The 2L Running of α in the SM

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Outline

- **Running of α in QED**

$\equiv$ **Re-organization of radiative corrections**

1. U(1) Ward identity in QED $\longrightarrow$ Renormalization equation $\alpha \Leftrightarrow \gamma$ self-energy
2. Dyson resummation $+$ Gauge invariance

QED $\neq$ SM

- 1. Running of α in SM
  2. Method (two loops)
  3. Numerical results

CALC 2006, JINR Dubna, 23 July 2006

Radiative Corrections at Two Loops

⇒ Reduction of experimental uncertainties

	now	LHC	LC	GigaZ
δM_W [MeV]	38	15	15	6
$\delta sin^2\theta^{lep}_{eff}(\times 10^{-5})$	15	21	6	1.3

http://pdg.lbl.gov S. Eidelman et al. Phys.Lett. B 592,1

see refs. in Erler-Heinemeyer-Hollik-Weiglein-Zerwas [hep-ph:0005024]

+ Knowledge of 2L SM radiative corrections

- Pole masses ⇔ $\overline{\text{MS}}$ masses [Jegerlehner-Kalmykov-Veretin]

* M_W [Hollik-Freitas-Walter-Weiglein] + [Awramik-Czakon-Onishchenko-Veretin]
* $sin^2\theta^{lep}_{eff}$ [Awramik-Czakon-Freitas-Weiglein] + [Hollik-Meier-Uccirati]
* ...

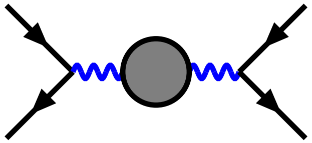
+ Reduction of input-data uncertainties

$2 \Rightarrow 2$ Scattering at Two Loops

- M_W $\rightarrow$ muon lifetime (static quantity) $\Rightarrow$ **2L tadpoles** $+$ **2L self-energies**

  $sin^2\theta^{lep}_{eff} \rightarrow$ $Z \Rightarrow l\bar{l}$ $\Rightarrow$ **2L vertices**

- $e\,\overline{e} \rightarrow f\,\overline{f}$ $\Rightarrow$ **2L SM corrections unknown** (scales)



(vector-boson exchange)

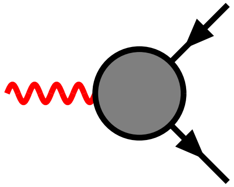
$\Rightarrow$ technically computable

$*$ **Running fine-structure constant** $\Rightarrow$ perturbative / non-perturbative physics

1. **low-energy (hadronization)**
2. **running** $\Rightarrow$ **resummation** $\Rightarrow$ **gauge-invariance ?**

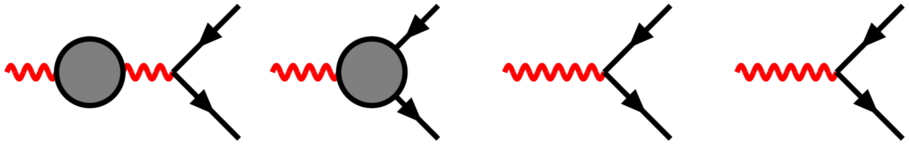
Definition of α

Thomson scattering
(t channel)



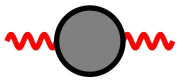
- Amplitude
- Pole at $p^2 = 0$
- Residue $\equiv \alpha$

α **input data**



$e = e(\alpha)$

- QED $\Rightarrow$ U(1) WI $\Rightarrow$ [vertex] $+$ [WF] $= 0$ $\quad (p^2 = 0)$
- $e = e(\alpha) \Leftarrow$ Dyson-resummed γ self-energy



$\Pi(s)\; s\; t_{\mu\nu} \qquad t_{\mu\nu} = \delta_{\mu\nu} + \frac{p_\mu p_\nu}{s}$

. . . using counterterms (i.e. OMS scheme)

- $\delta Z_e = -\delta Z_A - 2\delta Z_\psi - V(0)$
- $2\delta Z_\psi + V(0) = 0$ $\qquad \Rightarrow$ **universal**
- $\delta Z_e = -\delta Z_A$ $\qquad \Rightarrow$ **simple**

