Feynman Rules for Effective High-Energy Regge Theory

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Abstract

Starting from the gauge invariant effective action in the quasi-multi-Regge kinematics (QMRK), we evaluate the effective reggeon-gluon vertices: Rgg, RRg, $RRgg$, $Rggg$, $RRggg$, $Rgggg$ where the *on*-mass-shell particles g are gluons, or sets of gluons with small invariant masses. The explicit expressions satisfying the Bose-symmetry and gauge invariance conditions are obtained.

Plan

- Introduction: **Quasi-multi-Regge kinematics**

- High-energy **Effective action** and recurrence relations for the reggeon vertices

- Derivation of **effective vertices**: EV's "LEGO"

- **Cross sections** for jet production in QMRK

- Conclusions, applications and outlook

Quasi-multi-Regge kinematics: notations

Process under consideration: **Gluon-gluon collision** in the large-$\sqrt{s}$ regime (c.m. system).

The main contribution to the total cross section stems from QMRK of the final state particles. The final state particles compose several groups (clusters) with arbitrary number of partons with fixed masses M_i $(i = 1, 2, ..., n)$.

These clusters are produced in the **multi-Regge kinematics** with respect to each other:

$$P_A + P_B = Q_1 + Q_2 + ... + Q_n$$

$$s = 2P_A P_B = 4E^2 \gg s_i = 2Q_i Q_{i+1} \gg |t_i| = |q_i^2| \ , \ i = 1, 2, ..., n-1 \ ;$$

$$Q_i^2 = M_i^2 \ , \ Q_k = \sum_j p_j^{(k)} \ , \ k = 1, 2, ..., n$$

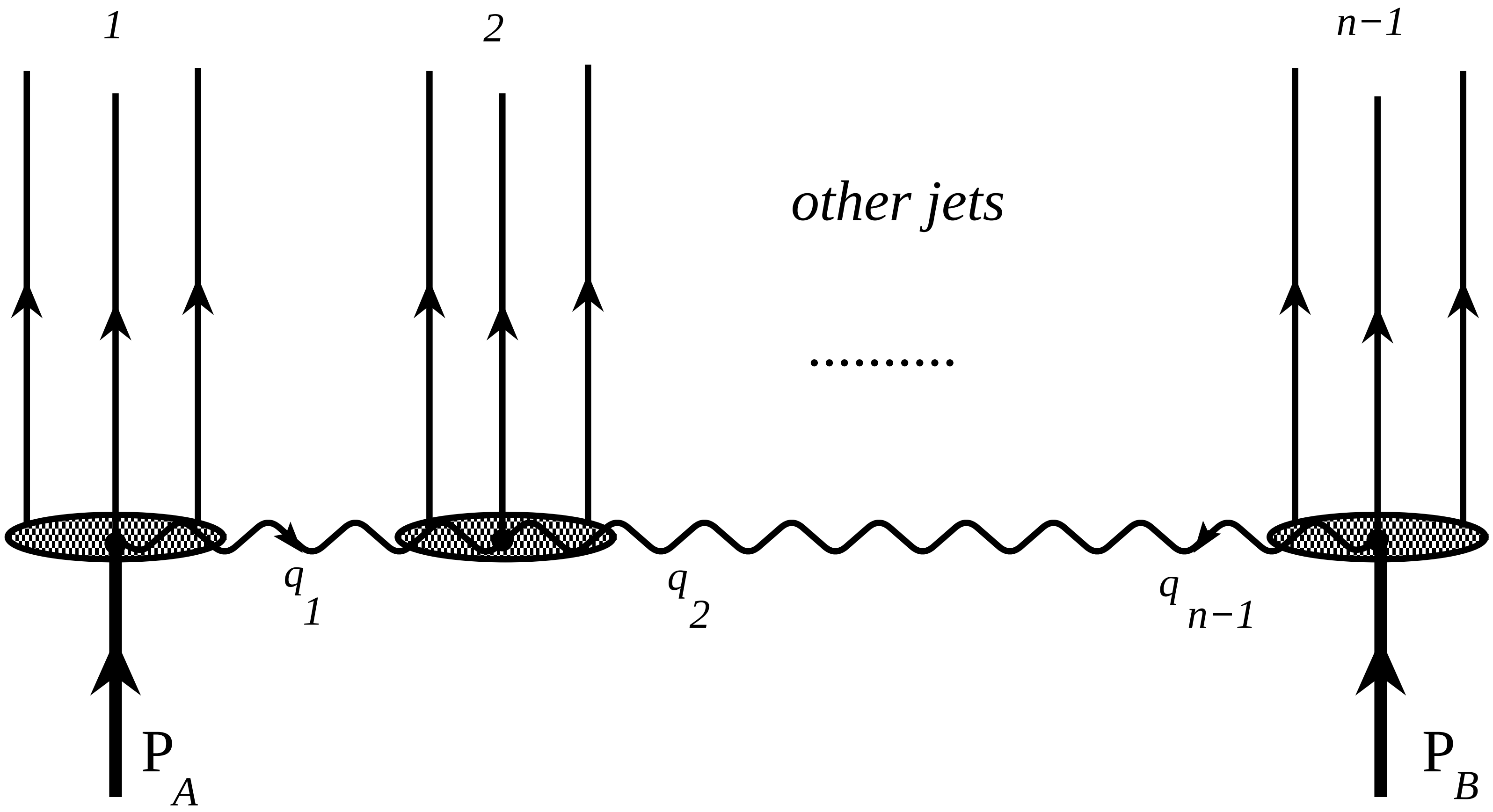

Figure 1: QMRK: notations. n-jet production.

Leading logarithmic approximation: BFKL

"Clusters" are single gluons:

$$A_n^{LLA} = A_n^{tree} \prod_i s_i^{\omega(t_i)}$$

The gluon **Regge trajectory** $j = 1 + \omega(t)$:

$$\omega(t) = -\frac{\alpha_s N_c}{4\pi^2} \int d^2\boldsymbol{k} \frac{\boldsymbol{q}^2}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} \; , \; t = -\boldsymbol{q}^2$$

Production amplitude in the **tree approximation**:

$$A_n^{tree} = 2s g_s T_{c_1 AA'} \Gamma_{AA'} \frac{1}{t_1} g_s T_{d_1 c_2 c_1} \Gamma^1_{2,1} \frac{1}{t_2} \cdots \cdot \frac{1}{t_{n-1}} g_s T_{c_{n-1} BB'} \Gamma_{BB'}$$

where $\Gamma_{NN'}$ and $\Gamma^{i-1}_{i,i-1}$ are **effective** Rgg and RRg vertices.

