Third-order potential corrections to the $t\bar{t}$ production near threshold

Kurt Schuller

Collaboration with Martin Beneke and Yuichiro Kiyo
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Overview

Motivation

Measurement of the top-quark mass m_t:

- Error in m_t has large impact on precision observables.
- One can 'see' physics beyond the SM.

[S.Heinemeyer,G.Weiglein '05]

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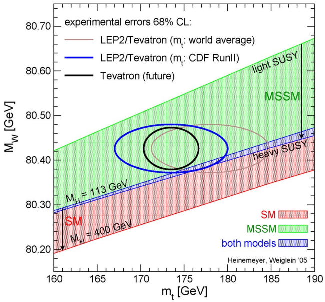

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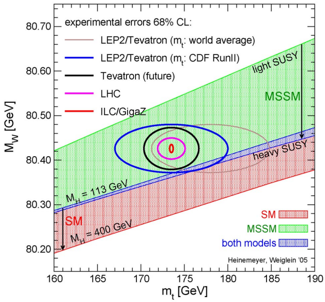

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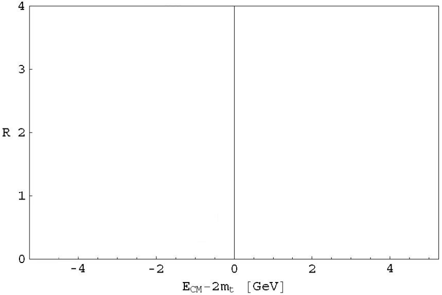

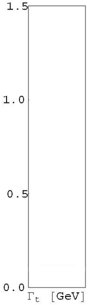

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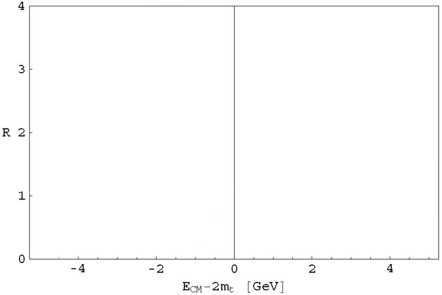

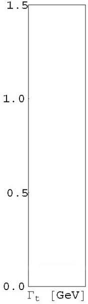

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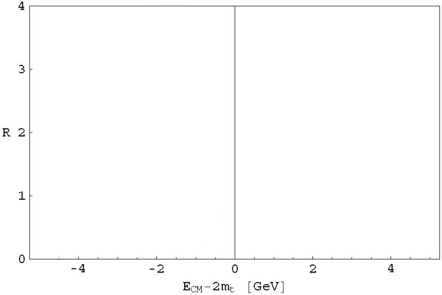

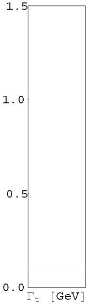

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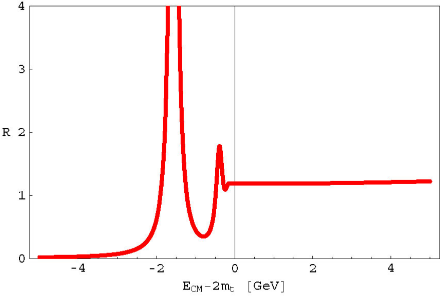

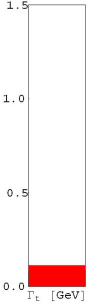

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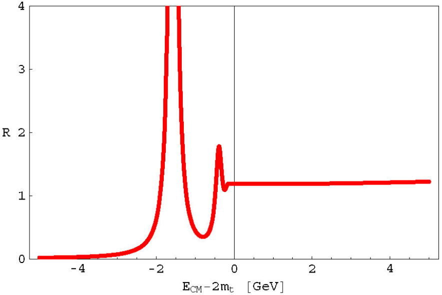

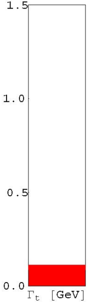

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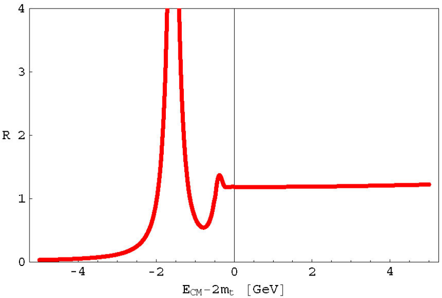

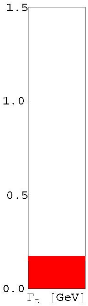

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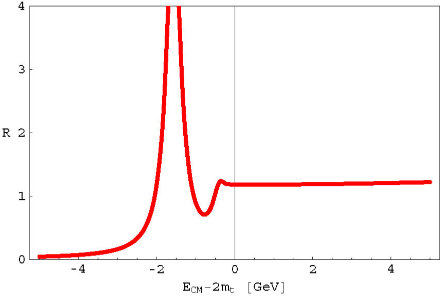

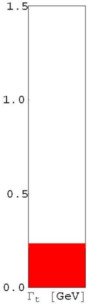

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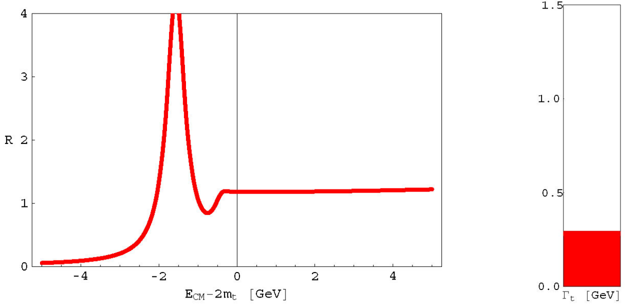