Running of α in QED

- Renormalization equation $\Rightarrow$ $e^2 = \frac{4\pi\alpha}{1-4\pi\alpha\Pi_1(0)}$

$\Rightarrow$ γ-exchange corrections to $e\bar{e} \to f\bar{f}$

$*$ $\Delta\alpha(s) \equiv 4\pi\alpha\,[\Pi_1(s) - \Pi_1(0)]$

$*$ Dyson resummation $\Rightarrow$ $\alpha(s) \equiv \frac{\alpha}{1+\Delta\alpha(s)}$

1L QED $\Rightarrow$ 3 contributions $\Delta\alpha(s) = \Delta\alpha_{lep}(s) + \Delta\alpha_{top}(s) + \Delta\alpha^{(5)}_{had}(s)$

- $\Delta\alpha_{lep}(s) = -\sum_l \frac{\alpha}{3\pi}[\ln(s/m_l^2) - \frac{5}{3} + \mathcal{O}(\frac{m_l^2}{s})]$ (large logs)
- $\Delta\alpha_{top}(s) \sim \frac{\alpha}{3\pi}\frac{4}{15}\frac{s}{m_t^2} \to 0$ (decoupling)
- $\Delta\alpha^{(5)}_{had}(s)$ $\leftrightarrow$ 5 light quarks $\Leftarrow$ **no** perturbation theory (**hadronization**)

Hadronic Contributions ($\longrightarrow$ see Jegerlehner lecture no 1)

Causality $\Rightarrow$ $\Pi(s) - \Pi(0) = \frac{s}{\pi}\int_{x_0}^{\infty} dx \frac{\text{Im}\Pi(x)}{x(x-s-i\epsilon)}$

Unitarity $\Rightarrow$ $\text{Im}\Pi(x) = \frac{x}{4\pi\alpha}\sigma(e\bar{e} \longrightarrow \gamma^* \longrightarrow hadrons)(x)$

$$\Delta\alpha^{(5)}_{had}(s) = \frac{s}{4\pi^2\alpha}\int_{4m_\pi^2}^{\infty} dx \frac{\sigma(e\bar{e} \longrightarrow \gamma^* \longrightarrow hadrons)(x)}{x - s - i\epsilon}$$

$\sigma(e\bar{e} \longrightarrow \gamma^* \longrightarrow hadrons)(x)$ $\Rightarrow$ exp. data / pQCD

$\Delta\alpha^{(5)}_{had}(M_Z^2) = -0.027896 \pm 0.000395$ [Jegerlehner 2001]

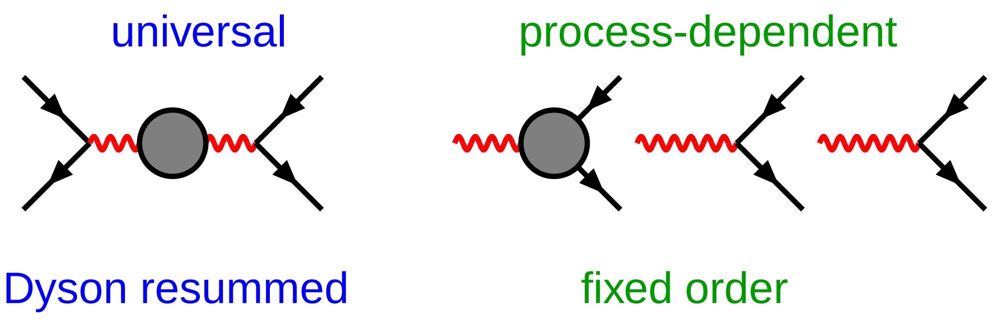
* exp. data $s < 5.5$ GeV 9.6 GeV $< s < 11$ GeV
* pQCD 5.5 GeV $< s <$ 9.6 GeV $s > 11$ GeV