LLA BFKL cross section grows rapidly

$$\sigma_t \sim s^{\omega} \quad , \quad \omega = \frac{4\alpha_s N_c}{\pi} \ln 2$$

$\Rightarrow$ violates Froissart bound! $\sigma_t \sim c \log^2 s$

One of the possible ways to unitarize LLA results—application of **Effective High-Energy Theory**: gauge invariant action describing gluon-reggeon interactions *(Lipatov, NPB(1995))*.

Kinematics

Introduce the **light-cone** vectors:

$$n^{+}=\frac{P_B}{E}\ ,\ \ n^{-}=\frac{P_A}{E}\ ,\ \ n^{+}n^{-}=2\ ,\ (n^{\pm})^2=0$$

$$k^{\pm}=\left(n^{\pm}\right)_{\mu}\cdot k^{\mu}\ ,\ \partial_{\pm}=\left(n^{\pm}\right)_{\mu}\cdot\partial^{\mu}$$

Derivatives:

$$\partial_{\pm}v(p)=-ip^{\pm}v(p)\ ,\ \frac{1}{\partial_{\pm}}v(p)=\frac{i}{p^{\pm}}v(p)\ ,\ \partial_{\sigma}^{2}A(q)=-q^2A(q)\ ,\ q^2=-\boldsymbol{q}_{\perp}^{2}$$

Momentum transfers:

$$q_1=P_A-P_{A'}\ \ ,\ \ q_{n-1}=P_B-P_{B'}$$

The **Sudakov decomposition**:

$$q_1 = q_{1\perp} + \frac{q_1^+}{2} n^- \ , \ q_{n-1} = q_{n-1\perp} + \frac{q_{n-1}^-}{2} n^+ \ , \ q_1^- = q_{n-1}^+ = 0$$

The **Sudakov variables** for produced particles are:

$$p_i = \frac{p_i^+}{2} n^- + \frac{p_i^-}{2} n^+ + p_{i\perp}$$

The **reggeized gluons** are *off*-mass-shell particles lying on **Regge trajectories**. Initial, final and produced particles are *on*-mass-shell (massless) particles.

Momentum conservation

- in the **fragmentation regions**:

$$P_A - q_1 \to p_1 + p_2 + ... + p_n \;:\; P_A^+ = \sum_{i=1}^{n} p_i^+ \;,\; |q_1^+| \ll p_i^+$$

$$P_B - q_{n-1} \to p_1 + p_2 + ... + p_n \;:\; P_B^- = \sum_{i=1}^{n} p_i^- \;,\; |q_{n-1}^-| \ll p_i^-$$

- in the **central regions**:

$$q_i + q_{i+1} \to \sum_{i=1}^{n} p_i \;,\; q_i^+ = \sum_{i=1}^{n} p_i^+ \;;\; q_{i+1}^- = \sum_{i=1}^{n} p_i^-$$

Interaction of the gluons and reggeons is described in terms of the **effective action** with fields:

gluon field: $v_\mu = -iT^a v_\mu^a$

reggeon field: $A_\pm = -iT^a A_\pm^a$

$$[T^a, T^b] = i f_{abc} T^c \quad , \quad \text{Tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab}$$

Action:

$$S = \int d^4x \left[\mathcal{L}_{gQCD} + \mathcal{L}_{ind}\right]$$

$$\mathcal{L}_{gQCD} = \frac{1}{2}\text{Tr}\, G_{\mu\nu}^2 = \mathcal{L}_k + \mathcal{L}_{int} \ , \quad D_\mu = \partial_\mu + g v_\mu \ , \quad G_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu]$$

$$\mathcal{L}_k = -\frac{1}{4}(\partial_\mu v_\nu^a - \partial_\nu v_\mu^a)^2$$

$$\mathcal{L}_{int} = \frac{g}{2} f_{abc}(\partial_\mu v_\nu^a) v_\mu^b v_\nu^c - \frac{g^2}{4} f_{abc} f_{ade} v_\mu^b v_\nu^c v_\mu^d v_\nu^e$$

Induced part

$$\mathcal{L}_{ind} = \mathcal{L}_{ind}^k + \mathcal{L}_{ind}^{gR} \quad , \quad \mathcal{L}_{ind}^k = -\partial_\mu A_+^a \cdot \partial_\mu A_-^a$$

Lipatov's Effective Lagrangian

$$\mathcal{L}_{ind}^{gR}(v_\pm, A_\pm) = -\mathrm{Tr}\left\{ \frac{1}{g}\partial_+ \left[\mathcal{P} \exp\left(-\frac{g}{2} \int_{-\infty}^{x^+} v_+(x') dx'^+ \right) \right] \cdot \partial_\sigma^2 A_-(x) + \right.$$

$$\left. + \frac{1}{g}\partial_- \left[\mathcal{P} \exp\left(-\frac{g}{2} \int_{-\infty}^{x^-} v_-(x') dx'^- \right) \right] \cdot \partial_\sigma^2 A_+(x) \right\} =$$

$$= \mathrm{Tr}\left\{ \left[v_+ - g v_+ \frac{1}{\partial_+} v_+ + g^2 v_+ \frac{1}{\partial_+} v_+ \frac{1}{\partial_+} v_+ - \ldots \right] \partial_\sigma^2 A_- + \right.$$

$$\left. + \left[v_- - g v_- \frac{1}{\partial_-} v_- + g^2 v_- \frac{1}{\partial_-} v_- \frac{1}{\partial_-} v_- - \ldots \right] \partial_\sigma^2 A_+ \right\}$$

