Lecture given by J.G. Körner at the Helmholz International School
Calc-2006, June 15 – 24, 2006, JINR, Dubna

Polar and azimuthal correlations in polarized top quark decays

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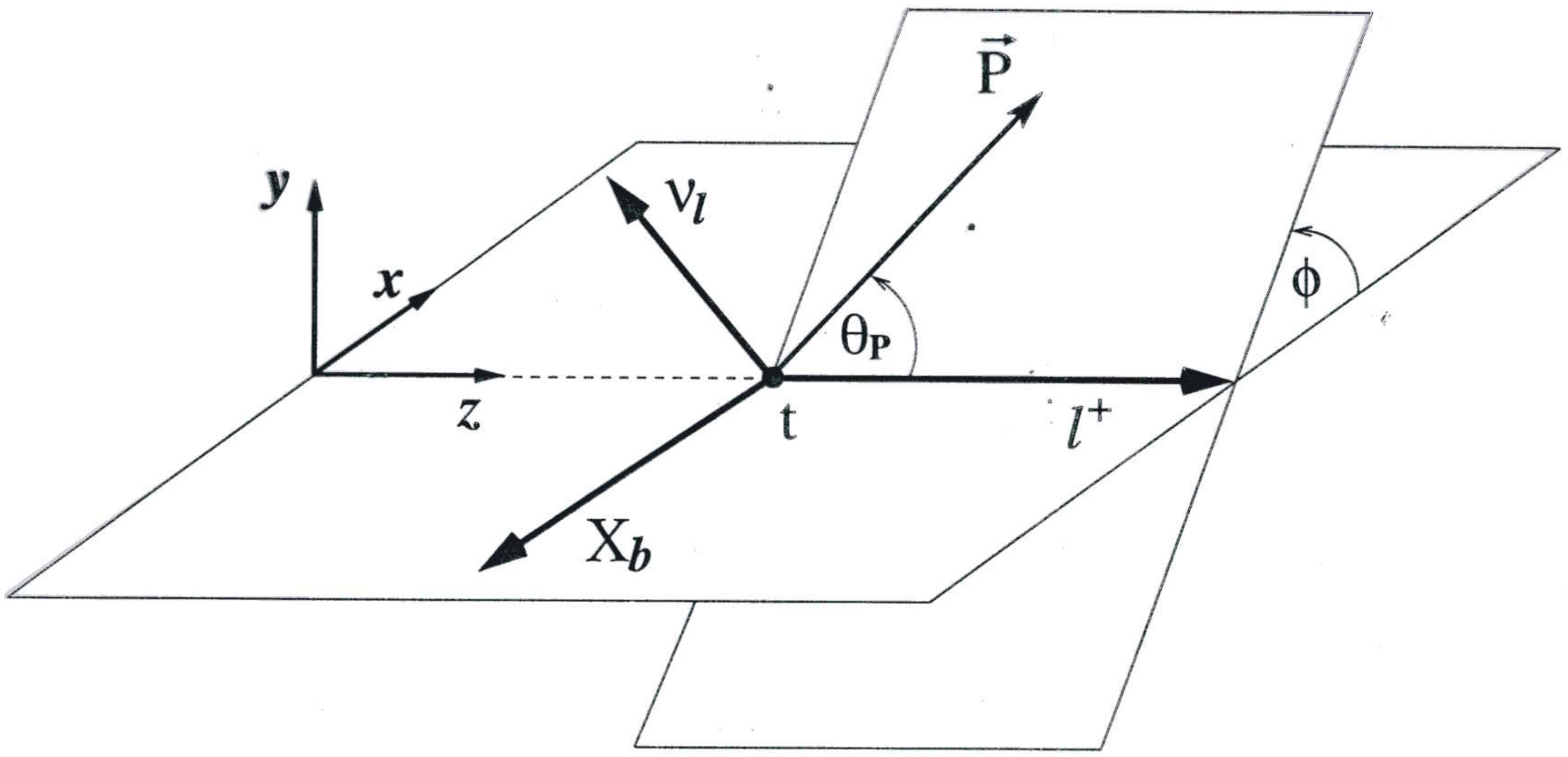
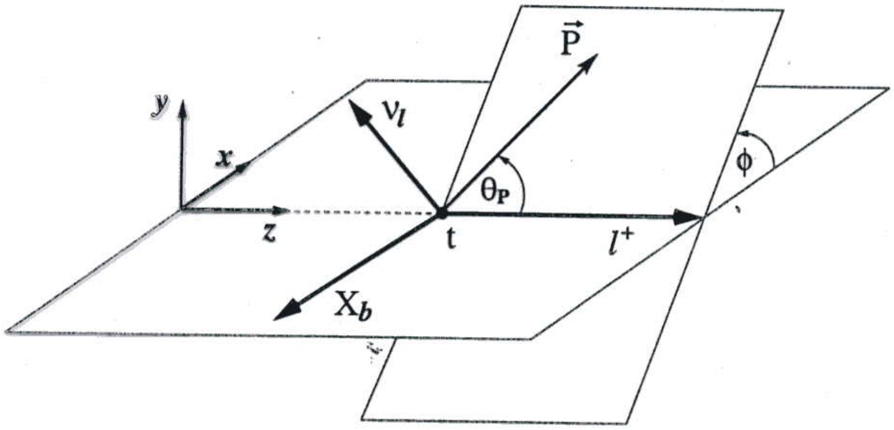