$\Delta\alpha_{lep}(M_Z^2) = -0.031498$ [Steinhauser 1998] 3 loops

$\Rightarrow$ **improvement** low-energy experimental data needed

Dyson Resummation in QED

running α 1L QED $\Leftrightarrow$ splitting radiative corrections



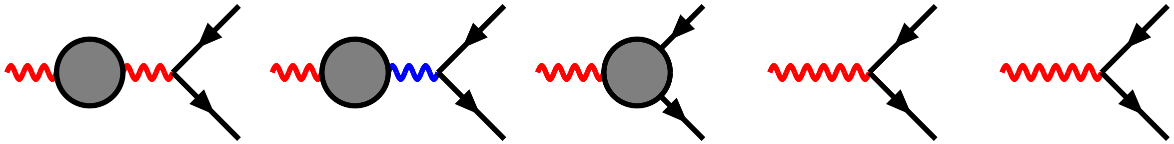
- $\Delta\alpha_{lep}(s) = -\sum_l \frac{\alpha}{3\pi}[\ln(s/m_l^2) - \frac{5}{3} + \mathcal{O}(\frac{m_l^2}{s})]$ **large logs** $\Rightarrow$ resummation
- $\Pi(s)$ 1L QED gauge invariant $\Rightarrow$ Dyson resummation preserving **gauge invariance**

Running α SM two loops

- no U(1) WI $\Rightarrow$ more than $\Pi(s)$
- $\Pi(s)$ 2L SM gauge invariant $p^2 = 0$ (Nielsen identities [Gambino-Grassi 1999])
  gauge dependent $p^2 \neq 0$
  $\Rightarrow$ Dyson resummation clashing with **gauge invariance**

Electric Charge Renormalization in the SM

SM $\sim$ QED $\Leftrightarrow$ electric-charge renormalization equation $\Pi(0)$



$+$ reducible

Diagonalization of the SM neutral sector

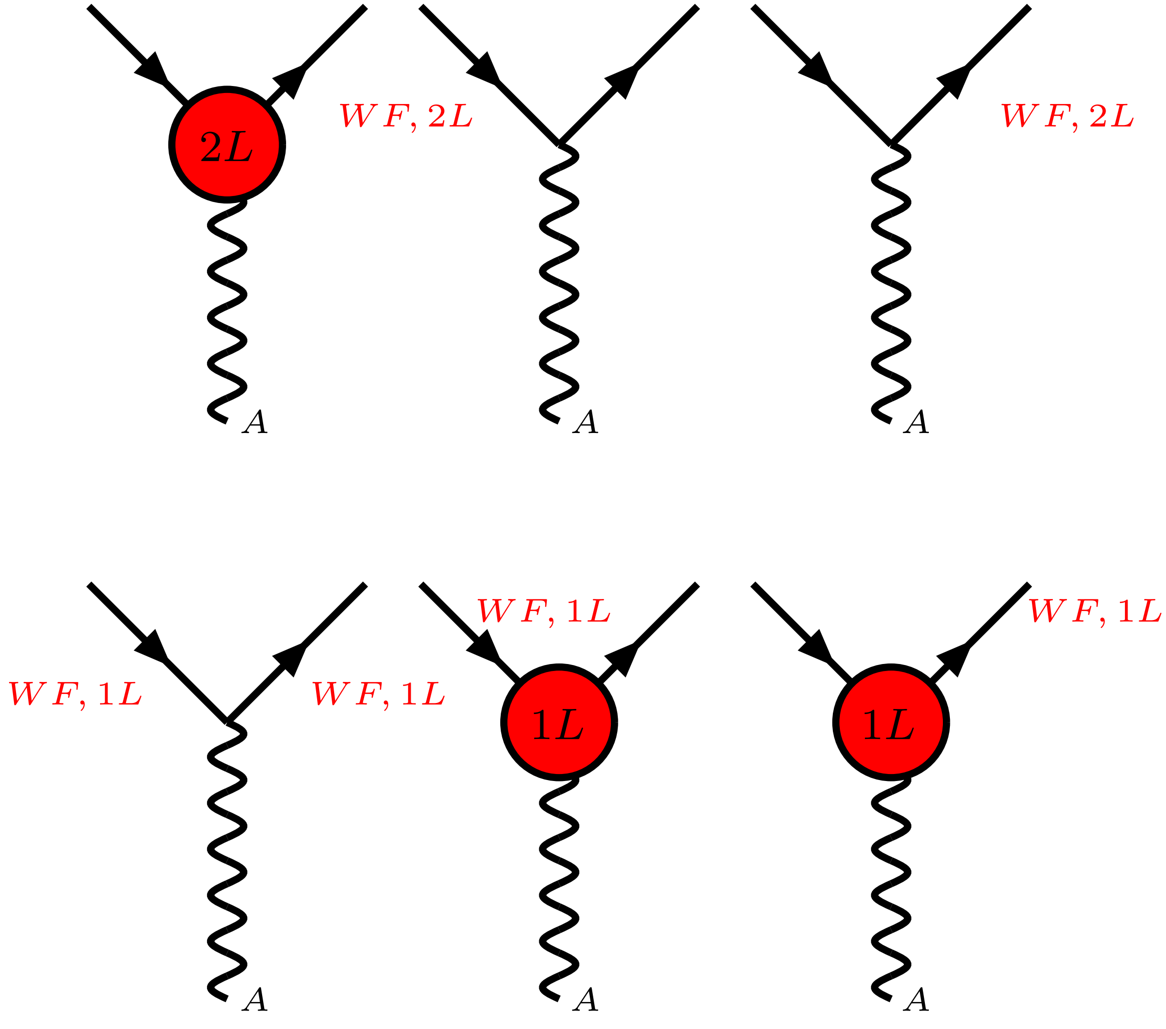
- $g = \overline{g}(1+\Gamma)$ $\Rightarrow$ see Passarino's lecture no.2
- $\Gamma \Leftrightarrow \Pi^{\mu\nu}_{AZ}(0) = 0$
- Feynman rules Γ dependent