In **QMRK**, momenta of reggeons are transverse $-q_i^2 \approx \boldsymbol{q_i}^2$ and the following kinematic relation is implied for the reggeon fields

$$\partial_- A_+ = \partial_+ A_- = 0$$

Induced vertices

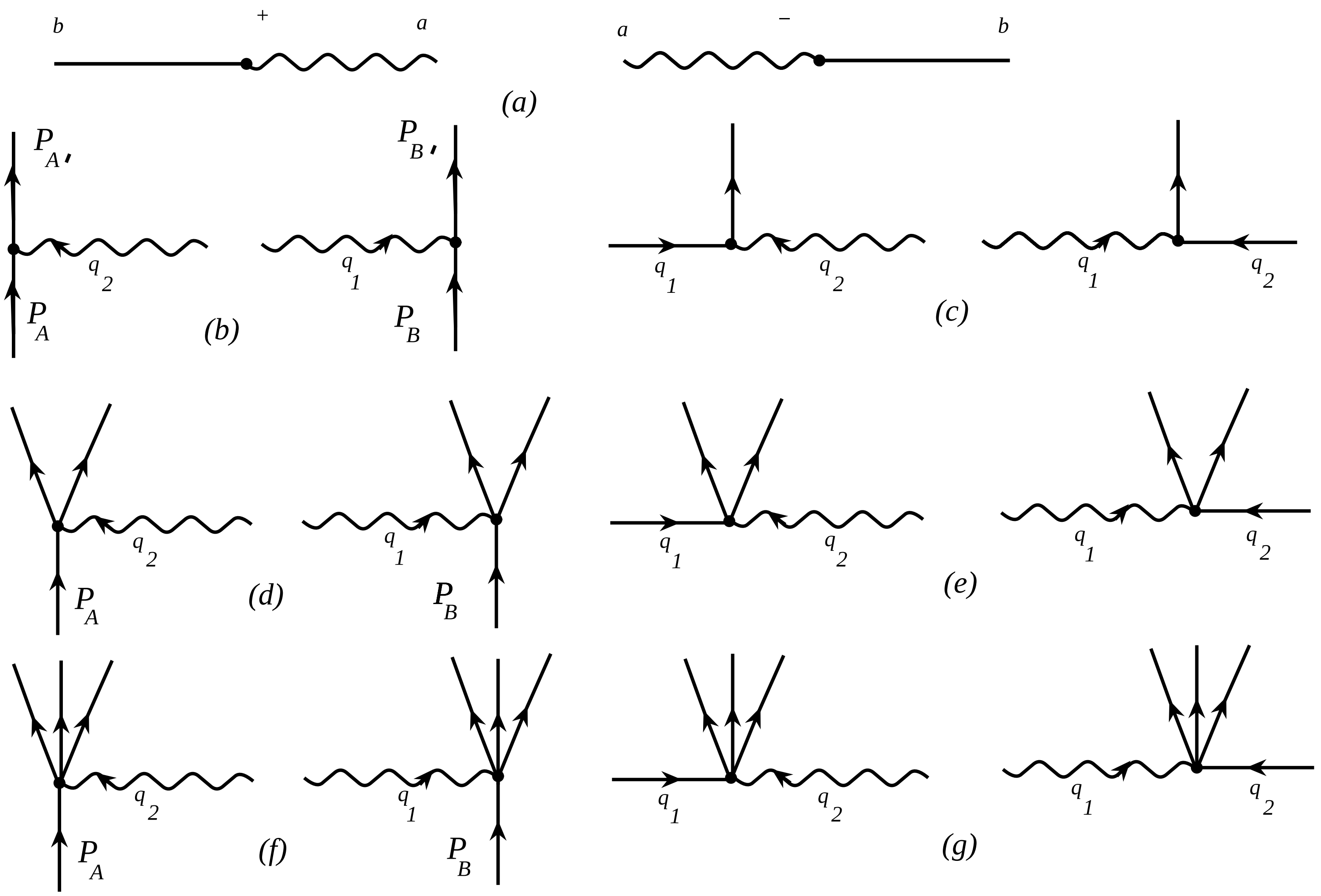

Figure 2: Induced $Rg, Rgg, Rggg, Rgggg$ vertices (up to four-gluon ones).

Explicit expressions for the induced vertices:

$$\text{Rg-vertex:}\quad \Delta^{\nu_0 \pm}_{a_0 c} = i\langle 0|\mathcal{L}^{gR}_{ind}|v^{\nu_0}_{a_0} A^c_{\pm}(q)\rangle = i\boldsymbol{q}^2_{\perp}(n^{\pm})^{\nu_0}\delta^{a_0 c}$$

$$\text{Rgg-vertex:}\quad \Delta^{\nu_0\nu_1+}_{a_0 a_1 c}(k^+_0, k^+_1) = i\langle 0|\mathcal{L}^{gR}_{ind}|v^{\nu_0}_{a_0}(k_0) v^{\nu_1}_{a_1}(k_1) A^c_+(q)\rangle =$$

$$= -g\boldsymbol{q}^2_{\perp}\, f_{a_0 a_1 c}\,(n^+)^{\nu_1}\frac{1}{k^+_0}(n^+)^{\nu_0} \quad , \quad k^+_0 + k^+_1 = 0$$

$$\text{Rggg-vertex:}\quad \Delta^{\nu_0\nu_1\nu_2+}_{a_0 a_1 a_2 c}(k^+_0, k^+_1, k^+_2) =$$

$$= \boldsymbol{q}^2_{\perp}\,(n^+)^{\nu_0}(n^+)^{\nu_1}(n^+)^{\nu_2}\left(\frac{f_{a_2 a_0 a}\, f_{a_1 a c}}{k^+_1 k^+_2} + \frac{f_{a_2 a_1 a}\, f_{a_0 a c}}{k^+_0 k^+_2}\right) \quad , \quad k^+_0 + k^+_1 + k^+_2 = 0$$

Recurrent relations:

$$\Delta^{\nu_0\nu_1\ldots\nu_r+}_{a_0 a_1\ldots a_r}(k^+_0, k^+_1, \ldots, k^+_r) = \frac{i}{k^+_r}(n^+)^{\nu_r}\cdot$$

$$\cdot\sum_{i=0}^{r-1} f_{a_r a_i a}\Delta^{\nu_0\nu_1\ldots\nu_{r-1}+}_{a_0 a_1\ldots a_{i-1} a a_{i+1}\ldots a_{r-1}}(k^+_0, k^+_1, \ldots, k^+_{i-1}, k^+_i + k^+_r, k^+_{i+1}, \ldots, k^+_{r-1})$$

Effective vertices' "LEGO"