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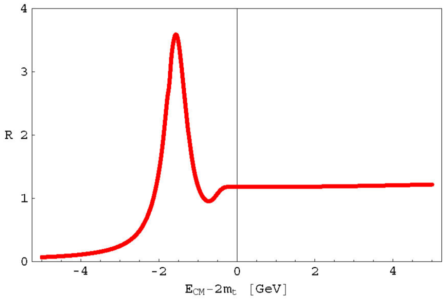

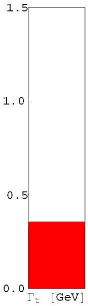

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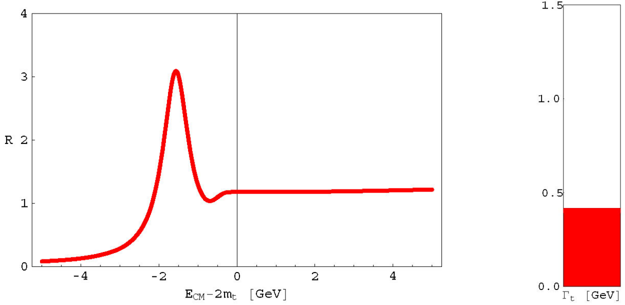

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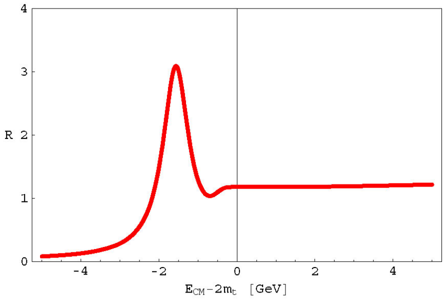

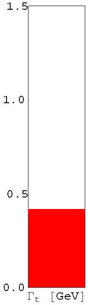

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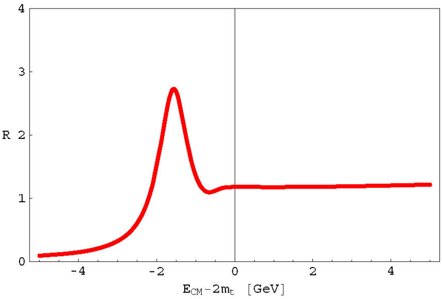

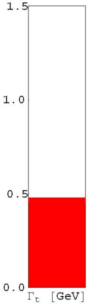

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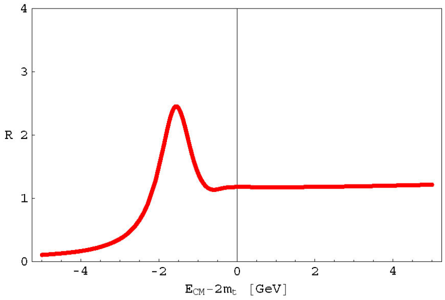

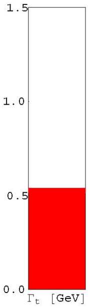

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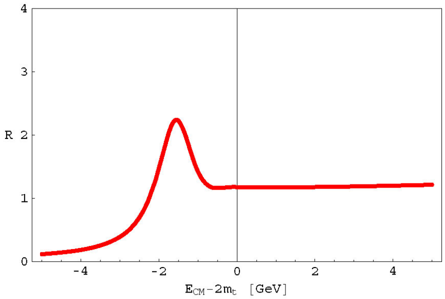

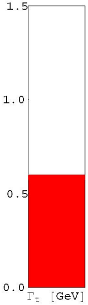

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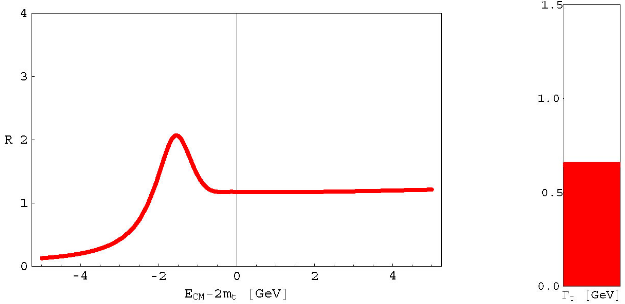

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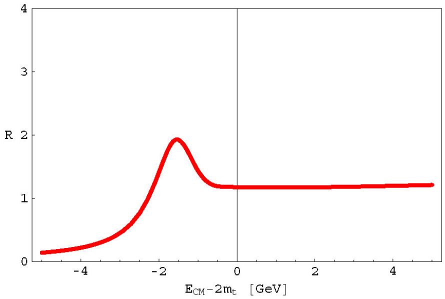

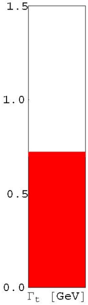

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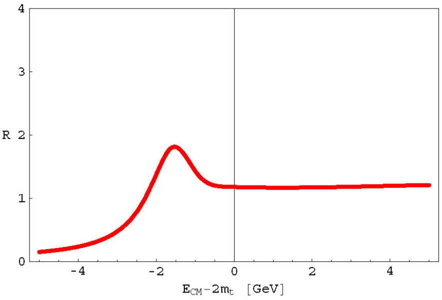