$\Rightarrow$ Thomson scattering $\longrightarrow \Pi(0)$
$\Rightarrow$ e=e(α) $\longrightarrow \Pi(0)$

$*$ SM two loops $\Rightarrow$ 4 classes of corrections

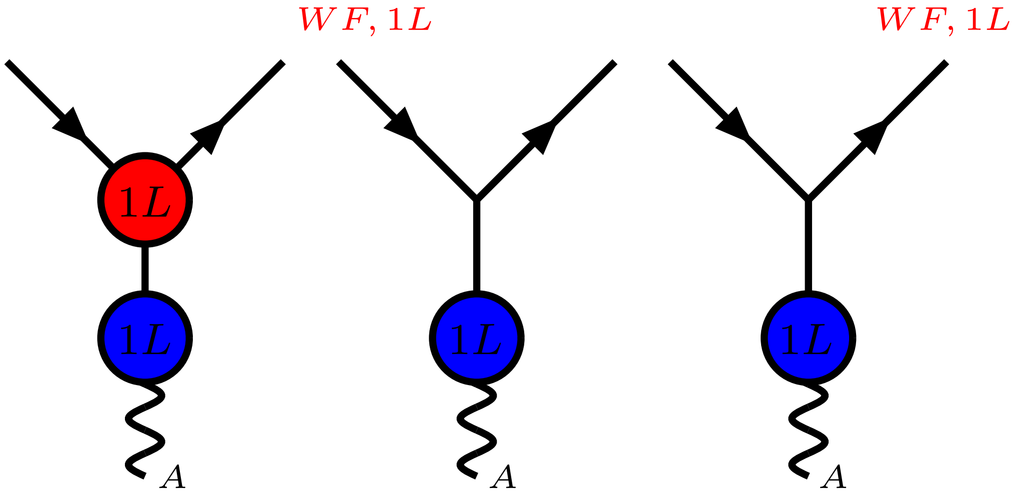
Class I

Entirely **process-dependent** corrections



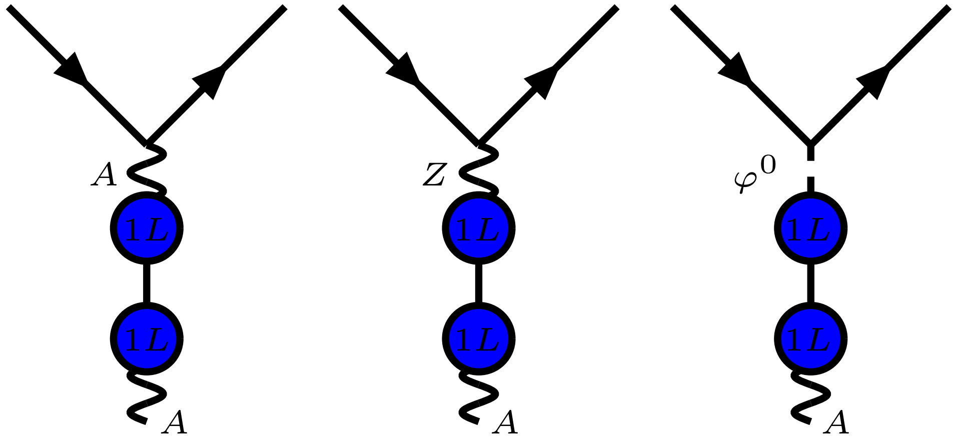
Class II

Self-energies $\times$ **process-dependent** corrections



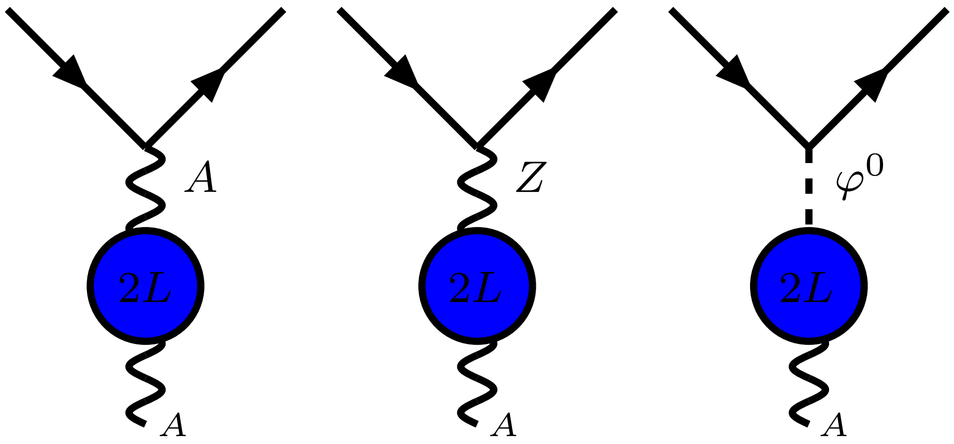
Class III

Reducible self-energy corrections



Class IV

Irreducible self-energy corrections



- Each class $\rightarrow$ (partial) scalarization [St.A.,Ferroglia,Passera,Passarino,Uccirati 04]

$\Rightarrow$ Algebraic cancellations $\Rightarrow$ **2L Red-IrRed γ self-energy**

$\Rightarrow$ QED-like renormalization equation $\Rightarrow$ $e^2 = \frac{4\pi\alpha}{1-4\pi\alpha\Pi_1(0)-(4\pi\alpha)^2\Pi_2(0)}$

diagonalization neutral sector (Γ) $\rightarrow$ **universal** $+$ **simple**

[Kennedy-Lynn 89, Passarino 91]

Photon-Exchange Corrections

- Renormalization equation $\Rightarrow$ $e^2 = \frac{4\pi\alpha}{1-4\pi\alpha\Pi_1(0)-(4\pi\alpha)^2\Pi_2(0)}$

$\Rightarrow$ Vector-boson exchange corrections to $e\bar{e} \to f\bar{f}$ $\Rightarrow$ Running couplings

$\Rightarrow$ γ-exchange corrections $\Rightarrow$ Running α $\Leftrightarrow$ QED

$*$ $\Delta\alpha(s) \equiv 4\pi\alpha\,[\Pi_1(s) - \Pi_1(0)] + (4\pi\alpha)^2\,[\Pi_2(s) - \Pi_2(0)]$

$*$ Dyson resummation $\Rightarrow$ $\alpha(s) \equiv \frac{\alpha}{1+\Delta\alpha(s)}$