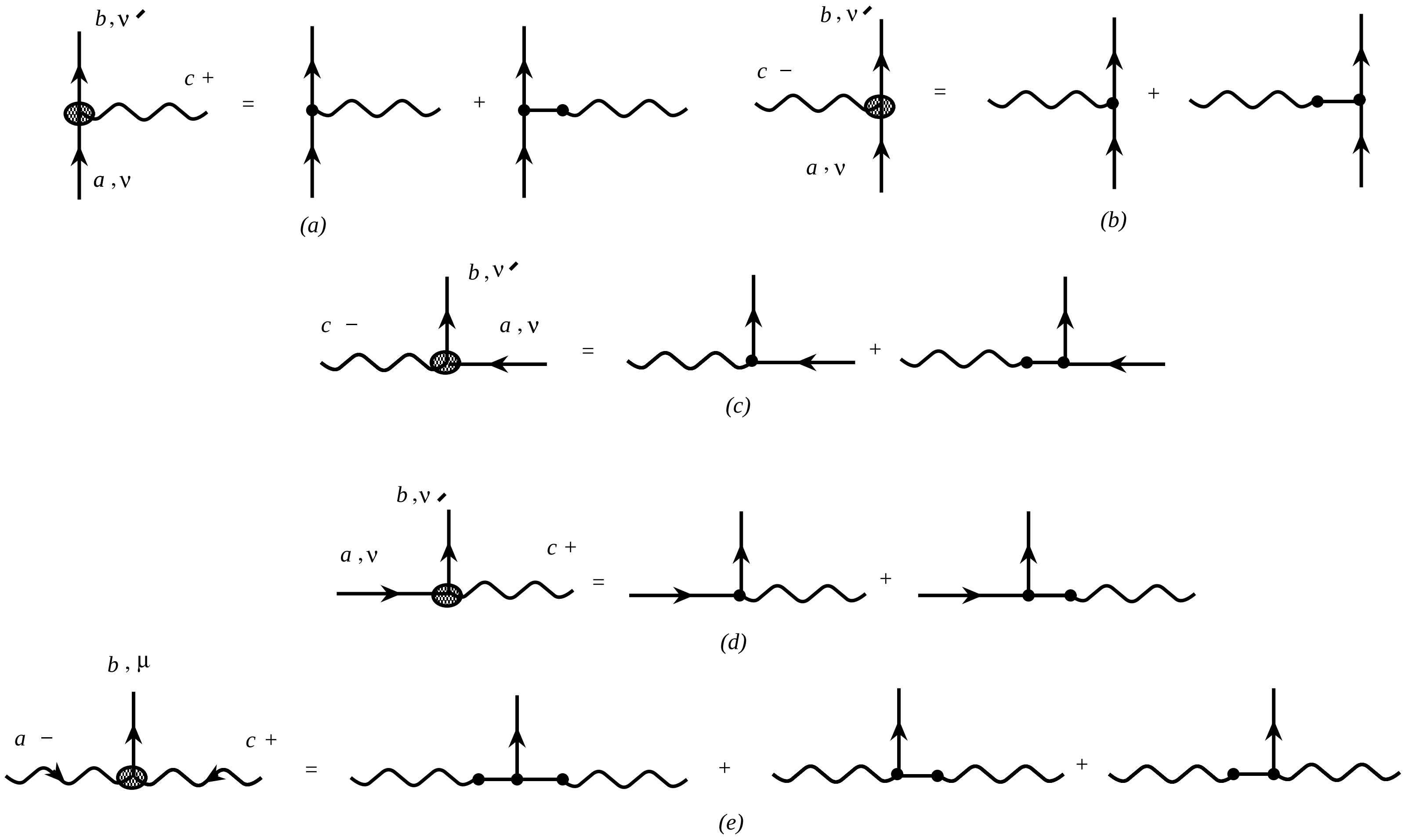

Figure 3: Construction of **effective** Rgg ((a, b, c, d)) and RRg (e) vertices: **Induced vertices** from the Lagrangian plus any allowed (within QMRK) combination of **propagators** in the t-channel.

Four types of Rgg vertices are possible:

- **Left margin:**

$$\gamma^{\nu\nu'+}_{||abc}\,(P_A, a; P_{A'}, b; q_2, c) = g f_{abc}\,\Gamma^{\nu\nu'+}\,(P_A, q_2),$$

$$\Gamma^{\nu\nu'+}\,(P_A, q_2) =$$

$$= 2P_A^+ g^{\nu\nu'} + \left(n^+\right)^\nu \left(-2P_A + P_{A'}\right)^{\nu'} + \left(n^+\right)^{\nu'} \left(-2P_{A'} + P_A\right)^{\nu} - \frac{q_2^2}{P_A^+}\left(n^+\right)^\nu \left(n^+\right)^{\nu'}$$

Gauge invariance:

$$\Gamma^{\nu\nu'+}(P_A, q_2)\cdot (P_{A'})_{\nu'} = P_A^\nu P_A^+ - P_A^2 (n^+)^\nu$$

$$\Gamma^{\nu\nu'+}(P_A, q_2)\cdot (P_A)_\nu = P_{A'}^{\nu'} P_A^+ - P_{A'}^2 \left(n^+\right)^{\nu'}$$

For *on*-mass-shell particle A: $P_A^2 = 0$, $(e(P_A)\cdot P_A) = 0$:

$$\Gamma^{\nu\nu'+}(P_A, q_2)\cdot (P_{A'})_{\nu'}\cdot e_\nu\,(P_A) = 0$$

- **Right margin:**

$$\gamma_{\|}^{\nu\nu'-}\left(P_B, a; P_{B'}, b; q_1, c\right) = -g f_{bac} \cdot$$

$$\cdot \left(2P_B^- g^{\nu\nu'} - \left(n^-\right)^{\nu'} \left(2P_{B'} - P_B\right)^{\nu} - \left(n^-\right)^{\nu} \left(2P_B - P_{B'}\right)^{\nu'} - \frac{q_1^2}{P_B^-} \left(n^-\right)^{\nu} \left(n^-\right)^{\nu'}\right)$$

- **Left central:**

$$\gamma_{\perp abc}^{\nu\nu'+}\left(q_1, a; k, b; q_2, c\right) = g f_{abc}\, \Gamma^{\nu\nu'+}(q_1, q_2)$$

$$\Gamma^{\nu\nu'+}(q_1, q_2) = 2q_1^+ g^{\nu\nu'} - \left(n^+\right)^{\nu} \left(q_1 - q_2\right)^{\nu'} - \left(n^+\right)^{\nu'} \left(q_1 + 2q_2\right)^{\nu} - \frac{q_2^2}{q_1^+} \left(n^+\right)^{\nu} \left(n^+\right)^{\nu'}$$

- **Right central:**

$$\gamma_{\perp abc}^{\nu\nu'-}\left(q_1, a; k, b; q_2, c\right) = -g f_{abc}\, \Gamma^{\nu\nu'-}(q_1, q_2)$$

$$\Gamma^{\nu\nu'-}(q_1, q_2) = 2q_2^- g^{\nu\nu'} + \left(n^-\right)^{\nu} \left(q_1 - q_2\right)^{\nu'} + \left(n^-\right)^{\nu'} \left(-q_2 - 2q_1\right)^{\nu} - \frac{q_1^2}{q_2^-} \left(n^-\right)^{\nu} \left(n^-\right)^{\nu'}$$