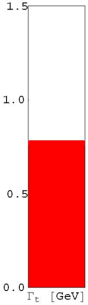

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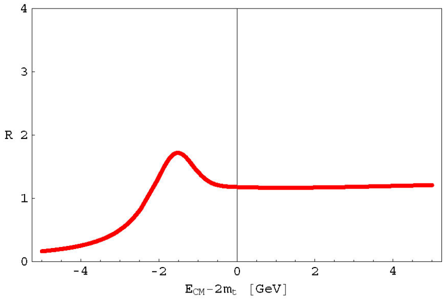

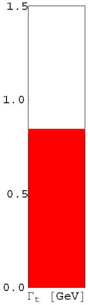

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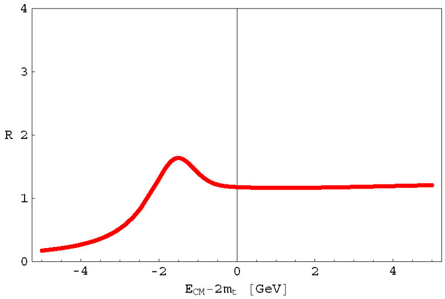

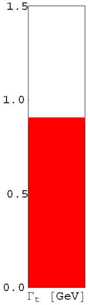

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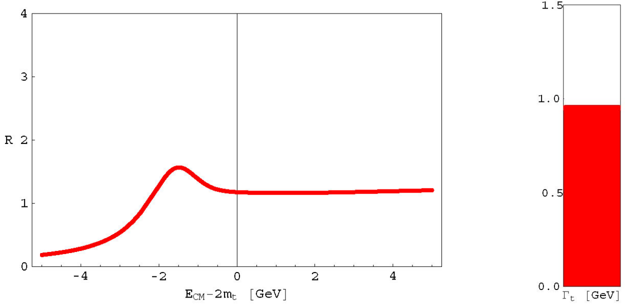

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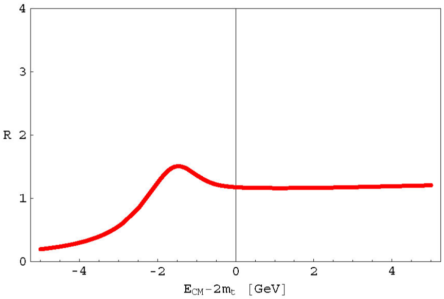

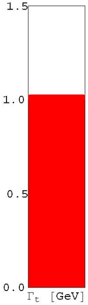

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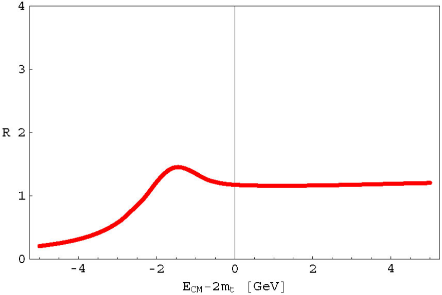

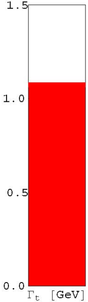

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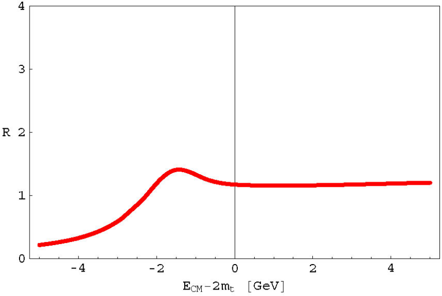

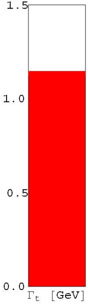

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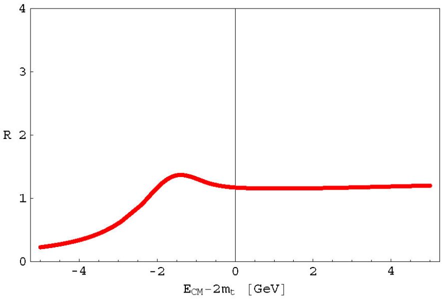

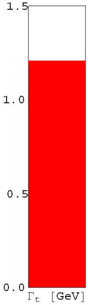

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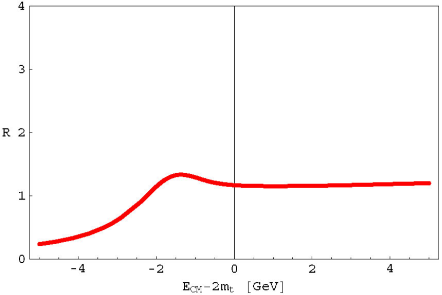

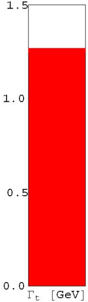

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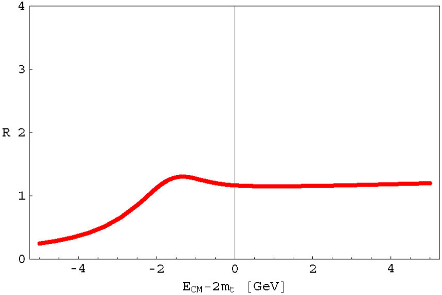

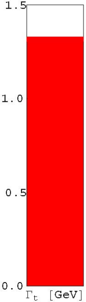

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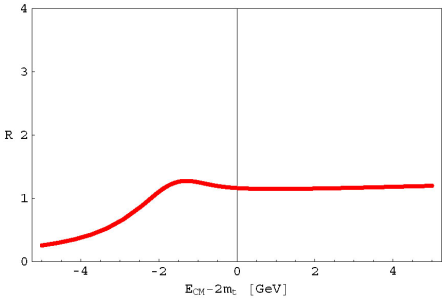