- $\Delta\alpha(s)$ $\Rightarrow$ 1L QED gauge invariant

  $\Rightarrow$ 2L SM gauge dependent

  $\Rightarrow$ **fixed-order computation** $\Leftrightarrow$ **large-logs effects**

  $\Rightarrow$ **Dyson resummation** $\Leftrightarrow$ **gauge-parameter independence**

Running(s) of α in the SM

1st possibility

- $\Pi(s)$ in R_ξ gauge (`arbitrary gauge parameters`)
- Dyson resummation ξ-independent part
- Fixed perturbative order ξ-dependent part

$\Rightarrow$ ξ dependence cancel $\Leftrightarrow$ $\Pi(s)_{\text{rest}}$ - process dependent corrections

$+$ **gauge-parameter independence**

$-$ **violates uniqueness**

2nd possibility $\Leftrightarrow$ $\overline{MS}$ definition of α running

[see Degrassi-Vicini 2L in full SM]

- $\Pi(0)$

$\Rightarrow$ $\frac{1}{\epsilon}$ γ $\ln \pi$

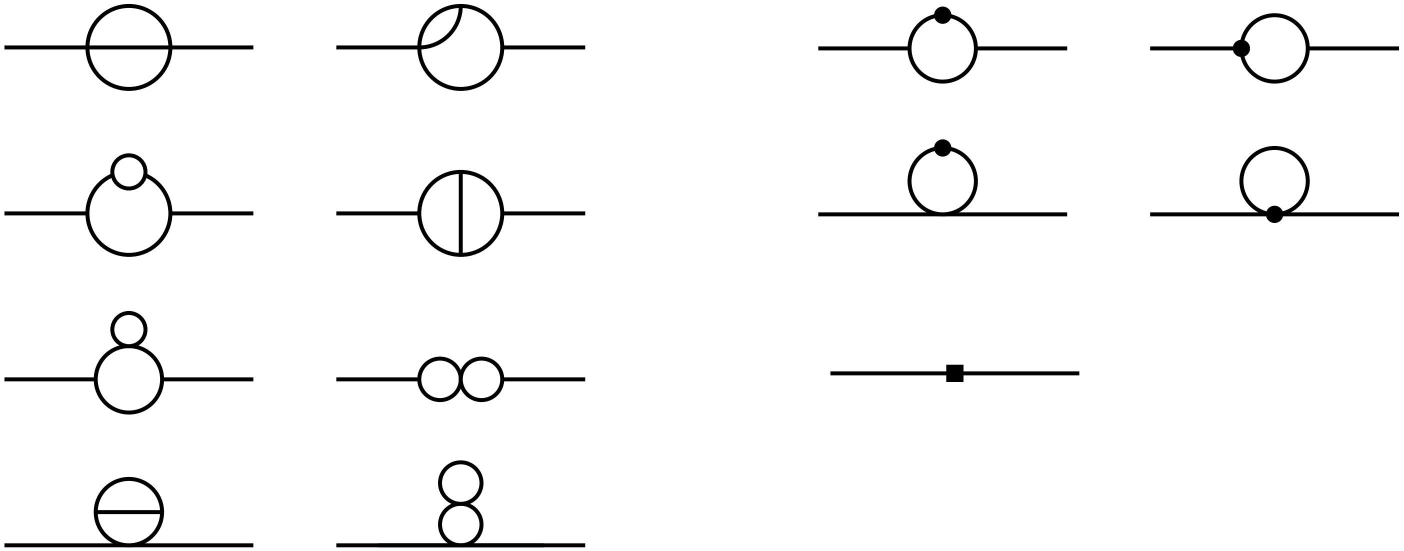
- $\mu^2 = s$

$+$ **gauge-parameter independence** ($p^2 = 0$)

$-$ **not the subtracted γ self-energy**

Method (see Uccirati's talk)

- **GraphShot** $\longrightarrow$ External legs $+$ Loops $\longrightarrow$ Diagrams
- Two-Loop $\quad + \quad$ Two-Leg