$\vec{P}$
y
ν_l
x
θ_P
ϕ
z
t
l^+
X_b


$\vec{P}$
ν_l
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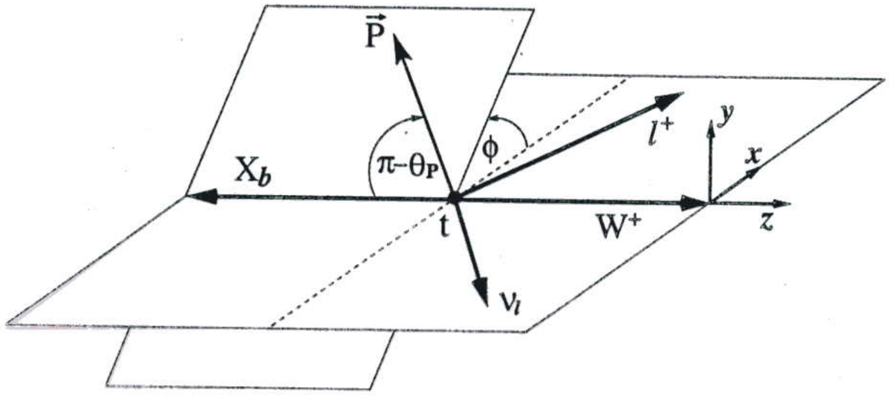
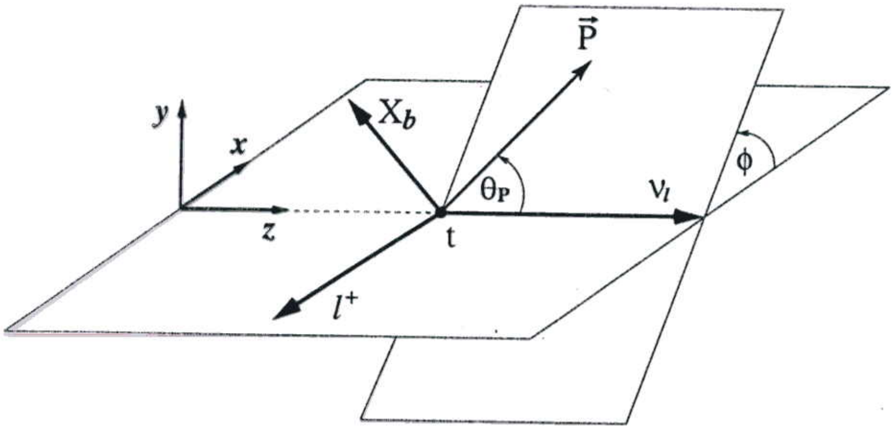
t
l^+
X_b

(1a)


$\vec{P}$
ϕ
l^+
y
x
X_b
$\pi-\theta_P$
t
W^+
z
ν_l

(2a')


$\vec{P}$
y
X_b
x
θ_P
ϕ
ν_l
z
t
l^+

(3a)

Covariant expansion of hadron tensor

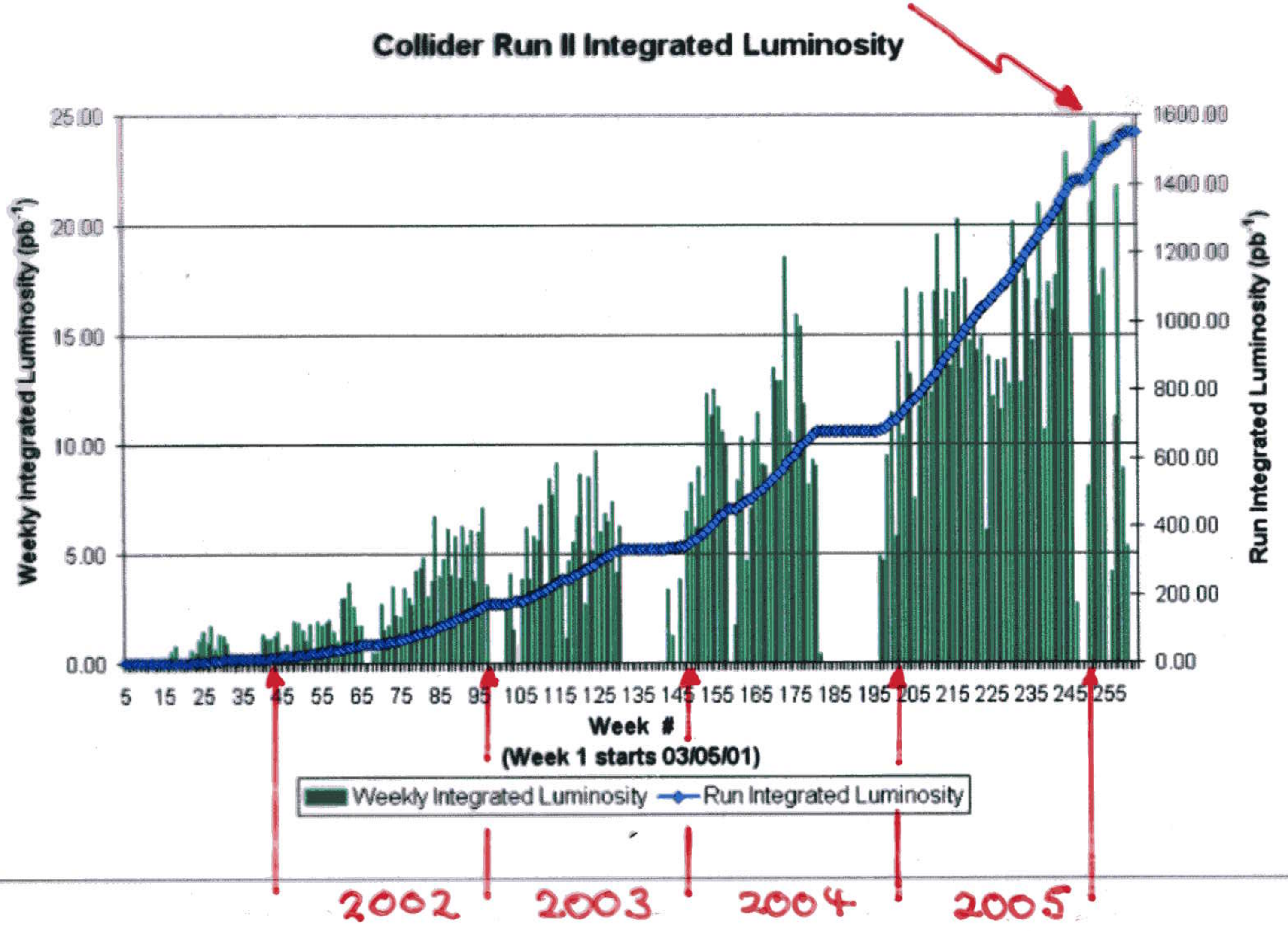
$$\begin{aligned} H^{\mu\nu} &= \Big(- g^{\mu\nu}\, H_1 + p_t^\mu p_t^\nu\, H_2 - i\epsilon^{\mu\nu\rho\sigma} p_{t,\rho} q_\sigma\, H_3 \Big) + \\ &- \ (q\cdot s_t)\Big(- g^{\mu\nu}\, G_1 + p_t^\mu p_t^\nu\, G_2 - i\epsilon^{\mu\nu\rho\sigma} p_{t,\rho} q_\sigma\, G_3 \Big) + \\ &+ \ \Big(s_t^\mu p_t^\nu + s_t^\nu p_t^\mu \Big)\, G_6 + i\epsilon^{\mu\nu\rho\sigma} p_{t\rho} s_{t\sigma}\, G_8 + i\epsilon^{\mu\nu\rho\sigma} q_\rho s_{t\sigma}\, G_9\,, \end{aligned}$$

- The aim of the game is to measure $W_i(q_0)$ and $G_i(q_0)$ or to compare measurements with Standard Model predictions.