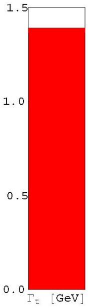

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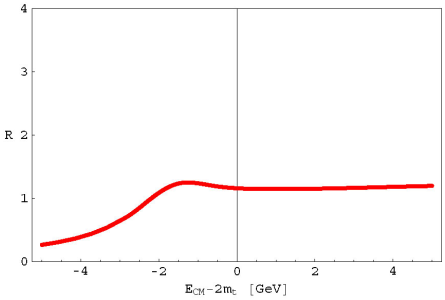

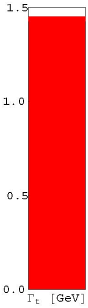

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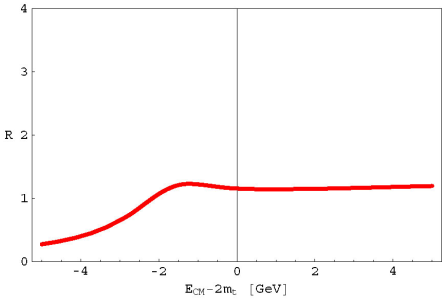

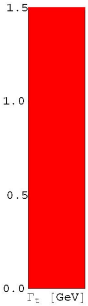

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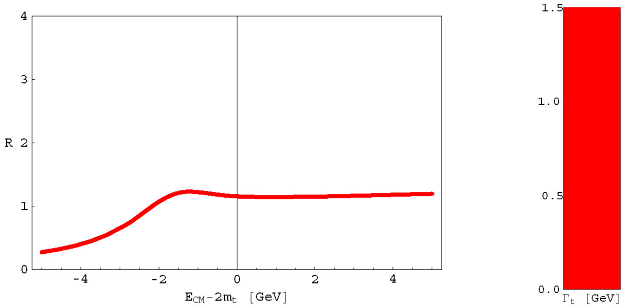

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Motivation for NNNLO

- The 2nd order corrections (NNLO) are large.

[M.Beneke,A.Signer,V.A.Smirnov '99]

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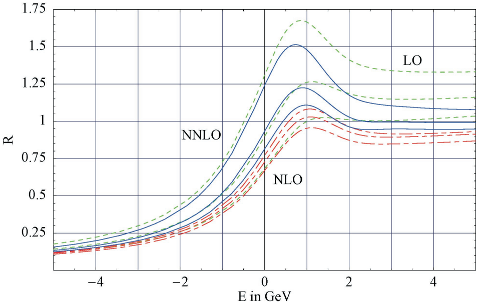
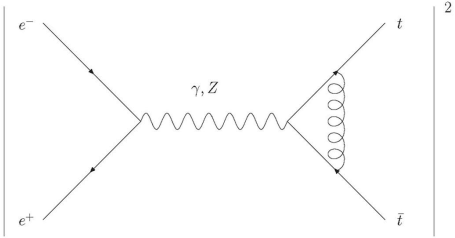


[M.Beneke,A.Signer,V.A.Smirnov '99]

Cross section and Green Function



The "Optical Theorem" connects the cross section and the Green Function:

$$R = \frac{\sigma_{t\bar{t}X}}{\sigma_{\mu^+\mu^-}} = \frac{18\pi e_t^2}{m_t^2}(1 + a_Z)\,\mathrm{Im}\,G(0,0;E + i\Gamma_t)$$

$$G(0,0,E + i\Gamma_t) = \sum_{n=1}^{\infty} \frac{|\phi_n(0)|^2}{E_n - (E + i\Gamma_t)} + \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{|\phi_{\mathbf{k}}(0)|^2}{\mathbf{k}^2/m_t - (E + i\Gamma_t)}$$

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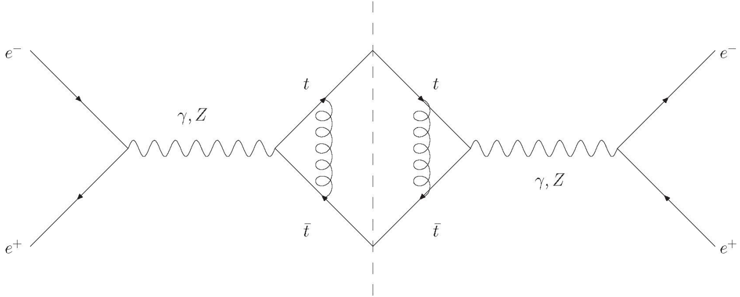

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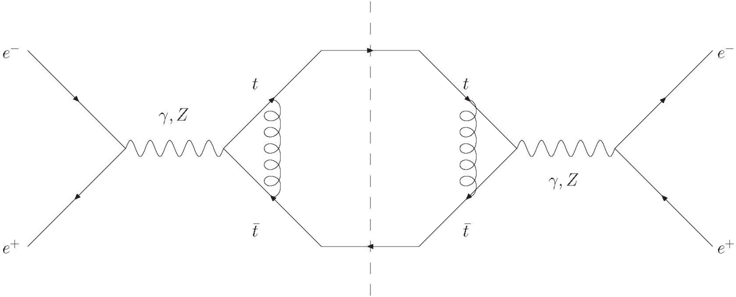