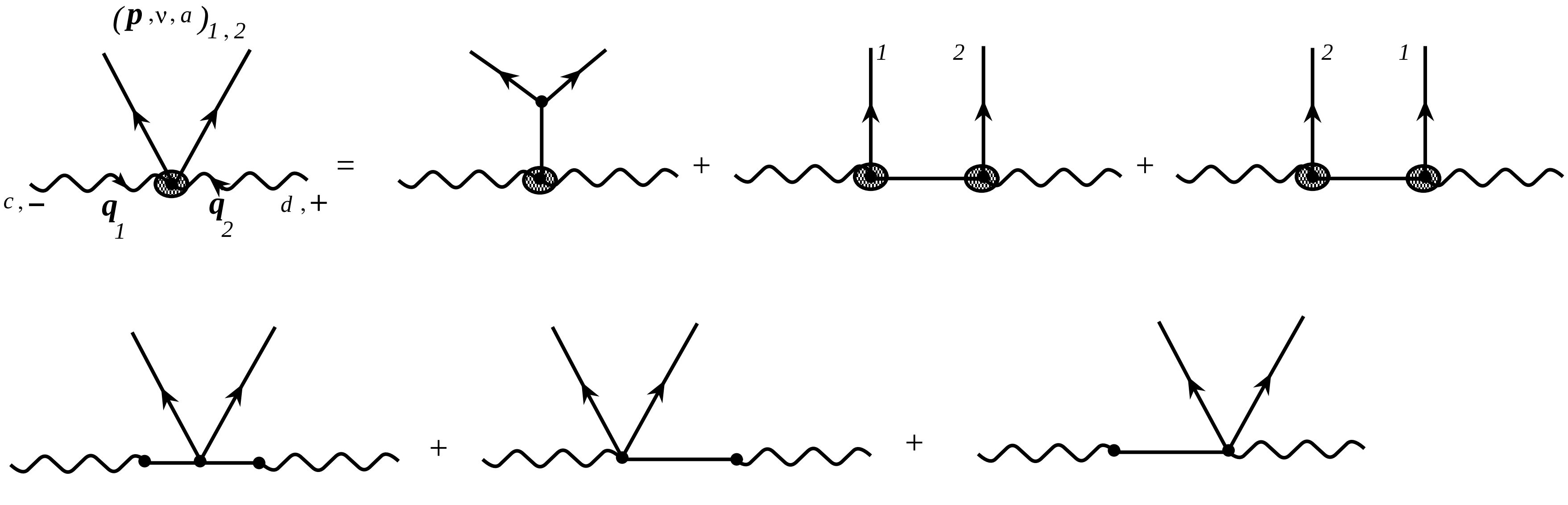

Figure 4: Effective vertex $RRgg$.

$$\frac{1}{ig^2}\Gamma^{-\nu_1\nu_2+}_{ca_1a_2d}\left(q_1;p_1,p_2;q_2\right)=$$

$$= \frac{T_1}{p_{12}^2} C^{\eta}\left(q_1, q_2\right) \gamma^{\nu_1 \nu_2 \eta}\left(-p_1, -p_2, p_{12}\right) + \frac{T_3}{\left(p_2 - q_2\right)^2} \Gamma^{\eta \nu_1 -}\left(q_1, p_1 - q_1\right) \Gamma^{\eta \nu_2 +}\left(p_2 - q_2, q_2\right) -$$

$$\frac{T_2}{\left(p_1 - q_2\right)^2} \Gamma^{\eta \nu_2 -}\left(q_1, p_2 - q_1\right) \Gamma^{\eta \nu_1 +}\left(p_1 - q_2, q_2\right) -$$

$$-T_1 \left[\left(n^-\right)^{\nu_1}\left(n^+\right)^{\nu_2} - \left(n^-\right)^{\nu_2}\left(n^+\right)^{\nu_1}\right] - T_2 \left[2 g^{\nu_1 \nu_2} - \left(n^-\right)^{\nu_1}\left(n^+\right)^{\nu_2}\right] -$$

$$T_3 \left[\left(n^-\right)^{\nu_2}\left(n^+\right)^{\nu_1} - 2 g^{\nu_1 \nu_2}\right] +$$

$$\Delta_{c a_1 a_2 d}^{\rho \nu_1 \nu_2 +}(q_1, p_1, p_2, q_2)(n^-)_\rho + \Delta_{c a_1 a_2 d}^{- \nu_1 \nu_2 \eta}(q_1, p_1, p_2, q_2)(n^+)_\eta$$

$$T_1 = f_{a_1 a_2 r} f_{c d r} \ , \ T_2 = f_{a_2 c r} f_{a_1 d r} \ , \ T_3 = f_{c a_1 r} f_{a_2 d r} \quad , \quad T_1 + T_2 + T_3 = 0$$

Gauge- and Bose-symmetries:

$$\Gamma_{c a_1 a_2 d}^{- \nu_1 \nu_2 +}\left(q_1; p_1, p_2; q_2\right) p_{1 \nu_1} = 0 \ , \ \Gamma_{c a_1 a_2 d}^{- \nu_1 \nu_2 +}\left(q_1; p_1, p_2; q_2\right) = \Gamma_{c a_2 a_1 d}^{- \nu_2 \nu_1 +}\left(q_1; p_2, p_1; q_2\right)$$

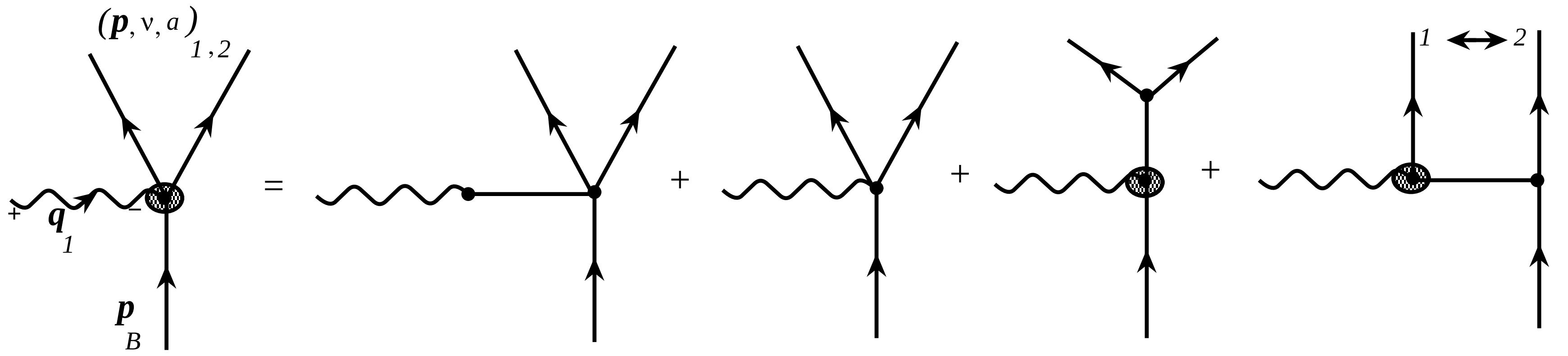

Figure 5: Margin *Rggg* right vertex.