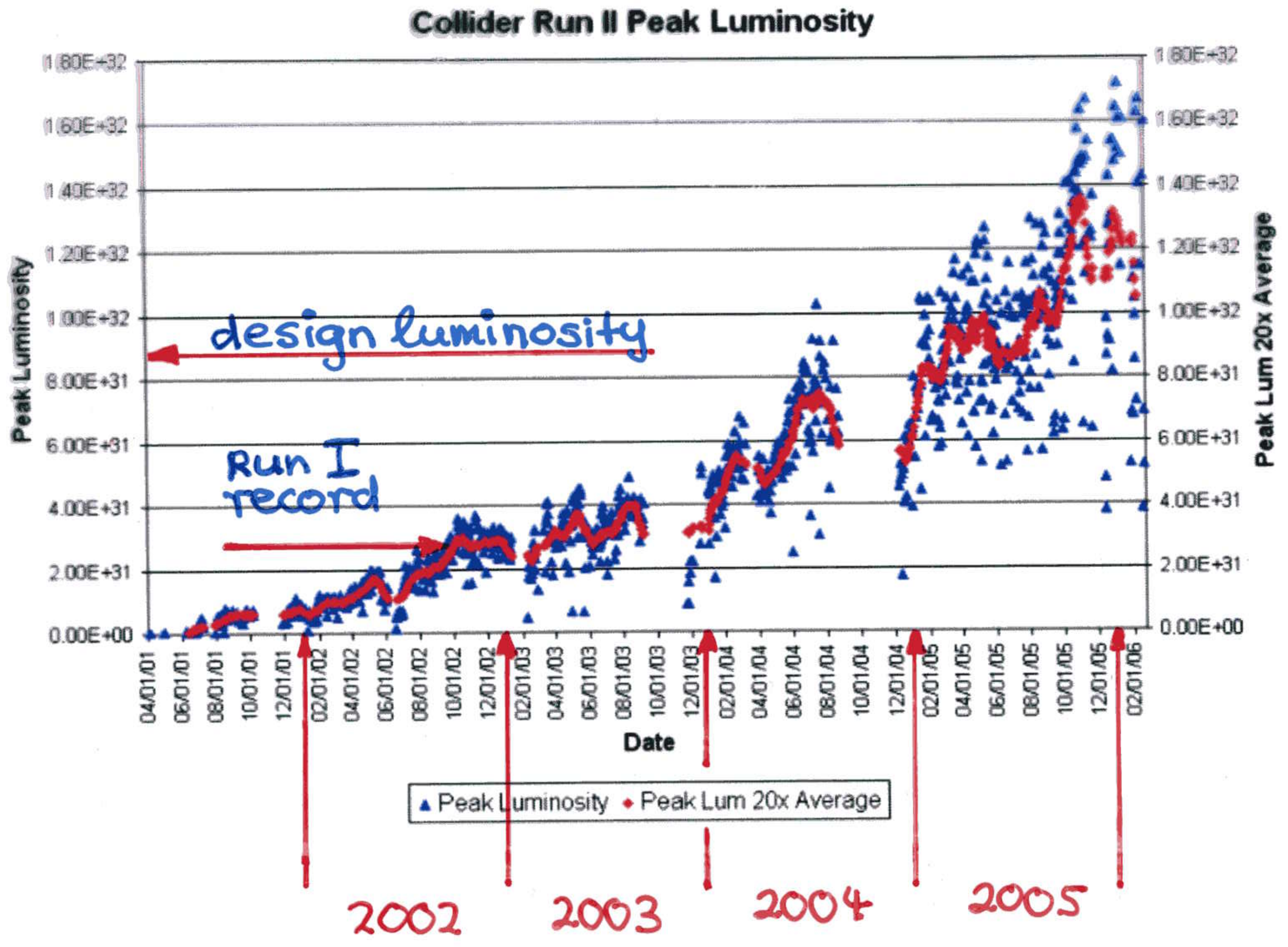
$$L^{\mu\nu} H_{\mu\nu}(G_1) = m_t q^2 G_1$$

$$\left\{ \begin{array}{l} \frac{x_l \hat{q}_0 - y^2}{x_l} \cos\theta_{P1} + \frac{y}{x_l}\sqrt{x_l(2\hat{q}_0 - x_l) - y^2} \sin\theta_{P1}\cos\phi \\ \sqrt{\hat{q}_0^2 - y^2} \cos\theta_{P2} \\ \frac{\hat{q}_0 - y^2}{2\hat{q}_0 - x_l} \cos\theta_{P3} + \frac{y}{2\hat{q}_0 - x_l}\sqrt{x_l(2\hat{q}_0 - x_l) - y^2} \sin\theta_{P3}\cos\phi \end{array} \right\},$$