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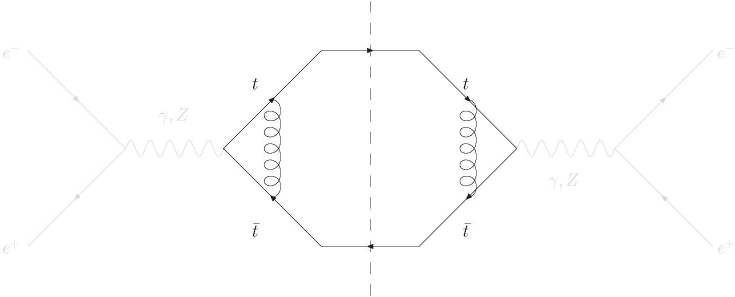

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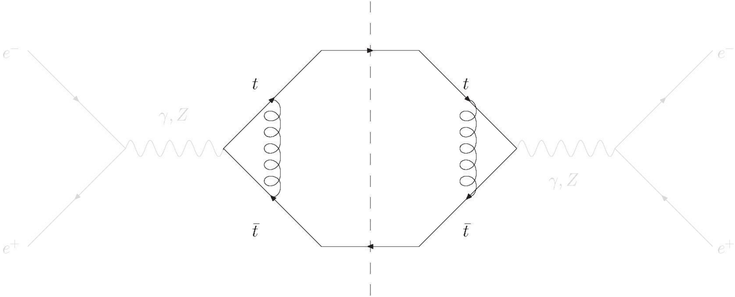

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Problems at Threshold

- Top-quark velocity v is small.
- Usual perturbation theory breaks down.
- Use of non-relativistic approach
  $\Rightarrow$ the expansion has to be done in α_S and v.
- So: terms of the order $\left(\frac{\alpha_S}{v}\right)^k$ have to be summed up for all powers k.

$$R = v \sum_k \left(\frac{\alpha_S}{v}\right)^k \left\{ \begin{array}{ll} 1 & (LO); \\ \\ \alpha_S, v & (NLO); \\ \alpha_S^2, \alpha_S v, v^2 & (NNLO); \\ \alpha_S^3, \alpha_S^2 v, \alpha_S v^2, v^3 & (NNNLO) \end{array} \right\}$$

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Summation in LO Green Function: $R = v \sum_k \left(\frac{\alpha_S}{v}\right)^k$

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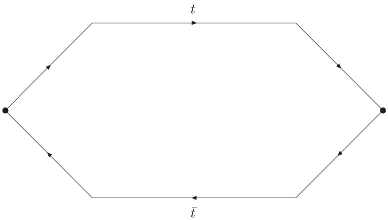

$\sim v$

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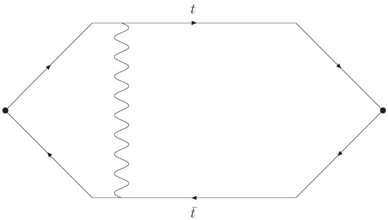

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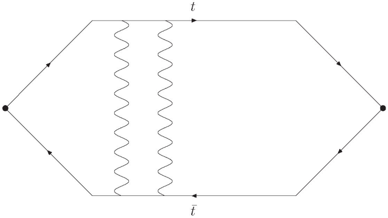

$\sim v \ \left(\frac{\alpha_S}{v}\right)^2$

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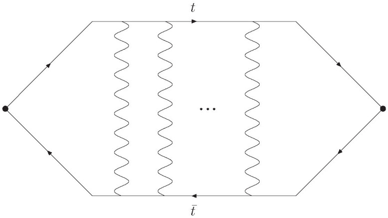

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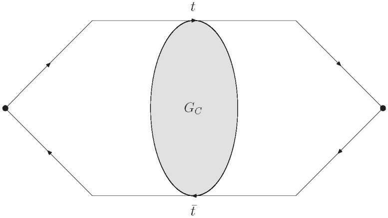

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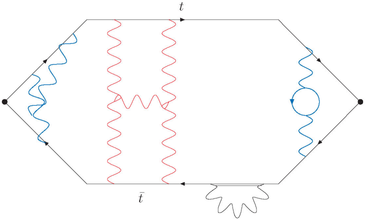
Higher order calculations

- The expansion of v and α_s is done systematically in the framework of effective theories



- Hard and soft modes integrated out (QCD - NRQCD - PNRQCD) [in analogy to PNRQED → Grozin's talk]
- PNRQCD Lagrangian:

$$\mathcal{L}_{\text{eff}} = \psi^\dagger(x)\left(i\partial^0 + \frac{\partial^2}{2m}\right)\psi(x) + \chi^\dagger(x)\left(i\partial^0 - \frac{\partial^2}{2m}\right)\chi(x)$$
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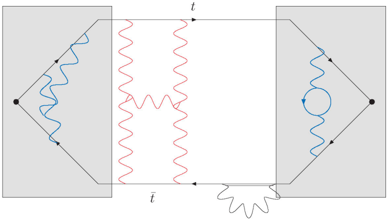

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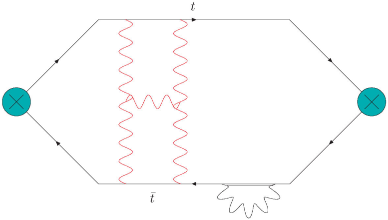

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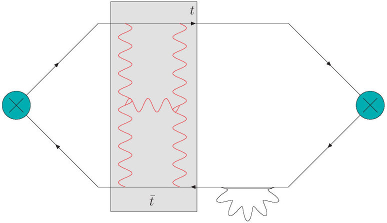