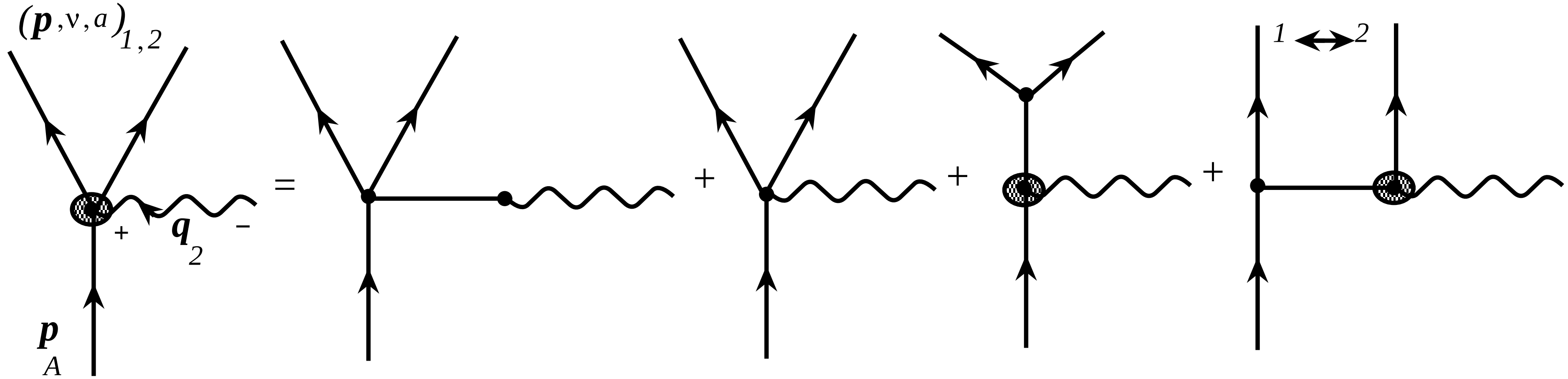

Figure 6: Margin $Rggg$ left vertex.

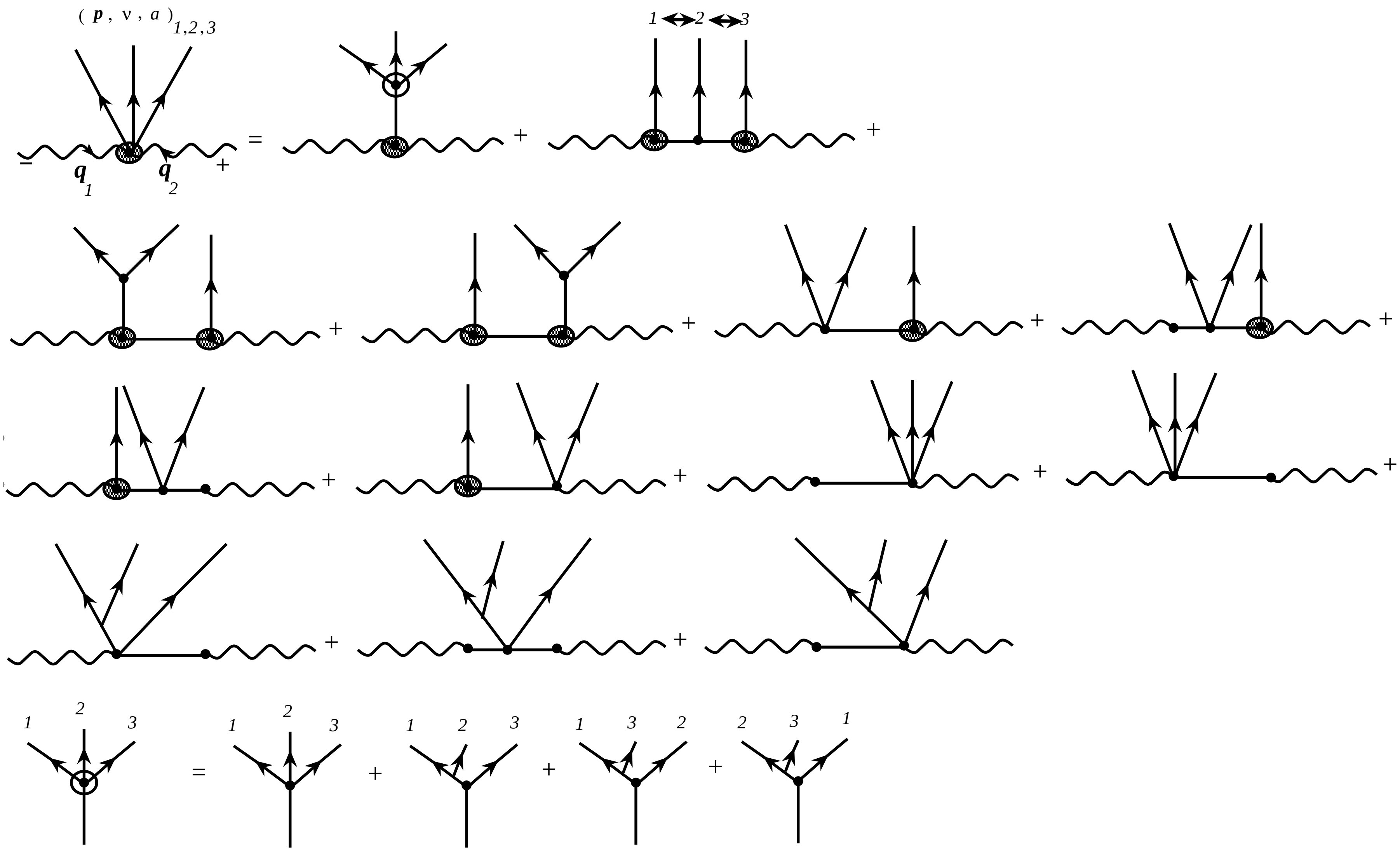

Figure 7: Central $RRggg$ vertex.