If Tevatron performs as here can reach 1.3 fb^{-1}/y



Collider Run II Integrated Luminosity


Collider Run II Peak Luminosity
Peak Luminosity
Peak Lum 20x Average
1.80E+32
1.60E+32
1.40E+32
1.20E+32
1.00E+32
8.00E+31
6.00E+31
4.00E+31
2.00E+31
0.00E+00
04/01/01
06/01/01
08/01/01
10/01/01
12/01/01
02/01/02
04/01/02
06/01/02
08/01/02
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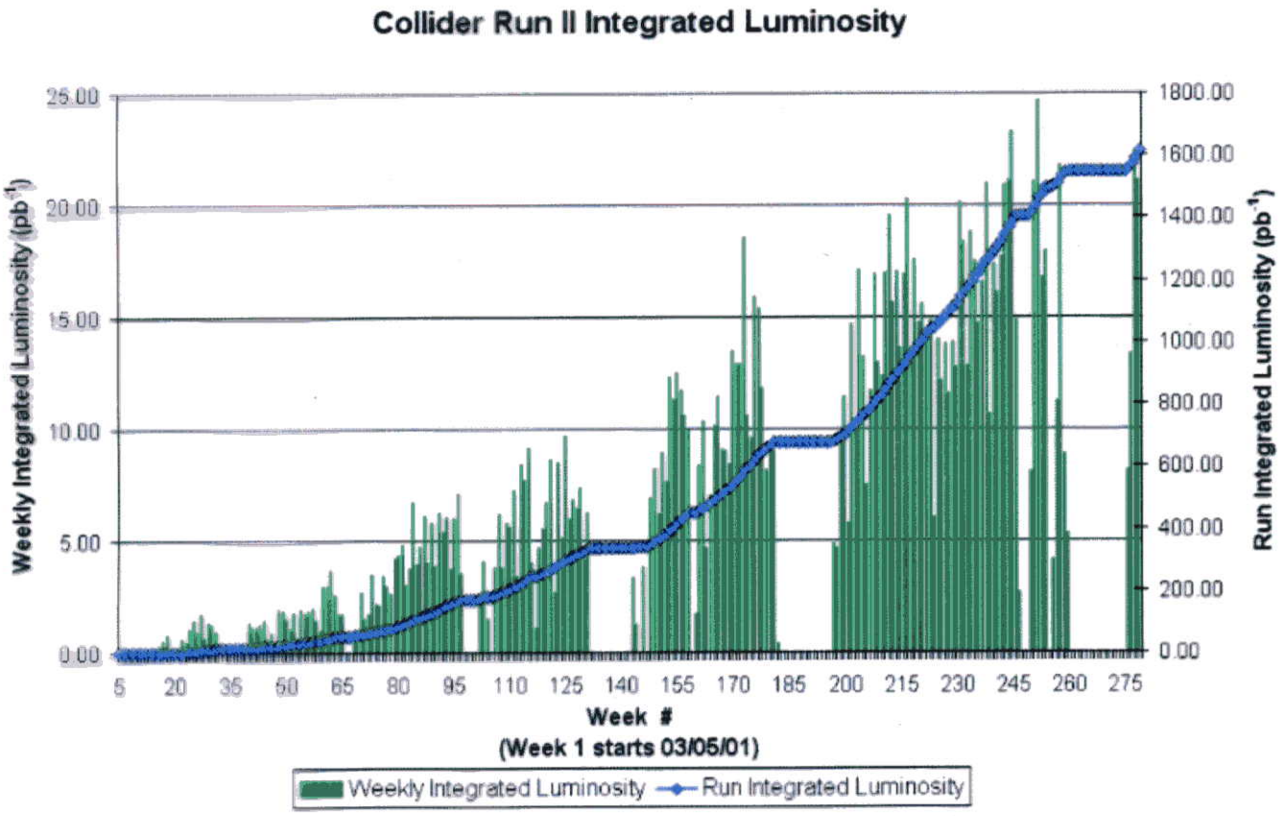
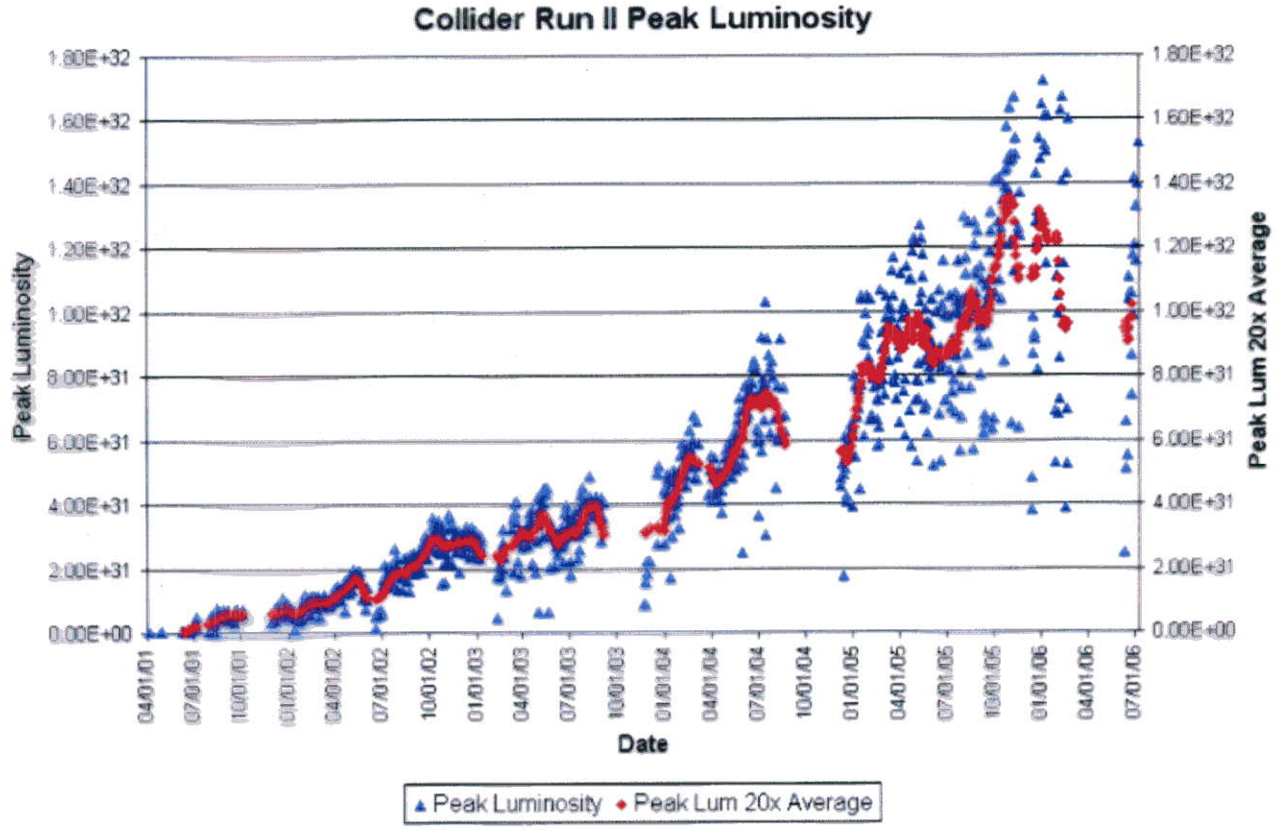
Date
▲ Peak Luminosity ◆ Peak Lum 20x Average
design luminosity
Run I record
2002
2003
2004
2005



