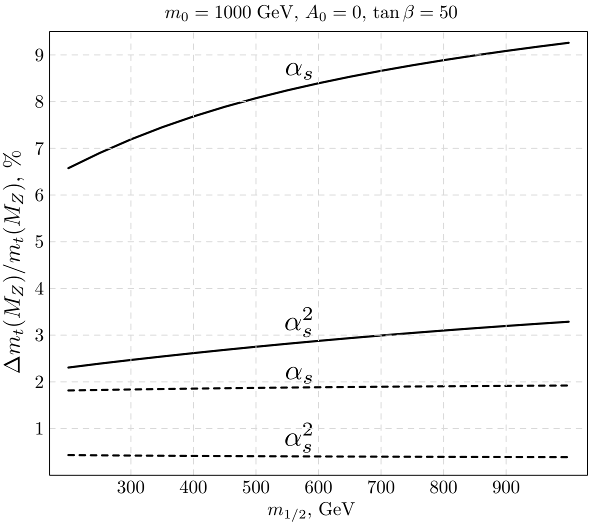
$$m_b \ll m_t, m_{h_0}, m_{H_0}, m_{H^+}, m_{\tilde{q}}, m_{\tilde{\chi}^0}, m_{\tilde{\chi}^+}, m_{G_0}, m_{G^+}$$

$$\frac{\Delta m_b}{m_b} \approx 1 + \alpha \sum_{n=-1}^{0} m_b^n \sigma_1^{(n)} + \alpha^2 \sum_{n=-1}^{0} m_b^n \sigma_2^{(n)}$$

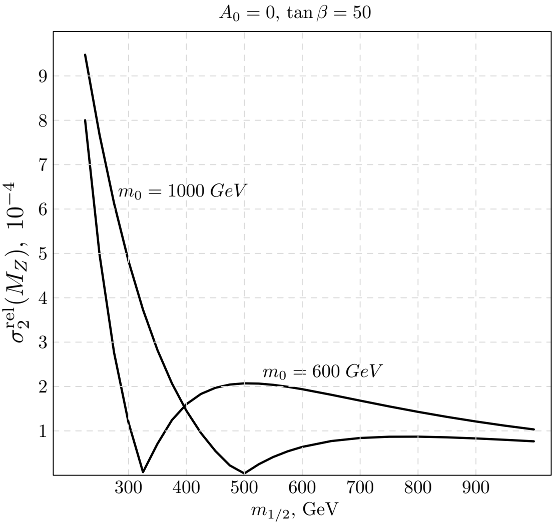


other subgraphs give $\mathcal{O}(m_b^2)$ contribution.

- Diagram generation: (slightly modified version of) *FeynArts*.
- Analytical and numeric calculations: *GiNaC* C++ library.
- Asymptotic expansion: *prop2exp* C++ library (based on GiNaC).
- Recursive reduction of two-loop vacuum integrals to a by integration by parts method, analytic and numeric evaluation of the master-integral: *bubblesii* C++ library (based on GiNaC too).
- Evaluation of running MSSM parameters and SUSY partner masses: *ffmssmsc* C++ library (~~forked from *SOFTSUSY*~~ written from scratch).

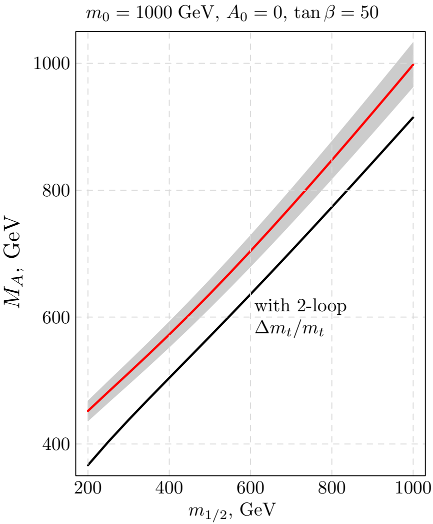


$\mathcal{O}(\alpha_s^2)$ corrections are $\approx 30\%$ of one-loop ones

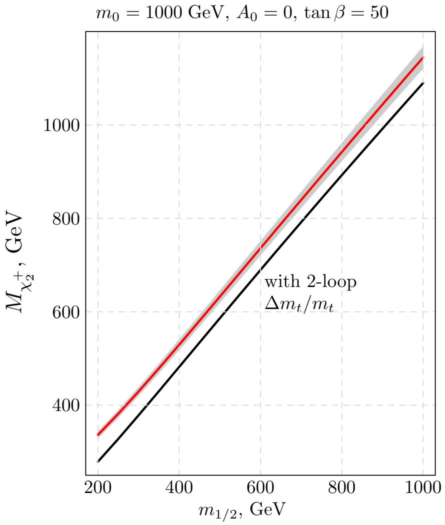


$$\sigma_2^{\rm rel} \equiv \left| \frac{\Delta^{(2)} m_t/m_t}{\Delta^{(0)} m_t/m_t} \right|$$

second-order terms of asymptotic expansion in m_t/M_{SUSY} give negligible contribution