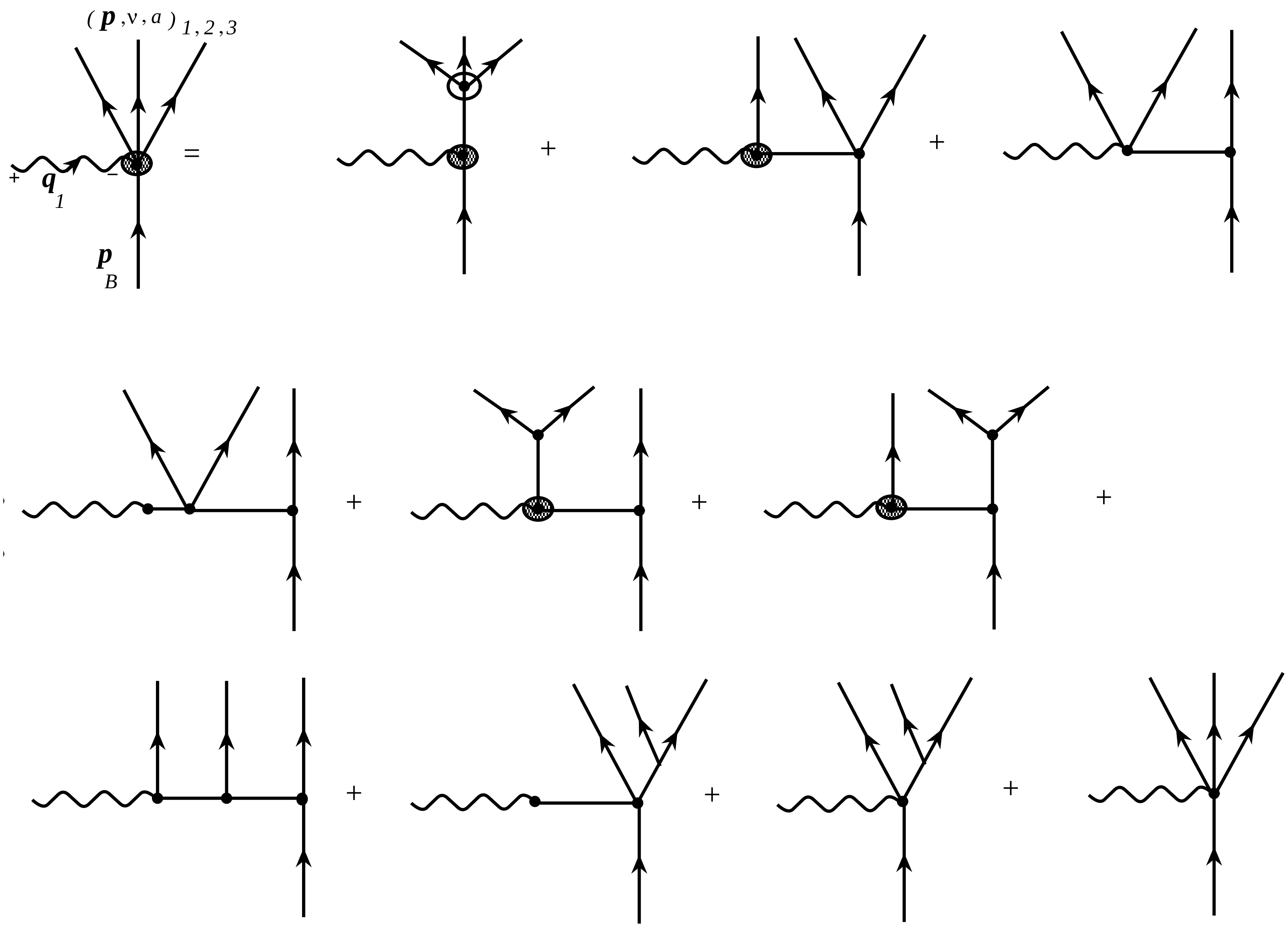

Figure 8: Margin *Rgggg* right vertex.