Collider Run II Integrated Luminosity



Collider Run II Peak Luminosity

TOP QUARK YIELD

PAST

- Top quark discovered by CDF and D0 (Tevatron) collaborations in 1995
- Run I Tevatron $\sqrt{s} = 1.8$ TeV
  approx 500 $t\bar{t}$ pairs at each detector (CDF, D0)

PRESENT

- Run II Tevatron $\sqrt{s} = 2.0$ TeV (started 2001)
- $\sigma(t\bar{t}) \simeq 6.8$ pb
  For integrated luminosity of $1\ fb^{-1}$ around
  7000 $t\bar{t}$ pairs expected
- $\sigma(t) \simeq 2.5$ pb (not yet seen)

  ↯ approx. 40% of $t\bar{t}$

FUTURE

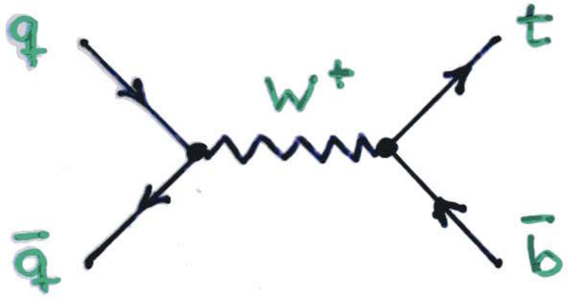
A. LHC to start in 2007

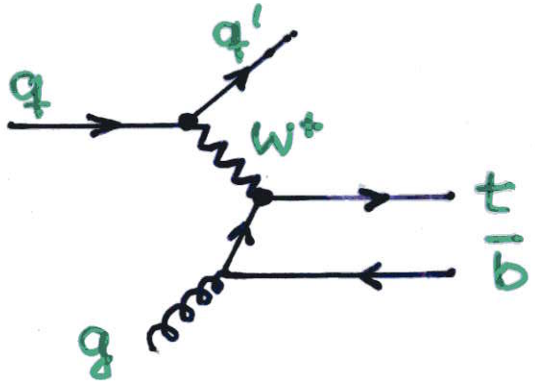
13% $q\bar{q} \to t\bar{t}$
87% $gg \to t\bar{t}$

- $\sigma(t\bar{t}) \simeq 800$ pb
  $\sigma(t) \simeq 300$ pb
- cross section 100 - fold increased
  luminosity 10 - fold increased (low luminosity run)

  ↷ $\simeq 10^7$ $t\bar{t}$-pairs per year
  (one $t\bar{t}$-pair every 4 s)
- later : high luminosity run
  - first years : low luminosity run $10\,fb^{-1}/y$
  - later : high luminosity run $100\,fb^{-1}/y$

- singly produced top quark expected to be almost 100% polarized (weak production!)





- top quark mass

  current world average $m_t = 172.7 \pm 2.9$ GeV

  expected uncertainty at end of run ($4-8\ fb^{-1}$):

$$\delta m_t \simeq 1.3 \text{ GeV}$$

B. International Linear Collider (ILC)

possibly starting in 2015

- $(1-4) \times 10^5$ $t\bar{t}$-pairs/y

  depending on $\sqrt{s}$

  $\sqrt{s} = 350, 500, 800$ GeV

- high degree of polarization of top quark through tuning of beam polarization

ANGULAR DECAY DISTRIBUTION

$$\frac{d\Gamma}{dx_e \, d\hat{q}_0 \, d\cos\Theta_p \, d\phi} =$$

$$\frac{1}{4\pi}\left(\frac{d\Gamma_A}{dx_e \, d\hat{q}_0} + P \left(\frac{d\Gamma_B}{dx_e \, d\hat{q}_0} \cos\Theta_p + \frac{d\Gamma_C}{dx_e \, d\hat{q}_0} \sin\Theta_p \cos\phi \right) \right)$$

A : unpolarized

B : polar correlation

C : azimuthal correlation

$x_e = 2E_\ell / m_t$ scaled lepton energy

$\hat{q}_0 = q_0 / m_t$ scaled energy of W^+

P : magnitude of top quark polarization

Polarization vector S_t^μ

$$S_t^\mu = (0 ; \sin\Theta_p \cos\phi , \sin\Theta_p \sin\phi , \cos\Theta_p)$$

generic momentum in decay plane

$$p^\mu = (E ; p_x , 0 , p_z)$$

INPUT

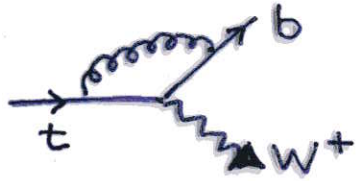
- Born term

$$B^{\mu\nu} = \frac{1}{4} \mathrm{Tr} \left\{ \not{p}_b \gamma^\mu (1-\gamma_5)(\not{p}_t + m_t) \frac{1}{2}(1+\gamma_5 \not{s}_t) \gamma^\nu (1-\gamma_5) \right\}$$

$$= \bar{p}_t^{\,\mu} p_b^{\,\nu} + \bar{p}_t^{\,\nu} p_b^{\,\mu} - \bar{p}_t \cdot p_b \, g^{\mu\nu} - i \varepsilon^{\mu\nu\alpha\beta} \bar{p}_{t,\alpha} p_{b,\beta}$$

$$\bar{p}_t^{\,\mu} = p_t^{\,\mu} - m_t s_t^{\,\mu}$$

- Loop contribution



$$\langle b(p_b)|J^V_\mu|t(p_t)\rangle = \bar{u}_b(p_b)\Big\{\gamma_\mu F^V_1 + p_{t,\mu}F^V_2 + p_{b,\mu}F^V_3\Big\}u_t(p_t),$$
$$\langle b(p_b)|J^A_\mu|t(p_t)\rangle = \bar{u}_b(p_b)\Big\{\gamma_\mu F^A_1 + p_{t,\mu}F^A_2 + p_{b,\mu}F^A_3\Big\}\gamma_5 u_t(p_t)$$