- Two-loop corrections to $\Delta m_t/m_t$ yields sizable change ($\approx 15\%$) of predicted masses of heavy Higgs bosons and chargino
- This change exceeds discrepancies (indicated by gray region on the plot) between different software for MSSM spectrum calculation



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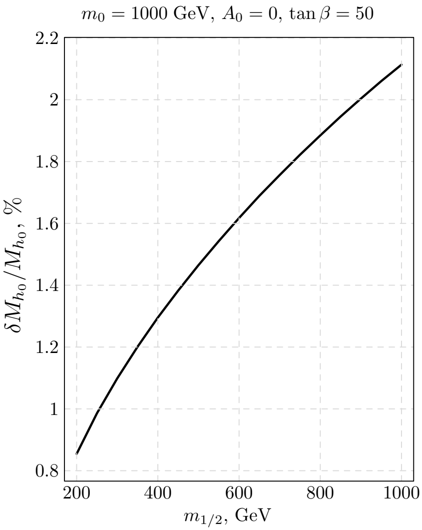
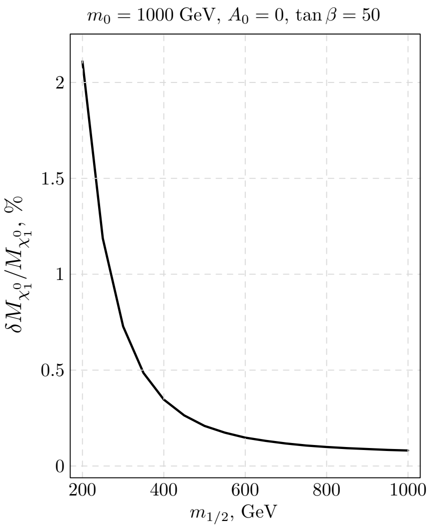


Masses of squarks, gluino, and relatively light particles (lightest neutralino, lightest Higgs boson) do not obtain any significant changes.



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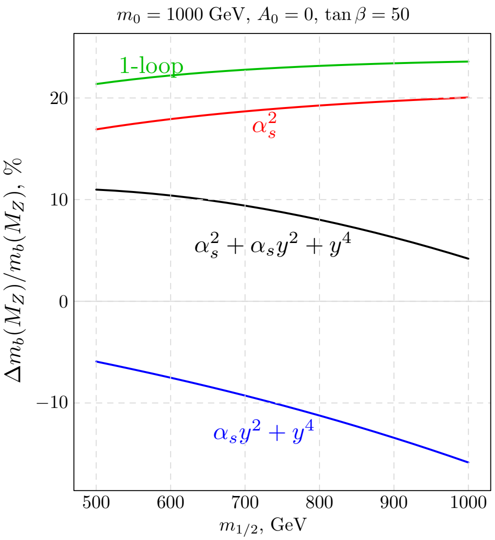

- $\mathcal{O}(\alpha_s^2)$ correction is positive and comparable to the one-loop MSSM correction
- $\mathcal{O}(\alpha_s y^2 + y^4)$ contribution has the opposite sign and partially compensate $\mathcal{O}(\alpha_s^2)$ corrections
- $\mathcal{O}(\alpha_s y^2 + y^4 + \alpha_s^2)$ corrections $\propto 30 - 40\%$ of the one-loop MSSM ones

- $\mathcal{O}(\alpha_s^2)$ MSSM corrections to the relation between pole and running masses of the t quark were evaluated.
- these corrections are $\approx 30\%$ of one-loop ones
- second-order terms of asymptotic expansion in m_t/M_{SUSY} give negligible contribution
- this correction yields $\gtrapprox 15\%$ change of predicted masses of heavy Higgs bosons and chargino

- $\mathcal{O}(\alpha_s y^2 + y^4)$ MSSM corrections to the relation between pole and running masses of the b quark were evaluated.
- $\mathcal{O}(\alpha_s^2)$ correction is positive and of the same order of magnitude as the one-loop MSSM correction
- $\mathcal{O}(\alpha_s y^2 + y^4)$ contribution has the opposite sign in most "interesting" regions of MSSM parameter space and partially compensate $\mathcal{O}(\alpha_s^2)$ corrections
- $\mathcal{O}(\alpha_s y^2 + y^4 + \alpha_s^2)$ corrections $\propto 30 - 40\%$ of the one-loop MSSM ones.

Our results can be used for

- calculation of the MSSM mass spectrum
- renormalization group analysis of the Yukawa coupling unification
- dark matter searches, relic density is sensitive to the masses of heavy quarks for large $\tan\beta$