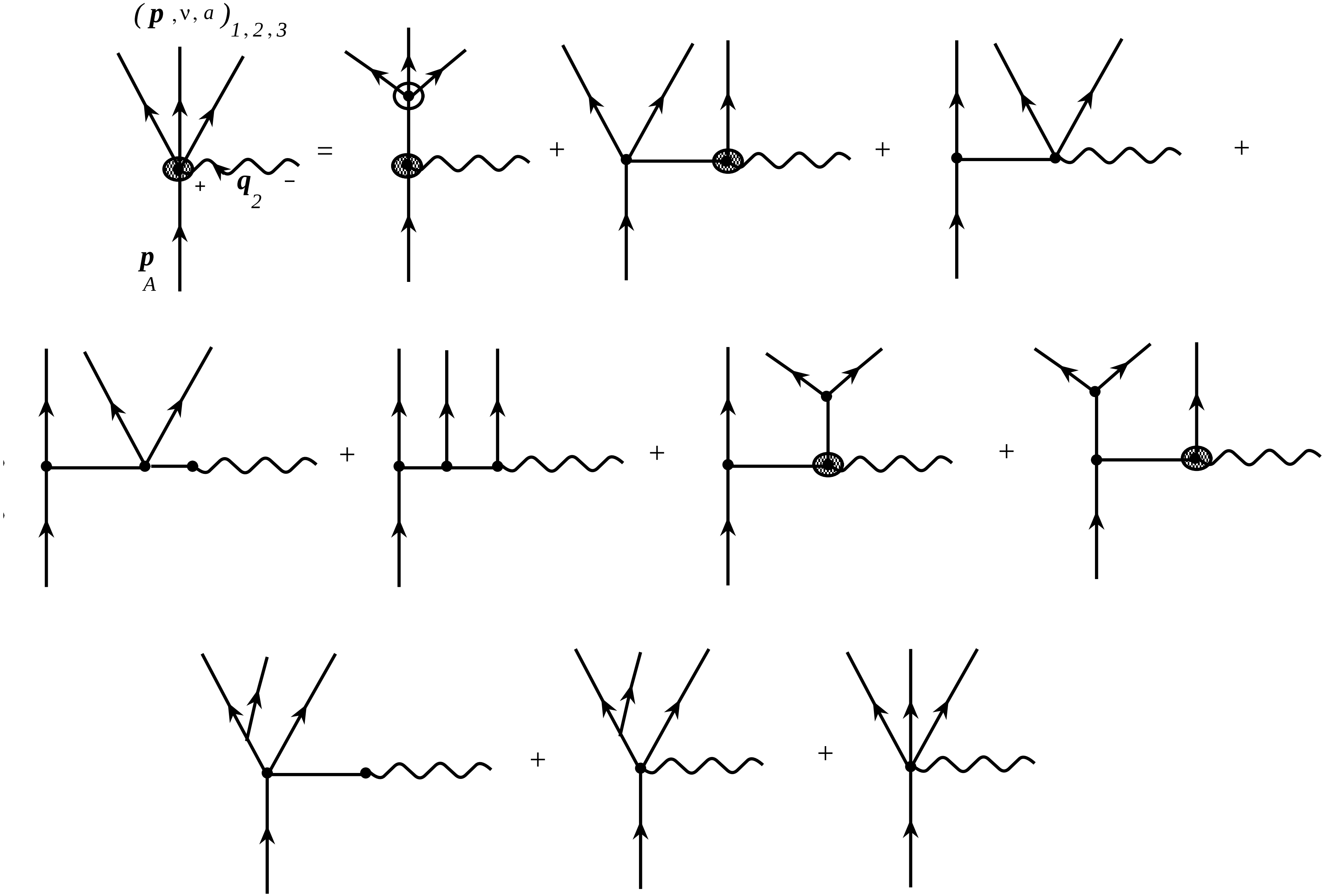

Figure 9: Margin $Rgggg$ right vertex.

Cross sections for jet production in QMRK

Sudakov decomposition

$$q_i = \alpha_i P_B + \beta_i P_A + \boldsymbol{q}_{i\perp} \ , \quad \boldsymbol{q}_{i\perp} \cdot P_A = \boldsymbol{q}_{i\perp} \cdot P_B = 0 \ , \ d^4 q_i = \frac{s}{2} d\alpha_i d\beta_i d^2 \boldsymbol{q}_i$$

Invariant masses of jets:

$$\begin{gathered}
(P_A - q_1)^2 = M_1^2 \approx -s\alpha_1 \ , \\
(q_1 - q_2)^2 = M_2^2 \approx -s\alpha_2\beta_1 \ , \\
\dots \\
(q_{n-1} + P_B)^2 = M_n^2 \approx s\beta_{n-1}
\end{gathered}$$

with the QMRK-ordering:

$$\alpha_1 \ll \alpha_2 \ll \dots \ll \alpha_n \ , \ \beta_{n-1} \ll \beta_{n-2} \ll \dots \ll \beta_1 \ , \ M_i^2 \sim M^2 \ll s \ ,$$

$$s_1 = (P_A - q_2)^2 \sim s_2 = (q_1 - q_3)^2 \sim \dots \sim s_{n-1} = (q_{n-2} + P_B)^2 \gg M^2$$

The **differential cross section** of the n-jet production:

$$d\sigma^{2\to n_{jet}} = \frac{1}{8s}|\mathcal{M}^{2\to n}|^2 d\Gamma_n^{(jet)}$$

The **phase volume** for the n jets containing n_i , $i = 1, ..., n$ produced particles in each jet:

$$d\Gamma_n^{(jet)} = \frac{(2\pi)^4}{s \cdot 2^{n-1}} \prod_{i=1}^{n-1} d^2\boldsymbol{q}_i \prod_{i=1}^{n} d\phi_i \int_{\frac{M^2}{s}}^{1} \frac{d\beta_1}{\beta_1} \int_{\beta_1}^{1} \frac{d\beta_2}{\beta_2} \cdots \int_{\beta_{n-3}}^{1} \frac{d\beta_{n-2}}{\beta_{n-2}} =$$

$$= \frac{(2\pi)^4}{s \cdot 2^{n-1}} \prod_{i=1}^{n-1} d^2\boldsymbol{q}_i \prod_{i=1}^{n} d\phi_i \cdot \frac{(\ln \frac{s}{M^2})^{n-2}}{(n-2)!}$$

$$d\phi_1 = dM_1^2 \, d\Gamma_1 (2\pi)^4 \delta^{(4)} \left(P_A - q_1 - \sum_{j=1}^{n_1} p_j^{(1)} \right) \; ,$$

$$d\phi_i = dM_i^2 \, d\Gamma_i (2\pi)^4 \delta^{(4)} \left(q_{i-1} - q_i - \sum_{j=1}^{n_2} p_j^{(i)} \right) \; , i = 2, ..., n-1 \; ,$$

$$d\phi_n = dM_n^2 \, d\Gamma_n (2\pi)^4 \delta^{(4)} \left(q_{n-1} + P_B - \sum_{j=1}^{n_n} p_j^{(n)} \right)$$

$$d\Gamma_i = \prod_{j=1}^{(n_i)} \frac{d^3 p_j^{(i)}}{2\varepsilon_j (2\pi)^3}$$

The **amplitude** for $2 \to n$:

$$\mathcal{M}^{2\to n} = 2^{-n+1} s \cdot g^n \; V_A V_B \left[\prod_{i=2}^{n-1} \frac{V_i^{i_1 \ldots i_{n_i}}(q_i, q_{i+1})}{q_i^2} \right]$$

The **differential cross section**:

$$d\sigma^{2\to n} = R(s)\alpha_s^n 2^{-5n+4}\pi^{-2n+3}\cdot V_A^2 V_B^2 \prod_{i=2}^{n-1}(V_i)^2 \cdot$$

$$\cdot \frac{\prod_{i=1}^{n-1} d^2\boldsymbol{q}_i}{\left(\prod_{i=1}^{n-1}\boldsymbol{q}_i{}^2\right)^2} \cdot \prod_{i=1}^{n} d\phi_i \frac{(\ln s/M^2)^{n-2}}{(n-2)!}$$

with the intermediate **gluons reggeization factor**:

$$R(s) = \prod_{i=1}^{n-1}\left(\frac{s_i}{M^2}\right)^{2(\omega(q_i^2)-1)}$$

Conclusions, applications and discussion

- A systematic self-consistent approach to **derivation of the Feynman rules directly from the effective reggeon-gluon action** is given; the results are presented in a convenient form for numerical simulations.

- All vertices satisfy the **Bose-symmetry and gauge invariance conditions**. The vertices of higher orders can be constructed, step by step, using the known ones given above.

- The obtained results can be used in analysis of **heavy flavors production** at high energy (e.g., *Kniehl, Vasin, Saleev: PRD (2006)*) and understanding of evolution, gluon saturation and unitarization phenomena at high energy (e.g., *Hatta: NPA (2006)*).

- Complete results and details of derivation: *NPB 721 (2005) 111*.