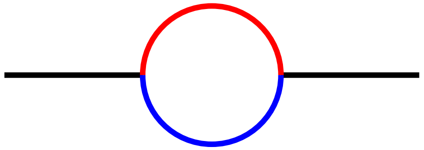
$*$ Standard PV **scalarization** $+$ sub-loop reduction *Böhm, Scharf, Weiglein*

$\Rightarrow$ Shifted dimensions $+$ recurrence relations *Tarasov*

$*$ **Bernstein-Sato + Tkachov algorithm** [see papers by Ferroglia-Passera-Passarino-Uccirati]

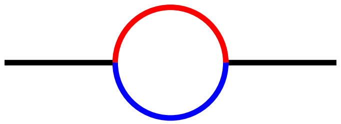
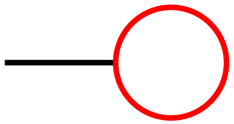
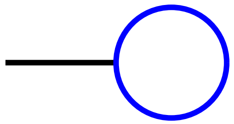
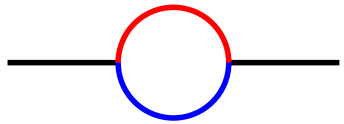
$\Rightarrow$ smooth representation $+$ singularity extraction **[LoopBack, Passarino-Uccirati]**

Standard Reduction



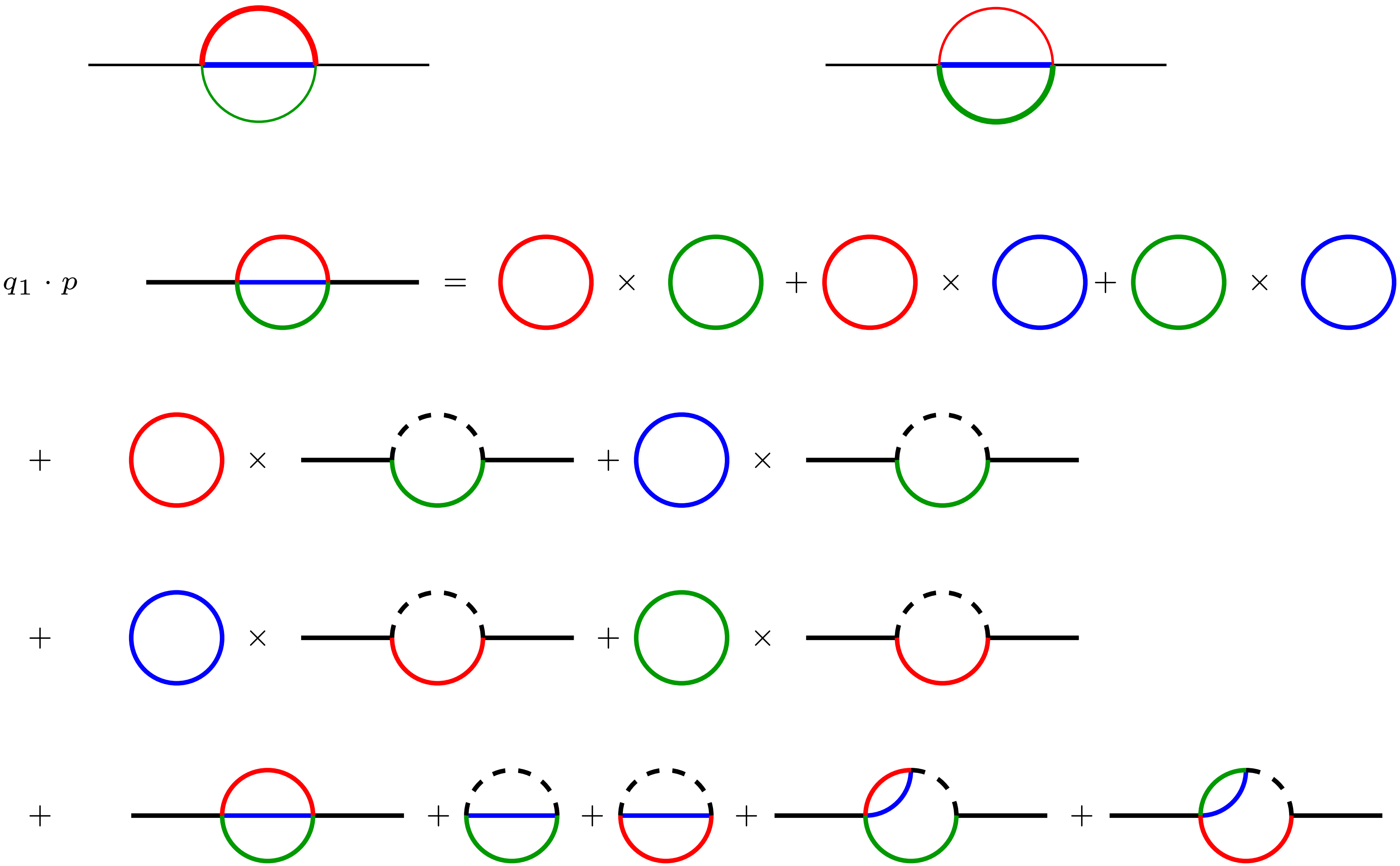
$$\int d^n q \frac{q_\mu}{[1][2]} = X p_\mu \qquad q\cdot p = \frac{1}{2}\left\{[2] - [1] - (p^2 - m_1^2 + m_2^2)\right\}$$