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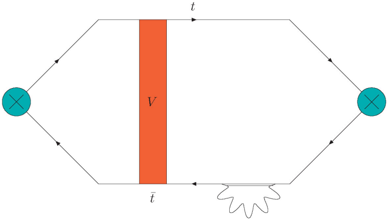

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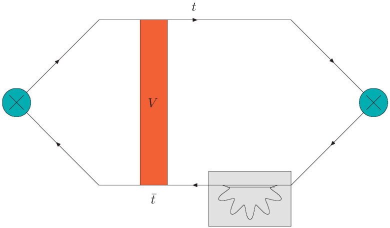

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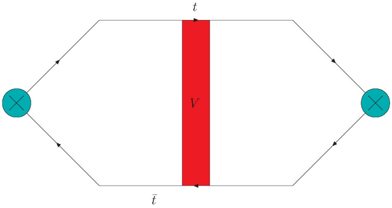

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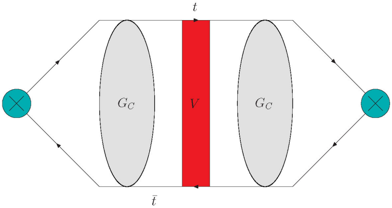

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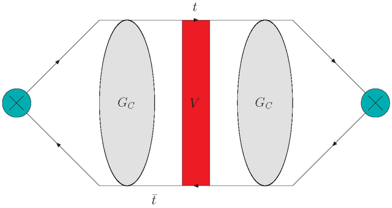

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The potentials in PNRQCD

In PNRQCD the $t\bar{t}$ interactions are described by potentials.

$$\widetilde{V}(\mathbf{q}) = -C_C(\alpha_S)\frac{4\pi C_F\alpha_S}{\mathbf{q}^2} - C_{1/m}(\alpha_S)\frac{2\pi^2 C_F\alpha_S^2}{m_t|\mathbf{q}|} + [C_\delta(\alpha_S) + C_S(\alpha_S)]\frac{\pi C_F\alpha_S}{m_t^2} + C_p(\alpha_S)\frac{C_F\alpha_S\mathbf{p}^2}{m_t^2\mathbf{q}^2}$$

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- Delta potential
- Spin dependent part
- p^2/q^2 potential
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Perturbation Theory

Calculation of the Green function in perturbation theory:

- Perturbative treatment of the potentials:

$$\begin{aligned} \delta V &= \delta V_1 + \delta V_2 + \delta V_3 + \ldots \\ \hat{G} &= \hat{G}_0 - \hat{G}_0 \delta V_1 \hat{G}_0 - \hat{G}_0 \delta V_2 \hat{G}_0 + \hat{G}_0 \delta V_1 \hat{G}_0 \delta V_1 \hat{G}_0 \\ &\quad - \hat{G}_0 \delta V_3 \hat{G}_0 + 2 \hat{G}_0 \delta V_1 \hat{G}_0 \delta V_2 \hat{G}_0 - \hat{G}_0 \delta V_1 \hat{G}_0 \delta V_1 \hat{G}_0 \delta V_1 \hat{G}_0 + \ldots \end{aligned}$$

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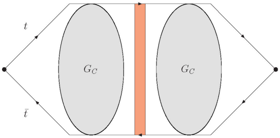

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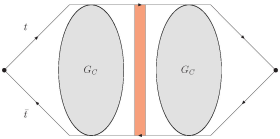

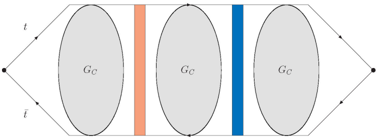

Singularities

Problems with the treatment of the Delta and $1/r^2$-Potentials:

- Insertion of the potentials creates divergencies.
- Other divergencies coming from hard vertex corrections.
- Final result is finite.
- We need order ϵ-correction to the potentials.

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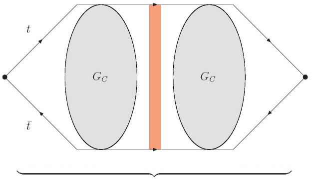

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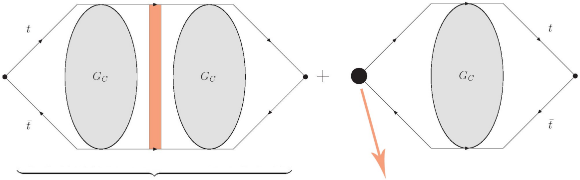

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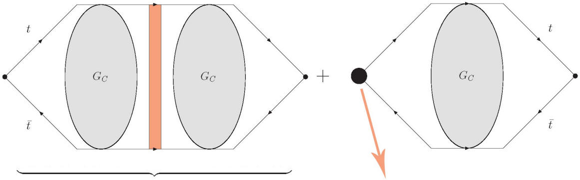

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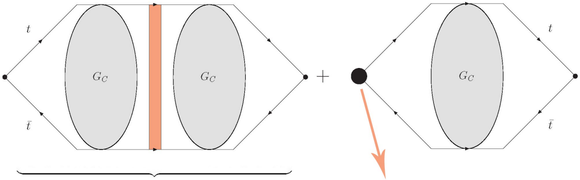

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Strategy:

- Identify the divergent structure.
- Divide the potential insertion into diagrams with the different divergent structures.

Calculated by Feynman parameters and IBP relations.