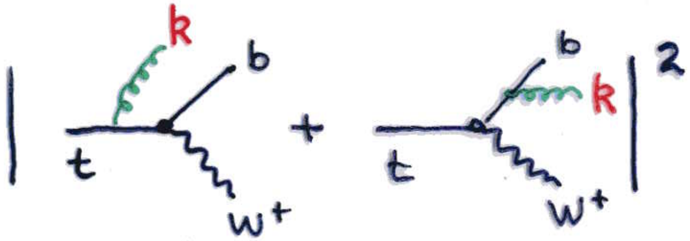
$\Lambda = m_g/m_t \qquad \varepsilon = m_b/m_t$

$$F^V_1 = F^A_1 = 1 - \frac{\alpha_s}{4\pi}C_F\Big(4 + \frac{1}{y^2}\ln(1-y^2) + \ln\Big(\frac{\boxed{\epsilon}}{1-y^2}\frac{\boxed{\Lambda^4}}{(1-y^2)^2}\Big) +$$
$$+2\ln\Big(\frac{\boxed{\Lambda^2}}{\boxed{\epsilon}}\frac{1}{1-y^2}\Big)\ln\Big(\frac{\boxed{\epsilon}}{1-y^2}\Big) + 2Li(y^2)\Big),$$
$$F^V_2 = -F^A_2 = \frac{1}{m_t}\frac{\alpha_s}{4\pi}C_F\frac{2}{y^2}\Big(+1 + \frac{1-y^2}{y^2}\ln(1-y^2)\Big),$$
$$F^V_3 = -F^A_3 = \frac{1}{m_t}\frac{\alpha_s}{4\pi}C_F\frac{2}{y^2}\Big(-1 + \frac{2y^2-1}{y^2}\ln(1-y^2)\Big)$$

$\boxed{\Lambda}$ infrared singularity

$\boxed{\varepsilon}$ collinear (mass) singularity

Tree contribution



$$\begin{aligned}
\mathcal{H}^{\mu\nu} \;=\; & -4\pi\alpha_s C_F \frac{8}{(k\cdot p_t)(k\cdot p_b)}\Bigg\{ \\
& - i\frac{k\cdot p_t}{k\cdot p_b}\Big(\epsilon^{\alpha\beta\mu\nu}\,(p_b-k)\cdot\bar{p}_t - \epsilon^{\alpha\beta\gamma\nu}(p_b-k)^{\mu}\,\bar{p}_{t,\gamma} + \epsilon^{\alpha\beta\gamma\mu}(p_b-k)^{\nu}\,\bar{p}_{t,\gamma}\Big)k_\alpha\,p_{b,\beta} + \\
& + \frac{k\cdot p_b}{k\cdot p_t}\Big[(\bar{p}_t\cdot p_t)\Big(k^\mu\,p_b^\nu + k^\nu\,p_b^\mu - k\cdot p_b\,g^{\mu\nu} - i\,\epsilon^{\alpha\beta\mu\nu}k_\alpha\,p_{b,\beta}\Big) + \\
& - (\bar{p}_t\cdot k)\Big((p_t-k)^\mu\,p_b^\nu + (p_t-k)^\nu\,p_b^\mu - (p_t-k)\cdot p_b\,g^{\mu\nu} - i\,\epsilon^{\alpha\beta\mu\nu}(p_t-k)_\alpha\,p_{b,\beta}\Big)\Big] + \\
& - (\bar{p}_t\cdot p_b)\Big(k^\mu\,p_b^\nu + k^\nu\,p_b^\mu - k\cdot p_b\,g^{\mu\nu} - i\,\epsilon^{\alpha\beta\mu\nu}k_\alpha\,p_{b,\beta}\Big) + (p_t\cdot p_b)\Big(k^\mu\,\bar{p}_t^\nu + k^\nu\,\bar{p}_t^\mu - k\cdot\bar{p}_t\,g^{\mu\nu}\Big) + \\
& - (k\cdot p_b)\Big(p_t^\mu\,\bar{p}_t^\nu + p_t^\nu\,\bar{p}_t^\mu - p_t\cdot\bar{p}_t\,g^{\mu\nu}\Big) + (k\cdot p_t)\Big((p_b+k)^\mu\,\bar{p}_t^\nu + (p_b+k)^\nu\,\bar{p}_t^\mu - (p_b+k)\cdot\bar{p}_t\,g^{\mu\nu}\Big) + \\
& + (k\cdot\bar{p}_t)\Big(2p_b^\mu\,p_b^\nu - p_b\cdot p_b g^{\mu\nu}\Big) + i\Big(\epsilon^{\alpha\beta\mu\nu}\,(k\cdot\bar{p}_t) + \epsilon^{\alpha\beta\gamma\mu}\,k^\nu\bar{p}_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu}\,k^\mu\bar{p}_{t,\gamma}\Big)p_{b,\alpha}\,p_{t,\beta} + \\
& + i\Big(\epsilon^{\alpha\beta\mu\nu}\,(p_t\cdot\bar{p}_t) + \epsilon^{\alpha\beta\gamma\mu}\,p_t^\nu\bar{p}_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu}\,p_t^\mu\bar{p}_{t,\gamma}\Big)k_\alpha\,p_{b,\beta}\Bigg\} + B^{\mu\nu}\cdot S_{SGF}
\end{aligned}$$

soft gluon factor

Born term structure

$$S_{SGF} \sim \left(\frac{p_b^\mu}{k\cdot p_b} - \frac{p_t^\mu}{k\cdot p_t}\right)\left(\frac{p_b^\nu}{k\cdot p_b} - \frac{p_t^\nu}{k\cdot p_t}\right) g_{\mu\nu}$$

some times $\bar{p}_t$

sometimes p_t

Generic structure of differential rate

$$\frac{d\Gamma_i^{NLO}}{dx_e\, dz} = \underbrace{M_L^i}_{\text{loop}}(x_e)\,\delta(z-\varepsilon^2) + \underbrace{M_T^i}_{\text{tree}}(x_e, z)$$

$(i = A, B, C)$

$$+ f(z, x_e)\Big[\underbrace{S_{SGF}}_{\text{subtracted soft gluon contribution}}(z)\Big]_+ + f(z, x_e)\,\delta(z-\varepsilon^2)\underbrace{A_{SGF}}_{\text{soft gluon contribution}}$$