- Tensor decomposition + Contraction + Reduction $\longrightarrow$ Scalarization
- One-loop $\longrightarrow$ **Reducible** scalar products

$2q\cdot p$  $=$  $-$  $-\,(p^2 - m_1^2 + m_2^2)$ 

- Two-loop $\longrightarrow$ **Irreducible** scalar products

Sub-Loop Reduction



Böhm, Scharf, Weiglein, hep-ph:9310358

$\overline{MS}$ Definition of α Running: Results

$$\alpha(s) = \frac{\alpha}{1 + \Delta\alpha(s)} \qquad \Delta\alpha(s) = \Pi(0)^{\mu^2=s}_{sub}$$

$m_t = 174.3$ GeV $\quad M_H = 150$ GeV

$\sqrt{s}$ [GeV]	M_Z	120	160	200	500
1 loop	128.105	127.974	127.839	127.734	127.305
2 loop	128.042	127.967	127.891	127.831	127.586

$m_t = 174.3$ GeV $\quad M_H = 300$ GeV

$\sqrt{s}$ [GeV]	M_Z	120	160	200	500
1 loop	128.105	127.974	127.839	127.734	127.305
2 loop	128.041	127.914	127.784	127.683	127.266

error dominated by experimental data on $\Delta\alpha^{(5)}_{had}$

$\xi = 1$ Definition of α Running: Results

$$\alpha(s) = \frac{\alpha}{1 + \Delta\alpha(s)} \qquad \Delta\alpha(s) = \Pi(s) - \Pi(0) \qquad \xi = 1$$

$m_t = 174.3$ GeV $\qquad M_H = 150$ GeV

$\Delta\alpha(s)$ 2L	$\sqrt{s}$ =200 GeV
EW (Re)	-0.003578
EW (Im)	+0.002156
pQCD (Re)	-0.0005522
pQCD (Im)	+0.0001178
1LRen (Re)	-0.0000977
1LRen (Im)	-0.0000998

Re $\alpha^{-1}(s) = 126.933$

Conclusions

$*$ SM $\sim$ QED electric-charge renormalization $\Pi(0)$

$*$ ... **Dyson resummation ?**

$\Rightarrow$ $\overline{MS}$ definition of α running $\Rightarrow$ check [Degrassi-Vicini hep-ph:0307122]

$\Rightarrow$ compare $\overline{MS}$ vs. $\xi = 1$

- talk F. Hofmann $\Rightarrow$ SUSY QCD 2L H $\rightarrow \gamma\,\gamma$ (expansions)

  $\Rightarrow$ apply BST smoothness algorithm to **2L decays**

  (IF configurations completed [Passarino-Uccirati hep-ph:0603121])

- QED process-dependent corrections 2L to Bhabha scattering

  (Mellin-Barnes technique, see Riemann and Smirnov lectures)