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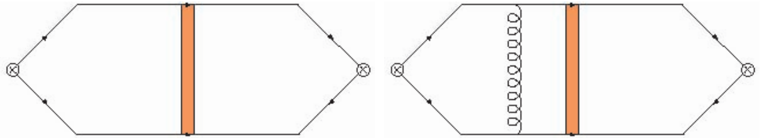

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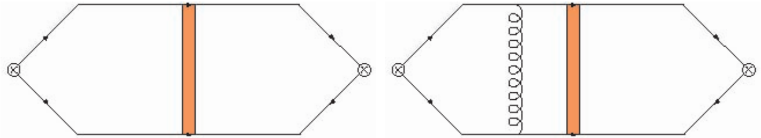

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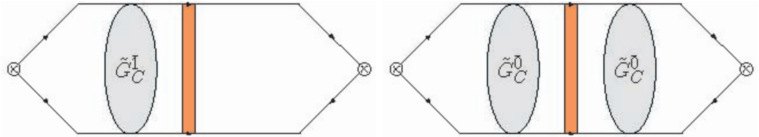

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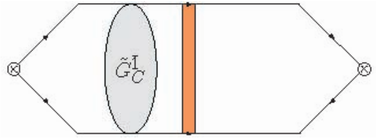

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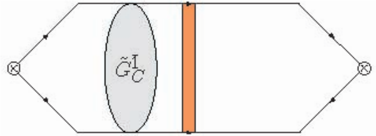

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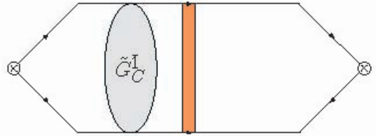

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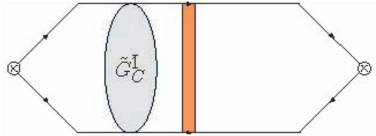

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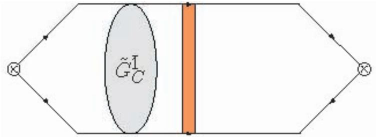

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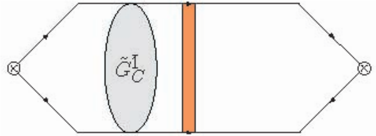

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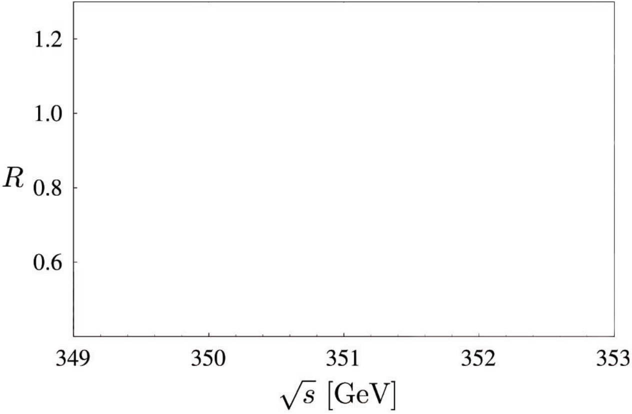

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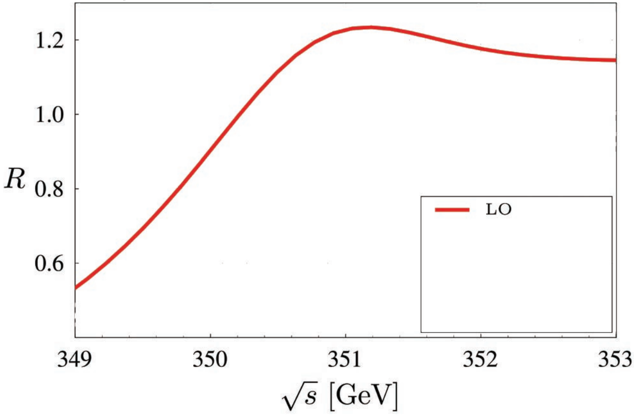

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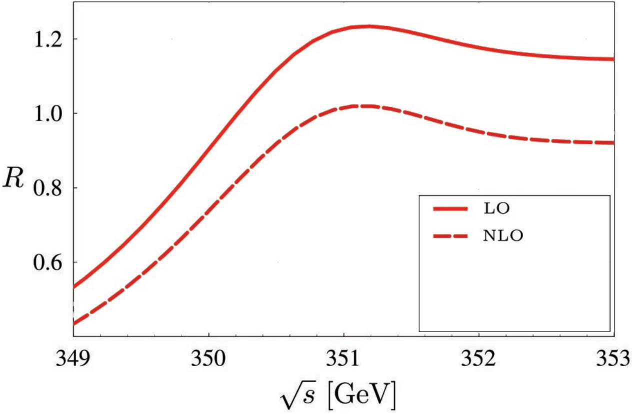

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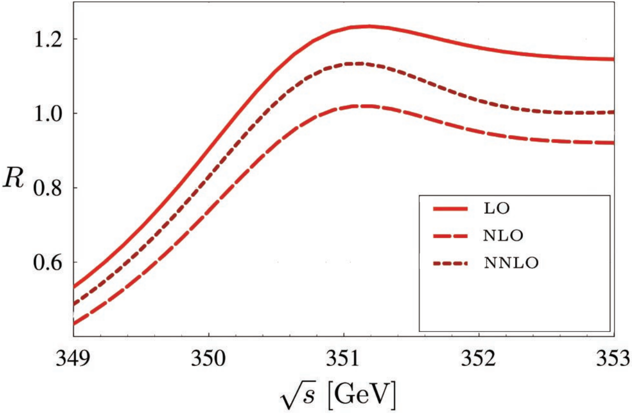