- $z = (p_b + k)^2/m_t^2 = 1 + y^2 - 2\hat{q}_0$ $\quad (y = m_W/m_t)$

  a shift in the energy variable

  $$\delta(\hat{q}_0 - \tfrac{1}{2}(1+y^2-z)) \longrightarrow \delta(z-\varepsilon^2)$$

- singularities occur when kinematics becomes Born term like $\quad z = (p_b + \cancel{k})^2/m_t^2 \rightarrow \varepsilon^2$

- "plus" prescription

$$\int_{\varepsilon^2} dz\, f(z,x_e)\left[S_{SGF}\right]_+ = \int_{\varepsilon^2} dz \left[f(z,x_e) - f(\varepsilon^2,x_e\right] S_{SGF}(z)$$

singular point

- soft gluon contribution

$$A_{SGF} = \int_{(\varepsilon+\Lambda)^2} dz\, S_{SGF}(z)$$

- subtraction

$$\int dz\, f(z,x_e)\, S_{SGF}(z)$$

$$= \int dz \left[f(z,x_e) - f(\varepsilon^2,x_e)\right] S_{SGF}(z) + \int dz\, f(\varepsilon^2,x_e)\, S_{SGF}(z)$$

SAMPLE RESULTS

A. Azimuthal correlation for z-axis along lepton

x_ℓ - spectrum

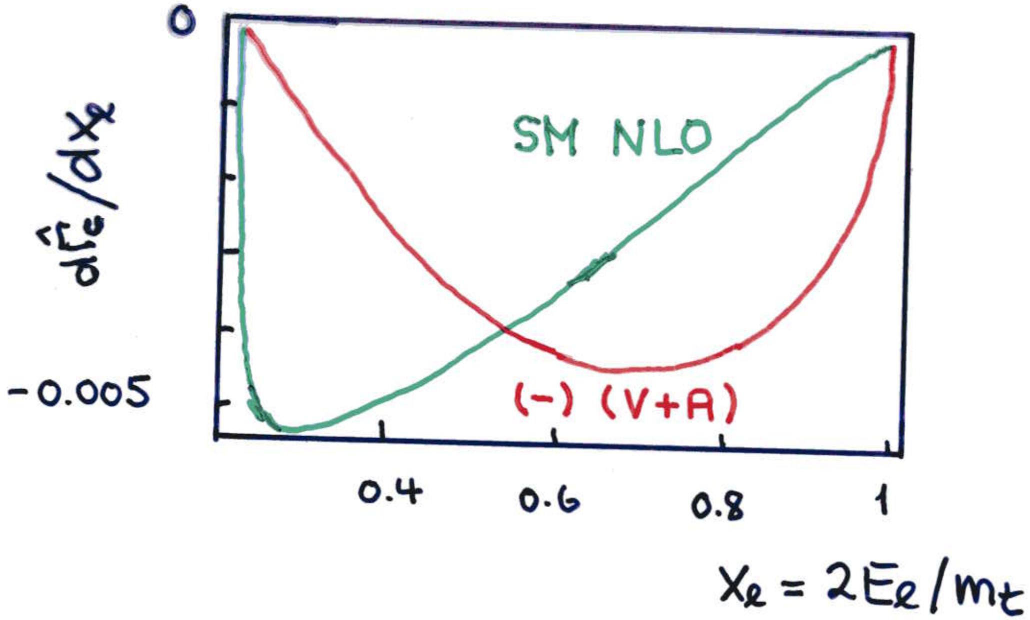
- azimuthal correlation is zero at Born term level

$$L_{\mu\nu} B^{\mu\nu} = 4\, p_\ell \cdot (p_t - m s_t)(p_\nu p_b)$$

$$p_\ell = E_\ell (1; 0, 0, 1)$$

$$s_t = (0; \sin\Theta_p \cos\phi, \sin\Theta_p \sin\phi, \cos\Theta_p)$$

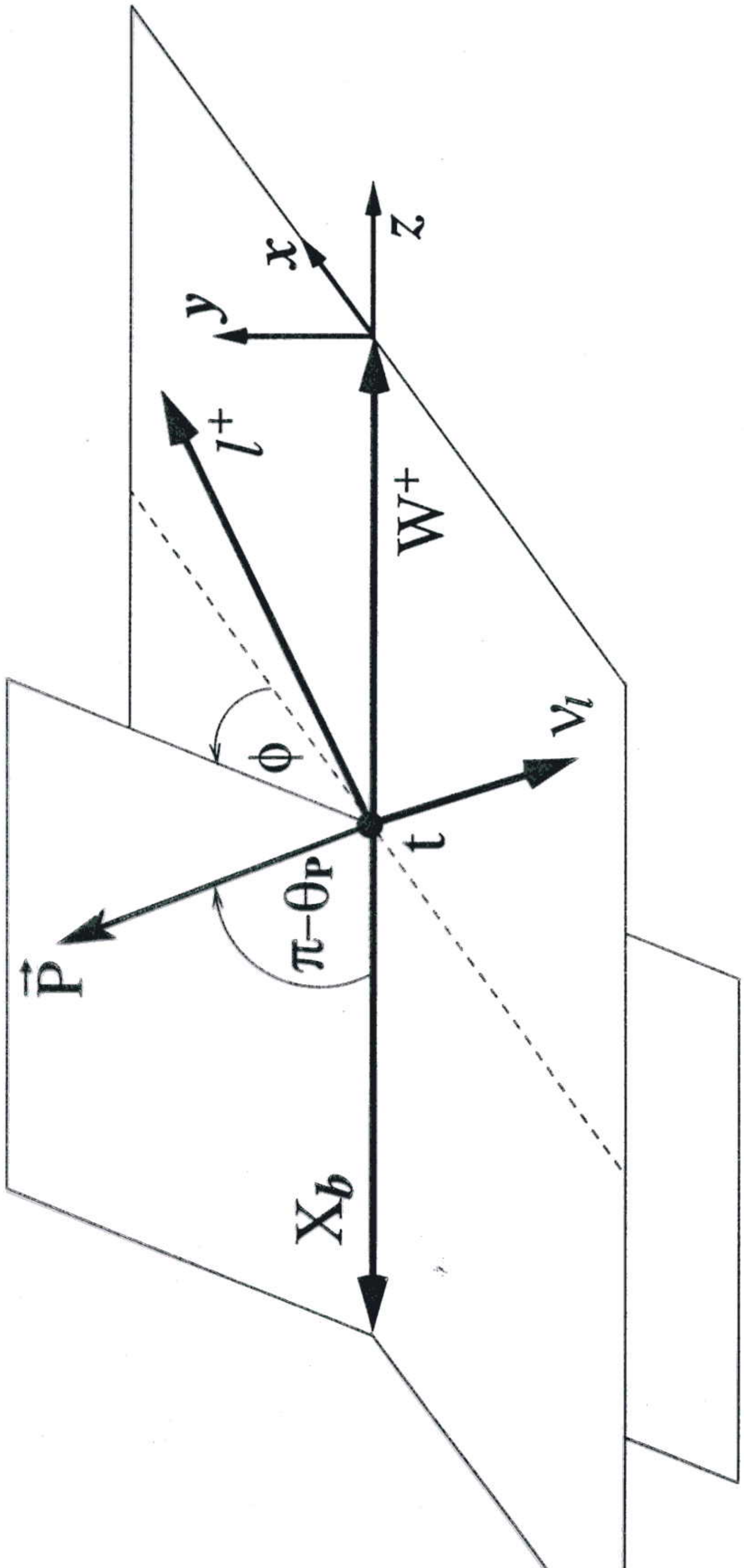
- vanishing of azimuthal correlation due to (V-A) structure of SM currents
- non-zero azimuthal correlation at NLO (or from right-chiral current)


0
-0.005
$d\hat{\Gamma}_e/dx_e$
SM NLO
(-) (V+A)
0.4
0.6
0.8
1
$x_e = 2E_e/m_t$

Integrated rates   z-axis along W^+

$$\Gamma_i = 2\pi\Gamma_F\frac{m_W}{\Gamma_W}y^2(1-y^2)^2(1+2y^2)\hat{\Gamma}_i$$

$$\begin{aligned}
\hat{\Gamma}_A &= 1+\frac{\alpha_s}{2\pi}C_F\frac{y^2}{(1-y^2)^2(1+2y^2)}\left\{\frac{(1-y^2)(5+9y^2-6y^4)}{2y^2}-\frac{2(1-y^2)^2(1+2y^2)\pi^2}{3y^2}\right.\\
&-\frac{(1-y^2)^2(5+4y^2)}{y^2}\ln(1-y^2)-\frac{4(1-y^2)^2(1+2y^2)}{y^2}\ln(y)\ln(1-y^2)-4(1+y^2)\times\\
&\left.\times(1-2y^2)\ln(y)-\frac{4(1-y^2)^2(1+2y^2)}{y^2}\mathrm{Li}_2(y^2)\right\},\\
\hat{\Gamma}_B &= \frac{1-2y^2}{1+2y^2}+\frac{\alpha_s}{2\pi}C_F\frac{y^2}{(1-y^2)^2(1+2y^2)}\left\{-\frac{(1-y)^2(15+2y-5y^2-12y^3+2y^4)}{2y^2}+\right.\\
&+\frac{(1+4y^2)\pi^2}{3y^2}-\frac{(1-y^2)^2(1-4y^2)}{y^2}\ln(1-y)-\frac{(1-y^2)(3-y^2)(1+4y^2)}{y^2}\ln(1+y)+\\
&\left.-\frac{4(1-y^2)^2(1-2y^2)}{y^2}\mathrm{Li}_2(y)+\frac{4(2+5y^4-2y^6)}{y^2}\mathrm{Li}_2(-y)\right\},\\
\hat{\Gamma}_C &= \frac{y}{\sqrt{2}(1+2y^2)}+\frac{\alpha_s}{2\pi}C_F\frac{y^2}{(1-y^2)^2(1+2y^2)}\left\{\frac{(1-y^2)(1+2y^2)}{\sqrt{2}y}-\frac{\pi^2}{6\sqrt{2}}\times\right.\\
&\times\frac{(3-5y^2+6y^4)}{y}-\frac{(1-y^2)^2(1+5y^2)}{2\sqrt{2}y^3}\ln(1-y^2)-\frac{y(5-11y^2)}{\sqrt{2}}\ln(y)+\\
&-\frac{(1-y)^2(3+7y+6y^2)}{\sqrt{2}y}\ln(y)\ln(1-y)-\frac{(1+y)^2(3-7y+6y^2)}{\sqrt{2}y}\ln(y)\ln(1+y)+\\
&\left.-\frac{(1-y)^2(7+15y+10y^2)}{\sqrt{2}y}\mathrm{Li}_2(y)-\frac{(1+y)^2(7-15y+10y^2)}{\sqrt{2}y}\mathrm{Li}_2(-y)\right\}
\end{aligned}$$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X_b
t
l^+
W^+
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y
x
z

Numerically:

System 1 z-axis along lepton

$$\frac{d\Gamma^{NLO}}{d\cos\theta_p\, d\phi} = \frac{\Gamma_A^{Born}}{4\pi}\left[(1-8.5\%)+(1-8.7\%)P\cos\theta_p - 0.24\% P\sin\theta_p\cos\phi\right]$$

System 2 z-axis along W^+ boson

$$\frac{d\Gamma^{NLO}}{d\cos\theta_p\, d\phi} = \frac{\Gamma_A^{Born}}{4\pi}\Big[(1-8.5\%)+(0.41-11.6\%)P\cos\theta_p-(0.76-8.2\%)P\sin\theta_p\cos\phi$$

System 3 z-axis along neutrino

$$\frac{d\Gamma^{NLO}}{d\cos\theta_p\, d\phi} = \frac{\Gamma_A^{Born}}{4\pi}\Big[(1-8.5\%)+(-0.32+1.0\%)P\cos\theta_p-(0.92-8.61\%)P\sin\theta_p\cos\phi$$

NOVELTY VALUE

	spectrum			rate		
	ℓ^+	W^+	ν_ℓ	ℓ^+	W^+	ν_e
unpolarized (A)	✓*	–	–	✓	–	–
polar (B)	✓*	new	✓*	✓*	✓**	✓*
azimuthal (C)	new	new	new	new	✓**	new

* Jezabek, Kühn (1989), Czarnecki, JGK

** Fischer, Groote, Mauser, JGK (2002)

new : Groote, Huo, Kadeer, Kubistin, JGK (2006)