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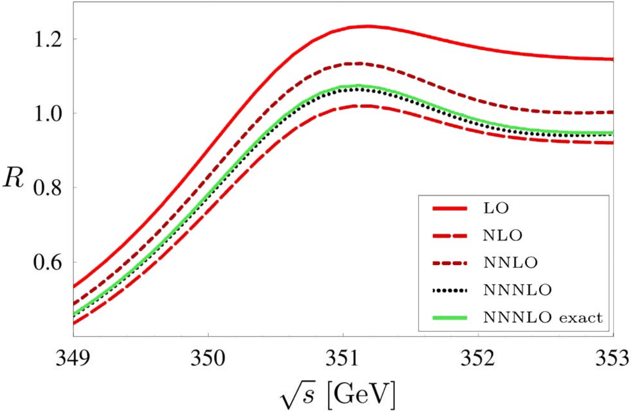

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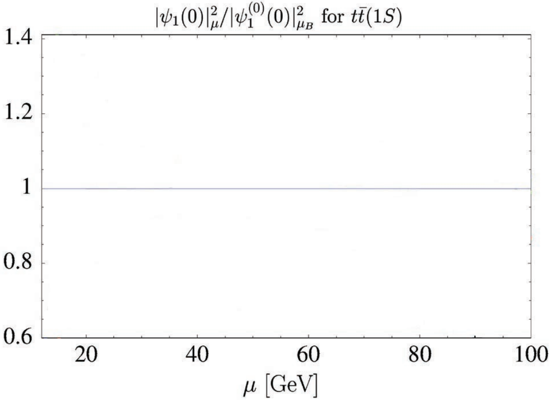

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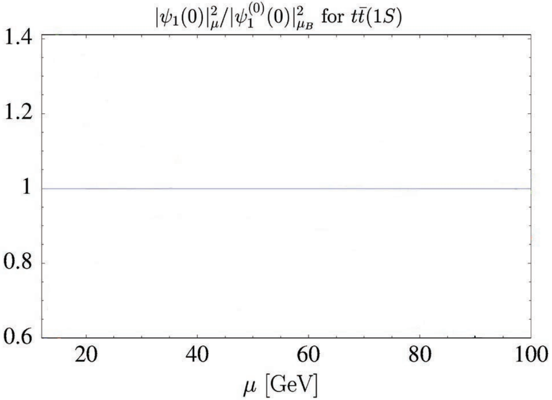

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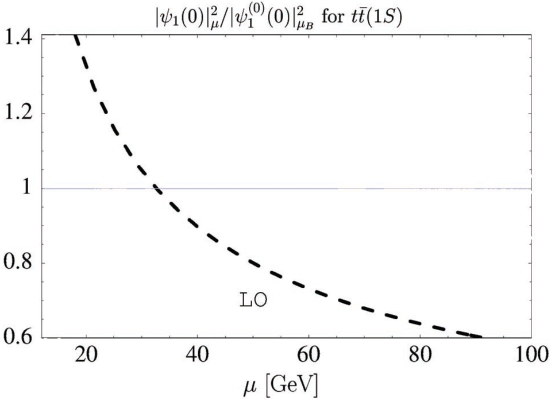

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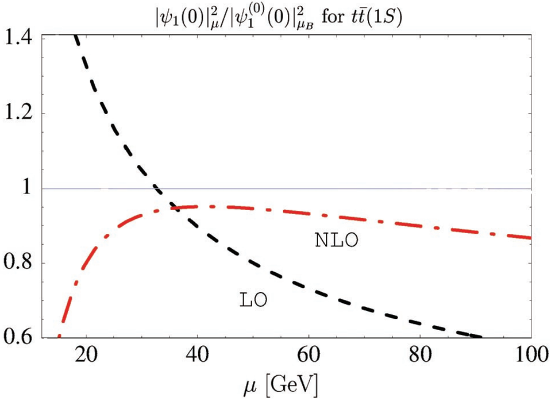

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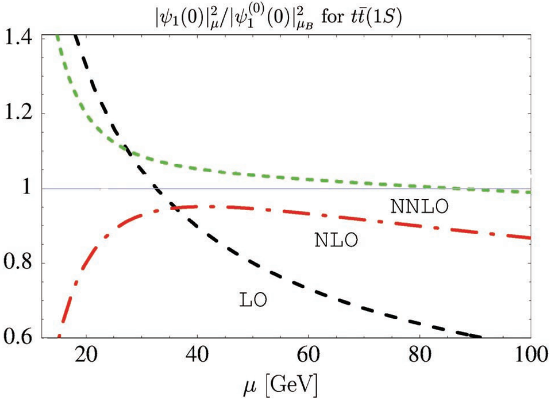

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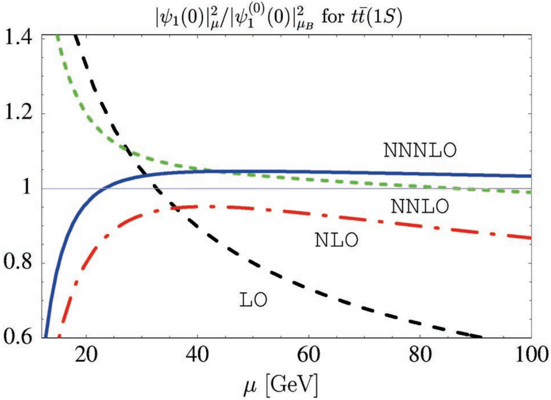

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